Victor Iliev Rizov

INHOMOGENEOUS STRUCTURAL COMPONENT WITH TWO LENGTHWISE CRACKS – A NONLINEAR FRACTURE ANALYSIS

NEHOMOGENA STRUKTURNA KOMPONENTA SA DVE PODUŽNE PRSLINE – NELINEARNA ANALIZA LOMA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 16.11.2020

Adresa autora / Author's address:
University of Architecture, Civil Engineering and Geodesy, Department of Technical Mechanics, 1 Chr. Smirnesky blvd., 1046 - Sofia, Bulgaria, email: v_rizov_fhe@uacg.bg

Keywords
- lengthwise crack
- inhomogeneous structural component
- nonlinear elastic behaviour
- fracture

Abstract

The paper analyses fracture of an inhomogeneous cylindrical structural component with two internal lengthwise cracks. The component is loaded in torsion and its cross-section is a circle. The structural component exhibits continuous material inhomogeneity in radial direction. The two cracks represent concentric circular cylindrical surfaces whose lengths differ. The structural component is made of a nonlinear elastic material and is treated as a structure of two degrees of internal static indeterminacy in order to obtain the torsion moments in the internal, interstitial, and external parts of the component (torsion moments in the internal and interstitial parts are taken as redundants). The lengthwise fracture is studied in terms of strain energy release rate. The solutions derived are verified by considering the balance of energy. Lengthwise fracture is analysed also for the case when a circular notch is cut-out in the external and interstitial parts of the component. The results obtained are compared with results for a structural component without a notch.

Ključne reči
- podužna prslina
- nehomogena strukturna komponenta
- nelinearno elastično ponašanje
- lom

Izvod

U radu se analizira lom nehomogene cilindrične strukturne komponente sa dve unutrašnje podužne prsline. Komponenta je opterećena na uvijanje. Poprečni presek strukturne komponente je kružnog oblika. U strukturnoj komponenti je neprekidna nehomogenost materijala u radijalnom pravcu. Dve prsline su predstavljene koncentričnim kružnim cilindričnim površinama različitih dužina. Strukturna komponenta je sačinjena od nelinearno elastičnog materijala i razmatra se kao konstrukcija sa dva stepena unutrašnje neodređenosti radi dobijanja momenata uvijanja u unutrašnjem delu, u međusloju, i u spoljnom delu komponente (momenti uvijanja u unutrašnjem delu i u međusloju se zanemaruju). Podužni lom se razmatra preko brzine oslobađanja deformacione energije. Dobijena rešenja se verifikuju preko balansa energije. Podužni lom se analizira i za slučaj kada postoji kružni zarez izveden na spoljnom delu i u međusloju komponente. Dobijeni rezultati se porede sa rezultatima za strukturnu komponentu bez zareza.

INTRODUCTION

Inhomogeneous materials are used in an increasing rate for various load-bearing structural applications in modern engineering. One type of these novel materials are inhomogeneous materials with continuously varying material properties in the solid, which are of interest in the present paper. The properties of these materials are continuous (smooth) functions of coordinates. This circumstance complicates the analysis of structural members and components made of inhomogeneous materials. The fact that the properties of inhomogeneous materials vary gradually eliminates interfacial stress concentrations (as one of the most important advantages of inhomogeneous materials over fiber reinforced composite materials). It should be mentioned that the strong interest towards inhomogeneous materials of continuously varying properties is due in a high degree to the wide application of functionally graded materials in aeronautics, nuclear reactors, microelectronics, chemical industry, and robotics. Functionally graded materials are a kind of inhomogeneous materials manufactured by mixing two or more constituent materials. Since the composition of the constituent materials can be changed gradually during the manufacturing process, smooth variations of macroscopic material properties along one or more coordinates can be obtained /1-8/. In this way, the properties of functionally graded materials can be formed technologically in order to get the best performance of structural members and components to externally applied loadings. The spatial variation of material properties can be tailored so as to satisfy specific exploitation requirements in different parts of a structure.

Fracture is one of the serious problems that affect the structural integrity and obstruct applications of functionally graded materials, especially in load-bearing structures /9-12/. Appearance of cracks abruptly deteriorates the operational capacity and shortens the life-time of the structures. Therefore, fracture analyses of functionally graded structural members and components under various loading conditions are much needed. Fracture analyses are of great importance for the design process to ensure fracture resistance and structural integrity.

The objective of the present paper is to analyse fracture behaviour of an inhomogeneous nonlinear elastic cylindrical structural component with two internal lengthwise circular cylindrical cracks loaded in torsion. It should be noted that previous works on lengthwise fracture deal with one crack located in the end of the beam /13-15/. When the lengthwise crack is in the end of the beam, the torsion moments in the crack arms can be obtained directly. However, when the crack is internal, the torsion moments in the crack arms which are needed to calculate the strain energy release rate cannot be found directly. In the present paper, the structural component with two internal lengthwise cracks is treated as a structure of two degrees of internal static indeterminacy. The torsion moments in the internal, interstitial, and external parts of the structural component obtained as a result of resolving the static indeterminacy are used in order to derive the strain energy release rate. The balance of energy is considered for verification of the solution of the strain energy release rate derived in the present paper.

FRACTURE ANALYSIS

The paper analyses the fracture behaviour of the inhomogeneous cylindrical structural component with two internal lengthwise concentric cracks shown in Fig. 1.

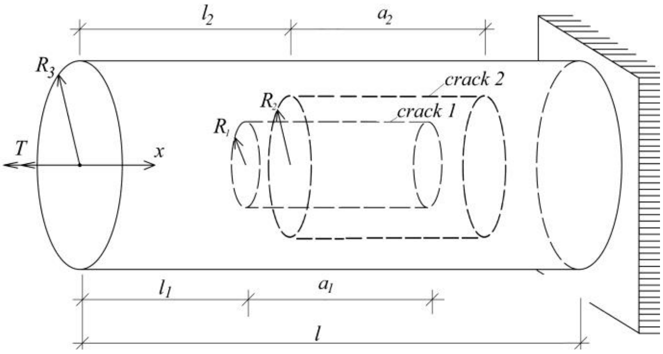

Figure 1. Inhomogeneous structural component of circular cross-section with two internal lengthwise circular cylindrical cracks loaded in torsion.

The cross-section of the structural component is a circle of radius R_3, and of length l. The component is clamped in its right-hand end. The loading consists of one torsion moment, T, applied at the free end. The structural component exhibits continuous material inhomogeneity in radial direction. The two cracks represent circular cylindrical surfaces. The length and radius of crack 1 are a_1 and R_1, respectively. The left-hand front of crack 1 is located at distance l_1 of the free end. The length and radius of crack 2 are denoted by a_2 and R_2, respectively. The distance between the left-hand front of crack 2 and the free end of the structural component is l_2. The component is made of nonlinear elastic material.

The fracture is studied in terms of strain energy release rate, G. First, a small increase in the length of crack 1 at the left-hand crack front is assumed. Strain energy release rate is expressed as, /13/:

$$G = \frac{1}{R_1}\left(\int_0^{R_1} u^*_{0a_{1r}} R dR + \int_{R_1}^{R_2} u^*_{0a_{2r}} R dR - \int_0^{R_3} u^*_{0r} R dR \right), \quad (1)$$

where: $u^*_{0a_{1r}}$, $u^*_{0a_{2r}}$ and u^*_{0r} are the complementary strain energy densities in internal and external crack arms behind the left-hand front of crack 1 and in the cross-section of the structural component ahead of the crack front. It should be mentioned that the cross-section of the internal crack arm is a circle of radius R_1. The external crack arm has a ring-shaped cross-section of internal and external radii, R_1 and R_3, respectively.

In the present paper, the mechanical behaviour of the material is treated by using the following nonlinear stress-strain relation, /16/:

$$\tau = P\gamma^m - Q\gamma^n , \quad (2)$$

where: τ is shear stress; γ is shear strain; P, Q, m and n are material properties. The distribution of material property, P, in radial direction is written as

$$P = P_0 + \frac{P_1 - P_0}{R_3} R , \quad (3)$$

where: $0 \le R \le R_3$. In Eq.(3), P_0 and P_1 are the values of P in the centre of cross-section at the periphery of the structural component.

Complementary strain energy density is equal to the area that supplements the area enclosed by the stress-strain curve to a rectangle. Thus, $u^*_{0a_{1r}}$ is expressed as

$$u^*_{0a_{1r}} = \tau\gamma - u_{0a_{1r}} , \quad (4)$$

where: $u_{0a_{1r}}$ is strain energy density. In principle, the strain energy density is equal to the area enclosed by the stress-strain curve. Thus, by integrating Eq.(2), one obtains

$$u_{0a_{1r}} = \frac{P\gamma^{m+1}}{m+1} - \frac{Q\gamma^{n+1}}{n+1} . \quad (5)$$

By combining Eqs.(2), (4) and (5), one derives

$$u^*_{0a_{1r}} = \frac{mP\gamma^{m+1}}{m+1} - \frac{nQ\gamma^{n+1}}{n+1} . \quad (6)$$

The distribution of shear strains is treated by applying the Bernoulli's hypotheses for plane section since cylindrical structural components of high length-to-diameter ratio are under consideration in the present paper. Thus, distribution of shear strains in the internal arm of crack 1 is written as

$$\gamma = \frac{\gamma_{\alpha l}}{R_1} R , \quad (7)$$

where: $\gamma_{\alpha l}$ is shear strain at the periphery of the internal crack. The following equation for the equilibrium of elementary forces in the cross-section of the internal crack arm is used to obtain $\gamma_{\alpha l}$:

$$\int_0^{R_1} \tau 2\pi R^2 dR = T_1 , \quad (8)$$

where: the torsion moment in the internal crack arm, T_1, is determined in the following manner. First, it should be noted that in portion $l_2 \le x \le l_1 + a_1$, the structural component is divided by the two cracks into three parts: internal, interstitial and external (Fig. 1). The cross-section of the internal part of the structural component is a circle of radius R_1. The interstitial and external parts of the component have ring-shaped cross-sections. The internal and external radii of the cross-section of interstitial part of the component are R_1 and R_2, respectively. The cross-section of the external part of the component has internal and external radii, R_2 and R_3, respectively. The torsion moments in the internal, interstitial and external part of structural component are denoted by T_1, T_2 and T_3, respectively. In order to determine T_1, T_2 and T_3, the structural component is treated as a structure of two degrees of internal static indeterminacy (torsion moments T_1 and T_2 are taken as internal redundants). Static indeterminacy is resolved by using the theorem of Castigliano for structures which exhibit material nonlinearity

$$\frac{\partial U^*}{\partial T_1}=0\,, \tag{9}$$

$$\frac{\partial U^*}{\partial T_2}=0\,, \tag{10}$$

where: complementary strain energy U^* is written as

$$U^* = U_1^* + U_2^* + U_{2B}^* + U_3^* + U_{3D}^* + U_4^* + U_5^*\,. \tag{11}$$

In Eq.(11), U_1^*, U_2^*, U_{2B}^*, U_3^*, U_{3D}^*, U_4^* and U_5^* are the complementary strain energies stored in the internal part of the structural component, portion $l_2 \le x \le l_1 + a_1$ of the interstitial part of the component, portion $l_1 + a_1 \le x \le l_2 + a_2$ of the interstitial part of the component, the external part of the component, portion $l_1 \le x \le l_2$, of the external part of the component, un-cracked portions $0 \le x \le l_1$ and $l_2 + a_2 \le x \le l$ of the component, respectively. It should be mentioned that since U_4^* and U_5^* do not depend on T_1 and T_2, only U_1^*, U_2^*, U_{2B}^*, U_3^* and U_{3D}^* are used to resolve the static indeterminacy by Eqs.(9) and (10).

The complementary strain energy in the internal part of the structural component is expressed as

$$U_1^* = a_1 \int_0^{R_1} u_{0a_{1r}}^* \, 2\pi R dR\,. \tag{12}$$

The complementary strain energy in the portion $l_2 \le x \le l_1 + a_1$ of the interstitial part of the component is written as

$$U_2^* = (l_1 + a_1 - l_2) \int_{R_1}^{R_2} u_{0I}^* \, 2\pi R dR\,, \tag{13}$$

where: complementary strain energy density u_{0I}^* is obtained by replacing γ with γ_I in Eq.(6). Here, γ_I is the distribution of shear strains in the cross-section the interstitial part of the component. Equation (7) is used to express γ_I. For this purpose, $\gamma_{\alpha l}$ and R_1 are replaced, respectively, with γ_{Il} and R_2, where γ_{Il} is the shear strain at the periphery of the interstitial part of the component. The following equation of equilibrium is applied to obtain γ_{Il} :

$$\int_{R_1}^{R_2} \tau_I \, 2\pi R^2 dR = T_2\,. \tag{14}$$

For this purpose, after substituting the shear stress in Eq.(14), the equation of equilibrium is solved with respect to γ_{Il} by using the MatLab® computer programme.

The complementary strain energy in the portion $l_1 + a_1 \le x \le l_2 + a_2$ of the interstitial part of the component is found as

$$U_{2B}^* = (l_2 + a_2 - l_1 - a_1) \int_0^{R_2} u_{0IB}^* \, 2\pi R dR\,. \tag{15}$$

Equation (6) is used to obtain the complementary strain energy density u_{0IB}^*. For this purpose, γ is replaced with γ_{IB}. The distribution of shear strains γ_{IB} is found by replacing $\gamma_{\alpha l}$ and R_1 with γ_{IIB} and R_2 in Eq.(7), where γ_{IIB} is the shear strain at the periphery of the interstitial part of the component in portion $l_1 + a_1 \le x \le l_2 + a_2$. The equation of equilibrium of the cross-section of the interstitial part of the component in portion $l_1 + a_1 \le x \le l_2 + a_2$ that is used to obtain γ_{IB} is written as

$$\int_0^{R_2} \tau_{IB} \, 2\pi R^2 dR = T_4\,. \tag{16}$$

By considering the equilibrium of torsion moments in the internal and interstitial parts of the structural component, the torsion moment T_4 is found as

$$T_4 = T_1 + T_2\,. \tag{17}$$

The complementary strain energy in the external part of the component is expressed as

$$U_3^* = a_2 \int_{R_2}^{R_3} u_{0E}^* \, 2\pi R dR\,. \tag{18}$$

Complementary strain energy density u_{0E}^* is obtained by replacing γ with γ_E in Eq.(6), where the distribution of shear strains γ_E is found by replacing $\gamma_{\alpha l}$ and R_1 with γ_{El} and R_3 in Eq.(7). Here, γ_{El} is the shear strain at the periphery of the external part of the component. The following equation for equilibrium is used to obtain γ_{El}:

$$\int_{R_2}^{R_3} \tau_E \, 2\pi R^2 dR = T_3\,, \tag{19}$$

where:

$$T_3 = T - T_1 - T_2\,. \tag{20}$$

The complementary strain energy in the portion $l_1 \le x \le l_2$ of the external part of the component is written as

$$U_{3D}^* = (l_2 - l_1) \int_{R_1}^{R_3} u_{0a_{2r}}^* \, 2\pi R dR\,. \tag{21}$$

Equation (6) is applied to obtain complementary strain energy density u_{0a2r}^*. For this purpose, γ is replaced with γ_{ED}. The distribution of shear strains γ_{ED} is obtained by replacing $\gamma_{\alpha l}$ and R_1 with γ_{EDl} and R_3 in Eq.(7). The shear strain at the periphery of the component γ_{EDl} is obtained by using the following equation for equilibrium of elementary forces:

$$\int_{R_1}^{R_3} \tau_{ED} \, 2\pi R^2 dR = T_5\,. \tag{22}$$

By considering the equilibrium of torsion moments, T_5 is found as

$$T_5 = T - T_1\,. \tag{23}$$

Complementary strain energy density u_{0r}^* is obtained by replacing γ with γ_H in Eq.(6). The distribution of shear strains γ_H is found by replacing $\gamma_{\alpha l}$ and R_1 with γ_{Hl} and R_3 in Eq.(7). Shear strain γ_{Hl} at the periphery of the structural component ahead of the left-hand front of crack 1 is determined by using the following equation of equilibrium of elementary forces in the cross-section of the structural component:

$$\int_0^{R_3} \tau_H 2\pi R^2 dR = T \,. \tag{24}$$

The strain energy release rate is obtained by substituting $u_{0a1r}{}^*$, $u_{0a2r}{}^*$ and $u_{0r}{}^*$ in Eq.(1). The integration is carried-out by using the MatLab® computer programme.

The strain energy release rate is determined also assuming a small increase of crack 1 length at the right-hand crack front. By using Eq.(1), the strain energy release rate is expressed as

$$G = \frac{1}{R_1}\left(\int_0^{R_1} u^*_{0a_{1r}} RdR + \int_{R_1}^{R_2} u^*_{0I} RdR - \int_0^{R_2} u^*_{0IB} RdR \right). \tag{25}$$

After substituting $u_{0a1r}{}^*$, $u_{0I}{}^*$ and $u_{0IB}{}^*$ in Eq.(25), the integration is carried-out by the MatLab® computer programme.

The following solution of the strain energy release rate is derived assuming a small increase of crack 2 length at the left-hand crack front:

$$G = \frac{1}{R_2}\left(\int_{R_1}^{R_2} u^*_{0I} RdR + \int_{R_2}^{R_3} u^*_{0E} RdR - \int_{R_1}^{R_3} u^*_{0a_{2r}} RdR \right). \tag{26}$$

Integration is performed by MatLab®.

When a small increase of crack 2 length at the right-hand crack front is assumed, the strain energy release rate is expressed as

$$G = \frac{1}{R_2}\left(\int_0^{R_2} u^*_{0IB} RdR + \int_{R_2}^{R_3} u^*_{0E} RdR - \int_0^{R_3} u^*_{0a_{2r}} RdR \right). \tag{27}$$

MatLab® is used to perform the integration in Eq.(27).

For verification, the strain energy release rate is derived also by considering the balance of energy. For this purpose, the following equation is used:

$$G = \frac{T}{2\pi R_1}\frac{\partial \phi}{\partial a} - \frac{1}{2\pi R_1}\frac{\partial U}{\partial a} , \tag{28}$$

where: φ is the angle of twist of the free end of the structural component; U is strain energy in the component. The angle of twist is determined by Maxwell-Mohr integrals. Equation (11) is applied to obtain U. For this purpose, complementary strain energies are replaced with corresponding strain energies. Strain energy densities are found by Eq.(5). The strain energy release rates obtained by Eq.(28) are exact matches of these determined by Eq.(1). This fact is a verification of the analysis of the strain energy release rate developed in the present paper.

NUMERICAL RESULTS

The solutions of the strain energy release rate derived in the previous section are applied to obtain numerical results in order to elucidate the peculiarities of the fracture behaviour of the inhomogeneous nonlinear elastic cylindrical structural component with two internal circular cylindrical cracks.

The strain energy release rate is presented in non-dimensional form by using the formula $G_N = G/(P_0R_3)$. Material inhomogeneity is characterized by P_1/P_0 ratio.

The influence of material inhomogeneity on lengthwise fracture behaviour of the structural component is illustrated in Fig. 2, where strain energy release rate in non-dimensional form is plotted against P_1/P_0 ratio at $R_1/R_3 = 0.3$, $R_2/R_3 = 0.8$ and $Q/P_0 = 0.5$. The strain energy release rate is obtained by using solutions Eq.(1) and Eq.(26) that are derived assuming increase of the cracks at their left-hand fronts. The curves in Fig. 2 indicate that strain energy release rate decreases with increasing of P_1/P_0 ratio (this behaviour is due to the increase of the component stiffness). Also, it is observed in Fig. 2 that the strain energy release rate obtained at increase of crack 2 is lower than that found at the increase of crack 1.

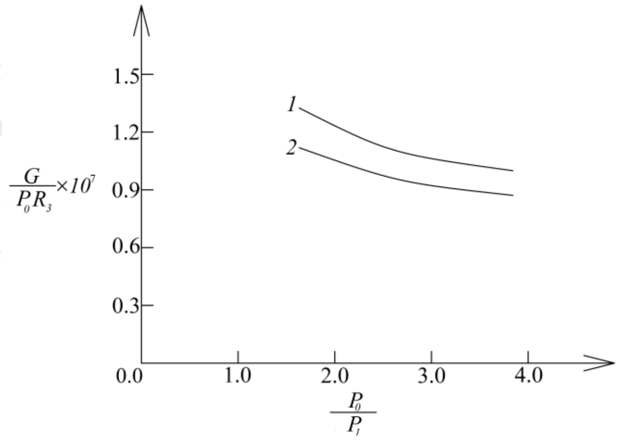

Figure 2. Strain energy release rate in non-dimensional form plotted against P_1/P_0 ratio (curve 1 - at increase of crack 1, curve 2 - at increase of crack 2).

In order to evaluate the effect of crack 1 location in the radial direction on lengthwise fracture behaviour, the strain energy release rate in non-dimensional form is plotted against R_1/R_3 ratio in Fig. 3. Solution Eq.(25) is used (it is obtained assuming the increase of crack 1 length at its right-hand front). The curves in Fig. 3 show that strain energy release rate decreases with increasing of R_1/R_3 ratio. The effect of material inhomogeneity is also evaluated. For this purpose, the strain energy release rate derived assuming linear-elastic behaviour of the inhomogeneous material is also plotted in Fig. 3 (the linear-elastic solution of strain energy release rate is obtained by substituting $m = 1$ and $Q = 0$ in the nonlinear solution since at $m = 1$ and $Q = 0$ the nonlinear stress-strain relation Eq.(2) transforms into Hooke's law). One can observe in Fig. 3 that the strain energy release rate derived by using the model with material nonlinearity is higher than that obtained by the linear-elastic model.

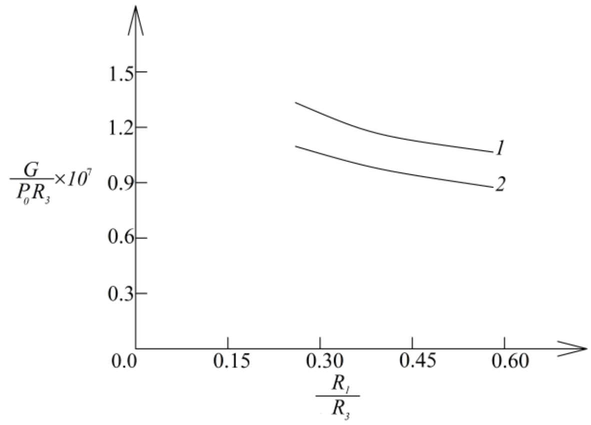

Figure 3. Strain energy release rate in non-dimensional form plotted against R_1/R_3 ratio (curve 1 - nonlinear elastic material behaviour, curve 2 - linear-elastic behaviour).

The locations of cracks 1 and 2 in the radial direction are characterized by R_1/R_3 and R_2/R_3 ratios, respectively. It is assumed that $R_3 = 0.005$ m, $T = 15$ Nm, $m = 0.8$ and 0.4.

It is interesting to analyse the lengthwise fracture behaviour of the inhomogeneous nonlinear elastic structural component assuming that a circular notch of depth $R_3 - R_1$ is cut-out in the external and interstitial parts of the component as shown in Fig. 4. The notch divides both external and interstitial parts of the structural component in two segments. Thus, the torsion moments in both external and interstitial parts of the component are zero. Therefore, the solutions to the strain energy release rate for the component with a circular notch are derived by substituting $T_2 = 0$ and $T_3 = 0$ in Eqs.(1), (25), (26) and (27). The strain energy release rate for the structural component with a circular notch is plotted in non-dimensional form against Q/P_0 ratio in Fig. 5. In order to perform a comparison, the strain energy release rate for the component without a circular notch (Fig. 1) is also plotted in Fig. 5. It can be seen in Fig. 5 that the strain energy release rate increases with increasing of Q/P_0 ratio. Also, one can observe in Fig. 5 that the strain energy release rate in the structural component with a circular notch is higher than that in the structural component without a notch (this is due to the fact that in the component with notch the torsion moment T loads only the internal part of the structural component).

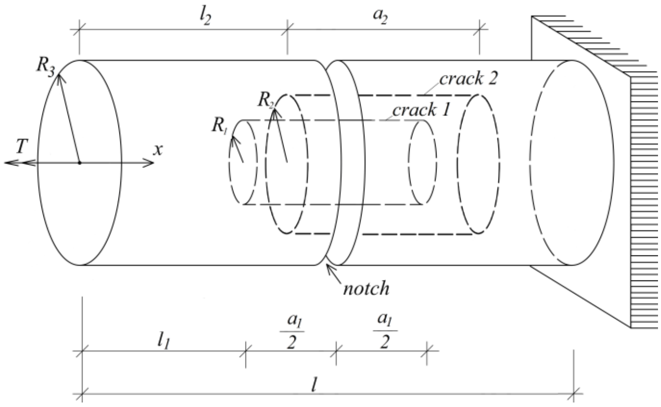

Figure 4. Inhomogeneous structural component with a circular notch in its external and interstitial parts.

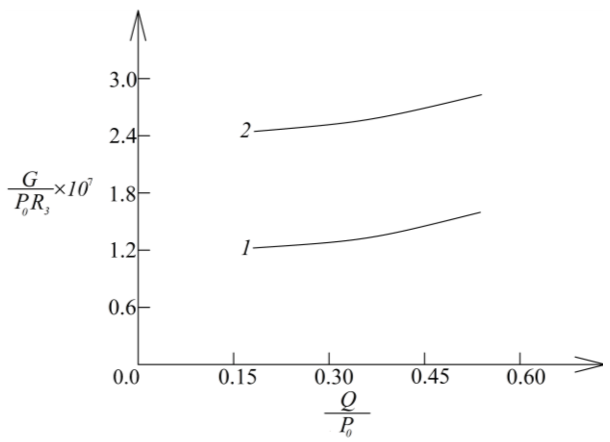

Figure 5. Strain energy release rate in non-dimensional form plotted against Q/P_0 ratio (curve 1 - structural component without notch, curve 2 - structural component with notch).

CONCLUSIONS

Fracture behaviour of an inhomogeneous cylindrical structural component with two internal lengthwise circular cylindrical cracks is analysed. The component is clamped at its right-hand end. The external loading consists of one torsion moment applied at the free end of the structural component. It is assumed that the component exhibits continuous material inhomogeneity in radial direction. Besides, the material has nonlinear elastic behaviour. The two lengthwise concentric cracks have different lengths and are located arbitrary in the radial direction. Fracture is studied in terms of strain energy release rate. Four solutions of strain energy release rate are derived by analysing the complementary strain energies at four crack fronts. The basic peculiarity of the analysis of internal lengthwise cracks is the fact that the torsion moments in the external, interstitial, and internal parts of the structural component cannot be determined directly. In order to determine these torsional moments, the inhomogeneous nonlinear elastic structural component is analysed as a statically indeterminate structure with two internal redundants. The strain energy release rate is verified by analysing the energy balance. Effects of material inhomogeneity in the radial direction, material nonlinearity, and the location of the two cracks, on the fracture behaviour of the structural component are investigated. It is found that the strain energy release rate decreases with increasing P_1/P_0 and R_1/R_3 ratios. The analysis reveals that strain energy release rate increases with increasing Q/P_0 ratio. Lengthwise fracture behaviour is analysed also for the case when a circular notch is cut-out in the external and interstitial parts of the component. It is found that the strain energy release rate in a structural component with a notch is higher than that in a structural component without a notch.

REFERENCES

1. Hedia, H.S., Aldousari, S.M., Abdellatif, A.K., Fouda, N. (2014), *New design of cemented stem using functionally graded materials (FGM)*, Biomed. Mater. Eng. 24(3): 1575-1588. doi: 10.3233/BME-140962
2. Hirai, T., Chen, L. (1999), *Recent and prospective development of functionally graded materials in Japan*, Mater Sci. Forum, 308-311: 509-514. doi: 10.4028/www.scientific.net/MSF.308-311.509
3. Neubrand, A., Rödel, J. (1997), *Gradient materials: an overview of a novel concept*, Zeit. f . Met. 88(1): 358-371.
4. Gasik, M.M. (2010), *Functionally graded materials: bulk processing techniques*, Int. J Mater. Prod. Technol. 39(1-2): 20-29. doi: 10.1504/IJMPT.2010.034257
5. Suresh, S., Mortensen, A., Fundamentals of Functionally Graded Materials, Processing and Thermomechanical Behavior of Graded Metals and Metal-Ceramic Composites, IOM Communications Ltd., London, 1998.
6. Yan, W., Ge, W., Smith, et al. (2016), *Multi-scale modeling of electron beam melting of functionally graded materials*, Acta Mater. 115: 403-412. doi: 10.1016/j.actamat.2016.06.022
7. Saiyathibrahim, A., Subramaniyan, R., Dhanapal, P. (2016), *Centrifugally cast functionally graded materials – A review*, In: Int. Conf. on Systems, Science, Control, Comm., Eng. and Technol., pp.68-73.
8. Uysal, M.U. (2016), *Buckling behaviours of functionally graded polymeric thin-walled hemispherical shells*, Steel Compos. Struct. 21(4): 849-862. doi: 10.12989/scs.2016.21.4.849

9. Erdogan, F. (1995), *Fracture mechanics of functionally graded materials*, Comp. Eng. 5(7): 753-770. doi: 10.1016/0961-9526 (95)00029-M
10. Mahamood, R.M., Akinlabi, E.T., Functionally Graded Materials, Springer Int. Publ., 2017. doi: 10.1007/978-3-319-53756-6
11. Tilbrook, M.T., Moon, R.J., Hoffman, M. (2005), *Crack propagation in graded composites*, Compos. Sci. Technol. 65(2): 201-220. doi: 10.1016/j.compscitech.2004.07.004
12. Carpinteri, A., Paggi, M., Pugno, N. (2006), *An analytical approach for fracture and fatigue in functionally graded materials*, Int. J Fract. 141(2): 535-547. doi: 10.1007/s10704-006-9 012-y
13. Rizov, V.I. (2017), *Analytical study of elastic-plastic longitudinal fracture in a functionally graded beam*, Strength, Fract. Complexity, 10(1): 11-22. doi: 10.3233/SFC-170197
14. Rizov, V.I. (2018), *Delamination in multi-layered functionally graded beams - an analytical study by using the Ramberg-Osgood equation*, Struct. Integ. and Life, 18(1): 70-76.
15. Rizov, V.I. (2018), *Delamination in nonlinear elastic multi-layered beams of triple graded materials*, Struct. Integ. and Life, 18(3): 163-170.
16. Lukash, P.A., Fundamentals of Nonlinear Structural Mechanics, Stroizdat, Moscow, 1998. (in Russian)