Anatolii N. Venediktov[1], Roman A. Sokolov[1], Roman J. Nekrasov[1], Dmitry P. Il'yaschenko[2], Elena V. Verkhoturova[3]*

DETERMINATION OF INTERNAL STRESSES IN AN ELECTROLYTIC COATING ARISING IN A SAMPLE DURING VIBRATIONS OF A TORSIONAL PENDULUM

ODREĐIVANJE UNUTRAŠNJIH NAPONA U ELEKTROLITIČKOJ PREVLACI NASTALIH U EPRUVETI TOKOM VIBRACIJA TORZIONOG KLATNA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 5.03.2021

Adresa autora / Author's address:
1) Tyumen Industrial Univers., Tyumen, Russian Federation
2) Yurga Institute of Technology National Research Tomsk Polytechnic University, Yurga, Russian Federation
email: mita8@rambler.ru
3) Irkutsk National Research Technical University, Irkutsk, Russian Federation

Keywords
- internal stresses
- torsional pendulum
- friction
- beam rotation
- electrolytic coatings

Ključne reči
- unutrašnja naprezanja
- torziono klatno
- trenje
- rotacija snopa
- elektrolitske prevlake

Abstract

In the course of work, on the example of a copper coating, it is shown that the relative difference between the tangential internal stresses on the surface of the coating deposited on the surface of a cylindrical sample and those near the substrate depend only on the diameter of the sample and the thickness of the coating. Using numerical methods with the help of the Comsol Multiphysics program, it has been established that the thickness of the coating layer, in which the influence of the substrate is felt, is about 5 μm. Based on the obtained experimental and calculated values, it has been shown that the minimum permissible coating thickness at which the mutual influence of the coating and substrate materials can be neglected is 25 μm.

Izvod

U ovom radu je na primeru bakarne prevlake, pokazano da je relativna razlika između tangencijalnih unutrašnjih napona, na površini prevlake nanete na površinu cilindričnog uzorka i onih u blizini podloge, zavisi samo od prečnika epruvete i debljine prevlake. Korišćenjem numeričkih metoda uz pomoć programa Comsol Multiphisics utvrđeno je da je debljina sloja prevlake, u kojem se oseća uticaj podloge, oko 5 μm. Na osnovu dobijenih eksperimentalnih i računskih vrednosti, pokazuje se da je minimalna dozvoljena debljina prevlake pri kojoj se može zanemariti međusobni uticaj materijala prevlake i podloge 25 μm.

INTRODUCTION

Measurement of the parameters of internal friction, that is, inelastic scattering of mechanical energy inside a solid /1, 2/, is widely used to study the processes occurring inside materials. Often, the information obtained by measuring internal friction - about diffusion /3, 4/, grain and subgrain boundaries, crystal structure defects, and dislocation structure is of a unique character, /5, 6/.

As a rule, scattering of mechanical vibration energy in a material is observed, and depending on the frequency of vibrations, internal friction is conventionally divided into low, medium, and high frequency. To measure low-frequency internal friction, a special device is usually used - a reverse torsion pendulum, /7, 8/.

The study of the internal friction of electrolytic coatings is associated with some difficulties, one of which is the impossibility to obtain sufficiently massive samples and test them independently of the substrate. Thus, the necessity of testing the coating in the coating-base system implies the need to take into account the mutual influence of two materials throughout the test /8, 9/.

One of the factors that determine the behaviour of the material of the tested sample are internal stresses arising in it /10/. However, due to the features mentioned above, internal stresses have to be calculated indirectly, taking into account the error associated with the influence of the substrate.

The purpose of the work is to determine internal stresses arising in a coating deposited on a sample of a circular cross-section during the internal friction test. One of the tasks is to establish the dependence of internal stresses on the angle of rotation of the torsional pendulum.

METHODS OF EXPERIMENTAL RESEARCH

The sample is a cylinder of diameter (d) with an electrolytic coating of thickness (h) applied to it, fixed between the stationary grip of the device and a torsional pendulum. Making oscillatory movements, the pendulum generates a twisting moment /8/ acting on the sample (Fig. 1).

Since only a twisting moment arises in the cross sections of the sample under the action of the pendulum, the stress-strain state of the sample can be considered within the framework of the theory of torsion of a cylindrical beam.

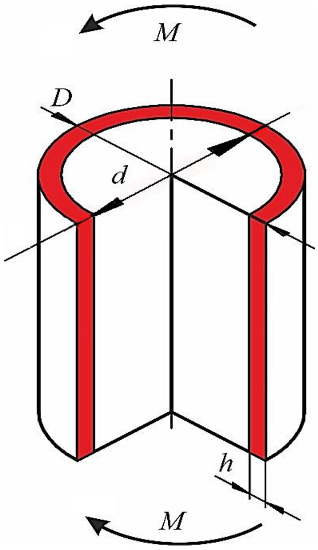

Figure 1. Twisting moments affecting the sample with a coating deposited on it (highlighted in colour).

Let's consider the torsional deformation of a cylindrical sample. Its characteristic is that sections perpendicular to the axis of the cylinder rotate in a plane relative to a certain longitudinal axis, which remains rectilinear and does not change its position. In this case, the sections do not change their shape, and their radii remain straight. In our case, this axis coincides with the axis of rotation of the cylinder.

If a force creating twisting moments is applied to the ends of a sample with a constant cross-section and length (l), then the stress-strain state of the sample will be uniform along its entire length.

It is known from the theory of material resistance that in the case of elastic deformation and a small angle of twist, an assumption can be made that after deformation the plane cross sections of the cylinder remain in-plane and the distances between them do not change. Thus, deformation of the cylinder can be considered as a result of rotation of cross sections around the axis of the cylinder in the direction of the twisting moment, and the sections do not bend and their radii remain straight. The angle of rotation of the section will obviously be greater the further it is from the fixed end, /11, 12/.

Such a deformation causes the appearance of tangential stresses (τ) in the cross-sectional plane, the direction of which is perpendicular to the circle radius of the cross-section of the cylindrical sample.

Deformations and tangential stresses (τ) corresponding to them, as it follows from experimental data, obey Hooke's law, /13/:

$$\tau = G\rho \frac{d\varphi}{dl} = G\rho \cdot \Theta , \qquad (1)$$

where: G is shear modulus of elasticity (GPa); ρ is the distance from the axis of rotation of the cylinder; φ is the angle of twist of the sample (the angle at which its free end will turn relative to the fixed one); $\Theta = \varphi / l$ is relative angle of twist.

The shear modulus of elasticity (G) and relative angle of twist (Θ) are constant for any cross section. As it can be concluded from the above formula, the tangential stresses (τ) in the section are directly proportional to the distance from the center (ρ) of the section /14/.

For practical calculations, we need to express the relationship of τ and M_K. For this purpose, we introduce polar coordinates ρ, α in the section of the sample. The centre of coordinates is located in the centre of the section, on the axis of sample rotation. The twisting moment can be calculated as an integral taken over the area of the section (S) of the moments of elementary tangential forces τdF:

$$M_K = \int_S \rho \tau dF . \qquad (2)$$

We then substitute Eq.(1) into Eq.(2). The shear modulus of elasticity G and Θ do not depend on the position of dF, therefore they can be taken out from under the integral sign, and then the twisting moment can be found by the formula:

$$M_K = G\Theta \int_S \rho^2 dF = G\Theta J_p , \qquad (3)$$

where: J_p is the polar moment of inertia that depends only on the geometry of the section. Given this formula, we obtain the relationship between M_K and Θ:

$$\Theta = \frac{M_K}{GJ_p} . \qquad (4)$$

Further, from Eq.(1) it follows that

$$\tau = \frac{M_K}{J_p} \rho . \qquad (5)$$

Let's move on to considering the stress state of our sample.

RESULTS OF RESEARCH

We will consider the case of application of a high-quality coating with good adhesion, then there will be no slippage between the coating and the base, and the stress-strain state of the coating can be considered as twisting of a circular tube.

The cross section of such a sample will be a ring with an outer diameter (D) and inner diameter (d), Fig. 1. The annular electrolytic coating has the same symmetry as a sample of a solid circular section. Therefore, considering the deformation of its flat sections, it is natural to accept all the same assumptions as for a solid cylindrical sample. Based on this, analysing the deformation of an elementary ring from the sample, we arrive at Eqs.(3)-(5).

The polar moment of inertia in these formulas, in a general form, which is also valid for a solid cylinder, is expressed as follows:

$$J_p = \int_S \rho^2 dS ,$$

where: S is the cross-sectional area.

For the polar moment of inertia of the cross section of the studied coating, which has the shape of a ring, by neglecting small values of the higher order in polar coordinates, then $dS = d\rho d\alpha$. Therefore, the double integral can be represented as follows:

$$J_p = \int_{d/2}^{D/2} \int_{0}^{2\pi} \rho^3 d\rho d\alpha .$$

Further, we can separate the variables in this integral and calculate it as the product of two single integrals:

$$J_p=\int_{d/2}^{D/2}\rho^3 d\rho\int_0^{2\pi}d\alpha=\frac{\pi}{32}(D^4-d^4)=\frac{\pi D^4}{32}(1-c^4)\,,$$

where: $c = d/D$.

We find the twisting moment from Eq.(4):

$$\Theta=\frac{M_K}{GJ_p}\,,\quad \text{from where}\quad M_K=\Theta GJ_p\,.$$

To calculate the max (τ_{max}) and min (τ_{min}) from Eq.(5) taking into account the obtained value for J_p and M_K, we obtain:

$$\tau_{max}=\frac{M_K}{J_p}\rho_{max}=\frac{M_K}{J_p}\frac{D}{2}\,,$$

$$\tau_{min}=\frac{M_K}{J_p}(\rho_{max}-h)=\frac{M_K}{J_p}\left(\frac{D}{2}-h\right).$$

As you can see, the tangential stresses in the coating change with depth. The maximum values are observed at the surface and the minimum values are reached at the substrate.

Using the example of a copper coating as presenting a particular interest, we will calculate the maximum (on the surface) and minimum tangential stresses in dependence of the twist angle in degrees. The shear modulus of elasticity G for copper is 45.5 GPa and the coating thickness h is taken to be 25 μm. The outer diameter of the sample D is taken to be 1 mm, the length of the sample l is taken to be 100 mm. The calculation results are shown in Fig. 2.

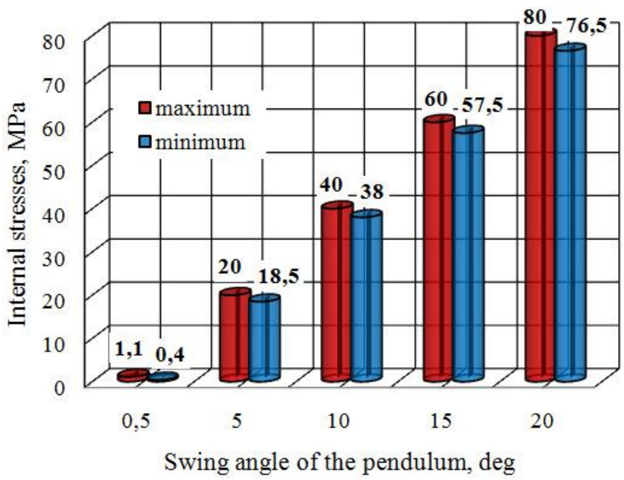

Figure 2. Changes in tangential stresses at the surface (maximum) and at the substrate (minimum) in dependence of the angle of twist of the torsional pendulum.

The graph shows that as the twist angle increases, the maximum and minimum values of internal stresses increase linearly and, simultaneously, the difference between them also increases. So, at a twist angle of 10 degrees, the difference between maximum and minimum tangential stresses is 2 MPa, or, in relative values, 4.96 % of the maximum. For a twist angle of 5 degrees, the difference is 1 MPa, and 4.96 %, respectively.

Let us find the relative difference δ between the internal stresses at the surface and at the substrate. We divide the minimum tangential stresses (τ_{min}) by the maximum ones (τ_{max}) and after small transformations we get the formula:

$$\delta=\frac{D-2h}{D}100\,\%\,.$$

As it can be seen, the sought value depends only on the diameter of the sample and the thickness of the coating.

The thickness of tested coatings rarely exceeds 0.1 mm, while the sample diameter is usually not less than 1 mm. In this case, the difference between minimum and maximum stresses will be small, about 5-15 %, and to use average stresses in the calculations will not be a significant mistake.

Stresses of interest are near the substrate, since they arise as a result of the mutual influence of coating and base materials. The influence of the stress state of the coating and the base on each other is shown on a computer model created in the Comsol Multiphysics program, since the analytical solution of this problem is associated with high computational capacity, /15/.

The same model sample with a diameter of 1 mm and a 25 μm thick copper coating applied to a steel base is used as a model.

A 3D model is used to solve the problem. The problem could be solved in a two-dimensional formulation, as well, since the deformations present are small and are located in the elastic zone. However, then it would not be possible to calculate volume deformations, which can play an important role in the contact zone of the coating with the base. The relative twist angle is taken equal to 10 degrees. The calculation results are shown in Fig. 3. According to simulation results, the minimum equivalent stresses (according to Mises) are 32.34 MPa, the maximum stresses are 36.0 MPa, which is in good agreement with the results obtained above, and the difference between them, about 4.96 %, confirms this.

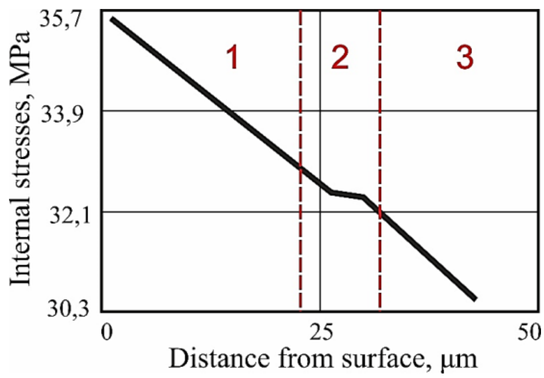

Figure 3. Distribution of stresses according to Mises in the cross section of the sample (distance is indicated from the surface): 1 - coating, 2 - transition layer, 3 - base.

On the graph, we can observe a jump in internal stresses in the contact zone of the coating with the substrate. The thickness of this zone is 5-10 μm, therefore, for satisfactory results of a test, in which this effect can be neglected, the thickness of the coating should be several times larger than the specified zone, and be at least 25 μm.

CONCLUSIONS

It has been established that the relative difference between tangential internal stresses on the surface of the coating and near the substrate depends only on the diameter of the sample and the thickness of the coating.

It has been established that the thickness of the coating layer, in which the influence of the substrate is detected, is about 5 μm.

It has been shown that the minimum permissible coating thickness at which the mutual influence of the coating and substrate materials can be neglected is 25 μm.

REFERENCES

1. Blanter, M.S. (1992), *Influence of interatomic interaction on internal friction spectrum in Nb-V-O alloys (computer simulation)*, Physica status solidi (a), 133(2): 317-323. doi: 10.1002/pssa.2211330214
2. Golovin, S.A., Golovin, I.S. (2012), *Mechanical spectroscopy of Snoek type relaxation*, Met. Sci. Heat Treat. 54: 208-216. doi: 10.1007/s11041-012-9483-6
3. Nowick, A.S., Anelastic Relaxation in Crystalline Solids, 1st Ed., Academic Press, Elsevier Inc., 1972. doi: 10.1016/B978-0-12-522650-9.X5001-0
4. Wert, C.A. (1986), *Internal friction in solids*, J Appl. Phys. 60(6): 1888-1895. doi: 10.1063/1.337237
5. Jiang, Z.C., Tian, Q.C., Ren, Z.M. et al. (2019), *Development and characterization of a MnCu-based high damping alloy plate*, IOP Conf. Series: Mater. Sci. Eng. 542: 012020. doi: 10.1088/1757-899X/542/1/012020
6. Gräfe, W. (2019), *Relation between internal friction and fatigue*, Adv. Eng. Forum, 31: 1-9. doi: 10.4028/www.scientific.net/AEF.31.1
7. Muvdi, B.B., McNabb, J.W. (1991), *Torsional stresses, strains, and rotations*, In: Engineering Mechanics of Materials, Springer, New York, NY : 159-197. doi: 10.1007/978-1-4612-3022-9_4
8. Dubigeon, S. (1992), *Torsional stiffness for circular orthotropic beams*, AIAA J, 30(9): 2355-2357. doi: 10.2514/3.11232
9. Sokolov, R.A., Novikov, V.F., Muratov, K.R., Venediktov, A.N. (2020), *Determination of the relationship between the factor of grain size factor and the corrosion rate of structural steel*, Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science, 22(3): 106-125. doi: 10.17212/1994-6309-2020-22.3-106-125 (in Russian)
10. Golovin, I.S., Sinning, H.R. (2003), *Damping in some cellular metallic materials*, J Alloys Compounds, 355(1-2): 2-9. doi: 10.1016/S0925-8388(03)00241-X
11. Muvdi, B.B., McNabb, J.W., Engineering Mechanics of Materials, Springer, New York, NY, 1991, 693 p. doi: 10.1007/978-1-4612-3022-9
12. Morrow, H., Kokernak, R., Statics and Strength of Materials, 7th Ed., Pearson, 2011, 528 p. ISBN: 978-0-135-03452-1
13. Khurmi, R.S., Khurmi, N., Strength of Materials, S. Chand and Co., Ram Nagar, New Delhi, 2008, 862 p. ISBN: 81-219-2822-2
14. Blanter, M.S., Golovin, I.S., Neuhäuser, H., Sinning, H.-R., Internal Friction in Metallic Materials: A Handbook. Springer Series in Materials Science, Publisher: Springer, 520 p., 2007. doi: 10.1007/978-3-540-68758-0
15. Blanter, M.S. (2012), *Computer simulation in mechanical spectroscopy*, Metal Sci. Heat Treat. 54(5-6): 217-220. doi: 10.1007/s11041-012-9484-5