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STRESS DISTRIBUTION ON AN ISOTROPIC PLATE WITH A RECTANGULAR OPENING UNDER BIAXIAL TENSION LOAD

RASPODELA NAPONA U IZOTROPNOJ PLOČI SA PRAVOUGAONIM OTVOROM POD DEJSTVOM DVOOSNOG ZATEZNOG OPTEREĆENJA

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Keywords
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- finite element method
- isotropic plate
- rectangular opening

Abstract

The paper analyses the stress distribution in an isotropic plate with a rectangular opening, subjected to tensile loads. The presence of such opening shape typically has a negative effect on the magnitude and distribution of stresses in parts of mechanical structures, that can influence their safety and reliability, and also has a negative impact on their service life. Hence, special attention is devoted to designing and manufacturing of parts that contain such openings. Thus, the effects of the presence of rectangular openings in parts of plates subjected to tension in two perpendicular directions is analysed. Analytical and numerical methods are used, and the materials are assumed to be isotropic.

Ključne reči
- koncentracija napona
- metoda konačnih elemenata
- izotropna ploča
- pravougaoni otvor

Izvod

U ovom radu se govori o raspodeli napona u dvoosno zategnutoj izotropnoj ploči u kojoj je izveden pravougaoni otvor. Pojava ovakvog oblika otvora uglavnom nepovoljno utiče na veličinu i raspodelu napona u delovima mašinskih konstrukcija što se može odraziti na njihovu sigurnost i pouzdanost u radu, pa čak i na životni vek. Stoga treba posvetiti posebnu pažnju projektovanju i konstruisanju delova koji u sebi sadrže ovakav oblik otvora. Zato je u ovom radu analiziran uticaj postojanja pravougaonog otvora u delovima tipa ploča napregnutih na zatezanje u dva međusobno upravna pravca. Korišćene su analitičke i numeričke metode, sa pretpostavkom izotropnih materijala.

INTRODUCTION

It is a common case, as confirmed in engineering practice, that various mechanical parts of different shapes and sizes, often contain certain geometric irregularities such as: openings, holes, grooves, notches etc. More often than not, the existence of such geometric irregularities in these parts is inevitable since they appear due to various design reasons such as: mass reduction, connection, oil transportation etc. These irregularities can result in stress concentration, a situation in which stress magnitudes increase considerably in these areas. Such extreme stress magnitudes can often cause various unwanted effects, including fracture, failure and disaster related to mechanical structures. For this reason, it is necessary to approach the design and manufacturing of parts containing these geometric irregularities in a thorough manner, in order to avoid unwanted consequences.

A non-negligible number of research studied this type of problems. For example, Savin /1/ uses the function of conform mapping for the purpose of obtaining results about stress distribution and anisotropic fields weakened by the presence of variously shaped openings. By applying the finite element method (FEM), Radojković /2/ obtains stress distribution results for such opening, in the case of uniaxial and biaxial loading for plate elements. Mushelishvili /3/ has introduced the complex variable method, successfully used in solving some of the basic problems related to mathematics elasticity theory. Elasticity theory equations for solving problems related to stress distribution can be found in the works of Timoshenko et al. /4/ and Rašković /5/. Bathe et al. /6/ and Zienkiewicz /7/ suggested numerical methods and finite element analysis for solving mathematics problems that would take too long to solve using conventional means due to their size and complexity. By introducing linear elasticity theory equations into the FEM, Nikolić /8/ has solved problems like these. Dolićanin et al. /9/ applied numerical methods in order to solve phenomena involving thin plates with variously shaped openings. The influence of material properties of residual stresses and the expansion of plastic zone in layered thermoplastic composite plates with a rectangular opening was studied by Ozben et al. /10/. Rao et al. /11/ determined the stress distribution around openings in symmetric laminate and isotropic plates, whereas Rezaeepazhand et al /12/ considered the effects of various parameters, such as opening geometry, load direction or opening orientation, on stress distribution and stress concentration factor in perforated plates. By applying the finite element method, Vanam et al. /15/ determined the optimal plate thickness in the case the plate is subjected to various loads and under various boundary conditions. Nagpal /16/ studied the optimisation of a rectangular plate with a middle square opening, subjected to planar loads, in order to mitigate the stress concentration factor, and Pan et al. /17/ studied the effects of different parameters on stress distribution in finite plates with rectangular openings under uniaxial tension using modified stress functions, in order to obtain the results. Analysis of the influence of special opening geometries in a finite plate on stress distribution and intensity factor is the goal of a paper by Watsar et al. /18/. Chauhan et al. /19/ studied the effect of plate size, opening geometry and location, material anisotropy, and load on stress concentration around polygonal openings. In their research, Shariati et al. /20/ deals with numerical and experimental analysis of rectangular plates with opening, while analysing the effect of opening location change on plate buckling, in addition to individual dimensions. Gokul et al. /21/ used ANSYS to perform a static analysis of thin plates with openings and to analyse the effect of opening size and shape on stress distribution in a plate. The influence of the ratio of geometry sizes and curvature radius of angles in a rectangular opening on stress distribution in finite cellular plates, weakened by the presence of rectangular openings, subjected to uniaxial loads is considered by Rahman /22/. Dehghani /23/ provided a general analytical solution for determining the stress distribution in a finite elastic plate with a circular or square opening, under biaxial loading. In the work of Jafari et al. /24/, the optimisation of parameters related to the analysis of perforated plates with square openings is analysed in order to decrease the stress concentration as much as possible, for the case of a finite isotropic plate. This analysis is based on metaheuristic optimisation. Ghajar et al. /25/ suggested a new conform mapping in order to analytically solve the stress intensity problem in an infinite plane with a square opening. Studying of various geometry parameters and shape effects on the stress intensity factor induced in an infinite plate with a rectangular opening with filleted angles is covered by Gunwanta /26/. In this paper, the finite element method (FEM) is used, via ANSYS software /27/.

ANALYTICAL RESULTS FOR STRESS DISTRIBUTION IN A PLATE WITH RECTANGULAR OPENING

The following text shows the analytically obtained results for stress distribution in a biaxially loaded isotropic plate with a rectangular opening /1, 2/. The isotropic plate, Fig. 1, has finite dimensions and a rectangular opening at its centre, with a side ratio $a/b = 3/2$ and a fillet radius of $r = (3/50)a$, wherein the opening side a is parallel to the x axis, and side b is parallel to the y axis. The plate is subjected to tensions in two perpendicular directions with surface forces q along the x direction, and p along the y direction.

During the mapping of the observed area with a rectangular opening to the interior of a unit circle with radius $\rho = 1$, according to the procedure given in /1, 2/, the expressions for stress components in polar coordinates, σ_ρ, σ_θ and $\tau_{\rho\theta}$, have the following form:

$$\sigma_\rho = \frac{\lambda_1+\lambda_2}{2}\sigma_T\left(1-\rho^2\right)-\lambda\sigma_T\left(1-4\rho^2+3\rho^4\right)\cos 2\theta\,,$$
$$\sigma_\theta = \frac{\lambda_1+\lambda_2}{2}\sigma_T\left(1+\rho^2\right)+\lambda\,\sigma_T\left(1+3\rho^4\right)\cos 2\theta\,, \tag{1}$$
$$\tau_{\rho\theta} = -\lambda\sigma_T\left(1+2\rho^2-3\rho^4\right)\sin 2\theta\,,$$

where: λ_1 and λ_2 are dimensionless parameters that determine the opening geometry; $\lambda = (\lambda_2 - \lambda_1)/2$ represents another dimensionless parameter; σ_T is yield stress of the material under tensile loading; θ is the polar angle, i.e. the angle between the positive x axis direction and direction of ρ, measured in the positive mathematical direction.

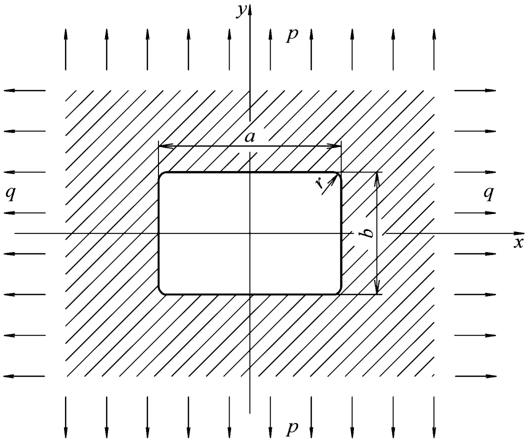

Figure 1. Biaxial tension load of a plate with a rectangular opening.

Maximal and minimal normal tensile stresses, $\sigma_{\max}$ and $\sigma_{\min}$, along with maximal tangential stress $\tau_{\max}$ are determined by the following formulas, according to /1, 2/, and shown in Fig. 2:

$$\sigma_{\max} = \frac{\sigma_\rho+\sigma_\theta}{2}+\sqrt{\left(\frac{\sigma_\rho-\sigma_\theta}{2}\right)^2+\tau_{\rho\theta}^2}\,,$$
$$\sigma_{\min} = \frac{\sigma_\rho+\sigma_\theta}{2}-\sqrt{\left(\frac{\sigma_\rho-\sigma_\theta}{2}\right)^2+\tau_{\rho\theta}^2}\,, \tag{2}$$
$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_\rho-\sigma_\theta}{2}\right)^2+\tau_{\rho\theta}^2}\,.$$

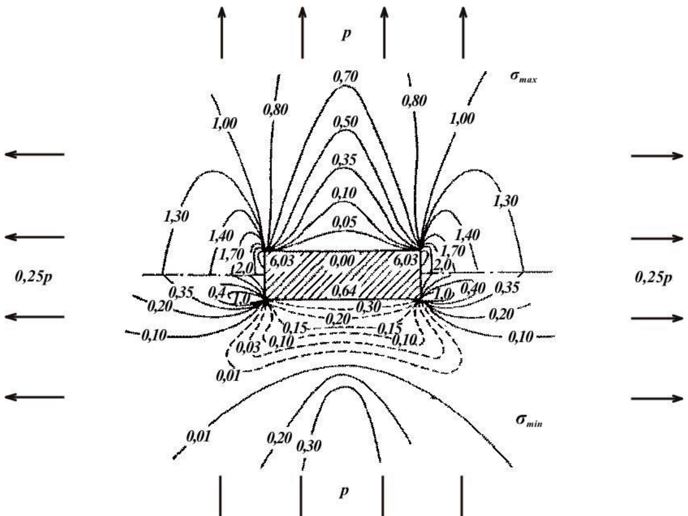

Figure 2. $\sigma_{\max}$ and $\sigma_{\min}$ distribution in an isotropic plate with rectangular opening subjected to biaxial tensile load.

Full lines in Fig. 2 depict the maximal tensile stresses (top half of figure), as well as minimal tensile stresses (bottom half of figure). Figure 2 also shows equal compressive stresses in the form of dotted lines.

NUMERICAL RESULTS FOR STRESS DISTRIBUTION IN A PLATE WITH A RECTANGULAR OPENING

Numerical results obtained in this paper are related to stress distribution in an isotropic plate with a rectangular opening. It is well known that numerical methods can be successfully used in order to obtain such results /6, 7, 9, 10, 13, 15, 20-22, 26/. Additionally, it is known that numerical methods have an advantage over analytical ones during the solving of more complex problems, that rely on complex mathematical apparatus and experimental methods, due to their costs. Finite element method (FEM) is among the most widely used numerical methods, and in this paper it is used in order to discretise a physical model into finite elements. However, the application of this method also involves the use of computers and corresponding software packages. Stress distribution in an isotropic plate with a rectangular opening here is obtained using ANSYS® software, /27/.

Using ANSYS® software for FEM calculations provided the results for maximal ($\sigma_{\max}$) and minimal ($\sigma_{\min}$) normal tensile stress distribution. Obtained results are related to biaxial tension in plate-type elements with rectangular opening in its centre. These plates are of dimensions 2×5×0.1 m, with the shorter side along the *x* direction, and the longer side along *y* direction. These plates are weakened by the presence of a rectangular opening located at the centre of the plate, whose side ratio is $a/b = 3/2$, wherein the length of side a is equal to 100 mm (along the *x* axis), and side b = 66.7 mm (along the *y* axis). Angles of the opening are filleted with a radius of $r = (3/50)a = 6$ mm. Plates are subjected to tensile forces $p = 1$ N/m^2 along the *x*, and $q = 0.25$ N/m^2 along *y* axes. The material is steel, with elasticity modulus $E = 2.1\times10^{10}$ N/m^2 and Poisson's ratio of $\mu = 0.33$. Discretisation of the model (plate) in these examples is performed using 2D triangular solid six-node finite elements.

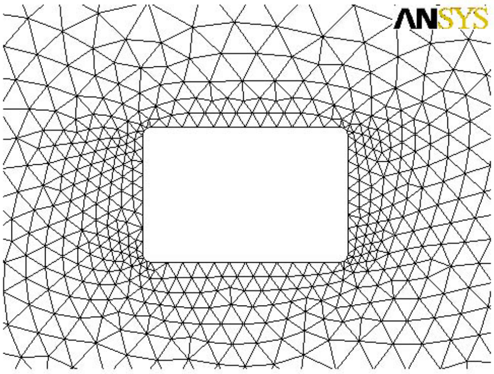

Figure 3. Part of a steel plate with a rectangular opening ($a\times b$ = 100×66.67 mm, r = 6 mm), discretised by 2D triangular finite elements.

Figure 3 shows a part of the steel plate with a rectangular opening and dimensions of a = 100 mm and b = 66.67 mm, and fillet radius of r = 6 mm, discretised using 2D triangular solid finite elements (since showing the whole model would make the figure unclear). In addition, it should be noted that the selection of the model, type and number of finite elements for the mesh resulted from personal experience, and that there are no exact criteria for this selection process.

Figure 4 shows the distribution of maximum normal tensile stress, $\sigma_{\max}$, indicating the highest values near the opening contour, at its angles (red colour in the figure), with maximum stress $\sigma_{\max}$ = 4.981 N/m^2.

Figure 5 shows distribution of minimum normal tensile stress $\sigma_{\min}$, indicating the highest values at the intersections of the opening contour and the *y* axis (blue fields in the figure), with the maximum stress $\sigma_{\max}$ = 0.5266 N/m^2.

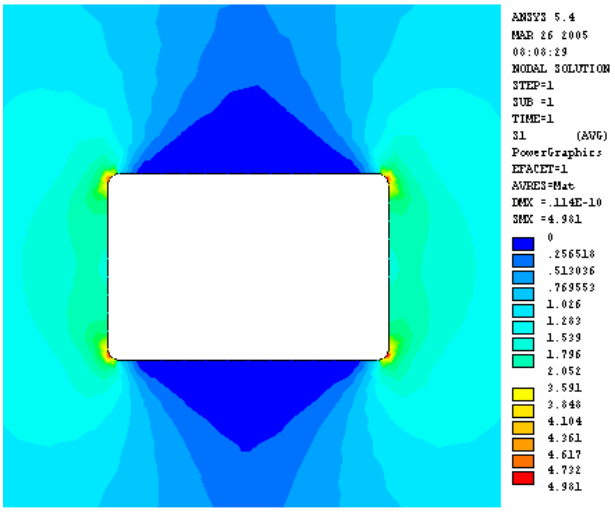

Figure 4. Maximum normal tensile stress distribution in a biaxially loaded steel plate with a rectangular opening

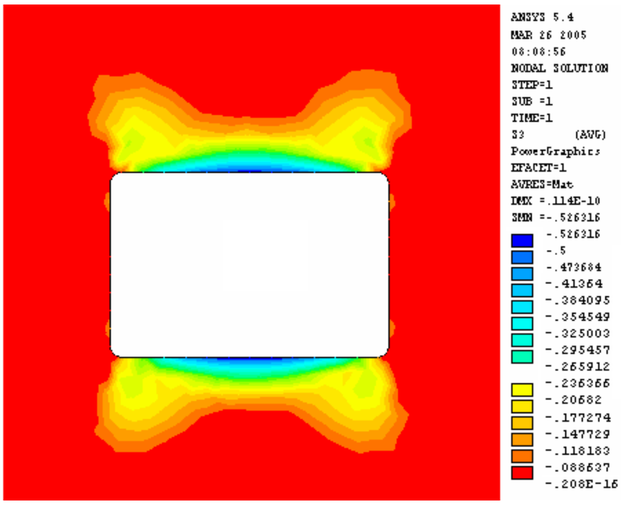

Figure 5. Minimum tensile stress distribution in a biaxially loaded steel plate with a rectangular opening.

COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS

Numerical results, obtained by FEM, are now compared to analytical results, obtained by the complex variable method, as shown in Table 1.

Table 1. Maximum ($\sigma_{\max}$) and minimum ($\sigma_{\min}$) tensile stress.

Method	$\sigma_{\max}$ (N/mm^2)	$\sigma_{\min}$ (N/mm^2)
Analytical	6.03	-0.64
FEM	4.981	-0.526316

By analysing the values obtained for maximum and minimum normal tensile stresses in the steel plate with a rectangular opening subjected to tensile forces $p = 1$ N/m^2 (y axis), and $q = 0.25$ N/m^2 (x axis), it can be concluded that the highest tensile stresses are located at the opening contour, specifically at its angles. Minimum normal tensile stress values are also located at the contour, specifically its intersection with the y axis.

CONCLUSION

Research shown in this paper aims to obtain certain results about the normal tensile stress distribution in isotropic plate elements with a rectangular opening, since these results are the basis of calculation of geometric stress intensity factor, whereas the minimum tensile stress distribution results are used in this case to justify the application of the selected numerical method (FEM). The rectangular shape of the opening may not be a common occurrence in mechanical structures, but it is still encountered in practice ever so often. Neglecting its effect on stress distribution and concentration can lead to unwanted consequences. Thus, calculations and design of parts containing such openings require special attention.

By analysing the values from Table 1, it can be concluded that under biaxial tension of an isotropic steel plate with a rectangular opening, in the case of surface forces acting in two mutually perpendicular directions, the highest maximal tensile stress values occur at the opening contour, in its angles. By comparing the values obtained under biaxial tension with those obtained in the case of uniaxial tension in a steel plate with identical dimensions, which can be found in /2, 13/, it can be concluded that the values in the biaxial case are slightly lower, due to the forces acting in an additional, perpendicular direction.

Results obtained in this paper can be of great significance in engineering practice and may be useful to engineers involved in calculation, design and manufacture of parts that contain such openings.

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