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THERMAL BEHAVIOUR IN A ROTATING DISC MADE OF TRANSVERSELY ISOTROPIC MATERIAL WITH RIGID SHAFT

TERMIČKO PONAŠANJE ROTIRAJUĆEG DISKA OD TRANSVERZALNO IZOTROPNOG MATERIJALA SA KRUTOM OSOVINOM

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Abstract

The present research deals with thermal behaviour in a rotating disc of transversely isotropic material with rigid shaft by using transition theory and generalized strain measure. It has been seen that the radial stress has a maximum at the internal surface of the disc made of transversely isotropic material. Transversely isotropic material required higher value of radial stress at the internal surface of measure n = 1/5 in comparison to the disc of isotropic material, meaning that the transversely isotropic material is more comfortable than that of isotropic material. By increasing the values of angular speed and thermal condition, the value of radial stress as well as tangential stress must be increased at the internal surface depicted graphically. With the introduction of thermal condition the creep strain rates increase at the internal surface of the disc of isotropic material.

Izvod

Opisano istraživanje se bavi termičkim ponašanjem rotirajućeg diska od transverzalno izotropnog materijala sa krutom osovinom primenom teorije prelaznih napona i generalisane mere deformacija. Pokazuje se da radijalni napon dostiže maksimum na unutrašnjoj površini diska od transverzalno izotropnog materijala. Transverzalno izotropni materijal zahteva veću vrednost radijalnog napona na unutrašnjoj površini sa merom n = 1/5 u odnosu na disk od izotropnog materijala, što znači da je transverzalno izotropni materijal pogodniji od izotropnog materijala. Povećanjem vrednosti ugaone brzine i termičkih uslova, vrednost radijalnog napona kao i tangencijalnog napona se povećavaju na unutrašnjoj površini diska što je ilustrovano grafički. Uvođenjem termičkog uslova, vrednosti brzine puzanja rastu na unutrašnjoj površini diska od izotropnog materijala.

INTRODUCTION

A lot of research has been carried out to predict creep deformation and thermal flow using thermoelasticity theories during the past few years. The process of creep has been observed for different materials. Under the conditions of constant load or stress with respect to time-dependent plastic flow of materials is defined as creep. Creep is usually concerned with engineers and metallurgists when evaluating components that operate under high stresses or high temperatures. Transition stresses in the rotating discs play a very important role for efficient design. Rotating discs have a wide range of applications such as rotors in rotating high speed gear engines, flywheels, turbines, computer disc drives, shrink fits, compressors, and machinery, etc. Such discs work under complex thermal and mechanical loads. For instance, gas turbine rotors engaged in power plants and aerospace engineering are subjected to centrifugal force and low and high temperature environment are highly strung to creep for a long time. The analytical studies of elastic-plastic rotating discs made of transversely isotropic materials can be found in many books /1-4/. The uses of rotating discs in engineering and scientific applications have generated interest in creep problems and have continued the research area for a long time. Swainger /5/ explained the analysis of deformation. Ghose /6/ discussed the thermal effect on the transverse vibration of a spinning disk of variable thickness. Murakami et al. /7/ obtained constitutive equation for transversely isotropic materials and its application to the bending of perforated circular plates. Furthermore, Thakur et al. /8/ applied Seth's transition theory to the problem of thermal creep transition stresses and strain rates by finitesimal deformation in a thin rotating disc with shaft and having variable density parameter. Gupta et al. /9/ investigated thermoelastic plastic transition in a thin rotating disc with inclusion by using Seth's theory. In this paper, thermal behaviour in a rotating disc made of transversely isotropic material with rigid shaft is discussed by using transition theory. The generalized principal measures in Cartesian coordinates may be written in the form /10, 11/:

$$\varepsilon_{ii} = \int_0^{\varepsilon_{ii}^A} \left[1-2\varepsilon_{ii}^A\right]^{\frac{n}{2}-1} d\varepsilon_{ii}^A = \frac{1}{n}\left[1-\left(1-2\varepsilon_{ii}^A\right)^{\frac{n}{2}}\right], \quad (1)$$

where: n is strain measure coefficient; ε_{ii}^A Almansi finite strain component; and i = 1, 2, 3, and n = -2, -1, 0, 1, 2, it gives Green, Cauchy, Hencky, Swainger, and Almansi, respectively.

MATHEMATICAL PROFILE AND GOVERNING BASIC EQUATIONS

We consider a homogeneous rotating disc mounted on a rigid shaft made of transversely isotropic material with constant density and having central bore of internal radius r_i and external radius r_0 ($r_0 > r_i$) as shown in Fig. 1. The disc rotates gradually increasing at angular speed ω around an axis perpendicular to its plane and passes through the centre. The thickness of disc is assumed very small so that the disc is effectively in a state of plane stress and the temperature Θ is applied to the central bore of the internal surface of the disc made of transversely isotropic/isotropic materials.

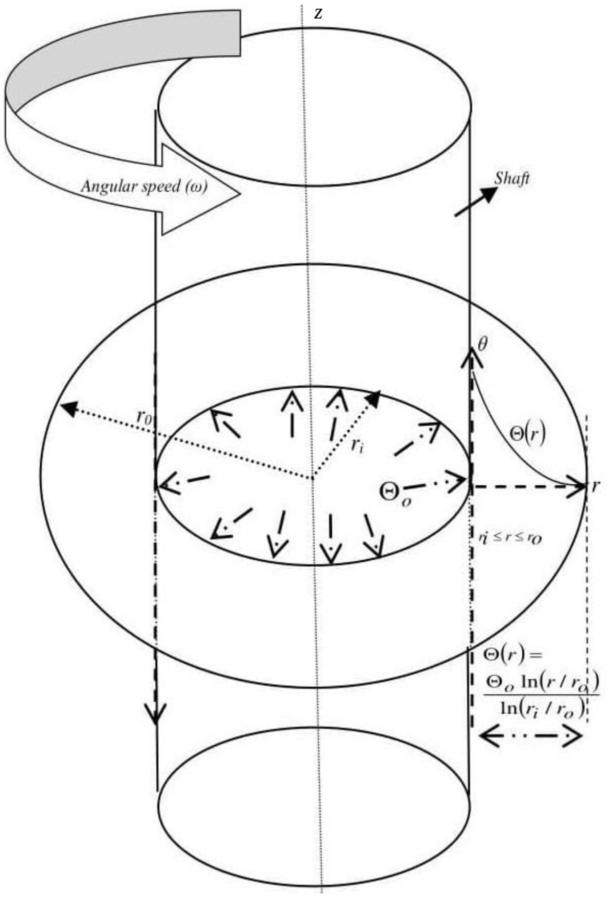


Figure 1. Geometry of rotating disc.

Boundary conditions: we consider internal surface of the rotating disc is fixed to a shaft and external surface is free from mechanical load. So, the boundary conditions of the problem are taken mathematically as:

$$u=0,\quad r=r_i;\quad \tau_{rr}=0,\quad r=r_0, \tag{2}$$

where: τ_{rr} and u are radial stress and displacement along the radial direction, respectively.

Displacement coordinates and strain measures: since the shaft is strained symmetrically, therefore, we can take the components of displacement in cylindrical coordinates as:

$$u=r(1-\eta);\quad v=0;\quad w=dz, \tag{3}$$

where: η is position function, depending on $\eta=\sqrt{x^2+y^2}$ only; and d is a constant.

Finite-strain components: the finitesimal components of strain are given by /10, 11/ as:

$$\varepsilon_{rr}^A \equiv \frac{\partial u}{\partial r}-\frac{1}{2}\left[\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{\partial w}{\partial r}\right)^2-v^2\right]=\frac{1}{2}\left[-(\eta+r\eta')^2+1\right], \tag{4}$$

$$\varepsilon_{\theta\theta}^A \equiv \frac{1}{r}\frac{\partial v}{\partial r}+\frac{u}{r}-\frac{1}{2r^2}\left[\left(\frac{\partial u}{\partial \theta}\right)^2+r^2\left(\frac{\partial v}{\partial \theta}\right)^2+\left(\frac{\partial w}{\partial \theta}\right)^2\right]-\frac{1}{2r^2}\times$$

$$\times\left[-vr^2\frac{\partial u}{\partial \theta}+u\frac{\partial v}{\partial \theta}-v\frac{\partial u}{\partial \theta}+ur^2\frac{\partial v}{\partial \theta}+u^2+r^2v^2\right]=\frac{1}{2}\left[-\eta^2+1\right], \tag{5}$$

$$\varepsilon_{zz}^A \equiv \frac{\partial w}{\partial z}-\frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2+r\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]=\frac{1}{2}\left[-(1-d)^2+1\right], \tag{6}$$

$$\varepsilon_{r\theta}^A=0=\varepsilon_{\theta z}^A=\varepsilon_{zr}^A, \tag{7}$$

where: u, v, w are the physical components of displacement and ε_{rr}^A, $\varepsilon_{\theta\theta}^A$, ε_{zz}^A, $\varepsilon_{r\theta}^A$, $\varepsilon_{\theta z}^A$ and ε_{zr}^A are the components of strain tensor ε_{ij}^A and superscript '^{A}' is the Almansi and $\eta' = d\eta/dr$.

Generalized strain components: using the generalized components of strain is given by /11/:

$$\varepsilon_{rr}=\frac{1}{n}\left[-(\eta+r\eta')^n+1\right], \tag{8}$$

$$\varepsilon_{\theta\theta}=\frac{1}{n}\left[-\eta^n+1\right], \tag{9}$$

$$\varepsilon_{zz}=\frac{1}{n}\left[-(1-d)^n+1\right], \tag{10}$$

$$\varepsilon_{r\theta}=0=\varepsilon_{\theta z}=\varepsilon_{zr}.$$

Stress-strain relations for transversely isotopic material: thermoelastic constitutive equations are given by /2, 12/:

$$\tau_{rr}=c_{11}\varepsilon_{rr}+(c_{11}-2c_{66})\varepsilon_{\theta\theta}+c_{13}\varepsilon_{zz}-\beta_1\Theta,$$
$$\tau_{\theta\theta}=(c_{11}-2c_{66})\varepsilon_{rr}+c_{11}\varepsilon_{\theta\theta}+c_{13}\varepsilon_{zz}-\beta_1\Theta,$$
$$\tau_{r\theta}=0=\tau_{\theta z}=\tau_{zr}=\tau_{zz}, \tag{11}$$

where: $\beta_1 = \alpha_1 c_{11} + 2\alpha_2 c_{12}$; α_1 is coefficient of linear thermal expansion across the axis of symmetry; α_2 is the corresponding quantities orthogonal to axis of symmetry; c_{ij} are elastic material parameters (constants); and Θ is the temperature change. Stresses are obtained as substitution in Eqs. (8)-(10), we get

$$\tau_{rr}=\frac{c_{11}}{n}\left[-(\eta+r\eta')^n+1\right]+(c_{11}-2c_{66})\frac{1}{n}\left[-\eta^n+1\right]+c_{13}\varepsilon_{zz}-\beta_1\Theta$$

$$\tau_{\theta\theta}=\frac{(c_{11}-2c_{66})}{n}\left[-(\eta+r\eta')^n+1\right]+\frac{c_{11}}{n}\left[-\eta^n+1\right]+c_{13}\varepsilon_{zz}-\beta_1\Theta$$

$$\tau_{r\theta}=0=\tau_{\theta z}=\tau_{zr}=\tau_{zz}. \tag{12}$$

The temperature field satisfying the heat equation is given /2/:

$$\nabla^2\Theta=\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Theta}{dr}\right)=0,$$

and $\Theta=\Theta_0$ at $r = r_i$, $\Theta=0$ at $r = r_0$,

where: Θ_0 is a constant. Solving this equation, we get

$$\Theta=\bar{\Theta}_0\ln(r/r_0), \tag{13}$$

where: $\bar{\Theta}_0 = \dfrac{\Theta_0}{\ln(r_i / r_0)}$.

Equations of equilibrium for stress-strain are satisfied except

$$\frac{d}{dr}(\tau_{rr}) + \frac{(\tau_{rr} - \tau_{\theta\theta})}{r} + \rho\omega^2 r = 0\,, \tag{14}$$

where: ρ is the constant density of the disc material.

Asymptotic solution at transition points: from Eq.(12) in Eq.(14), we get a nonlinear differential equation in η as:

$$c_{11} n\eta^{n+1} T(1+T)^{n-1} \frac{dT}{d\eta} = -c_{11} n\eta^n T(1+T)^n - (c_{11} - 2c_{66}) n\eta^n T +$$

$$+2c_{66}\left[1-(1+T)^n\right]\eta^n - n\beta_1\bar{\Theta}_o + n\rho\omega^2 r^2\,, \tag{15}$$

where: $r\eta' = T\eta$. From Eq.(13), the transitional points of η are $T = -1$ and $T \to \pm\infty$.

ANALYTICAL CREEP SOLUTION OF THE PROBLEM

Gupta et al. /9/, Seth /10, 11/, and Thakur et al. /13-35, 37, 38/ have shown the asymptotic solution which leads to creep state at transition point $T \to -1$ via the principal stress differences. We define the transition function Ψ to find the creep stresses as:

$$\Psi = \tau_{rr} - \tau_{\theta\theta} \equiv \frac{2}{n} c_{66}\eta^n\left[-(1+T)^n + 1\right]. \tag{16}$$

Taking the logarithmic differentiation of Eq.(16) with respect to r, we get:

$$\frac{d}{dr}(\ln\Psi) = \frac{nT\left\{1-(1+T)^n - \eta(1+T)^n \dfrac{dT}{d\eta}\right\}}{r\left[-(1+T)^n + 1\right]}\,. \tag{17}$$

Using Eq.(15) into Eq.(17) and by taking asymptotic value $T \to -1$, we get

$$\frac{d}{dr}(\ln\Psi) = \frac{1}{r}\left\{-2n - \frac{n\rho\omega^2 r^2}{c_{11}\eta^n} + \frac{n}{\eta^n c_{11}}\beta_1\bar{\Theta}_o + c_1(n-1)\right\} \tag{18}$$

where: $c_1 = 2c_{66}/c_{11}$. As $T \to -1$, the asymptotic value of η is M/r, M is a constant. Now integrating Eq.(18), we get

$$\Psi = \tau_{rr} - \tau_{\theta\theta} \equiv A_1 r^{-2n+c_1(n-1)} \exp(f)\,, \tag{19}$$

where: A_1 is a constant of integration and

$$f = \frac{n\beta_1\bar{\Theta}_o}{c_{11}}\int \frac{1}{\eta^n r}dr - \frac{n\rho\omega^2}{c_{11}}\int \frac{r}{\eta^n}dr\,.$$

From Eq.(19) and Eq.(14), we get

$$\tau_{rr} = -A_1\int r^{-1-2n+c_1(n-1)} \exp(f)dr - \frac{\rho\omega^2 r^2}{2} + A_2\,, \tag{20}$$

where: A_2 is a constant of integration. Inserting Eq.(4) into Eq.(20), we have

$$A_2 = A_1 \int_{r=r_0} r^{-1-2n+c_1(n-1)} \exp(f)dr - \frac{\rho\omega^2 {r_o}^2}{2}\,. \tag{21}$$

By using Eq.(21) into Eq.(20), we get

$$\tau_{rr} = A_1 \int_r^{r_0} r^{-1-2n+c_1(n-1)} \exp(f)dr - \frac{\rho\omega^2(r^2 - r_0^2)}{2}\,. \tag{22}$$

Now, from Eq.(19) by making use of Eq.(22), we get

$$\tau_{\theta\theta} = A_1\left[\int_r^{r_0} r^{-1-2n+c_1(n-1)} \exp(f)dr - r^{-2n+c_1(n-1)} \exp(f)\right] -$$

$$-\frac{\rho\omega^2(r^2 - r_0^2)}{2}\,. \tag{23}$$

To investigate the displacement of the rotating disc by combining Eq.(16) and Eq.(19), we get

$$\frac{2}{n} c_{66}\eta^n\left[1-(1+T)^n\right] = A_1 r^{-2n+c_1(n-1)} \exp(f)\,. \tag{24}$$

As $T \to -1$, Eq.(24), we get

$$\eta = \left[\frac{n}{2c_{66}} A_1 r^{-2n+c_1(n-1)} \exp(f)\right]^{\frac{1}{n}}. \tag{25}$$

Inserting Eq.(25) into Eq.(3), the component of displacement becomes:

$$u = r - r\left[\frac{n}{2c_{66}} A_1 r^{-2n+c_1(n-1)} \exp(f)\right]^{\frac{1}{n}}. \tag{26}$$

Therefore, the constant A_1 is obtained by applying Eq.(2) into Eq.(26), we get

$$A_1 = \frac{2c_{66} r_i^{2n-c_1(n-1)}}{n\exp(f)\big|_{r=r_i}}\,. \tag{27}$$

Now, using the value of A_1 in Eqs.(22), (23) and (26), we get

$$\tau_{rr} = \frac{2c_{66} r_i^{2n-c_1(n-1)}}{n\exp(f)\big|_{r=r_i}} \int_r^{r_0} r^{-1-2n+c_1(n-1)} \exp(f)dr -$$

$$-\frac{\rho\omega^2(r^2 - r_0^2)}{2}\,, \tag{28}$$

$$\tau_{\theta\theta} = \frac{2c_{66} r_i^{2n-c_1(n-1)}}{n\exp(f)\big|_{r=r_i}}\left[\int_r^{r_0} r^{-1-2n+c_1(n-1)} \exp(f)dr -\right.$$

$$\left. - r^{-2n+c_1(n-1)} \exp(f)\right] - \frac{\rho\omega^2(r^2 - r_0^2)}{2}\,, \tag{29}$$

$$u = r - r\left[\left(\frac{r}{r_i}\right)^{-2n+c_1(n-1)} \frac{\exp(f)}{\exp(f)\big|_{r=r_i}}\right]^{\frac{1}{n}}. \tag{30}$$

Non-dimensional components: non-dimensional quantities are introduced as follows:

$$R = \frac{r}{r_0}\,,\ R_0 = \frac{r_i}{r_0}\,,\ \tau_r = \frac{\tau_{rr}}{c_{66}}\,,\ \tau_\theta = \frac{\tau_{\theta\theta}}{c_{66}}\,,\ U = \frac{u}{r_0}\,,$$

$$\Omega^2 = \frac{\rho\omega^2 r_0^2}{c_{66}}\,,\ \beta_4 = \frac{\beta_1\bar{\Theta}_0}{c_{66}}\,.$$

Creep stress distribution and displacement

Creep stress distribution and displacement in non-dimensional form become from Eqs.(28)-(30), we get

$$\tau_r = \frac{2R_0^{2n-c_1(n-1)}}{n\exp(f)\big|_{R=R_0}} \int_R^1 R^{-1-2n+c_1(n-1)} \exp(f)dR - \frac{\Omega^2(R^2-1)}{2}\,, \tag{31}$$

$$\tau_\theta = \frac{2R_0^{2n-c_1(n-1)}}{n\exp(f)\big|_{R=R_0}}\left[\int_R^1 R^{-1-2n+c_1(n-1)}\exp(f)dR - \right.$$

$$\left. -R^{-2n+c_1(n-1)}\exp(f)\right] - \frac{\Omega^2(R^2-1)}{2}, \qquad (32)$$

$$U = R - R\left[\left(\frac{R}{R_0}\right)^{-2n+c_1(n-1)}\frac{\exp(f)}{\exp(f)\big|_{R=R_0}}\right]^{\frac{1}{n}}, \qquad (33)$$

where: $f = \frac{\beta_4 c_1}{2}\left(\frac{r_0}{M}\right)^n R^n - \frac{n\Omega^2 c_1}{2(n+2)}\left(\frac{r_0}{n}\right)^n R^{n+2}$.

Creep strain rates: creep strain rates are given /34/:

$$\dot{\varepsilon}_{rr} = \zeta\left[\tau_r - k\tau_\theta + \beta_4(1-k)\left(1-\frac{c_{13}}{c_{11}}\right)\right],$$

$$\dot{\varepsilon}_{\theta\theta} = \zeta\left[\tau_\theta - k\tau_r + \beta_4(1-k)\left(1-\frac{c_{13}}{c_{11}}\right)\right],$$

$$\dot{\varepsilon}_{zz} = \zeta\left[-(1-k)(\tau_r+\tau_\theta) + \beta_4\left\{\frac{1}{2}\left(1-\frac{c_{12}}{c_{11}}\right) + \frac{c_{13}(1-k)}{c_{11}} - \right.\right.$$

$$\left.\left. -\frac{c_{13}}{c_{11}}\left(1-\frac{2c_{13}}{c_{11}}\right)\right\}\right], \qquad (34)$$

where: $k = 1 - \frac{c_{11}(c_{11}-c_{12})}{c_{11}{}^2 - c_{13}{}^2}$; and $\zeta = \left[\frac{n(\tau_r - \tau_\theta)}{2}\right]^{\frac{1}{n}-1}$.

Equations (31)-(34) are the same as given by Thakur et al. /31/ when we neglect the thermal condition.

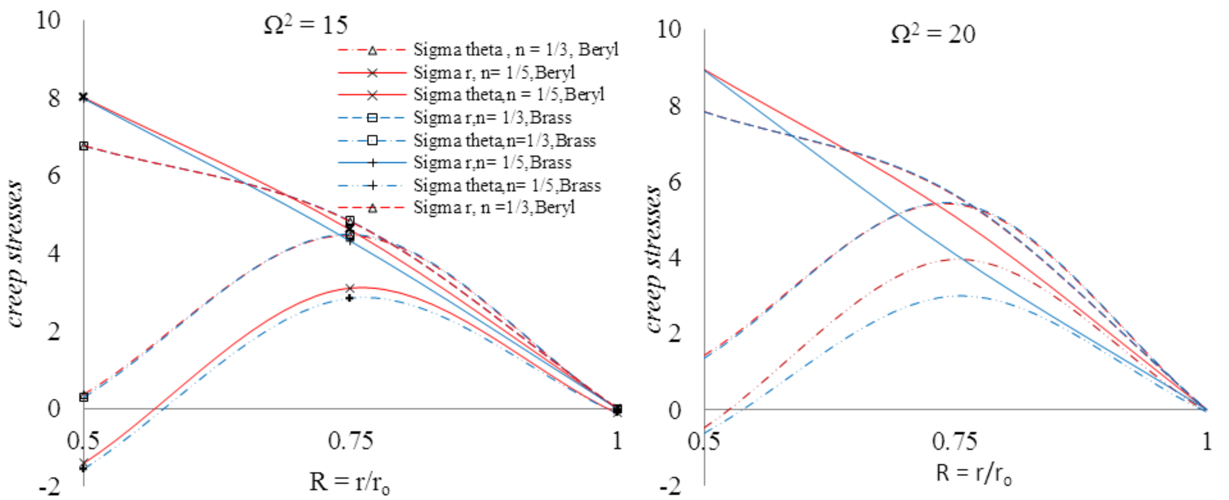
NUMERICAL RESULTS AND DISCUSSION

Simpson's 1/3 rule of numerical integration has been used to approximate the values of the definite integrals in Eqs.(31) and (32). To illustrate, the following analysis of creep stresses and strain rates and displacement based on the below analysis, the following values have been taken: $\Omega^2 = \rho\omega^2 r_0^2 = 15.20$; elastic constants c_{ij} (10^{10} N/m) for transversely isotropic material (say beryl: $c_{11} = 2.746$, $c_{12} = 0.980$, $c_{13} = 0.674$) and isotropic material (say brass: $c_{11} = 3.000$, $c_{12} = 1.000$, $c_{13} = 1.000$) /31/; $\beta_4 = 0, 25, 50$; $n = 1/N$ (measure in classical theory) /36/. Figure 2 depicts the creep stress distribution along the radii ratio $R = r/r_0$ at $\Omega^2 = 15$, 20; and temperature $\beta_4 = 0, 25, 50$; and $n = 1/3, 1/5$ (i.e. $N = 3, 5$). It has been observed that from Fig. 2a-c the radial stress is maximum at the internal surface of the disc made of beryl material as compared to the brass material. Beryl material disc required higher value of radial stress at the internal surface to measure $n = 1/5$ in comparison to the disc made of brass material, meaning that beryl material disc is more comfortable than that of brass material. With increasing the values of angular speed and thermal condition, the value of radial stress as well as tangential stress must be increased on the internal surface.

From Fig. 3, curves are drawn between the strain rates and radii ratio $R = r/r_0$ at $\Omega^2 = 15$, 20; $\beta_4 = 0, 25, 50$, and $n = 1/3, 1/5$ (i.e. $N = 3, 5$). It has been observed that the rotating disc made of brass material has a maximum value at the internal surface as compared to beryl material for measures $n = 1/3, 1/5$ (i.e. $N = 3, 5$) at $\Omega^2 = 15$. The values of strain rates further are increased at the internal surface with increasing value of $\Omega^2 = 20$ for measures $n = 1/3, 1/5$ (i.e. $N = 3, 5$). With the introduction of thermal condition, the values of creep strain rates increase at the internal surface of the disc made of brass material in comparison to beryl material.

(notation: Sigma $r \equiv \sigma_r$, Sigma theta $\equiv \sigma_\theta$)

(a) $\beta_4 = 0$



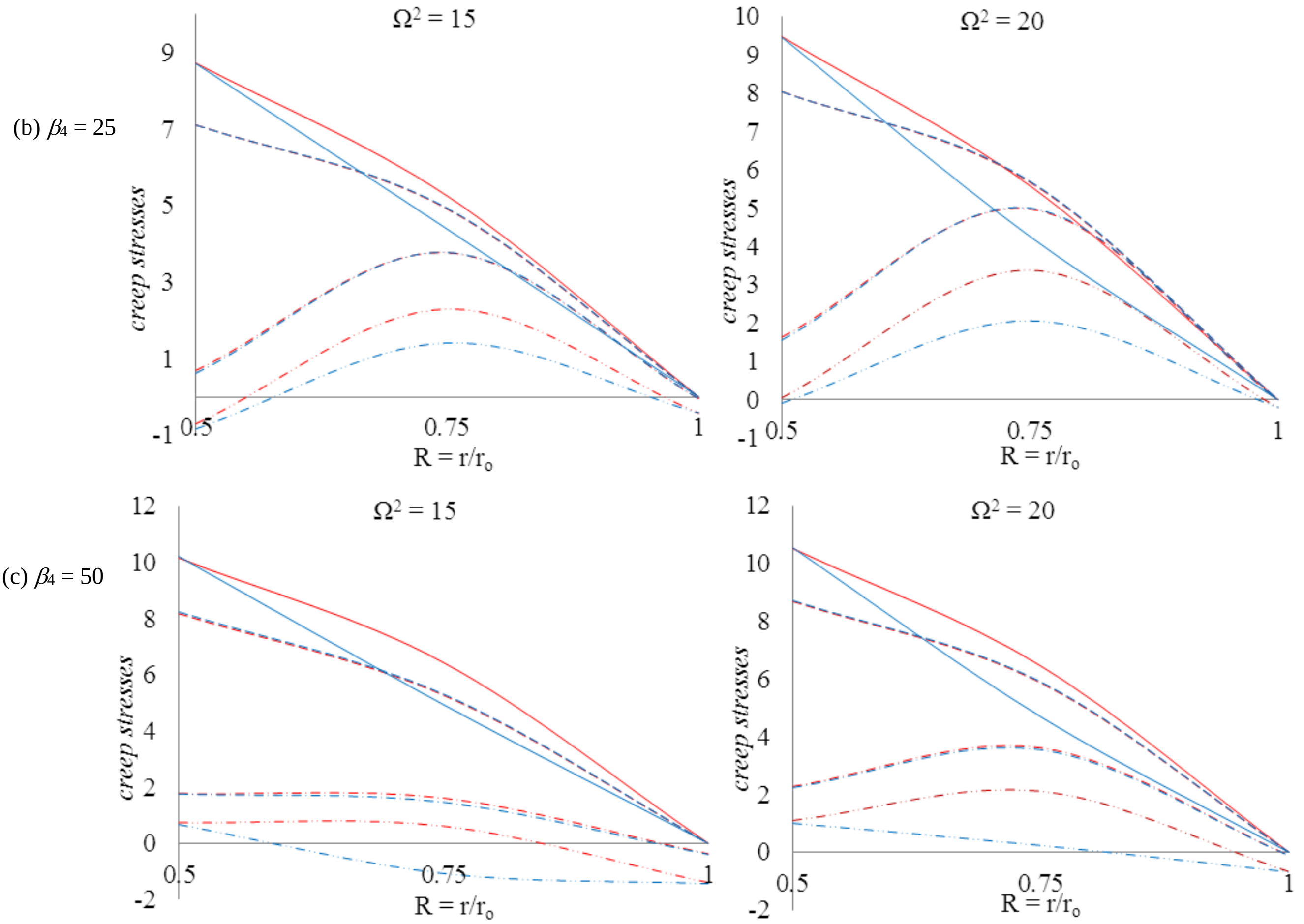
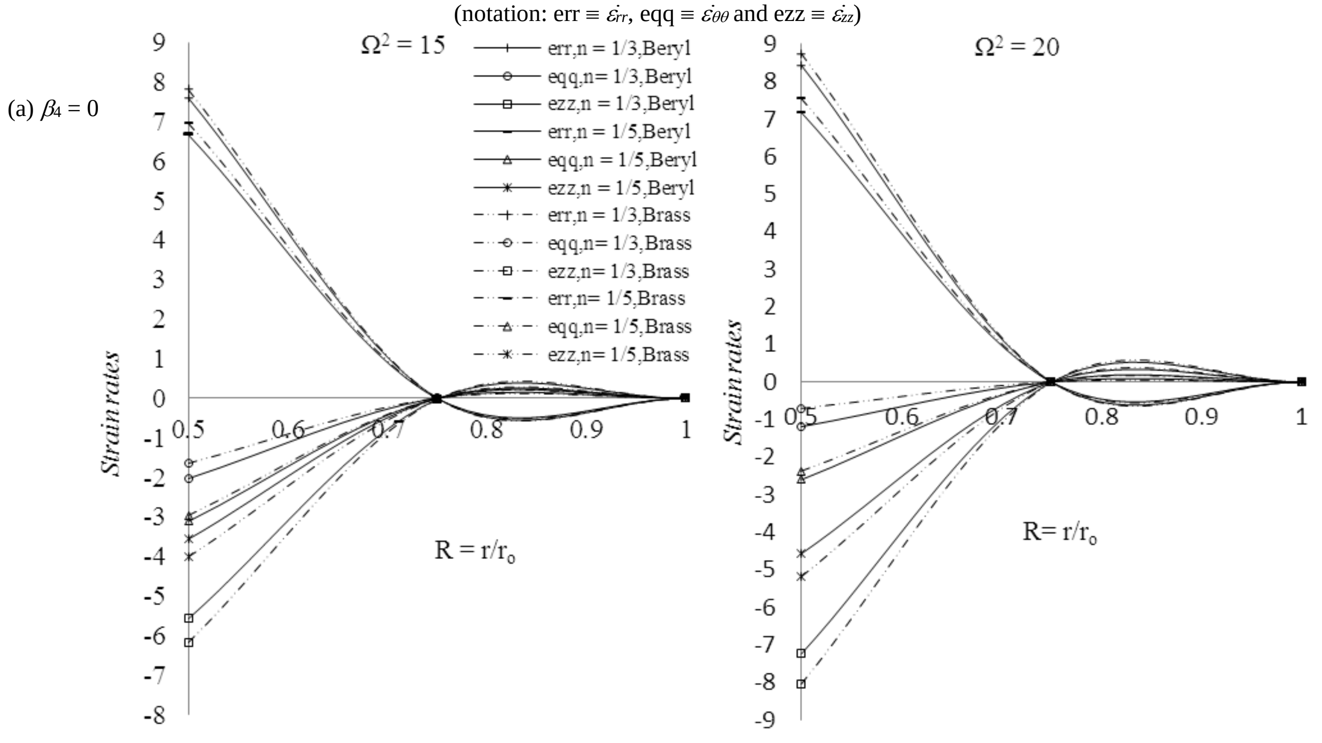


Figure 2. Creep stresses along the radii ratio $R = r/r_0$ at: a) $\beta_4 = 0$; b) $\beta_4 = 25$; and c) $\beta_4 = 50$.

(notation: err ≡ $\dot{\varepsilon}_{rr}$, eqq ≡ $\dot{\varepsilon}_{\theta\theta}$ and ezz ≡ $\dot{\varepsilon}_{zz}$)



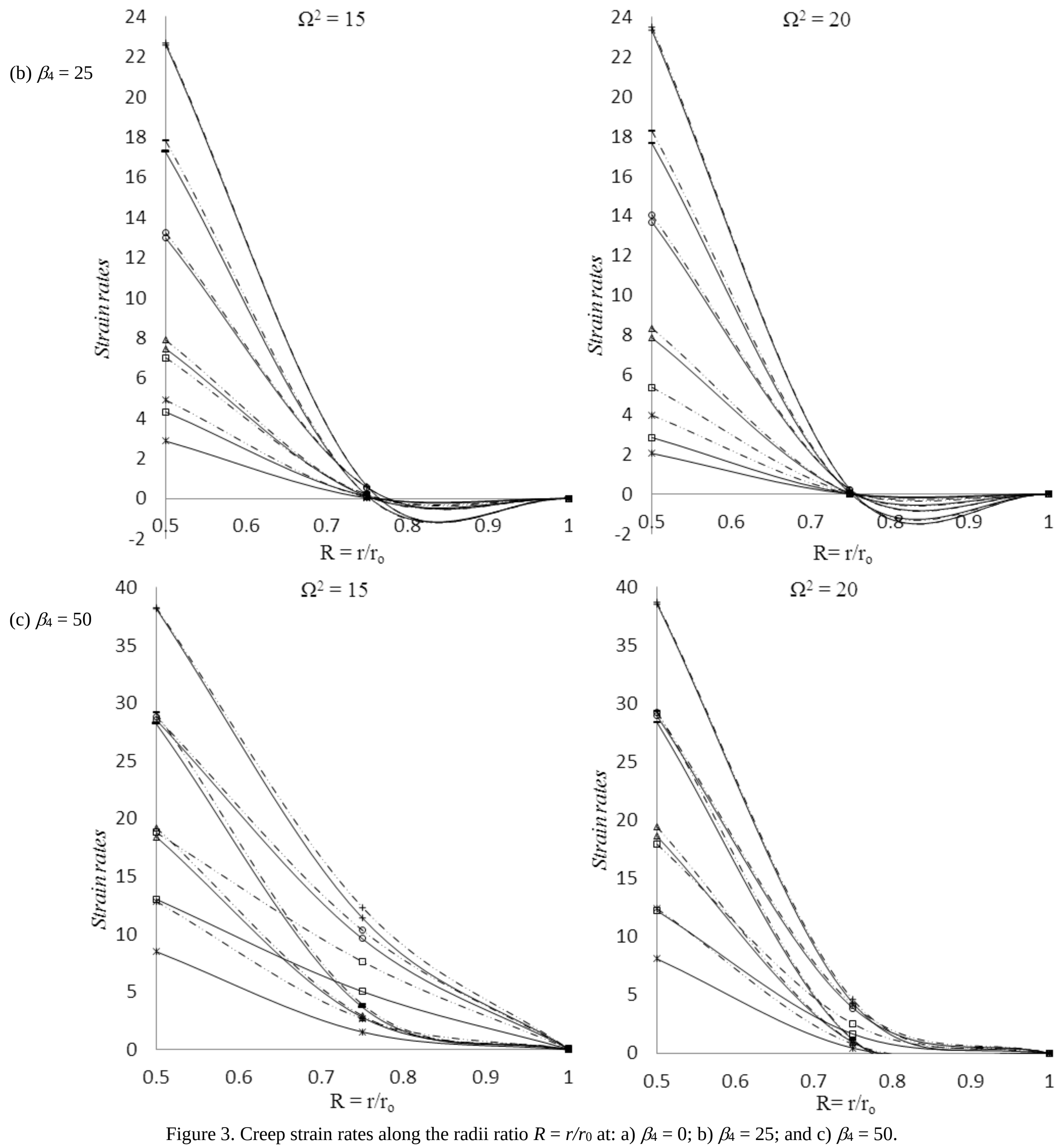


Figure 3. Creep strain rates along the radii ratio $R = r/r_0$ at: a) $\beta_4 = 0$; b) $\beta_4 = 25$; and c) $\beta_4 = 50$.

CONCLUSIONS

Solutions are calculated that consider the plane symmetry to achieve a goal for reliable rotating disc analysis and to obtain the stress distribution and strain rates under the application of the thermal condition. To gain such type of goal, transversely isotropic disc or an isotropic disc with shaft is considered. It is observed that the radial stress has a maximum at the internal surface of the disc made of beryl material as compared to the disc of brass material. Beryl material disc required higher value of radial stress at the internal surface for measure $n = 1/5$ in comparison to the disc of brass material which means that the beryl material disc is more comfortable than that of the brass material. With increasing the values of angular speed and thermal condition the radial, as well as tangential stress must be increased on the internal surface. It is also observed that the rotating disc of brass material has a maximal value at the internal surface as compared to the beryl material for measures n = 1/3, 1/5 (i.e. N = 3, 5) at $\Omega^2 = 15$. With the introduction of thermal condition, the values of creep strain rates increase at the internal surface of the disc made of brass material.

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