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OSCILLATORY MOTIONS IN AN ELECTROTHERMAL-CONVECTION IN SHEAR-THINNING VISCOELASTIC NANOFLUID LAYER IN A POROUS MEDIUM

OSCILATORNA KRETANJA KOD ELEKTROTERMALNE KONVEKCIJE U PSEUDO-PLASTIČNOM VISKOELASTIČNOM NANOFLUIDNOM SLOJU POROZNE SREDINE

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Abstract

In this paper we study electrothermal convection in a horizontal layer of Maxwellian dielectric nanofluid saturating a porous medium. Darcy-Maxwellian fluid model is used to describe rheological behaviour of nanofluid. The used model for nanofluid incorporates the effects of thermophoresis and Brownian diffusion. The Navier-Stokes equations of motion are modified due to the presence of applied AC electric field by the inclusion of dielectrophoretic force and Coulomb force. By applying linear stability analysis based upon perturbation theory and one-term Galerkin method, we derive the expression for thermal Rayleigh number for cases of stationary convection and oscillatory motion. Effects of Vadasz number, AC electric Rayleigh number, Lewis number, modified diffusivity ratio, nanoparticle Rayleigh number and medium porosity have been discussed for the case of stationary and oscillatory convection.

Izvod

U radu se istražuje elektrotermalna konvekcija u horizontalnom sloju dielektričnog Maksvelovskog nanofluida koji je zasićen u poroznoj sredini. Model tipa Darsi-Maksvel fluida se primenjuje u opisivanju reološkog ponašanja nanofluida. Primenjeni model nanofluida sadrži uticaje termoforeze i Braunove difuzije. Jednačine kretanja Navije-Stoksa su modifikovane usled prisustva električnog polja naizmenične struje, dodavanjem dielektroforetičke i Kulombove sile. Primenom linearne analize stabilnosti, na bazi teorije perturbacije i jednočlanog metoda Galerkina, izveden je izraz za termički Rejlejev broj za slučajeve stacionarne konvekcije i oscilatornog kretanja. Uticaji Vadazovog broja, Rejlejevog broja naizmeničnog polja, Luisovog broja, modifikovanog koeficijenta difuzivnosti, Rejlejevog broja nanočestice i poroznosti sredine, su diskutovani za slučaj stacionarne i oscilatorne konvekcije.

INTRODUCTION

Maxwel /1/ was the first who proposed a model for viscoelastic fluid having an immense storage space of energy. Owing to the reticence of shear-thinning rheological in performance of blood, we are concerned with Maxwell model seeing as blood behaves in view of the fact that a viscoelastic liquid is connected to shear charge. Maxwell model is the simplest rate type of fluid model or uniformly as an integral depiction of stress that represents the properties of relaxation-time which cannot be concluded in the differential type of viscoelasitc fluids (see, Bland /2/). The examples of viscoelastic fluids are polymer liquids, paints, certain oils, lubricants, colloidal and suspension solutions, clay coating and find applications in electronic chips, movement of biological fluids, food processing paper productions, nuclear waste repository, grain storage, mantle convection, geothermal energy utilization and oil reservoir modelling etc. /3-10/. The activities of blood manifesting its shear-thinning are payable to stress-relaxation properties of stress, which has four autonomous unique accounting parameters, namely, elasticity, plasma viscosity, the formed rouleaus and their outcome in the viscosity of blood, and how does the shear-thinning take place in the flow motion.

A comprehensive study of thermal convection in a horizontal layer of viscous fluid saturating a porous medium is largely studied by Ingham and Pop /11-12/, Vafai /13-14/, Nield and Bejan /15/ etc. During last few years, convective instability of a horizontal nanofluid layer saturating a porous layer by means of Buongiorno /16/ model has been largely examined by different authors /17-24/. But the study of thermal convection of viscoelastic nanofluids in porous media is very limited. The thermal instability in a porous layer saturated with viscoelastic nanofluid fluid is analysed by Rana and Chand /25/, Chand and Rana /26-27/, Umavathi et al. /28/, and Chand et al. /29/.

In recent times, attention has been given to the electrohydodynamics in the study of thermal instability of viscoelastic nanofluid in a porous medium. The functional electric force of fluid motion is a very effectual technique in receiving extremely supportive motivating consequences in the cooling of laptops and strategy of the flight in space discretely, ionization, prepared on nanoscale being used at a huge level in the current epoch. The effect of electrohydrodynamic in thermal instability of differnet types of an elastico-viscous fluid has been analysed by different authors /30-35/. They found that the vertical AC electric field destabilized the stationary convection. Sharma et al. /36/ have deliberated electro-thermal convection in dielectric Maxwellian nanofluid layer and observed that viscoelasticity hastens the existence of oscillatory modes and the thermal Prandtl number delayed the existence of oscillatory modes.

In the present chapter we examine the influence of rheological behaviour and a vertical AC electric field on the stationary and oscillatory convection of non-Newtonian nanofluid in a porous medium. The Maxwell fluid model is applied to depict the rheological behaviour of the nanofluid sheet of restricted depth *d*, for the stress-free margins. We analyse the solidity by using a Galerkin approximation and numerical computations have been approved with the software MATHEMATICA® version-11.3.

FORMULATION OF THE PROBLEM AND MATHEMATICAL MODEL

Consider an infinitely horizontal layer of Maxwellian electrically conducting nanofluid in a porous medium heated from below of thickness *d* acted upon the vertical gravity force $\mathbf{g}(0,0,-g)$ (Fig. 1). This nanofluid layer is bounded between two parallel planes $z = 0$ and $z = d$, which are controlled at temperatures and nanoparticle volume fraction T_0, φ_0 of lower fluid layer and T_1, φ_1 of upper fluid layer, $T_0 > T_1$.

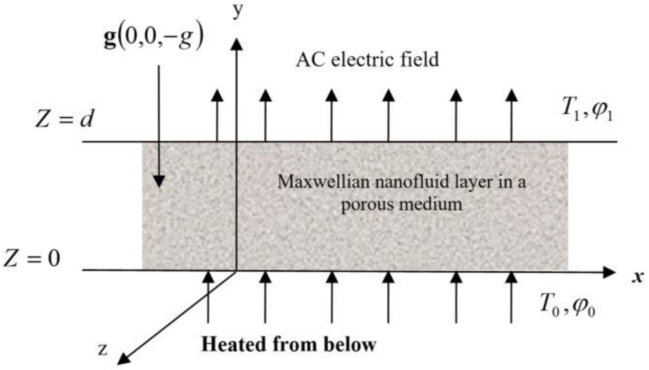

Figure 1. Physical configuration of the problem.

GOVERNING EQUATIONS

The governing conservation equations for Darcy-Maxwell nanfluid under the influence of vertical AC electric field leading to the physical system under study by means of Boussinesq approximation (Maxwel /1/, Buongiorno /16/, Nield and Kuznetsov /19/, Rana and Chand /25/ and Sharma et al. /36/) are

$$\nabla \cdot \mathbf{q_D} = 0, \tag{1}$$

$$\frac{\rho_{bf}}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left\{\frac{\partial \mathbf{q_D}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q_D}\cdot\nabla)\mathbf{q_D}\right\}=$$

$$=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla p-\mathbf{f_e}-\frac{\mu}{k_1}\mathbf{q_D}\right]+\left(1+\lambda\frac{\partial}{\partial t}\right)\times$$

$$\times\left[\left\{\phi\rho_{np}+(1-\phi)\rho_{nf}(1-\alpha(T-T_1))\right\}g\right], \tag{2}$$

where: $\mathbf{q_D}$ is the velocity of nanofluid; ρ_{bf} is fluid density; p is fluid pressure; T is fluid temperature; μ is fluid viscosity; k_1 is medium permeability; ε is porosity; and λ is stress relaxation parameter (accounting for viscoelasticity); $\mathbf{f_e}$ is the electrical origin force given by

$$\mathbf{f_e} = \rho_e\mathbf{E}-\frac{1}{2}\mathbf{E}^2\nabla K+\frac{1}{2}\left(\rho\frac{\partial K}{\partial t}\mathbf{E}^2\right), \tag{3}$$

where: ρ_e is density of charge; K is electric constant; $\mathbf{E}$ is electric field. The term $\rho_e\mathbf{E}$ is the force produced due to a free charge after the name of Coulomb and the second term $-\mathbf{E}^2\nabla K/2$ depends on the gradient of K. The bulk of the dielectric fluid remains uninfluenced with the electrical force $\mathbf{f_e}$. Due to the dielectric constant K and electrical conductivity σ, the built up free charge is prevented for a long-time due to the sufficient relaxation appearing in the presence of electric field in most dielectric fluids at standard power-line frequencies. Thus, dielectric loss produced at these frequencies becomes very low so as to make the temperature field unchanged at the same time. Therefore, the first term $\rho_e\mathbf{E}$ is neglected as compared to the di-electrophoretic force term $-\mathbf{E}^2\nabla K/2$ for most dielectric fluids.

It is also assumed that the dielectric constant, K can be expressed (Yadav et al. /31/) as

$$K = K_0[1-\gamma_0(T-T_0)], \tag{4}$$

where: $\gamma_0 > 0$ is the coefficient of dielectric constant with temperature relative variations, assumed to be small $0 < \gamma_0\Delta T << 1$.

The modified pressure term, using Eq.(3) yields

$$P = p-\frac{1}{2}\left(\rho\frac{\partial K}{\partial t}\mathbf{E}^2\right). \tag{5}$$

Assuming free charge density to be very small, the relevant Maxwell equations are

$$\nabla.(K\mathbf{E}) = 0, \tag{6}$$

$$\nabla\times\mathbf{E} = 0. \tag{7}$$

In view of Eq.(7), $\mathbf{E}$ can be expressed as $\mathbf{E} = -\nabla\varphi$.
The conservation equation for the nanoparticles is

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{q_D}\cdot\nabla}{\tilde{\varepsilon}}\right]\phi = -\frac{1}{\rho_p}\nabla.j_p\,. \tag{8}$$

Here, φ is the nanoparticle volumetric fraction, ρ_p is the density of nanoparticles and j_p, the nanoparticles diffusion mass flux, given by

$$j_p = -\rho_p D_B\nabla\phi-\rho_p\frac{D_T}{T_1}\nabla T, \tag{9}$$

where: D_B (Brownian diffusion coefficient) and D_T (thermophoretic diffusion coefficient) are given as

$$D_B = \frac{k_B T}{3\pi\mu_{nf}d_{np}},\ \tilde{D}_T = \frac{\mu_{bf}}{\rho_{nf}}\frac{0.26k_{nf}}{(2k_{bf}+k_{np})}\phi, \tag{10}$$

where: k_B is Boltzmans constant; μ_{bf} is base fluid viscosity; d_{np} is the diameter of nanoparticle; ρ_{bf} is base fluid density; k_{bf} and k_{np} are thermal conductivities of base fluid and nanoparticles, respectively. Using the value of j_p from Eq.(9) into Eqs.(8), the conservation equation of nanoparticles yields

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{q_D}.\nabla}{\varepsilon}\right]\phi = D_B\nabla^2\phi+\frac{D_T}{T_1}\nabla^2 T\ . \quad (11)$$

The heat energy equation is

$$(\rho_{nf}c)_{bm}\left\{\frac{\partial T}{\partial t}+\mathbf{q_D}.\nabla T\right\}=k_m\nabla^2 T+$$
$$+\varepsilon(\rho_{nf}c)_{np}\left\{D_B\nabla\phi.\nabla T+\frac{D_T}{T_0}\nabla T.\nabla T\right\}, \quad (12)$$

where: c is specific heat of the material constituting the nanoparticles; $(\rho_{nf}c)_{bm}$ is effective capacity; $(\rho_{nf}c)_{bf}$ is the heat capacity of the nanofluid.

Here both bounding surfaces of the fluid are assumed to be stress-free and the medium adjoining the nanofluid is a perfect conductor, the appropriate boundary conditions are

$$\tilde{w}=\frac{\partial^2\tilde{w}}{\partial z^2}=\tilde{T}=\tilde{\phi}=\frac{\partial\tilde{\varphi}}{\partial\tilde{z}}=0 \quad \text{at} \quad \tilde{z}=0 \quad \text{and} \quad \tilde{z}=\tilde{d}\ . \quad (13)$$

BASIC SOLUTIONS

Primary flow representing the basic state is assumed to be quicsent /2, 7, 9, 12/, no settling of suspended nanoparticles and is assumed to be stationary. Initially, no motions are present in the nanofluid flow and the physical quantities vary in the vertical direction z-axis only. Therefore, the velocity, pressure, temperature, dielectric constant, electric field, electric potential and nanoparticle volume fraction are given by

$$q_D=q_b=0,\quad \mathrm{P}=P_b(z),\quad K=K_b(z),\quad T=T_b(\tilde{z}),$$
$$\phi=\phi_b(z),\quad E=E_b(z),\quad T_b=T_0-\frac{\Delta T}{d}\tilde{z},$$
$$\phi_b=\phi_0+\left(\frac{D_T\Delta T}{D_B T_1 d}\right)z,\quad K_b=K_0\left(1+\frac{\gamma_0\Delta T}{d}z\right)k, \quad (14)$$
$$E_b=\frac{E_0}{\left(1+\frac{\gamma_0\Delta T}{d}z\right)}\hat{k},\quad \varepsilon_b=\varepsilon_0\left(1+\frac{\gamma_0\Delta T}{d}z\right)k,$$

where: subscript 'b' denotes the basic state and $\hat{k}$ is the unit vector along z-axis.

Also we have

$$V_b(z)=-\frac{E_0 d}{\gamma_0\Delta T}\log\left(1+\frac{\gamma_0\Delta T}{d}\right)k\ ,$$

where: $E_0=-\dfrac{V\left(\frac{\gamma_0\Delta T}{d}\right)}{\log(1+\gamma_0\Delta T)}$ is the root mean square value of the electric field at $z = 0$.

PERTURBATION SOLUTIONS

Let the primary flow be slightly disturbed from the equilibrium position so as to examine the stability of the perturbed modes with respect to the involved, physical variables by superimposing infinitesimal disturbances to the basic state flow. It is assumed that

$$q_D=q_D^*,\quad T=T_b+T^*,\quad K=K_b+K^*,$$
$$P=P+P^*,\quad E=E_b+E^*,\quad V=V_b+V^*, \quad (15)$$

where: q_D^*, T^*, K^*, P^*, E^*, and V^* are perturbations superimposed into the physical quantities of the equilibrium state.

On substituting these perturbations and using the solutions of primary flow Eq.(14) the Eqs.(2), (7), (11) and (12) in the non-dimensional linearized perturbed form using linear theory (neglecting the products and higher orders of perturbed quantities) and Boussinesq approximation yields

$$\left[1+\frac{1}{V_a^*}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]\nabla^2 w^*-R_a\nabla_1^{\,2}T^*-R_{ea}\nabla_1^{\,2}T^*+$$
$$+R_N\nabla_1^{\,2}\phi^*+R_{ea}\nabla_1^{\,2}\frac{\partial\varphi^*}{\partial z}\left(1+\lambda\frac{\partial}{\partial t}\right)=0\ , \quad (16)$$

$$\frac{\partial T^*}{\partial t}-w^*=\nabla^2 T^*+\frac{N_B}{L_e}\left(\frac{\partial T^*}{\partial z}-\frac{\partial\phi^*}{\partial t}\right)-\frac{2N_A N_B}{L_e}\frac{\partial T^*}{\partial z} \quad (17)$$

$$\frac{1}{\sigma_1}\frac{\partial\phi^*}{\partial t}+\frac{w^*}{\varepsilon}=\frac{1}{L_e}\nabla^2\phi^*+\frac{N_A}{L_e}\nabla^2 T^*\ , \quad (18)$$

$$\frac{\partial T^*}{\partial z}-\nabla^2\varphi^*=0\ , \quad (19)$$

where:

$$(x,y,z)=\frac{1}{d}(x^*,y^*,z^*),\quad t=\frac{t\kappa_{bm}}{\sigma d^2},$$
$$(u,v,w)=\frac{1}{\kappa_{bm}}(u^*,v^*,w^*)d,$$
$$\phi=\frac{(\phi-\phi_0)}{(\phi_1-\phi_0)},\quad T=\frac{(T-T_1)}{T_0-T_1}, \quad (20)$$
$$\varphi=\gamma_0 E_0\beta\varphi d,\quad \sigma_1=\frac{(\rho_{nf}c_{np})_{bm}}{(\rho_{nf}c_{np})_{bf}},$$
$$\kappa_{bm}=\frac{k_m}{(\rho_{nf}c_{np})_{bf}}$$

are non-dimensional variables. These are defined as:

$p_1=\dfrac{\mu}{\rho_0\alpha}$ is the Prandtl number, (21)

$D_a=\dfrac{k_1}{d^2}$ is the Darcy number, (22)

$V_a=\dfrac{\varepsilon P_r}{D_a}$ is the Vadasz number, (23)

$R_a=\dfrac{\rho_{nf}g\alpha dk(T_0-T_1)}{\mu_{bf}\kappa_{bm}}$ is the Rayleigh number, (24)

$R_{ea}=\dfrac{\gamma_0^{\,2}KE_0^{\,2}d^2(\Delta T)^2}{\mu\kappa_{bm}}$ is the electric Rayleigh number, (25)

$R_n=\dfrac{(\rho_{np}-\rho_{bf})(\phi_1-\phi_0)gk_1 d}{\mu\alpha}$ is the nano particles Rayleigh number, (26)

$N_B=\dfrac{\rho_{np}c_{np}}{(\rho c)_{bf}}(\phi_1-\phi_0)$ is modified paticle density increment, (27)

$L_e = \frac{\kappa_{bm}}{D_B}$ is the Lewis number of the nanofluid, (28)

$N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\phi_1 - \phi_0)}$ is the modified diffusivity ratio, (29)

$R_m = \frac{\{\rho_{np}\phi_0 + \rho_{bf} + (1-\phi_0)\} g k_1 d}{\mu\alpha}$ is the basic density Rayleigh number, (30)

$\lambda_1 = \frac{\lambda \kappa_{bm}}{d^2}$ is the parameter accounting for stress-relaxation time, (31)

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is a Laplacian operator,

$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

Equation (16) is derived by the use of the identity curlcurl = graddiv – ∇^2.

Boundary conditions Eq.(13), in non-dimensional form become

$$w^* = \frac{\partial^2 w^*}{\partial z^2} = T^* = \phi^* = \frac{\partial \varphi^*}{\partial z} = 0 \quad \text{at} \quad z^* = 0 \quad \text{and} \quad z^* = 1. \tag{32}$$

Normal mode analysis

Now an arbitrary perturbation is analysed into a complete set of normal modes and then the stability of each of these modes is examined individually. For the system of Eqs. (16)-(19), the analysis can be made in terms of two-dimensional periodic wave numbers. Thus, we ascribe to the quantities describing the dependence on x, y and t of the form $\exp(ilx + \omega t)$, where l, m are the wave numbers in the x and y-direction, respectively; and ω is the growth rate of the disturbances, which in general is a complex constant.

Above consideration allows to suppose that the perturbations quantities w^*, T^*, φ^* and V^* are of the form

$$\begin{aligned} w^*(x,y,z,t) &= W(z)\exp(ilx + imy + \omega t), \\ T^*(x,y,z,t) &= \Theta(z)\exp(ilx + imy + \omega t), \\ \phi^*(x,y,z,t) &= \Phi(z)\exp(ilx + imy + \omega t), \\ V^*(x,y,z,t) &= \Psi(z)\exp(ilx + imy + \omega t). \end{aligned} \tag{33}$$

With the help of Eq.(33) and boundary conditions Eq.(32), the non-dimensional differential Eqs.(16)-(19) after linearization turn into

$$\left\{1 + \frac{\omega}{V_a}(1+\lambda_1\omega)\right\}\left(D^2 - a^2\right)W - (1+\lambda_1\omega)\times \times\left\{-a^2(R_a + R_{ea})\Theta + a^2 R_n \Phi + a^2 R_{ea} D\Psi\right\} = 0, \tag{34}$$

$$\frac{W}{\varepsilon} - \frac{N_A}{L_e}\left(D^2 - a^2\right)\Theta - \left\{\frac{1}{L_e}\left(D^2 - a^2\right) - \frac{\omega}{\sigma}\right\}\Phi = 0, \tag{35}$$

$$W + \left\{\frac{N_B}{L_e}D + \left(D^2 - a^2\right) - 2\frac{N_A N_B}{L_e} - \omega\right\}\Theta - \frac{N_B}{L_e}D\Phi = 0, \tag{36}$$

$$D\Theta - \left(D^2 - a^2\right)\Psi = 0. \tag{37}$$

By employing the Eq.(34), the boundary conditions Eq.(33) transform to

$$W = D^2 W = \Theta = \Phi = D\Psi = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \tag{38}$$

The set of differential Eqs.(34)-(37) in addition to the boundary conditions Eq.(38) comprise a characteristic-value problem for Rayleigh number R_a and known values of the supplementary parameters λ_1, R_n, R_{ea}, ε, L_e, N_A, whose solutions ought to be obtained.

Linear stability analysis and dispersion relation

The precise logical solutions for the set of ordinary differential Eqs.(34)-(37) using one-term Galerkin approximation of lowest mode satisfying boundary conditions Eq.(38) yields,

$$W = A\sin\pi z, \quad \Theta = B\sin\pi z, \quad \Phi = C\sin\pi z, \quad \Psi = D\cos\pi z, \tag{39}$$

where: A, B, C and D are constants.

Prevailing the solutions, conferred by Eqs.(39) into Eqs. (34)-(37), through the orthogonality of trial functions and boundary conditions Eqs.(33), a structure of linear consistent equations is drafted as:

$$\begin{bmatrix} \frac{a^2+\pi^2}{1+\lambda_1\omega} + \frac{\omega}{V_a}\left(a^2+\pi^2\right) & -a^2(R_a + R_{ea}) & a^2 R_n & -a^2\pi R_{ea} \\ 1 & -\left(a^2+\pi^2\right) - \omega & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{N_A\left(a^2+\pi^2\right)}{L_e} & \frac{a^2+\pi^2}{L_e} + \frac{\omega}{\sigma} & 0 \\ 0 & -\pi & 0 & -\left(a^2+\pi^2\right) \end{bmatrix} \begin{bmatrix} W \\ \Theta \\ \Phi \\ \Psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{40}$$

The non-trivial solution of the matrix Eqs.(40) is obtained by equating the determinant of the coefficient matrix to zero and after some algebraic simplifications gives the value of the thermal Rayleigh number, R_a as:

$$-\left(a^2+\pi^2\right)a^2 R_n(1+\lambda_1\omega)\left\{\frac{a^2+\pi^2+\omega}{\varepsilon} + \frac{\left(a^2+\pi^2\right)N_A}{L_e}\right\} - a^2\left(a^2+\pi^2\right)(R_a + R_{ea})(1+\lambda_1\omega)\left\{\frac{\omega}{\sigma} + \frac{a^2+\pi^2}{L_e}\right\} +$$

$$+\left(a^2+\pi^2\right)\left(a^2+\pi^2+\omega\right)\left\{\frac{\omega}{\sigma}+\frac{a^2+\pi^2}{L_e}\right\}\left\{\left(a^2+\pi^2\right)+\frac{\left(a^2+\pi^2\right)\omega(1+\lambda_1\omega)}{V_a}\right\}+a^2\pi^2\left\{\frac{\left(a^2+\pi^2\right)}{L_e}+\frac{\omega}{\sigma}\right\}R_{ea}(1+\lambda_1\omega)=0\,, \qquad (41)$$

Equation (41) is the dispersion relation representing the effect of Lewis number, kinematic visco-elasticity parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio on thermal electro instability in a layer of Maxwell nanofluid in a porous medium under the influence of vertical AC electric field.

STATIONARY CONVECTION

The stationary motion is identified by putting $\omega = 0$ in Eq.(41), we get the thermal Rayleigh number of stationary modes as:

$$R_a^S=\frac{\left(a^2+\pi^2\right)^2}{a^2}-\frac{a^2}{\left(a^2+\pi^2\right)}R_{ea}-\left(N_A+\frac{L_e}{\varepsilon}\right)R_n\,. \qquad (42)$$

It is obvious from Eq.(42) that stationary Rayleigh number (R_a^S) is independent of stress-relaxation time (λ_1), Vadasz number (V_a) and ratio of specific heat (σ) for stationary modes, since these vanish with the vanishing of ω.

The minimum value of stationary Rayleigh number (R_a^S) is stated by putting $\partial R_a^S/\partial a^2 = 0$, which reveals

$$a^4\left(1+\frac{\pi^2}{a^2}\right)^3\left(1-\frac{\pi^2}{a^2}\right)=\pi^2 R_{ea}\,. \qquad (43)$$

It is noted from Eq.(43) that the values of the critical wave number do not confide in the parameters secretarial for nanoparicles, though depend upon $\tilde{R}_{ea}$ only.

Therefore, the effects of various non-dimensional parameters namely, electric field (R_{ea}), nanofluid Lewis number (L_e), modified diffusivity ratio (N_A), and the concentration Rayleigh number (R_n) on the stability of stationary modes have been investigated analytically by examining the behaviour of $\frac{\partial R_a^S}{\partial R_{ea}}$, $\frac{\partial R_a^S}{\partial L_e}$, $\frac{\partial R_a^S}{\partial N_A}$, $\frac{\partial R_a^S}{\partial R_n}$ and $\frac{\partial R_a^S}{\partial \varepsilon}$.

It is depicted from Eq.(42) that,

$$\frac{\partial R_a^S}{\partial R_{ea}}=-\frac{a^2}{\left(a^2+\pi^2\right)}\,, \qquad (44)$$

which is always negative for all wave numbers, thereby lessening the Darcy Rayleigh number with increment in R_{ea} (electric Rayleigh number). Thus, R_{ea} has always a destabilizing effect on the system.

Equation (42), shows that

$$\frac{\partial R_a^S}{\partial L_e}=-\frac{R_n}{\varepsilon}\,, \qquad (45)$$

and

$$\frac{\partial R_a^S}{\partial N_A}=-R_n\,. \qquad (46)$$

It is noteworthy from Eqs.(45) and (46) for bottom-heavy particles (i.e. for negative value of R_n) both the nanofluid Lewis L_e and the modified diffusivity ratio N_A stabilize the system.

Equation (42) moreover gives that

$$\frac{\partial R_a^S}{\partial R_n}=-\left(N_A+\frac{L_e}{\varepsilon}\right), \qquad (47)$$

which is always negative for $\left(N_A+\frac{L_e}{\varepsilon}\right)>0$, since the value of N_A is taken in the range of -1 to -25 and L_e in the range of 100-400. Thus, thermophoresis reduces with an increase in negative values of N_A which means that thermophoresis push the heavier nanoparticles upwards, which strengthens the stabilizing effects of particle distributions,

$$\frac{\partial R_a^S}{\partial \varepsilon}=\frac{L_e R_n}{\varepsilon^2}\,. \qquad (48)$$

If $R_n > 0$, the right hand side of Eq.(48) is positive and it is negative if $R_n < 0$. Thus, medium porosity has stabilizing/destabilizing effect. These results are in good accord with the results derived by Nield and Kuznetsov /19/, Rana et al. /30, 35/, and Chand et al. /28/.

OSCILLATORY MOTION

Now, the growth rate, $\omega = \omega_r + i\omega_i$, where: ω_r and ω_i are real. For oscillatory convection, $\omega \neq 0$ and $\omega_r = 0$, i.e., $\omega = i\omega_i \neq 0$. Here, the critical Darcy Rayleigh number for the onset of instability is examined via a state of pure oscillations of growing amplitude by putting $\omega = i\omega_i$ in Eq.(41) and after some arithmetical simplifications, we find

$$R_a=\Delta_1+i\omega_i\Delta_2\,. \qquad (49)$$

where: Δ_1 and Δ_2 are framed as follows

$$\Delta_1=\frac{\left(a^2+\pi^2\right)\left(a^2+\pi^2+\lambda_1\omega_I^2\right)}{a^2\left(1+\lambda_I^2\omega_i^2\right)}-\frac{\omega_i{}^2}{V_a a^2}\left(a^2+\pi^2\right)-\frac{a^2\omega_i{}^2 L_e{}^2 R_e}{\left(a^2+\pi^2\right)^3\sigma^2+\left(a^2+\pi^2\right)L_e{}^2\omega_i^2}-\frac{\sigma R_n L_e\left\{\left(a^2+\pi^2\right)^2\sigma+L_e\omega_i^2\right\}}{\varepsilon\left\{\left(a^2+\pi^2\right)^2\sigma^2+L_e^2\omega_i^2\right\}}-$$

$$-\frac{\left(a^2+\pi^2\right)\sigma^2\left\{a^2R_{ea}+\left(a^2+\pi^2\right)^2 N_A R_n\right\}}{\left(a^2+\pi^2\right)^2\sigma^2+L_e^2\omega_i^2}\,, \qquad (50)$$

and

$$\Delta_2 = \left(a^2+\pi^2\right)\lambda_1\left(V_a-\sigma\omega_i^2\right)+\left(a^2+\pi^2\right)L_e{}^3a^2\sigma V_a\left\{a^2\sigma V_a R_n\left(1+\lambda_1\omega_i^2\right)+V_a+\left(a^2+\pi^2\right)\left(-V_a\lambda_1+\lambda_1{}^2\omega_i^2\right)\right\}-\left(a^2+\pi^2\right)^3\varepsilon\sigma^3V_aL_ea^2\times$$
$$\times\left\{-\left(a^2+\pi^2\right)-V_a-\left(a^2+\pi^2\right)\left(a^2\sigma V_aL_eR_n\left(\sigma-\varepsilon N_A\right)\right)\right\}\left(1+\lambda_1{}^2\omega_i^2\right)+\varepsilon\omega_i{}^2\left(a^2+\pi^2\right)\left(a^2+\pi^2\right)^3\varepsilon\sigma^3V_aL_ea^2\times$$
$$\times\left\{-\left(a^2+\pi^2\right)-V_a-\left(a^2+\pi^2\right)\left(a^2\sigma V_aL_eR_n\left(\sigma-\varepsilon N_A\right)\right)\right\}. \qquad (51)$$

On comparing real and imaginary parts of Eq.(41), we have, $R_a = \Delta_1$, which is implicit on simplication of Darcy Rayleigh number of oscillatory modes as:

$$R_a^{osc}=\frac{\left(a^2+\pi^2\right)\left(a^2+\pi^2+\lambda_1\omega_i^2\right)}{a^2\left(1+\lambda_1{}^2\omega_i^2\right)}-\frac{\omega_i{}^2}{V_aa^2}\left(a^2+\pi^2\right)-\frac{a^2\omega_i{}^2L_e{}^2R_{ea}}{\left(a^2+\pi^2\right)^3\sigma^2+\left(a^2+\pi^2\right)L_e^2\omega_i^2}-\frac{\sigma R_nL_e\left\{\left(a^2+\pi^2\right)^2\sigma+L_e\omega_i^2\right\}}{\varepsilon\left\{\left(a^2+\pi^2\right)^2\sigma^2+\tilde{L}_e^2\omega_i^2\right\}}-$$
$$-\frac{\left(a^2+\pi^2\right)\sigma^2\left\{a^2R_{ea}+\left(a^2+\pi^2\right)^2N_AR_n\right\}}{\left(a^2+\pi^2\right)^2\sigma^2+L_e^2\omega_i^2}, \qquad (52)$$

and $$i\omega_i\Delta_2=0. \qquad (53)$$

Since for oscillatory modes, $i\omega_i \neq 0$, therefore Eq.(41) gives that $\Delta_2 = 0$, which subscribes a dispersion relation (relation between growth rate $\tilde{\omega}$ and wave number a) of the form

$$a_1(\omega_i^2)^2+a_2(\omega_i^2)+a_3=0\,,\quad \frac{a_3}{a_1}>0\,, \qquad (54)$$

where, $$a_1=\varepsilon\lambda_1^2L_e^2(a^2+\pi^2)^2\,, \qquad (55)$$

$$a_2=\sigma V_aL_eR_na^2\lambda_1^2(a^2+\pi^2)[L_e-(\sigma-\varepsilon N_A)]+\varepsilon L_e^2(a^2+\pi^2)^2+\varepsilon V_aL_e^2(a^2+\pi^2)[1-\lambda_1(a^2+\pi^2)]+$$
$$+\varepsilon\sigma^2\lambda_1^2(a^2+\pi^2)^4\,, \qquad (56)$$

$$a_3=\sigma V_aL_eR_na^2(a^2+\pi^2)[L_e-(\sigma-\varepsilon N_A)]+\varepsilon\sigma^2(a^2+\pi^2)^3[V_a+(a^2+\pi^2)]-$$
$$-\varepsilon V_a\lambda_1\sigma^2(a^2+\pi^2)^4\,. \qquad (57)$$

Equations (52) and (54) have to be satisfied for the occurance of oscillatory modes for a wave number corresponding to various non-dimensional parameters L_e, V_a, λ_1, R_n, R_{ea}, ε, N_A and $\tilde{\sigma}$.

For oscillatory motion, $\tilde{\omega}$ is real and so there must be one variation of sign in Eq.(53) implying thereby that the Eq.(54) has at most one positive root for which the critical Darcy Rayleigh number for oscillatory modes is attained for different values of non-dimensional wave number from Eq.(52).

Since ω is real, the values of ω_i^2 have to be positive. Furthermore, there will be no change of sign in Eq.(54) for $\sigma > \varepsilon N_A$ and $\lambda_1(a^2 + \pi^2) > 1$. Therefore, for $\sigma < \varepsilon N_A$ and $\lambda_1(a^2 + \pi^2) < 1$, oscillatory modes can occur, the violation of which necessarily implies non-occurrence of oscillatory motion.

It is observed from Eq.(54) that existence of oscillatory modes is uninfluenced due to the presence of vertical AC electric field. However, these modes depend on other non-dimensional parameters accounting for nanoparticles, porous medium and viscoelasticity.

Validation of results

In the deficiency of electric field that is, $R_{ea} = 0$, the Eqs. (52) and (42) diminish to

$$R_a^{osc}=\frac{(a^2+\pi^2)(a^2+\pi^2+\lambda_1\omega_i^2)}{a^2(1+\lambda_i^2\omega_i^2)}-\frac{\sigma R_nL_e\{(a^2+\pi^2)^2+L_e\omega_i^2\}}{\varepsilon\{(a^2+\pi^2)\sigma^2+L_e^2\omega_i^2\}}-$$
$$-\frac{\omega_i^2}{V_aa^2}(a^2+\pi^2)-\frac{(a^2+\pi^2)^3\sigma^2N_AR_n}{(a^2+\pi^2)^2\sigma^2+L_e^2\omega_i^2}\,, \qquad (58)$$

and $$R_a^S=\frac{(a^2+\pi^2)^2}{a^2}-\left(N_A+\frac{L_e}{\varepsilon}\right)R_n\,, \qquad (59)$$

which are in an excellent concurrence with the earlier results given by Umavathi et al. /29/ for the limiting case of stress-free boundaries.

When simultaneously, the stress-relaxation-time parameter and the nanoparticles are not embedded, that is λ_1, $R_n = 0$, and $N_A = 0$, the Eqs.(58) and (59) reduce to

$$R_a^{osc}=\frac{(a^2+\pi^2)^2}{a^2}-\frac{\omega_i{}^2}{V_aa^2}(a^2+\pi^2)\,, \qquad (60)$$

and $$R_a^S=\frac{(a^2+\pi^2)^2}{a^2}\,, \qquad (61)$$

which are in good agreement with the prior results of Chand et al. /28/.

It is praiseworthy to depict that instability sets in through bottom-heavy pattern of nanoparticles. Since for bottom-heavy nanoparticle configuration, the convection in rheological nanofluids is through oscillatory modes. Hence, for small negative values of both R_n and N_A, oscillatory Rayleigh number R_a^{osc} takes negative values, however, the stationary Rayleigh number R_a^S gains negative values only for variations in R_{ea}.

NUMERICAL RESULTS AND DISCUSSIONS

To probe the outcome of different parameters on linear thermal instability in a porous layer saturating a nanofluid in the existence of electric field, the Eq.(42) for stationary and Eq.(52) satisfying Eq.(54) for oscillatory convection are analysed numerically with the software Mathematica® v.11.3 for bottom-heavy configuration. The linear stability theory exposes the criterion of stability in the form of critical Darcy-Rayleigh number under which the system is stable and unstable above.

The tentative values and fixed acceptable values of the dimensionless parameters are alike as those used by Buongiorno /16/, Yadav /34/, and Sharma et al. /36/, which are given as: $\lambda_1 = 0.6$; $V_a = 3$; $R_n = -0.1$; $N_A = -5$; $L_e = 200$, $R_{ea} = 100$; $\varepsilon = 0.6$; and $\sigma = 1.5$.

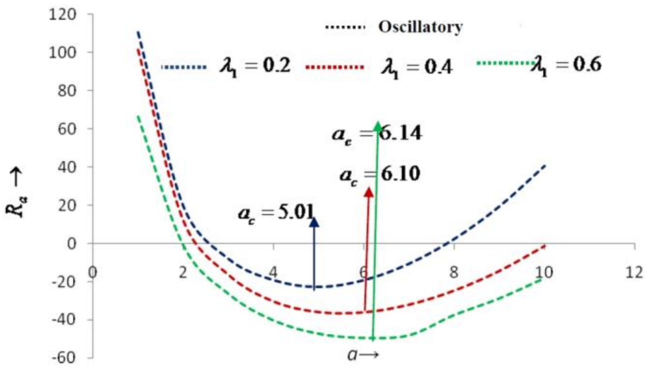

Figure 2. Variation of oscillatory Rayleigh number (R_a) with respect to wave number (a) for diff. values of stress-relax. time parameter.

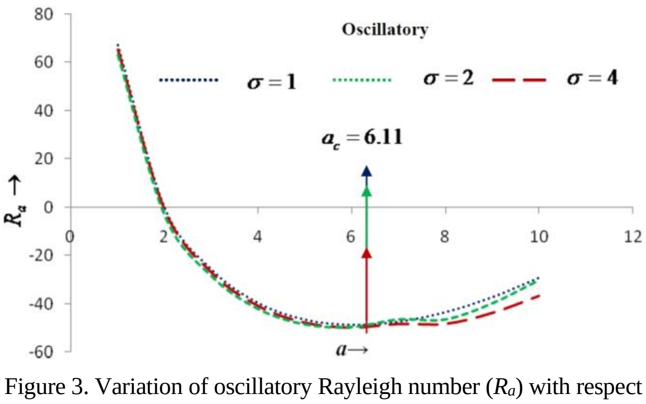

Figure 3. Variation of oscillatory Rayleigh number (R_a) with respect to wave number (a) for diff. values of capacity ratio parameter σ.

From Figs. 2 and 3 it is noted that the oscillatory thermal Rayleigh (R_a^{osc}) increases with the decrease in capacity ratio parameter (σ) and stress-relaxation time parameter (λ_1) depicting thereby that σ and λ_1 advances the onset of oscillatory motion. It is also clear from the graphs that the critical wave number does not change with the variation in σ, that is $a_c = 6.11$, whereas it increases with increase in λ_1, that is, $a_c = 5.01, 6.10, 6.14$.

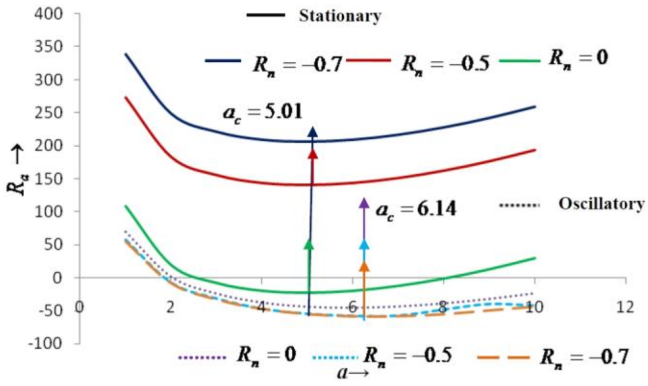

Figure 4. Variation of oscillatory and stationary Rayleigh number (R_a) with respect to wave number (a) for diff. values of nanoparticles Rayleigh number (R_n).

Figure 4 displays the variation of Darcy Rayleigh number (R_a) for both stationary and oscillatory modes vs. wave number a for different values of nanoparticles R_n = 0, -0.5, -0.7 (bottom-heavy case).

It is depicted from the figure that R_a for stationary mode decreases as R_n increases, which advances the onset of convection, whereas a slight stabilizing effect is observed for oscillatory mode. This happens so because strengthening of volumetric fraction of nanoparticles, the Brownian motion of nanoparticles increases, implying thereby the destabilizing effect on the stability of the system, which is in confirmation with the analytical result.

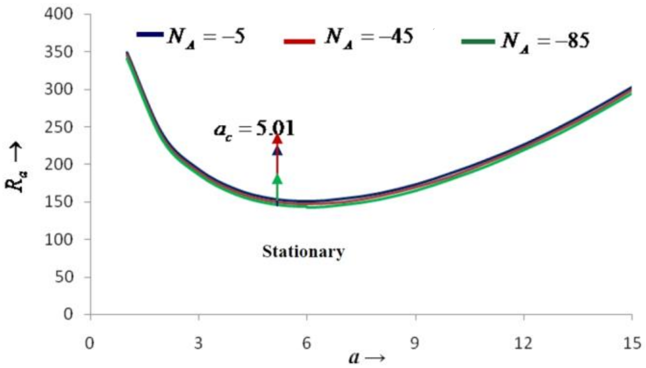

Figure 5. Variation of stationary Rayleigh number (R_a) with respect to wave number (a) for different values of modified diffusivity ratio (N_A) and N_A = -5, -45, -85 (bottom-heavy case) respectively.

Figure 5 asseses the variation of Darcy Rayleigh number (R_a) for stationary modes vs. wave number a for different values of nanoparticles N_A = -5, -45, -85 (bottom-heavy case). From the figure, it is noticed that N_A has slightly stabilizing influence on stationary modes, thereby delaying the onset of stationary convection.

The effect of medium porosity on the stability of stationary and oscillatory modes is displayed in Fig. 6. The thermal Rayleigh number, R_a, decreases with increase in porosity for stationary modes, implying thereby, it has destabilizing effect of medium porosity. It is also depicted from the graph that the critical wave number increases with increase in medium porosity for oscillatory modes. It transpires so as the volume engaged by the solid matrix increases with increase in the value of medium porosity, which in turn has a tendency to expidite the fluid flow.

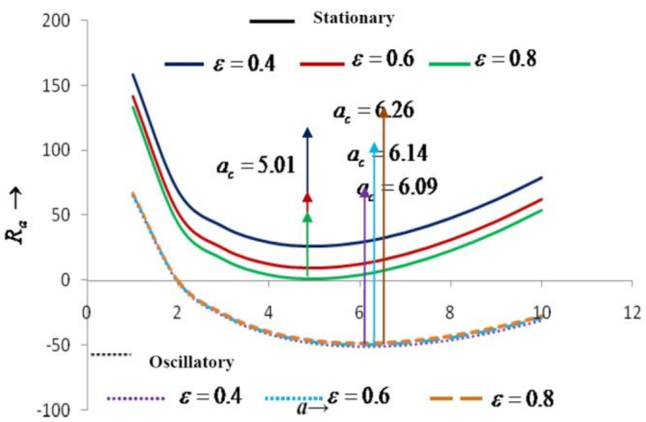

Figure 6. Variation of stationary and oscill. Rayleigh num. (R_a) with respect to wave number (a) for diff. values of medium porosity (ε).

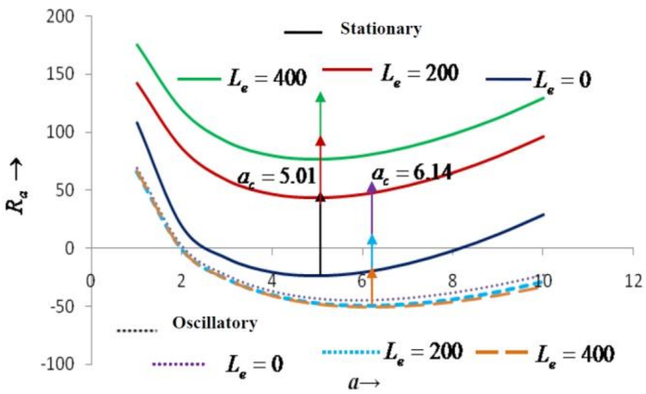

Figure 7. Variation of stationary and oscill. Rayleigh num. (R_a) with respect to wave number (a) for diff. values of Lewis number (L_e).

The plot of R_a against wave number a for various values of Lewis number, L_e = 0, 200, 400 is illustrated in Fig. 7. Figure 7 reveals that R_a increases with increase in Lewis number for stationary modes. However, the value of R_a decreases with increase in L_e depicting thereby the destabilizing effect on the oscillatory modes. This situation occurs so for the Brownian motion of the nanoparticles decreases with increase in Lewis number.

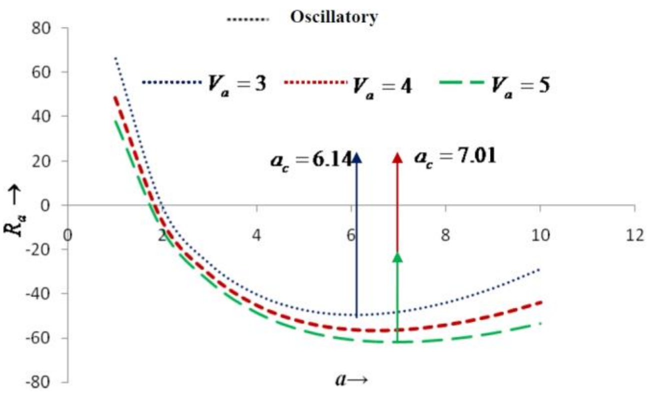

Figure 8. Variation of oscill. Rayleigh number (R_a) with respect to wave number (a) for different values of Vadasz number (V_a).

Figure 8 illustrates the effect of V_a on the R_a and it is observed that with increase in V_a, Darcy Rayleigh number decreases, implying thereby the destabilizing effect of Vadasz number on the system. The critical wave numbers increase with decrease in Vadasz number that is, 6.14 and 7.01.

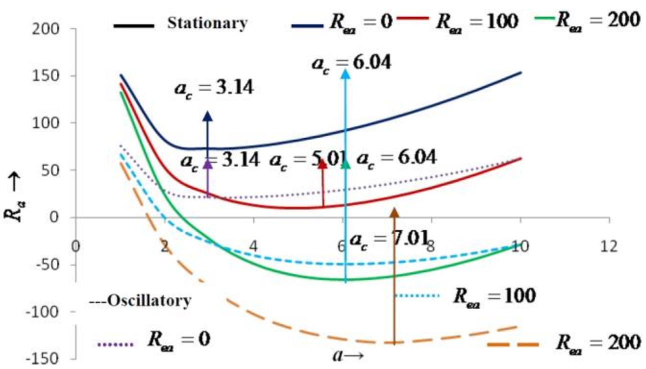

Figure 9. Variation of stationary and oscill. Rayleigh num. (R_a) with respect to wave num. (a) for diff. values of elec. Rayleigh num. (R_{ea}).

The changes of R_a vs. wave number a for different value of electric Rayleigh number R_{ea} = 0, 100, 200 are plotted in Fig. 9. It is depicted from the graphs that R_a (stationary) and R_a (oscillatory) decrease with increase in R_{ea}. This appears so because the destabilizing electrostatic energy to the system enhances a less stable system because of higher electric field.

It is worth mentioning that the effect of the variation in parameters: Lewis number; modified diffusivity ratio; concentration Rayleigh number; and porosity is very small due to large value of L_e and very small values of N_A and R_n. To verify the numerical results derived to compute the critical wave number and corresponding critical Darcy Rayleigh number to discuss the stability of the system, the results are calculated under the limiting case of nanoparticle and electric field in Eq.(42) (i.e. $R_n = N_A = 0$, $R_{ea} = 0$). It is noted that in the absence of nanoparticles and electric field, the critical Darcy Rayleigh number is equal to $4\pi^2$ and the corresponding critical wave number is π = 3.14 which is the precisely identical outcome by Lapwood, /3/. Thus, exactness of the numerical method applied is confirmed.

It is observed from Figs.(2) to (9) that for bottom-heavy nanoparticle distribution (negative value of R_n), oscillatory convection sets earlier than stationary convection. Consequently, oscillatory convection is possible only for negative value of R_n for a saturated porous medium. It is notable that R_a for stationary convection is always higher than that of oscillatory modes, which can be described as: the restoring forces motivated at the onset of convection due to prevalence of stationary motion, are not sufficient to inhibit the system from leaning away from steadiness. Therefore, the values of R_n and N_A are taken to be negative.

CONCLUSIONS

Linear stability analysis in a horizontal porous medium saturated with a Maxwell dielectric nanofluid in a verticlal AC electric field heated from below is investigated. The modified Darcy-Maxwell model is used to incorporate the effect of Brownian motion along with thermophoresis. The normal mode technique and one-term Galerkin approximation are used to derive the thermal Rayleigh number for both the cases of stationary convection and oscillatory motion. The principal results drawn are as follows:

- The concentration Rayleigh number, porosity and electric Rayleigh number tend to destabilize the system, whereas Lewis number stabilizes the system towards stationary modes and it is independent of stress-relaxation time, capacity ratio and Vadasz number.

- It is found that the size of convection cells depends only on AC electric Rayleigh number and decreases with increasing electric Rayleigh number.
- For oscillatory convection stress-relaxation time, capacity ratio, the Vadasz number, Lewis number, and electric Rayleigh number destabilize the system, whereas porosity stabilizes the system for bottom-heavy distribution.
- The modified diffusivity ratio has no significant effect on the system for both stationary and oscillatory mode.
- For oscillatory motions, it is found that the critical wave number increases with increase in stress-relaxation, porosity, Vadasz number, and electric- Rayleigh number, whereas the critical wave number remains uninfluenced with the increase in capacity ratio, concentration Rayleigh, modified diffusivity ratio, and Lewis number.

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