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COMPARISON OF MATERIAL RESPONSE FOR THERMOMECHANICAL STRESSES IN FUNCTIONALLY GRADED ROTATING CYLINDERS

POREĐENJE ODZIVA U MATERIJALU ZA TERMOMEHANIČKE NAPONE KOD ROTACIONIH CILINDARA OD FUNKCIONALNIH KOMPOZITA

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Abstract

In this paper, an attempt is made for modelling of nonlinear thermoelastic stress analysis for cylinders made up of innovatively graded material with ceramic at inner surface and metal at outer surface. Material properties of functionally graded cylinder namely, Young's modulus, density, and coefficient of thermal expansion are varied in radial direction in a power law form. Using equation of equilibrium for an axisymmetric cylinder, the relation between stress-strain and strain-displacement, effect of elasticity modulus, coefficient of thermal expansion, temperature, density on rotating functionally graded cylinder under internal-external pressure and centrifugal body force is analysed. The study provides solution for radial and tangential stress, and radial displacement in closed form. The objective of our analysis is to conduct material comparison for the performance of functionally graded cylinders with different material combinations under different pressure conditions. Results for thermomechanical radial and tangential stress, and displacement are discussed with their graphical presentation. The outcome of our analysis leads to an observation that functionally graded rotating cylinder made up of Al-SiC or Al-ZrO$_2$ has a better response to thermomechanical loading conditions.

Ključne reči
- funkcionalni kompozitni materijali (FGMs)
- debelozidi cilindar
- napon-deformacija
- modul elastičnosti
- toplotna provodljivost
- termička ekspanzija

Izvod

U ovom radu je izveden pokušaj modeliranja nelinearnom analizom termoelastičnog napona za cilindre od novijih kompozitnih materijala sa keramikom na unutrašnjoj strani, i metalom na spoljnoj strani. Osobine materijala cilindra od funkcionalnog kompozita, na pr. Jungov modul elastičnosti, gustina i koeficijent termičkog širenja su promenljive veličine u radijalnom pravcu, koje se menjaju po eksponencijalnom zakonu. Primenom jednačine ravnoteže za aksijalno simetričan cilindar, analiziraju se: veza napon-deformacija i deformacija-pomeranje, uticaj modula elastičnosti, koeficijenta termičkog širenja, temperatura i gustina na rotirajući cilindar od funkcionalnog materijala, pod dejstvom unutrašnjeg-spoljašnjeg pritiska i centrifugalne sile. Istraživanje daje rešenje za radijalni i tangencijalni napon, i za radijalno pomeranje u zatvorenom obliku. Cilj analize je poređenje materijala u smislu performansi cilindra od funkcionalnog materijala sa različitim kombinacijama vrste materijala pod različitim uslovima pritiska. Data je diskusija grafički predstavljenih rezultata termomehaničkih radijalnih i tangencijalnih napona, kao i pomeranja. U zaključku primećujemo da cilindar od funkcionalnog materijala tipa Al-SiC ili Al-ZrO$_2$ ima bolji odziv na uslove termomehaničkog opterećenja.

INTRODUCTION

Over the last few years, an innovative class of materials, known as functionally graded materials (FGMs) have gained a tremendous popularity in the community of material scientists due to its peculiar property of providing a smooth gradation of material properties between the metal and ceramic interface. Functionally graded materials are made up of metal matrix reinforced with ceramic as function of position. Due to such a heterogeneous mixture of metal and ceramic from one surface to the other, a smooth variation of material properties can be achieved. The motive behind such a variation of material properties from metal to ceramic phase is to enhance the overall material performance under high temperature conditions. The enhanced material properties can avoid the delamination process arising due to thermal expansion mismatch in the material body with two functional surfaces, namely, ceramic and metal. Recently, Mehta et al. /1/ investigated the effect of material properties mismatch in sandwich cylinder with inner layer of functionally graded material and outer layer made up of composite material. The study provided numerical solution for variation of thermomechanical stresses under the effect of varying material properties namely, elasticity modulus, thermal conductivity, and coefficient of thermal expansion on pressurized sandwich cylinder. Dini et al. /2/ provided exact solutions for a rotating sandwich disk subjected to magnetic and thermal loading. The study outlined the effects of internal heat generation, convection heat transfer and angular velocity on functionally graded sandwich disk. As ceramic surface provides thermal insulation, corrosion, oxidation resistance and metal surface provide high toughness and conductivity, together, functionally graded materials provide the best of metal and ceramic properties with controlled variation from one surface to the other. These materials play an important role in designing of aircraft structures and pressure vessels that can sustain their structure in high temperature environment and under high thermal gradient. With the advent of functionally graded materials, material scientists have been able to design penetration resistant armour plates and bullet proof vests. These materials have applications in various fields such as chemical engineering, automobiles and railway industry, medicine and biosciences, energy, optoelectronics, etc. Depending on the nature of loading and working conditions, choice of metal or ceramic matrix can be carried out. Ceramic matrix increases the fracture toughness and strength of the functionally graded material. Functionally graded cylinders having ceramic composition at inner radius and metal at outer radius are used in engine components as cylinder blocks and pistons due to its enhanced thermal barrier and wear resistance. Cylinder made up of ceramic rich inner surface and metallic rich outer surface can be effectively used to sustain the engineering component under high temperature loading and temperature gradient over a short period of time. This situation where the structure can be under a thermal shock occurs in the field of aerospace industry during re-entry of space vehicles, experiencing temperature gradients from -73°C to about 1100°C in few minutes of time interval. Kacar /3/ studied plane elasticity for pressurized, rotating hollow cylinders, spheres and thin disks. The author presented closed form solution for mechanical stresses for structures made up of different materials with power law material gradation.

Lin /4/ studied the effect of mechanical load on a rotating thin-walled functionally graded annular disk with varying thickness, elasticity, density, and provided closed form solution for generalized confluent hypergeometric differential equation. Hosseini et al. /5/ considered nano disk made up of functionally graded material with nonlinear thickness in radial direction and presented numerical solution for elastic stresses and displacement. Researchers (Maheshwari and Sharma /6/, Mehta and Sahni /7/) have analysed thermomechanical stresses in functionally graded rotating disk and have found that for disk made up of functionally material is a better option than composite materials. Madan et al. /8/ presented limit elastic speed and corresponding stresses in a sigmoid functionally graded disk. The study highlighted the importance of limit elastic speed for designing of functionally graded disk as it affects the onset of yielding in the disk. Yıldırım /9/ conducted a detailed study on effect of anisotropy in polar orthotropic rotating discs for different materials and presented complementary function method solution for elastic stresses under free-free, fixed free and fixed guided constraints. Some researchers have investigated internally pressurized functionally graded hollow cylinder with uniform heat generation and radially varying material properties (in power law form) namely, modulus of elasticity, coefficient of thermal expansion and thermal conductivity. Evci and Gülgeç /10/ presented an analytical solution for the stress field in the above cylinder, whereas Sharma and Kaur /11/ focused on numerical stress analysis of the above cylinder using finite element method.

Kamdi and Lamba /12/ studied FG hollow cylinder made up of alumina ceramic and nickel metal from inner to outer radius. They provided an analytical solution for an inverse thermoelastic problem and evaluated thermomechanical stresses in isotropic FG hollow cylinder under uniform temperature loading. Researchers (Paul and Sahni /13, 14/, Delouei et al. /15/) have also studied 2D mechanical, thermal problems for functionally graded materials with cylindrical and spherical geometry. The outcome of these investigations helps in understanding and tailoring of design parameters of functionally graded materials. Yarimpabuç /16/ formulated a mathematical model for analysing nonlinear thermomechanical stresses in functionally graded thick cylinders and spheres under the effect of high temperature loading. The material properties, namely, Young's modulus, thermal conductivity and coefficient of thermal expansion considered in the study are graded in terms of both, temperature and radius of the cylindrical/spherical body. Arslan et al. /17/ investigated spherical containers and pressure vessels made up of metal/metal composites. Their study focused on steel/aluminium functionally graded material with power law variation of Young's modulus and coefficient of thermal expansion in terms of radius of the body and temperature loading on the body. Bouiadjra et al. /18/ investigated thermodynamic loading response of functionally graded beam with exponentially varying material properties in thickness direction by using Navier solution to derive an analytical solution for the displacement field. Attia and Abo-Bakr /19/ presented a parametric study to exhibit the effects of material and geometrical parameters on FG nanoactuators and proposed a solution for improving mechanical design of micro/nanoactuators made of functionally graded materials. Functionally graded materials have been also explored for their sound absorption capabilities by using polymer-based epoxy as matrix material and jute fiber or hemp fiber as reinforcement. Fiber reinforced advanced composite materials are prospective materials for various engineering applications in automotive, rail, construction and aerospace industries /20/. An analytical solution is derived for the mechanical response of functionally graded cylinders made of five different material combinations and compared the results with finite element method using ANSYS APDL package, /21/.

A detailed review of literature lead to an observation that a comprehensive analysis of functionally graded materials made up of different ceramic-metal combinations under severe working conditions can address the issues of life cycle and performance for engineering components. As observed from the literature, considerable amount of research has been carried out on development of mathematical models and analytical/numerical solutions for thermomechanical stresses in disc, cylinder and spherical bodies made up of functionally graded material. So, in order to analyse the behaviour of the engineering components under severe working conditions, the selection of the material becomes an important criterion. Hence, in our study, an effort has been made to provide material comparison for functionally graded cylinders under nonlinear thermal loading, centrifugal body force and internal-external pressure conditions. A mathematical model for thermomechanical stresses in functionally graded materials with different ceramic-metal combinations, namely, aluminium-silicon carbide, aluminium-zirconia, nickel-alumina and steel-zirconia is developed, and an analytical solution for radial and tangential stresses and displacement in functionally graded rotating cylinders under nonlinear thermal loading is presented.

MATHEMATICAL FORMULATION FOR THERMO-MECHANICAL STRESS ANALYSIS IN FG ROTATING CYLINDER

We consider an axisymmetric functionally graded cylinder exposed to radially varying nonlinear thermal loading, centrifugal body force in radial direction and under internal-external pressure. The equation of equilibrium for such an axisymmetric rotating FG cylinder with steady state temperature distribution and radially varying loading condition is given as /7/,

$$r\frac{d\sigma_r}{dr}+\sigma_r-\sigma_\theta+\rho(r)\omega^2r^2=0\,, \tag{1}$$

where: σ_r, σ_θ, $\rho(r)$, ω are radial stress, tangential stress, density, and angular speed, respectively.

Applying stress function *S(r)*, an analytical solution is derived that satisfies the equation of equilibrium given by Eq.(1), and the relation between stresses and stress function is given as,

$$\sigma_r=\frac{S(r)}{r}\quad\text{and}\quad\sigma_\theta=\frac{dS(r)}{dr}+\rho(r)\omega^2r^2\,. \tag{2}$$

The relation between strain-displacement in functionally graded cylinder /2/ is given as,

$$\varepsilon_r=\frac{du}{dr}\quad\text{and}\quad\varepsilon_\theta=\frac{u}{r}\,, \tag{3}$$

where: ε_r, ε_θ, and u are radial and tangential strain, and displacement, respectively.

In order to obtain the compatibility condition between strains and displacement in an FGC, we eliminate u from above set of Eq.(3), and can be written as,

$$\varepsilon_r=\varepsilon_\theta+r\frac{d\varepsilon_\theta}{dr}\,. \tag{4}$$

Radial and tangential thermomechanical strain components in FG cylinder under plain strain can be expressed in terms of Young's modulus and stresses using Hooke's law /3/, given as

$$\varepsilon_r=\frac{1}{Y(r)}[\sigma_r-\nu\sigma_\theta]+\alpha(r)T(r)\,, \tag{5}$$

$$\varepsilon_\theta=\frac{1}{Y(r)}[\sigma_\theta-\nu\sigma_r]+\alpha(r)T(r)\,, \tag{6}$$

where: *Y(r)*, *α(r)*, *T(r)*, and ν are Young's modulus, coef. of thermal expansion, temperature, Poisson's ratio, in respect.

Using Eqs.(2), (5) and (6) in Eq.(4), we obtain,

$$r^2\frac{d^2S}{dr^2}+r\frac{dS}{dr}\left(1-r\frac{Y'(r)}{Y(r)}\right)+S\left(\nu r\frac{Y'(r)}{Y(r)}-1\right)=-\rho(r)\omega^2r^3\times$$
$$\times\left(3+\nu-r\frac{Y'(r)}{Y(r)}\right)-\rho'(r)\omega^2r^4-Y(r)r^2\alpha'(r)T(r)-$$
$$-r^2Y(r)\alpha(r)T'(r)\,. \tag{7}$$

Consider internal (r_i) and external (r_o) radii of the cylinder as 20 cm and 50 cm, respectively. The radially varying material properties of FG rotating cylinder are defined as,

$$Y(r)=Y_o\left(\frac{r}{r_o}\right)^{t_1}, \tag{8}$$

$$\rho(r)=\rho_o\left(\frac{r}{r_o}\right)^{t_2}, \tag{9}$$

$$\alpha(r)=\alpha_o\left(\frac{r}{r_o}\right)^{t_3}, \tag{10}$$

where: $t_1=-\frac{\log\left[\frac{Y_o}{Y_i}\right]}{\log\left[\frac{r_i}{r_o}\right]}$, $t_2=-\frac{\log\left[\frac{\rho_o}{\rho_i}\right]}{\log\left[\frac{r_i}{r_o}\right]}$, $t_3=-\frac{\log\left[\frac{\alpha_o}{\alpha_i}\right]}{\log\left[\frac{r_i}{r_o}\right]}$,

are material gradient parameters. Also, Y_o, ρ_o and α_o are Young's modulus, density, and coefficient of thermal expansion at external radius, respectively. Figure 1 graphically depicts the behaviour of material properties for functionally graded cylinder. As seen from Fig. 1, for functionally graded cylinder, decreasing Young's modulus but with increasing coefficient of thermal expansion causes decrease in density property of the material. Radially varying nonlinear thermal loading *T(r)* for FG cylinder is defined as /7/,

$$T(r)=T_{r_i}-T_{ref}+(T_{r_o}-T_{r_i})\left(\left(\frac{r-r_i}{r_o-r_i}\right)^{k_1}+\left(\frac{r-r_i}{r_o-r_i}\right)^{k_2}\right), \tag{11}$$

where: T_{ref}, T_{ri} and T_{ro} are reference, inner and outer temperature, respectively. Also, k_1 and k_2 are temperature gradient parameters, respectively.

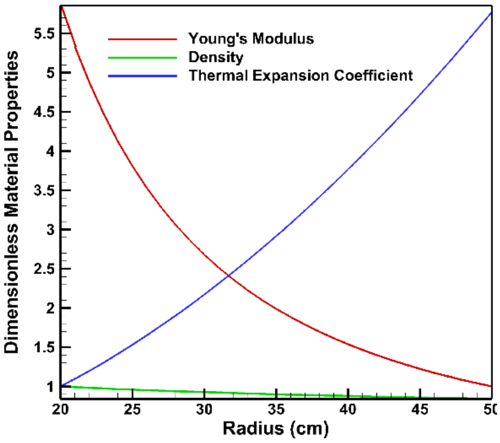

Figure 1. Dimensionless material properties.

The temperature loading with a nonlinear distribution profile over the radius of FG cylinder is depicted in Fig. 2. Here, $k_1 = 1$ and $k_2 = 2$, provides a complete quadratic thermal distribution profile. The pressure boundary conditions in FG cylinder can be described as given below:

$$\sigma_r(r_i)=-q_i\quad\text{and}\quad\sigma_r(r_o)=-q_o\,. \tag{12}$$

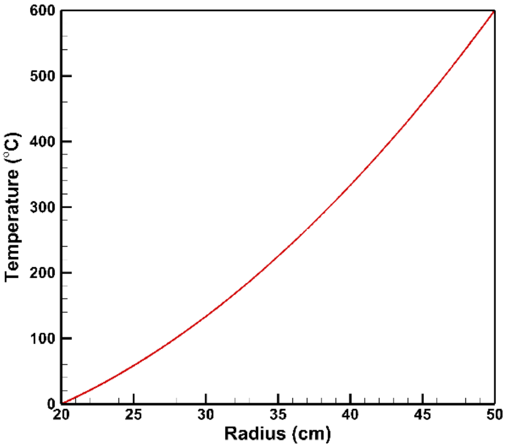

Figure 2. Temperature distribution profile.

Using Eqs.(8)-(11) in Eq.(7), we get the stress function, $S(r)$ as,

$$S(r)=C_1 r^{\frac{t_1+m}{2}}+C_2 r^{\frac{t_1-m}{2}}+C_3 r^{t_2+3}+C_4 r^{t_1+t_3+1}+$$
$$+C_5 r^{t_1+t_3+2}+C_6 r^{t_1+t_3+3}\,. \tag{13}$$

Here, C_1 and C_2 are constants of integration. The other constants C_3, C_4, C_5, and C_6 are given as,

$$C_3=-\frac{\rho_o\omega^2(3+\nu-t_1+t_2)}{r_o^{t_2}(t_2^2+6t_2-t_1t_3-3t_1+\nu t_1+8)}\,,$$

$$C_4=-\frac{Y_o\alpha_o t_3}{r_o{}^{t_1+t_3}(t_3^2+t_1t_3+2t_3+t_1+\nu t_1)}\times$$
$$\times\left(T_{r_i}-T_{ref}-\left(\frac{T_{r_o}-T_{r_i}}{r_o-r_i}\right)r_i-\left(\frac{T_{r_o}-T_{r_i}}{(r_o-r_i)^2}\right)r_i{}^2\right),$$

$$C_5=-\frac{Y_o\alpha_o t_3}{r_o{}^{t_1+t_3}(t_3^2+t_1t_3+4t_3+2t_1+\upsilon t_1+3)}\times$$
$$\times\left(\frac{T_{r_o}-T_{r_i}}{r_o-r_i}-2\left(\frac{T_{r_o}-T_{r_i}}{(r_o-r_i)^2}\right)r_i\right),$$

$$C_6=-\frac{Y_o\alpha_o t_3}{r_o{}^{t_1+t_3}(t_3^2+t_1t_3+6t_3+3t_1+\upsilon t_1+8)}\left(\frac{T_{r_o}-T_{r_i}}{(r_o-r_i)^2}\right)r_i\,.$$

Using Eq.(2), thermomechanical radial stress σ_r can be obtained as,

$$\sigma_r(r)=C_1 r^{\frac{t_1+m-2}{2}}+C_2 r^{\frac{t_1-m-2}{2}}+C_3 r^{t_2+2}+C_4 r^{t_1+t_3}+$$
$$+C_5 r^{t_1+t_3+1}+C_6 r^{t_1+t_3+2}\,. \tag{14}$$

Here, C_1 and C_2 are constants of integration obtained from boundary condition Eq.(12), given as

$$C_1=\frac{\beta_1}{y_1}-\frac{\beta_2 y_1 y_2-\beta_1 z_1 y_2}{y_1{}^2 z_2-z_1 y_1 y_2}\quad\text{and}\quad C_2=\frac{\beta_2 y_1-\beta_1 z_1}{y_1 z_2-z_1 y_2}\,, \tag{15}$$

where:

$$\beta_1=-q_i-C_3 r_i{}^{t_2+3}-C_4 r_i{}^{t_1+t_3}-C_5 r_i{}^{t_1+t_3+1}-C_6 r_i{}^{t_1+t_3+2}\,;$$

$$\beta_2=-q_o-C_3 r_o{}^{t_2+2}-C_4 r_o{}^{t_1+t_3}-C_5 r_o{}^{t_1+t_3+1}-C_6 r_o{}^{t_1+t_3+2}\,;$$

$$y_1=r_i^{\frac{t_1+m-2}{2}}\ ;\ y_2=r_i^{\frac{t_1-m-2}{2}}\ ;\ z_1=r_o^{\frac{t_1+m-2}{2}}\ ;\ z_2=r_o^{\frac{t_1-m-2}{2}}\ ;$$

and $m=\sqrt{t_1^2-4\nu t_1+4}$.

Using Eqs.(3) and (6), radial displacement, radial and tangential components of thermomechanical strains can be obtained. Tangential stress under thermomechanical loading can be calculated as,

$$\sigma_\theta=C_1\left(\frac{t_1+m}{2}\right)r^{\frac{t_1+m-2}{2}}+C_2\left(\frac{t_1-m}{2}\right)r^{\frac{t_1-m-2}{2}}+C_3(t_2+3)r^{t_2+2}+$$
$$+C_4(t_1+t_3+1)r^{t_1+t_3}+C_5(t_1+t_3+2)r^{t_1+t_3+1}+C_6(t_1+t_3+3)r^{t_1+t_3+2}+$$
$$+\frac{\rho_o\omega^2}{r_o{}^{t_2}}r^{t_2+2}\,. \tag{16}$$

RESULTS AND DISCUSSION

In our analysis, functionally graded cylinder is rotating at angular speed $\omega = 650$ rad/s, under the effect of internal pressure q_i and external pressure q_o conditions. The cylinder is subjected to radially varying temperature as given by Eq.(11) in which temperature parameters are given as $T_{ref} = 0$°C, $T_{ri} = 0$°C, and $T_{ro} = 300$°C. Table 1 presents material properties: Young's modulus, density, and coefficient of thermal expansion at inner and outer surfaces of FG cylinder.

Table 1. Material properties for FGMs, /2/.

Material	Y_i (GPa)	Y_o (GPa)	ρ_i (g/cm^2)	ρ_o (g/cm^2)	α_i $\times 10^{-6}$ (1/°C)	α_o $\times 10^{-6}$ (1/°C)	ν
Al-SiC	410	70	3.2	2.7	4	23.1	0.25
Al-ZrO$_2$	151	70	5.7	2.7	10	23.1	0.3
Ni-Al$_2$O$_3$	380	180	3.8	8.5	7.4	13	0.3
SUS304-ZrO$_2$	151	210	5.7	7.8	10	12	0.3

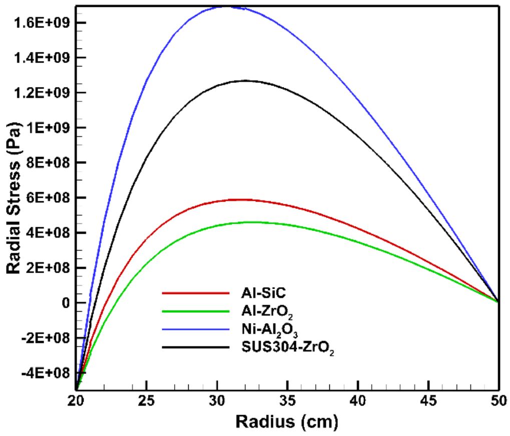

Figure 3. Radial stress: $q_i = 0.5$ GPa, $q_o = 0$, $T_{ref} = 0$, $T_{ri} = 0$, $T_{ro} = 300$°C, $\omega = 650$ rad/s.

Figure 3 presents variation of radial stress under internal pressure, $q_i = 0.5$ GPa in FG cylinder made up of different material combinations of ceramic and metal properties at inner and outer radial surfaces, respectively. It can be observed that radial stress constrained to pressure boundary condition is compressive at inner radial points and becomes tensile over the radius of the cylinder. This tensile nature of radial stress can be attributed due to radially varying body force $\rho(r)\omega^2 r$. Moreover, it is evident from Fig. 3 that cylinder made up of aluminium-zirconia (Al-ZrO$_2$) has the lowest magnitude of radial stress, whereas cylinder of nickel-alumina (Ni-Al$_2$O$_3$) is exposed to highest magnitude of radial stress among the other material combinations.

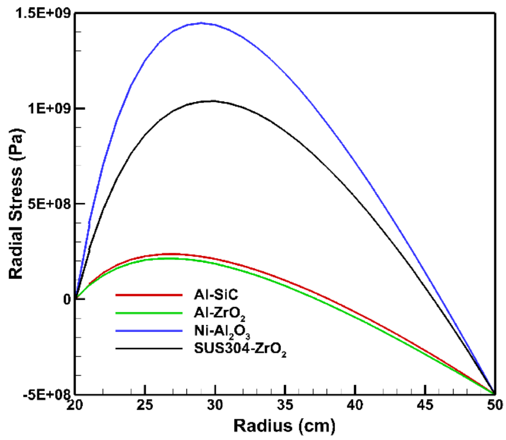

Figure 4. Radial stress: $q_i = 0$, $q_o = 0.5$ GPa, $T_{ref} = 0$, $T_{ri} = 0$, $T_{ro} =$ 300°C, $\omega = 650$ rad/s.

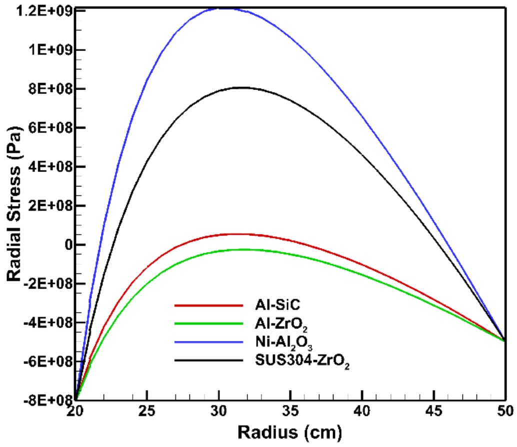

Figure 5. Radial stress: $q_i = 0.8$ GPa, $q_o = 0.5$ GPa, $T_{ref} = T_{ri} = 0$, $T_{ro} = 300$°C, $\omega = 650$ rad/s.

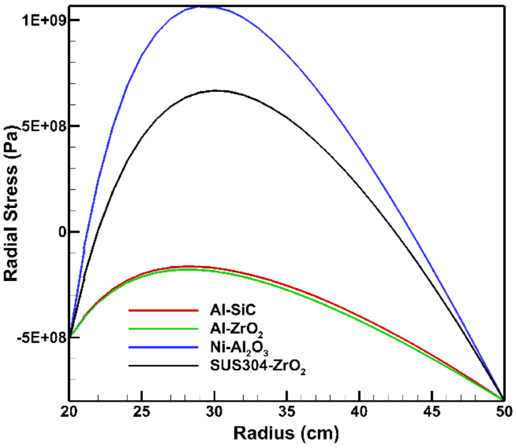

Figure 6. Radial stress: $q_i = 0.5$ GPa, $q_o = 0.8$ GPa, $T_{ref} = T_{ri} = 0$, $T_{ro} = 300$°C, $\omega = 650$ rad/s.

As observed from Fig. 4, under the effect of external pressure, $q_o = 0.5$ GPa, radial stress is compressive towards outer radial surface but importantly, FG cylinder made up of Al-SiC and Al-ZrO$_2$ have extremely small difference of magnitude of radial stress in comparison to their response under internal pressure case.

Figure 7 presents tangential stress under internal pressure, $q_i = 0.5$ GPa, in FG cylinder with ceramic at inner and metal at outer surface. It can be seen that tangential stress in FG cylinder is tensile for all material combinations and follows a decreasing trend from inner to outer radial points. Tangential stress in FG cylinder graded with aluminium-zirconia has lowest magnitude at inner radial points, but along outer radius of the cylinder, it has higher magnitude in comparison with cylinder made up of aluminium-silicon carbide FG material combination. As observed from Fig. 7, FG cylinders graded with nickel-alumina and steel-zirconia are exposed to higher magnitude of tangential stress. A high tensile nature of tangential stress at inner radial surface as compared to outer surface is due to compressive behaviour of radial stress at inner radial surface of the cylinder.

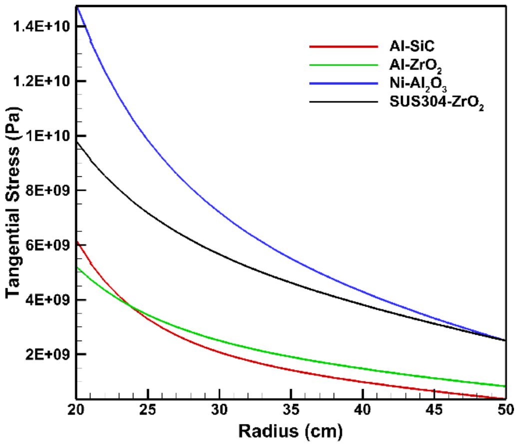

Figure 7. Tangential stress: $q_i = 0.5$ GPa, $q_o = 0$, $T_{ref} = T_{ri} = 0$, $T_{ro} = 300$°C, $\omega = 650$ rad/s.

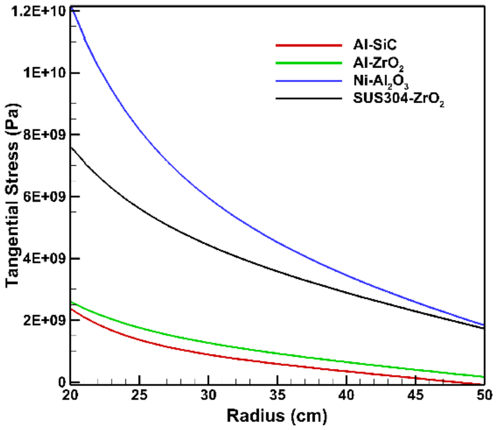

Figure 8. Tangential stress: $q_i = 0$, $q_o = 0.5$ GPa, $T_{ref} = T_{ri} = 0$, $T_{ro} = 300$°C, $\omega = 650$ rad/s.

Physically, it can be interpreted that tangential stress in FG cylinder is countering the effect of radial stress along the radius of cylinder, together with rotating body force. Hence, rotation of cylinder at high value of angular speed will cause higher magnitude of tangential stress along the radius of the cylinder. Tangential stress in FG cylinder under external pressure, $q_o = 0.5$ GPa is presented in Fig. 8. Here, the tangential stress in FG cylinder made up of Al-SiC is of the lowest magnitude throughout the radius of the cylinder.

Under the pressure difference at radial ends, i.e. high internal pressure - low external pressure, and low internal pressure - high external pressure (Figs. 9-10), FG cylinder is exposed to lower magnitude of tangential stress as compared to internal and external pressure cases, respectively.

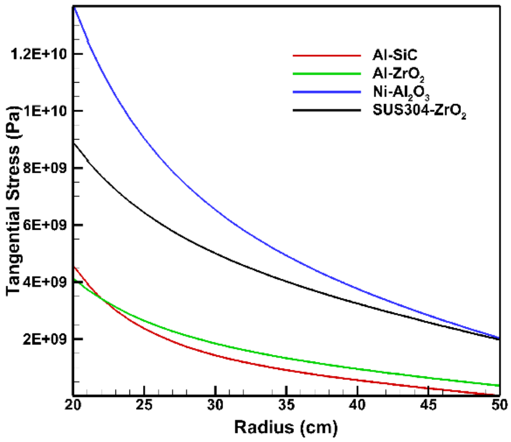

Figure 9. Tangential stress: q_i = 0.8 GPa, q_o = 0.5 GPa, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

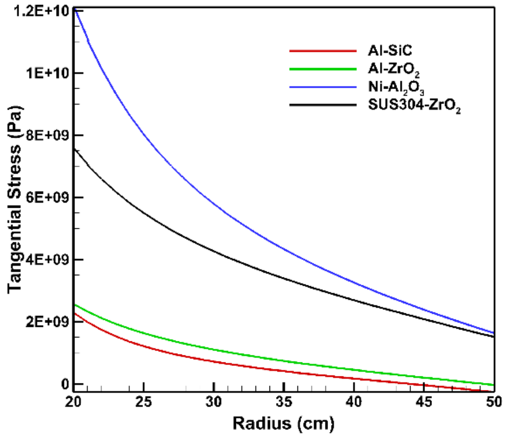

Figure 10. Tangential stress: q_i = 0.5 GPa, q_o = 0.8 GPa, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

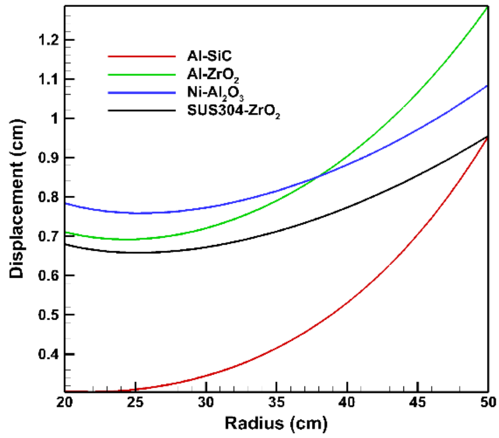

Figure 11. Radial displacement: q_i = 0.5 GPa, q_o = 0, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

As observed from Fig. 11, displacement under internal pressure, q_i = 0.5 GPa in rotating FG cylinder under nonlinear temperature loading and compressive internal pressure, increases from inner radial points to outer radial points. It can be noted from Fig. 11 that FG cylinder graded with aluminium-silicon carbide has the lowest magnitude of displacement along the radius of the cylinder as compared to other material combinations. At outer surface of FG cylinder made up of aluminium-silicon carbide and steel-zirconia, displacement is found to have lowest magnitude. As seen from Fig. 12, under the effect of external pressure, q_o = 0.5 GPa, FG cylinder made up of Al-SiC performs better as displacement at outer radial point (u = 0.717932 cm) and throughout the radius of the cylinder is found to be lowest. Also, as depicted from Fig. 12, FG cylinder made up of Ni-Al_2O_3 is exposed to higher displacement throughout the radius of the cylinder. Figures 13-14 present displacement in FG cylinder under high internal - low external, and high

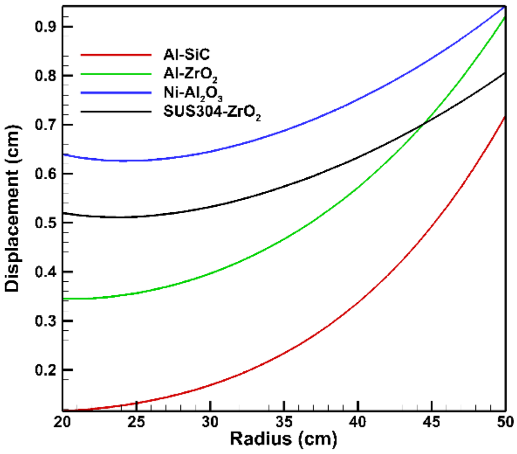

Figure 12. Radial displacement: q_i = 0, q_o = 0.5 GPa, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

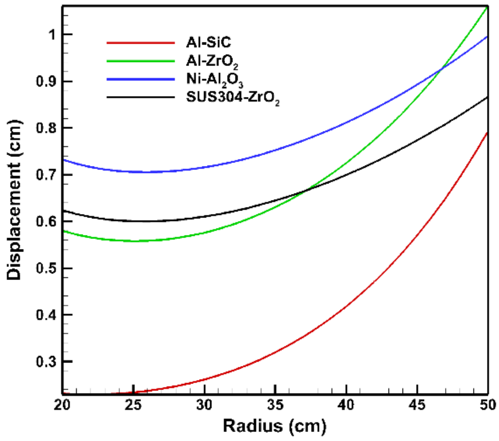

Figure 13. Radial displacement: q_i = 0.8 GPa, q_o = 0.5 GPa, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

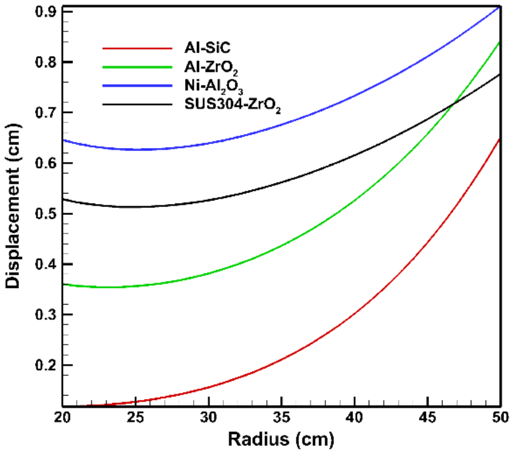

Figure 14. Radial displacement: q_i = 0.5 GPa, q_o = 0.8 GPa, $T_{ref} = T_{ri}$ = 0, T_{ro} = 300°C, ω = 650 rad/s.

external - low internal pressure conditions. FG cylinders made up of aluminium zirconia, nickel-alumina, and steel-zirconia experience in high radial displacement at inner radius under high pressure even at lower temperature loading at inner surface as compared to outer radial surface.

CONCLUSION

In our study, analysis of functionally graded cylinder with inner ceramic and outer metal surface, subjected to internal/external pressure, nonlinear temperature loading, and centrifugal body force is conducted. Material properties, namely, Young's modulus, density, and coefficient of thermal expansion are varying in power law form from inner to outer radius of cylinder. Thermomechanical stresses and displacement for cylinder with different material combinations are evaluated in closed form and presented graphically. Some of the outcomes from our study are as below:

- Functionally graded cylinder of aluminium-zirconia undergoes lower magnitude of radial stress in comparison to other material combinations analysed in our study.
- Functionally graded cylinder made up of aluminium-silicon carbide material is exposed to lower magnitude of tangential stress and displacement in comparison to other material combinations analysed in our study.
- Physically, it can be interpreted that a high resistivity of external force, i.e., Young's modulus plays a significant role in the designing of functionally graded cylinder together with thermomechanical loading conditions as it is observed from the analysis that material with higher value of Young's modulus is exposed to lower magnitude of radial and tangential stresses.
- Radial stress is high under rotating body force, but under temperature loading it decreases. It further decreases under internal pressure due to pressure difference at inner radius.
- Radial stress is high under internal pressure but when temperature loading is introduced, magnitude of radial stress decreases towards inner radial surface.

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