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STRESS CONCENTRATION STATE OF PIEZOELECTRICITY IN CYLINDRICAL SHELL BASED ON THE NON-CLASSICAL THEORY

STANJE PIJEZOELEKTRIČNOSTI USLED KONCENTRACIJE NAPONA U CILINDRIČNOJ LJUSCI NA BAZI NEKLASIČNE TEORIJE

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- cilindrična ljuska
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- granični sloj

Abstract

This paper presents the stress concentration at the ends of piezoelectricity cylindrical shell. The shell is subjected to electric potential and mechanical loading applied in the top and bottom surfaces. The mathematical model of electro-elasticity behaviour is based on the non-classical theory and Lagrange's principle. The governing differential equations are reduced to ordinary differential equations by means of trigonometric function expansion for displacement and electric potential. Electromechanical state of the smart cylindrical shell for axisymmetric loads with clamped support is provided. Numerical results are presented showing the effect of boundary layer on local stress concentration at clamped edges. Comparison between the obtained numerical results to classical theory of the Kirchhoff-Love type is accurate.

Izvod

U radu je predstavljena koncentracija napona na pijezo-električnim krajevima cilindrične ljuske. Ljuska je podvrgnuta električnoj potencijalnoj razlici i mehaničkom opterećenju na gornjoj i donjoj površini. Matematički model elektroelastičnog ponašanja se bazira na neklasičnoj teoriji i na principu Lagranža. Odgovarajuće diferencijalne jednačine su redukovane na obične diferencijalne jednačine putem razvoja trigonometrijskim funkcijama za pomeranja i za električni potencijal. Opisano je elektromehaničko stanje pametne cilindrične ljuske kod osnosimetričnog opterećenja sa uklještenjem u osloncu. Prikazani su numerički rezultati uticaja graničnog sloja na lokalnu koncentraciju napona u uklještenim krajevima. Poređenje daje precizno poklapanje dobijenih numeričkih rezultata sa klasičnom teorijom tipa Kirchhoff-Love.

INTRODUCTION

Piezoelectric plates and shells are active smart materials used in both sensor and actuator applications. In the aerospace industries, piezoelectric elements are used on aircraft because of low cost, low power consumption, low weight, and ease in embedding or bonding with the structure. Use of piezoelectric elements helps aircraft improve the quality of aerodynamics and the effective management of their deformations, /9, 18/. So, the research and development of smart structures promises new design opportunities for next generation high performance mechanical and structural systems.

Stress-deformed state of thin plates and shells is calculated on the assumptions of Kirchhoff-Love /1/, Timoshenko and Gol'Denveizer, /2, 3/. They are called classical deformation theory (CDT) and can be used to represent three-dimensional equations of elasticity theory in two-dimensional form. Based on CDT, Tzou /5/, Patron /13/, and Carrera /16/, presented an electroelasticity model of piezoelectric plates and shells by joining elasticity equations with Maxwell' equations. Mindlin /6/ and Reissner /17/ provided first order shear deformation theory (FSDT); and Reddy /4/ developed third order shear deformation theory (TSDT) and high order shear deformation theory (HSDT) /14/, for not only isotropic but also for anisotropic and piezoelectric magnetic laminated materials.

Santosh Kapuria /10/ presented a three-dimensional solution of cylindrical shell for axisymmetric load for simply-supported edges. Benjeddou /11/ calculated electroelasticity behaviour of beams, based on finite element modelling. Kumar /22/ and Alibeigloo /23/ introduced finite element formulation to 9 nodes to model the static and dynamic analysis of laminated composite shells subjected to electrical, mechanical and thermal loadings. Arshid and Khorshidvand /7/ used Hamilton's variational principle to derive the governing motion equations for a circular plate made up of a porous material integrated by piezoelectric actuator patches. Then, the free vibration analysis was carried out in order to evaluate the effect of some conditions on stress-deformed state. Akbari Alashti /12/ provided three-dimensional thermo-elastic analysis of a functionally graded cylindrical shell with piezoelectric layers under the effect of asymmetric thermo-electro-mechanical loads by differential quadrature method. Mitchell and Reddy /8/ have published a power series solution for axisymmetric composite cylinder with surface-bonded or embedded piezoelectric laminae under axial load.

However, in these papers, boundary conditions only for fully simply-supported cases are studied, and stress concentration has not been mentioned much. In this work, a non-classical theory is presented to model the electromechanical state of piezoelectric smart cylindrical shells. The displacement and potential field, in this case, satisfy the energy compatibility conditions proposed by Lurie and Vasiliev, /15/. The generalized Lagrange principle is utilized to derive the governing equations of equilibrium. The Laplace transform is used to analyse the stress concentration state of cylindrical shells due to axisymmetric electric potential and mechanical loading applied at the ends with clamped edges. The comparison of results of the cylindrical shell with the classical theory is given.

THE GOVERNING EQUATIONS

Consider a circular cylindrical shell in the orthogonal curvilinear coordinate system $0\xi\theta z$. The thickness, length and radius of the shell are assumed to be $2h$, L and R, in respect.

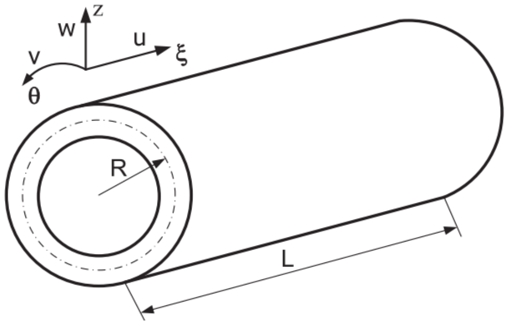


Figure 1. Coordinate systems of cylindrical shell.

Linear constitutive equations of piezoelectric materials /5/ are presented by Eq.(1):

$$\begin{aligned}\{\sigma\}&=[C]\{\varepsilon\}-[e]^T\{E\},\\ \{D\}&=[e]\{\varepsilon\}+[\mu]^T\{E\},\end{aligned} \tag{1}$$

where: $\sigma = [\sigma_\xi, \sigma_\theta, \sigma_z, \tau_{\xi\theta}, \tau_{\theta z}, \tau_{z\xi}]$ is the stress vector; $D = [D_\xi, D_\theta, D_z]$ is electric displacement vector; $E = [E_\xi, E_\theta, E_z]$ is electric field vector; $C = C_{ij}$ ($i = \overline{1,6}$, $j = \overline{1,6}$) is elastic stiffness matrix; $\varepsilon = [\varepsilon_\xi, \varepsilon_\theta, \varepsilon_z, \gamma_{\xi\theta}, \gamma_{\theta z}, \gamma_{z\xi}]$ is strain vector; $e = e_{ij}$ ($i = \overline{1,3}$, $j = \overline{1,6}$) is piezoelectric matrix; and $\mu = \mu_{ij}$ ($i = \overline{1,3}$, $j = \overline{1,3}$) is dielectric matrix.

The strain components in above equations are related to displacement components /19-21/, which are determined by Eq.(2):

$$\begin{aligned}&\varepsilon_\xi=\frac{1}{R}\frac{\partial u}{\partial\xi},\ \varepsilon_\theta=\frac{1}{R+z}\left(\frac{\partial v}{\partial\theta}+w\right),\ \gamma_{\xi\theta}=\frac{1}{R}\frac{\partial v}{\partial\xi}+\frac{1}{R+z}\frac{\partial u}{\partial\theta},\\ &\varepsilon_z=\frac{\partial w}{\partial z},\ \gamma_{\xi z}=\frac{1}{R}\frac{\partial w}{\partial\xi}+\frac{\partial u}{\partial z},\ \gamma_{\theta z}=\frac{1}{R+z}\frac{\partial w}{\partial\theta}+\frac{\partial v}{\partial z}-\frac{v}{R+z}.\end{aligned} \tag{2}$$

To describe the electromechanical state of a piezoshell, enthalpy /16/ H is used:

$$H=\frac{1}{2}\{\varepsilon\}^T[C]\{\varepsilon\}-\{\varepsilon\}^T[e]^T\{E\}-\frac{1}{2}\{E\}^T[e]\{E\}. \tag{3}$$

Then the components of mechanical stress and electric displacement of the shell are related to the enthalpy by formulas:

$$\sigma=\frac{\partial H}{\partial\varepsilon},\quad D=\frac{\partial H}{\partial E}. \tag{4}$$

Based on energy compatibility conditions proposed by Lurie and Vasiliev /15/, the displacement and electric potential fields are presented by the system of equations, Eq.(5),

$$\begin{aligned}u(\xi,\theta,z)&=u_0(\xi,\theta)+u_1(\xi,\theta)z+u_2(\xi,\theta)\frac{z^2}{2!}+u_3(\xi,\theta)\frac{z^3}{3!},\\ v(\xi,\theta,z)&=v_0(\xi,\theta)+v_1(\xi,\theta)z+v_2(\xi,\theta)\frac{z^2}{2!}+v_3(\xi,\theta)\frac{z^3}{3!},\\ w(\xi,\theta,z)&=w_0(\xi,\theta)+w_1(\xi,\theta)z+w_2(\xi,\theta)\frac{z^2}{2!},\\ \varphi(\xi,\theta,z)&=\varphi_0(\xi,\theta)+\varphi_1(\xi,\theta)z+\varphi_2(\xi,\theta)z^2.\end{aligned} \tag{5}$$

Electrostatic equations neglecting magnetic effects /5/ can be provided as:

$$E_\xi=-\frac{\partial\varphi}{A_1\partial\xi},\ E_\theta=-\frac{\partial\varphi}{A_2\partial\theta},\ E_z=-\frac{\partial\varphi}{\partial z}, \tag{6}$$

where: $A_1 = R$, $A_2 = 1 + z/R$ are the Lame parameters of the cylindrical shell.

The piezoelectric shell is subjected to a prescribed surface traction $q_{i3} = q_{i3}^{\pm}(\xi,\theta)$, ($i$ = 1,2,3) and electric potential $\varphi_i = \varphi^{\pm}(\xi,\theta)$ at the top and bottom surface $z = \pm h$ of the shell, in respect. Combining the fourth equation in Eq.(5), we have

$$\varphi_1=\frac{\varphi^+-\varphi^-}{2h},\quad \varphi_2=-\frac{\varphi_0}{h^2}+\frac{\varphi^++\varphi^-}{2h^2}. \tag{7}$$

To establish the equilibrium equation, the Lagrange principle is used. The total potential energy must be a minimum value, thus, /4/:

$$\delta U-\delta A=0, \tag{8}$$

where: U is the elastic potential energy; A is work done by external forces and electric charges Q at the upper and lower surfaces. The variation of the electroelastic potential energy is defined in the following Eq.(9),

$$\begin{aligned}\delta U-\delta A&=\int(\sigma\delta\varepsilon+D\delta E)dV-\\ &-\sum\int(q_{i3}(\delta u_i+\delta v_i+\delta w_i)dS-\int Q\delta\varphi dS.\end{aligned} \tag{9}$$

Equilibrium equations are governed by integrating separately Eq.(9) according to the potential and displacement components, then taking independently the possible potential and displacement equal zero. From there, we get the following equilibrium equations system as follows, Eq.(10),

$$\begin{aligned}&\frac{\partial N_{11}^{(0)}}{\partial\xi}+\frac{\partial N_{21}^{(0)}}{\partial\theta}=0,\\ &\frac{\partial N_{11}^{(i)}}{\partial\xi}+\frac{\partial N_{21}^{(i)}}{\partial\theta}-RN_{13}^{(i-1)}=0,\quad (i=\overline{1,3}),\\ &\frac{\partial N_{12}^{(0)}}{\partial\xi}+\frac{\partial N_{22}^{(0)}}{\partial\theta}+N_{23}^{(0)}=0,\end{aligned} \tag{10}$$

$$\frac{\partial N_{12}^{(i)}}{\partial \xi}+\frac{\partial N_{22}^{(i)}}{\partial \theta}+N_{23}^{(i)}-RN_{23}^{(i-1)}-iN_{23}^{(i)}=0, \quad (i=\overline{1,3}),$$

$$\frac{\partial N_{13}^{(0)}}{\partial \xi}+\frac{\partial N_{23}^{(0)}}{\partial \theta}-N_{22}^{(0)}+Rp_z^{(0)}=0,$$

$$\frac{\partial N_{13}^{(j)}}{\partial \xi}+\frac{\partial N_{23}^{(j)}}{\partial \theta}-N_{22}^{(j)}-RN_{33}^{(j-1)}+Rp_z^{(j)}=0, \quad (j=\overline{1,2}),$$

$$\frac{\partial\left(h^2 ND_1^{(0)}-ND_1^{(2)}\right)}{\partial \xi}+\frac{\partial\left(h^2 ND_2^{(0)}-ND_2^{(2)}\right)}{\partial \theta}-2ND_3^{(1)}=0.$$

where: $N_{ij}^{(k)}(i=\overline{1.3}, j=\overline{1.3}, k=\overline{0.3})$ are mechanical forces and moments; $ND_i^{(k)}$ $(i=\overline{1.3}, k=\overline{0.2})$ are electrical forces and moments, and the following designations are adopted:

$$\left(N_{11}^{(i)},N_{12}^{(i)},N_{13}^{(i)}\right)=\int_{-h}^{h}(\sigma_\xi,\tau_{\xi\theta},\tau_{\xi z})\frac{z^i}{i!}dz, \quad (i=\overline{0,3}),$$

$$\left(N_{22}^{(i)},N_{21}^{(i)},N_{23}^{(i)}\right)=\int_{-h}^{h}(\sigma_\theta,\tau_{\xi\theta},\tau_{\theta z})\frac{z^i}{i!}dz, \quad (i=\overline{0,3}),$$

$$N_{33}^{(j)}=\int_{-h}^{h}\sigma_z\frac{z^j}{j!}dz, \quad (j=\overline{0,2}), \tag{11}$$

$$ND_1^{(i)}=\int_{-h}^{h}D_1\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz, \quad ND_2^{(i)}=\int_{-h}^{h}D_2\frac{z^i}{i!}dz, \quad (i=\overline{0,2}),$$

$$ND_3^{(i)}=\int_{-h}^{h}D_3\left(1+\frac{z}{R}\right)\frac{z^i}{i!}dz, \quad (i=\overline{0,2}),$$

$$p_z^{(i)}=q_{33}^{+}(\xi,\theta)\left(1+\frac{h}{R}\right)\left(\frac{h^i}{i!}\right)-q_{33}^{-}(\xi,\theta)\left(1-\frac{h}{R}\right)\left(\frac{(-h)^i}{i!}\right), \quad (i=\overline{0,3}).$$

The boundary conditions at the edges are:

- for clamped supported:

$$u_i=v_i=w_i=0, \quad \varphi_j=0, \quad i=\overline{0,3}, \quad j=\overline{0,2},$$

- for simply supported:

$$\sigma_\xi^i=v_i=w_i=0, \quad \varphi_j=0, \quad i=\overline{0,3}, \quad j=\overline{0,2},$$

- for free from supported:

$$\sigma_\xi^i=\sigma_{\xi\theta}^i=\sigma_{\xi z}^i=0, \quad \varphi_j=0, \quad i=\overline{0,3}, \quad j=\overline{0,2}.$$

ELECTROELASTICITY STATE OF CYLINDRICAL SMART SHELL FOR AXISYMMETRIC LOAD

Consider a circular cylindrical shell subjected to axisymmetric mechanical load $q_{i3}(\xi)$ and electronic potential $\varphi(\xi)$ at top and bottom surfaces with fully clamped edges. In this case displacements are $v_{ij}(\xi,\theta)=v(\xi,\theta,z)=0$ and boundary conditions are

$$u_i=v_i=w_i=0, \quad \varphi_j=0, \quad i=\overline{0,3}, \quad j=\overline{0,2}. \tag{12}$$

The solution of the boundary value problem, satisfying boundary conditions Eq.(12) is taken in the separable form of the following Fourier series:

$$q(\xi)=\sum_{m=1}^{\infty}Q_m\sin(m\xi)+Q_0(\xi),$$

$$u_k(\xi)=\sum_{m=1}^{\infty}U_k\sin(m\xi)+U_{k0}(\xi), \quad k=\overline{0,3}, \tag{13}$$

$$w_l(\xi)=\sum_{m=1}^{\infty}W_l\sin(m\xi)+W_{l0}(\xi), \quad l=\overline{0,2},$$

$$\varphi(\xi)=\sum_{m=1}^{\infty}\varphi_m\sin(m\xi)+\varphi_0(\xi).$$

Substituting Eq.(13) into Eq.(10) we obtain differential equations to determine $u_i(\xi,\theta)$, $w_j(\xi,\theta)$, $\varphi_j(\xi,\theta)$, $i=\overline{0.3}$, $j=\overline{0.2}$ functions as follows,

$$Ki_{d1}^{\varphi_0}\frac{\partial}{\partial\xi}\varphi_0+\sum_{j=0}^{3}\left(Ki_{d2\xi}^{uj}\frac{\partial^2}{\partial\xi^2}+Ki^{uj}\right)u_j+\sum_{j=0}^{2}Ki_{d1\xi}^{wj}\frac{\partial}{\partial\xi}w_j+$$

$$+\left(k_i^{q_{13}^+}q_{13}^{+}+k_i^{q_{13}^-}q_{13}^{-}\right)=0, \quad (i=\overline{1,4}),$$

$$Ki_{d2\xi}^{\varphi}\frac{\partial^2}{\partial\xi^2}\varphi_0+\sum_{j=0}^{3}Ki_{d1\xi}^{uj}\frac{\partial}{\partial\xi}u_j+\sum_{j=0}^{2}\left(Ki_{d2\xi}^{wj}\frac{\partial^2}{\partial\xi^2}+Ki^{wj}\right)w_j+$$

$$+\left(k_i^{q_{33}^+}q_{33}^{+}+k_i^{q_{33}^-}q_{33}^{-}\right)+\left(k_i^{\varphi^+}\varphi^{+}+k_i^{\varphi^-}\varphi^{-}\right)=0, \quad (i=\overline{5,7}),$$

$$\sum\left(K_{d2\xi}^{\varphi_0}\frac{\partial^2}{\partial\xi^2}+K^{\varphi_0}\right)\varphi_0+\sum_{j=0}^{3}K_{d1\xi}^{uj}\frac{\partial}{\partial\xi}u_j+$$

$$+\sum_{j=0}^{2}K_{d1\xi}^{wj}w_j+\left(k^{\varphi^+}\varphi^{+}+k^{\varphi^-}\varphi^{-}\right)=0, \tag{14}$$

where: coefficients K_i are constants that depend on geometric parameters, elastic and electrical constants of the shell material.

To solve the system of ordinary differential Eqs.(14), an operational method based on the Laplace transform is used and the algebraic equation system is obtained.

NUMERICAL RESULTS

In numerical simulations, the smart circular shell is assumed to be made of PZT-4 with material constants given /16/: Young's moduli $Y_1=Y_2=81.3$ GPa and $Y_1=64.5$ GPa; Poisson ratios $\nu_{12}=0.329$ and $\nu_{13}=\nu_{23}=0.432$; shear moduli $G_{13}=G_{23}=25.6$ GPa and $G_{12}=30.6$ GPa; dielectric coefficients $\mu_{11}=\mu_{22}=13060$ pC/Vm and $\mu_{33}=11510$ pC/Vm; piezoelectric coefficients $e_{15}=e_{24}=12.72$ C/m^2, $e_{31}=e_{32}=-5.20$ C/m^2 and $e_{33}=15.08$ C/m^2. Geometrical parameters of shell are used: radius $R=1.0$ m, length $L=4R$, relative length $h/R=1/100$.

Two cases of circular cylinder shell are considered:

– the shell is under the action of axisymmetric mechanical loading at the top surface of the shell;

– the shell is under the action of axisymmetric electric potential at the top surface of the shell.

In the first case, mechanical loads are presented as a sinusoidal function $q_{33}^{+}(\xi,\theta)=Q_0\cos(m\xi)$ and as linear function $q_{33}^{+}(\xi,\theta)=Q_0c\xi$, $(q_{13}^{\pm}=q_{23}^{\pm}=q_{33}^{-}=0)$ with $Q_0=const$.

Electromechanical state of shell is shown in Figs. 2-7. In Figs. 2-3 and in Figs. 5-6 we notice that the transverse stress $\sigma_z=0$ at a distance away from edge zone. So, the classical theory and calculation program in this paper meet at very good agreement. But at the boundary position for clamped support, the maximal stress σ_z, negligible in classical theory, and according to the revised theory, accounts for about 42%

of the maximum normal stress σ_ξ for sinusoidal load and 45 % for linear load.

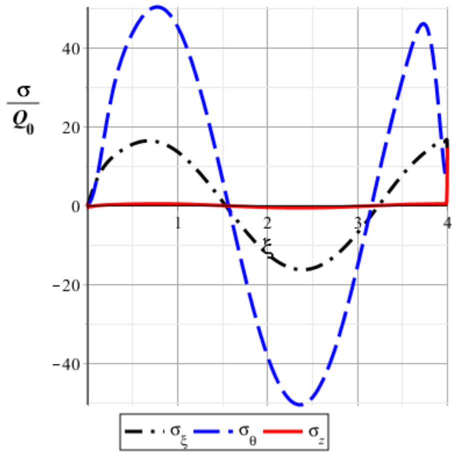


Figure 2. Stress distribution at boundary position by the length at middle plane for $q_{33} = Q_0\sin(m\xi)$, $\varphi = 0$.

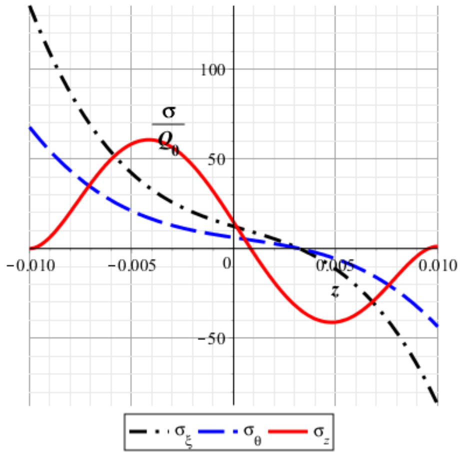


Figure 3. Stress distribution at boundary position by the thickness for $q_{33} = Q_0\sin(m\xi)$, $\varphi = 0$.

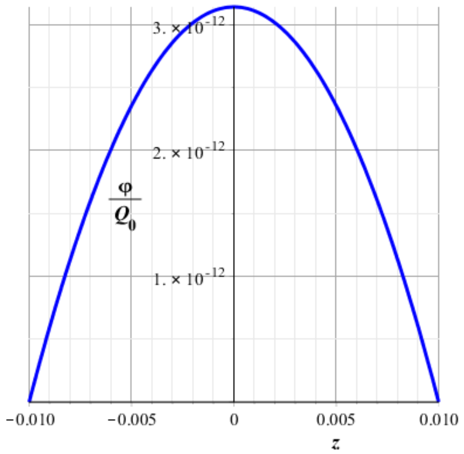


Figure 4. Electric potential distribution at position $\xi = 2$ by the thickness for $q_{33} = Q_0\sin(m\xi)$, $\varphi = 0$.

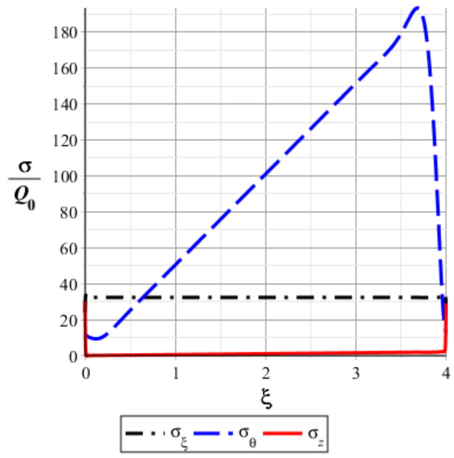


Figure 5. Stress distribution at boundary position by the length at middle plane for $q_{33} = Q_0\xi$, $\varphi = 0$.

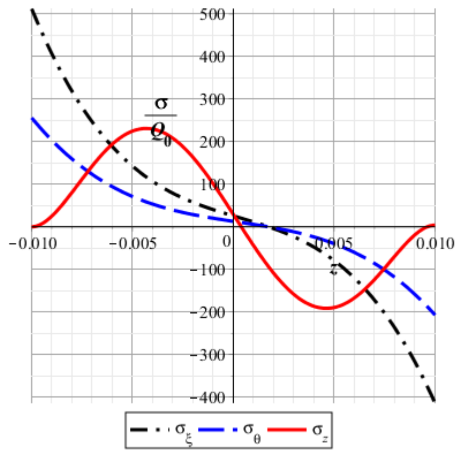


Figure 6. Stress distribution at boundary position by the thickness for $q_{33} = Q_0\xi$, $\varphi = 0$.

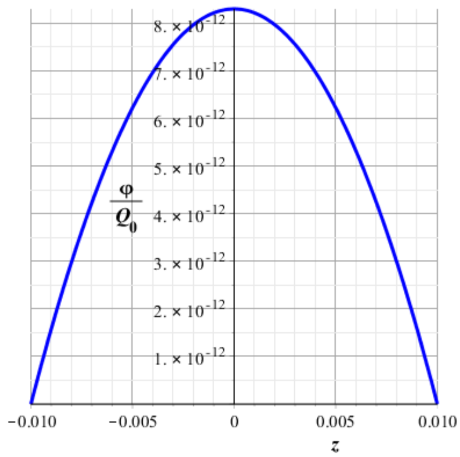
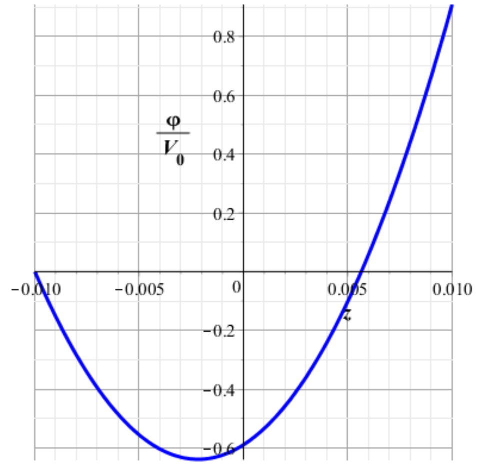


Figure 7. Electric potential distribution at position $\xi = 2$ by the thickness for $q_{33} = Q_0\xi$, $\varphi = 0$.

In the second case, electric potentials are considered as sinusoidal function $\varphi^+(\xi,\theta) = V_0\sin(m\xi)$ and linear function $\varphi^+(\xi,\theta) = V_0\xi$, $(\varphi^-(\xi,\theta) = 0)$, with $V_0 = const$.

The electromechanical state of the shell is presented in Figs. 8-11.



Figurer 8. Electric potential distribution at position $\xi = 2$ by the thickness for $q_{33} = 0$, $\varphi = V_0\sin(m\xi)$.

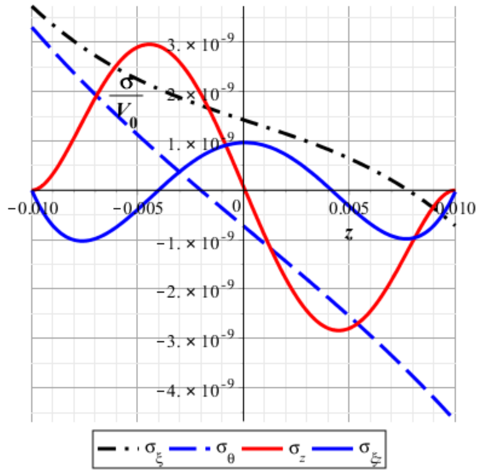


Figure 9. Stress distribution at boundary position by the thickness for $q_{33} = 0$, $\varphi = V_0\sin(m\xi)$.

Similar to first case, we can see that at the boundary position maximal transverse stress σ_z, negligible in the classical theory, according to the non-classical theory in the paper, accounts for about 56 % of maximal normal stresses σ_ξ for sinusoidal potential, and 54 % for linear function.

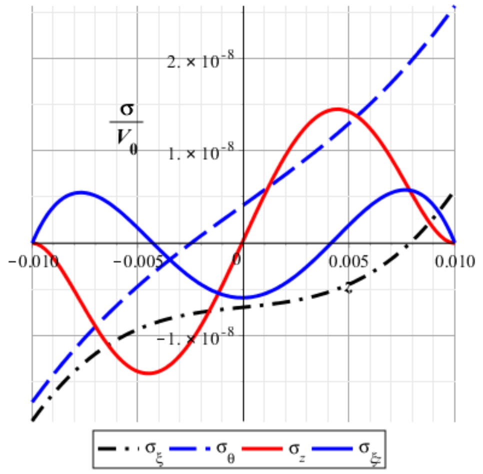


Figure 10. Stress distribution at boundary position by the thickness, for $q_{33} = 0$, $\varphi = V_0\xi$.

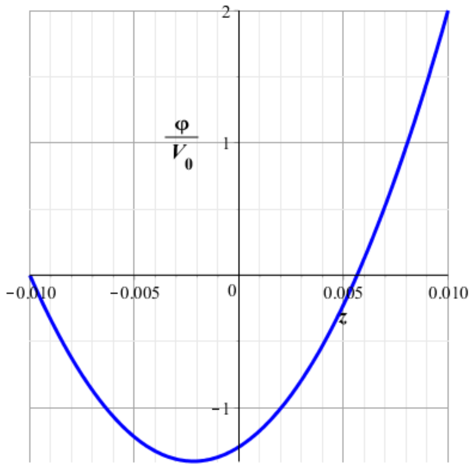


Figure 11. Electric potential distribution at position $\xi = 2$ by the thickness for $q_{33} = 0$, $\varphi = V_0\xi$.

CONCLUSION

A non-classical theory model on the basic governing equations and corresponding natural boundary conditions for piezoelectric circular shells has been derived. The model given here can be used in the research of sensors and actuators, based on piezoelectric effects.

With given formulations, electromechanical characteristics of a fully clamped support of circular cylinder shell for axisymmetric mechanical loads and electric potentials is provided. The results obtained in this work show there are stress concentration at the clamped edges. The transverse normal near clamped edges, which are neglected in classical theory of the Kirchhoff-Love type, must be calculated when studying the strength of elements in various structural joint designs.

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