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WATER WAVE TRAPPING BY FLOATING ⊥-SHAPED POROUS BREAKWATER

BLOKIRANJE VODENIH TALASA PLUTAJUĆOM POROZNOM KONSTRUKCIJOM ⊥ OBLIKA

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Keywords
- wave trapping
- integral equation
- BEM (Boundary Element Method)
- reflection coefficient

Abstract

A BEM based numerical model is presented to investigate the water trapping by a floating ⊥-shaped porous breakwater using small amplitude wave theory. The flow through the permeable medium is analysed using the Sollitt and Cross model, /10/. The efficiency of the breakwater system is analysed by evaluating the reflection coefficient for a broad variety of structural and wave parameters. The study illustrates that the wave reflection is more in the long-wave regime and less in the short-wave regime for larger values of the structural porosity. Also is noticed that the minimum wave reflection is obtained in the presence of porous breakwater having 35% porosity with relative width = 2.0.

Ključne reči
- razbijanje talasa
- integralna jednačina
- metoda graničnog elementa (BEM)
- keoficijent refleksije

Izvod

Numerička metoda na bazi BEM je predstavljena za istraživanje razbijanja talasa plutajućom poroznom konstrukcijom obilka ⊥, primenom teorije talasa male amplitude. Protok kroz propusnu sredinu je analiziran modelom Solita i Krosa, /10/. Efikasnost sistema za razbijanje talasa je analiziran određivanjem koeficijenta refleksije za širok raspon konstrukcionih i talasnih parametara. U radu je ilustrovano kako refleksija talasa pripada pre dugotalasnom režimu nego kratkotalasnom režimu za velike vrednosti poroznosti konstrukcije. Takođe se uočava da se minimalna refleksija talasa postiže u prisustvu porozne konstrukcije sa 35% poroznosti i relativne širine = 2.0.

INTRODUCTION

Over the last few years, there is a great interest in using the floating and flexible porous breakwaters in coastal regions to reduce the amplitude of the incoming waves, also to protect the shoreline or offshore structures from wave-attacks. In general, the amplitude of the reflected waves from the breakwater system should be small. Otherwise, there is a possibility of occurrences of wave resonance in the harbours. In this regard, permeable floating breakwaters are introduced as they can effectively reduce the reflected wave amplitudes. Therefore, the study of water wave interactions with floating or flexible porous breakwaters is of utmost important in ocean engineering problems. The eigenfunction expansion method /1/ is used in analysis of the action of waves on a freely floating body with a permeable wall. The study concluded that the porosity of the sidewall, dimensions, and position of the porous regions have great effects on the planer components of the wave loads. However, the impact on the vertical components of the forces is insignificant. Yip et al. /2/ presented the wave trapping by permeable and elastic structures. The study concludes that the full-wave reflection always takes place when the gap between the end-wall and obstacle is an integer multiple of half of the incident wavelength irrespective of the flexibility and porosity. An analytical solution to wave diffraction by a submerged floating body using the eigenfunction expansion method is given by Zheng et al. /3/. Further, using the aforementioned solution technique Zhou et al. /5/ studied the wave diffraction by a floating rectangle shaped barrier with a symmetric gap in its bottom. It is found that the gap in the bottom has little impact on the horizontal component of force acting on the barrier. However, it has a significant impact on the perpendicular forces. Zheng et al. /4/ used the BEM to study the geometrical parameters effect on wave loads and hydrodynamic characteristics when waves interact with the submerged circular floating breakwaters. Koley et al. /6/ used surface-piercing and bottom-standing permeable breakwaters for oblique wave trapping. It is identified that surface-piercing porous structures effectively dissipate incident waves rather than bottom standing porous structures. Vijay and Sahoo /7/ used the multi-domain BEM to examine the wave scattering by a double floating pervious box. The study reveals that with a proper combination of porosity, these boxes can effectively dissipate incoming water waves for intermediate water depths. For porous breakwaters, one of the important tasks is to find appropriate shape of the breakwater to optimise wave diffraction phenomena. In this regard, T and ⊥ -shaped breakwater can serve the purpose. Neelamani and Rajendran /8-9/ presented an experimental study to examine the hydrodynamic characteristics of T-shaped and ⊥-shaped breakwater.

In this study, a BEM based numerical method is adopted to examine the water wave trapping by ⊥-shaped porous breakwater. The flow in the porous zone is analysed using Sollitt and Cross model /10/. The advantage of considering the BEM solution technique is that in the BEM method, the zeros of the complex dispersion relation in a permeable zone are not needed. Finally, the breakwater system is analysed by evaluating the reflection coefficient for a variety of structural and wave parameters.

PROBLEM FORMULATION

Here, a comprehensive mathematical formulation is described for wave trapping by floating ⊥-shaped porous breakwater using linear wave theory in constant water depth h. A 2D system with x-axis and z-axis are perpendicular to each other is adopted, as shown in Fig. 1. Here, it is to be noted that the potential flow theory is applicable. Thus, the velocity potentials exist and are given by $\Phi_j(x,z,t) = \mathrm{Re}\{\phi_j(x,z)\exp(-i\omega t)\}$, where ϕ_j satisfy the Laplace equation in every region R_j, $j = 1,2$,

$$\nabla^2\phi_j = 0\,. \qquad (1)$$

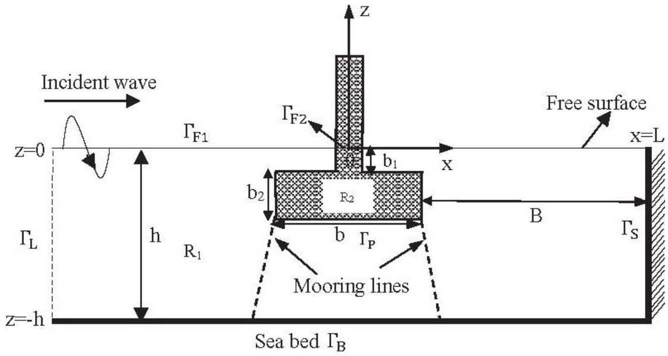

Figure 1. Schematic representation of the physical problem.

The imperable boundary condition (bc) on the bottom bed i.e., on $z = -h$, and the rigid wall bc at $x = L$ is given by

$$\frac{\partial\phi_1}{\partial n} = 0\,, \qquad (2)$$

where: n denotes outward drawn normal. The free-surface bc on $z = 0$ is written as (see /10/)

$$\frac{\partial\phi_j}{\partial n} - \left(\frac{\omega^2\Delta}{g}\right)\phi_j = 0 \quad \text{on} \quad \Gamma_{Fj},\ j = 1,2 \qquad (3)$$

with $\Delta = (m + if)$. In which m is inertia coefficient and f is friction coefficient. It is to be noted that $m = 1$ and $f = 0$ for fluid region R_1.

The fluid-porous region interface bcs are given by, /11/,

$$\begin{cases} \phi_1 = \Delta\phi_2\,, \\ \dfrac{\partial\phi_1}{\partial n} = -\varepsilon\dfrac{\partial\phi_2}{\partial n}\,, \end{cases} \qquad (4)$$

where: ε is structural porosity. The far-field bcs are expressed as, /6/,

$$\phi_1(x,z) = \left(\frac{-igH}{2\omega}\right)\left(e^{ik_0x} + R_0e^{-ik_0x}\right)\left(\frac{\cosh k_0(z+h)}{\cosh k_0h}\right), \qquad (5)$$

as $x \to -\infty$, where k_0 is progressive wave mode satisfying $\omega^2 - gk\tanh kh = 0$; with H being incident wave height; R_0 is the unknown associated with the reflected wave. Further, it is to be that the floating body is stable at SWL, the effects of mooring lines on the breakwater system are neglected. In order to get a closed region, an auxiliary boundary is placed at $x = -L$, and rewriting Eq.(5) as follows

$$\frac{\partial(\phi_1-\phi_0)}{\partial n} - ik_0(\phi_1-\phi_0) = 0 \quad \text{on} \quad \Gamma_L. \qquad (6)$$

SOLUTION USING BEM

The efficiency of the breakwater system is analysed by solving the aforementioned BVP using BEM. Applying the Green's identity to ϕ and G (Green's function), we get

$$\phi(\bar{z}) = \int_\Gamma\left(\phi(z)\frac{\partial G(z,\bar{z})}{\partial n} - G(z,\bar{z})\frac{\partial\phi(z,\bar{z})}{\partial n}\right)d\Gamma\,,$$
$$\bar{z} = (\xi,\eta)\in\Gamma\,. \qquad (7)$$

Here, $z = (x,z)$, $\bar{z}$ are the point and field points, respectively, and

$$G = \frac{1}{2\pi}\ln r\,,\quad \frac{\partial G}{\partial n} = \frac{1}{2\pi r}\frac{\partial r}{\partial n}\,,\quad r = dist(z,\bar{z})\,. \qquad (8)$$

Substituting Eqs.(2)-(6) into Eq.(7) over each boundary of R_j, $j = 1,2$, we get

$$-\frac{1}{2}\phi_1 + \int_{\Gamma_L}\left(\frac{\partial G}{\partial n} - ik_0G\right)\phi_1 d\Gamma + \int_{\Gamma_B}\phi_1\frac{\partial G}{\partial n}d\Gamma + \int_{\Gamma_S}\phi_1\frac{\partial G}{\partial n}d\Gamma +$$
$$+\int_{\Gamma_{F1}}\left(\frac{\partial G}{\partial n} - KG\right)\phi_1 d\Gamma + \int_{\Gamma_P}\left(\phi_1\frac{\partial G}{\partial n} - G\frac{\partial\phi_1}{\partial n}\right)d\Gamma =$$
$$= \int_{\Gamma_L}\left(\frac{\partial\phi_0}{\partial n} - ik_0\phi_0\right)Gd\Gamma\,,\quad (K = \omega^2/g), \qquad (9)$$

$$-\frac{1}{2}\phi_2 + \int_{\Gamma_P}\left(\frac{1}{\Delta}\phi_1\frac{\partial G}{\partial n} + \frac{1}{\varepsilon}G\frac{\partial\phi_1}{\partial n}\right)d\Gamma +$$
$$+\int_{\Gamma_{F2}}\left(\frac{\partial G}{\partial n} - K\Delta G\right)\phi_2 d\Gamma = 0\,. \qquad (10)$$

The discretized form of Eqs.(9) and (10) are as follows

$$\sum(H^{ij} - ik_0G^{ij})\phi_{1j}\Big|_{\Gamma_L} + \sum H^{ij}\phi_{1j}\Big|_{\Gamma_B} + \sum H^{ij}\phi_{1j}\Big|_{\Gamma_S} +$$
$$+\sum(H^{ij} - KG^{ij})\phi_{1j}\Big|_{\Gamma_{F1}} + \sum\left(\phi_{1j}H^{ij} - G^{ij}\frac{\partial\phi_{1j}}{\partial n}\right)\Bigg|_{\Gamma_P} =$$
$$= \sum\left(\frac{\partial\phi_{0j}}{\partial n} - ik_0\phi_{0j}\right)G^{ij}\Bigg|_{\Gamma_L}\,, \qquad (11)$$

$$\sum\left(\frac{1}{\Delta}H^{ij}\phi_{1j} + \frac{1}{\varepsilon}G^{ij}\frac{\partial\phi_{1j}}{\partial n}\right)\Bigg|_{\Gamma_P} + \sum\left(H^{ij} - K\frac{1}{\Delta}G^{ij}\right)\phi_{2j}\Bigg|_{\Gamma_{F2}} = 0 \qquad (12)$$

Here, the summations are carried out over N constant elements by assuming ϕ and $\partial\phi/\partial n$ are constants. In Eqs. (11) and (12), expressions H^{ij} and G^{ij} are defined as, /12/,

$$H^{ij} = -\frac{1}{2}\delta_{ij} + \int_{\Gamma_j}\frac{\partial G}{\partial n}d\Gamma\,,\quad G^{ij} = \int_{\Gamma_j}Gd\Gamma\,, \qquad (13)$$

where: δ_{ij} takes 1 when variables are the same, otherwise 0.

For evaluating integrals in H^{ij} and G^{ij}, the Gauss quadrature method is adopted. However, the integrals need to be evaluated analytically as $r \to 0$. This is due as the singularity arises in the fundamental solution when the distance $r \to 0$ (see /12/). Finally, Eqs.(11) and (12) are transformed into matrix equations and the same are solved for unknown velocity potentials. The reflection coefficient is given by

$$K_r = |R_0|\,. \qquad (14)$$

RESULTS AND DISCUSSION

This section discusses the effect of structural porosity ε, friction coefficient f, width of the breakwater b, thickness of the breakwater b_2, and submergence depth of the upper rectangular cross-section b_1 (from mean free surface $z = 0$) on the wave reflection by floating porous breakwater. For computational purpose, it is assumed that h = 10 cm, $b_1 = 0.5h$, $b_2 = 0.5h$, $b = 2.0h$, $f = 1$, $\varepsilon = 0.35$, unless otherwise mentioned.

In Figs. 2a and 2b K_r versus k_0h is plotted for various ε and f, respectively. Figure 2a shows that structural permeability ε has insignificant effect in the long-wave regime $k_0h < 0.5$, whereas, K_r increases with increase in ε for $0.5 < k_0h < 1.0$. Further, it is also noticed that the reflection coefficient K_r attains minimum for $\varepsilon = 0.35$. Moreover, the reflection coefficient K_r reduces for higher ε in intermediate and short-wave regimes. This is because of the energy loss of incident waves by the permeable breakwater. Figure 2b depicts that K_r decreases in the long-wave regime irrespective of change in f. Further, K_r is oscillatory in nature in the intermediate and short-wave regime. This oscillatory pattern reduces as f increases. This oscillatory pattern is owing to the interaction of reflected waves with the porous barrier as well as leeside wall.

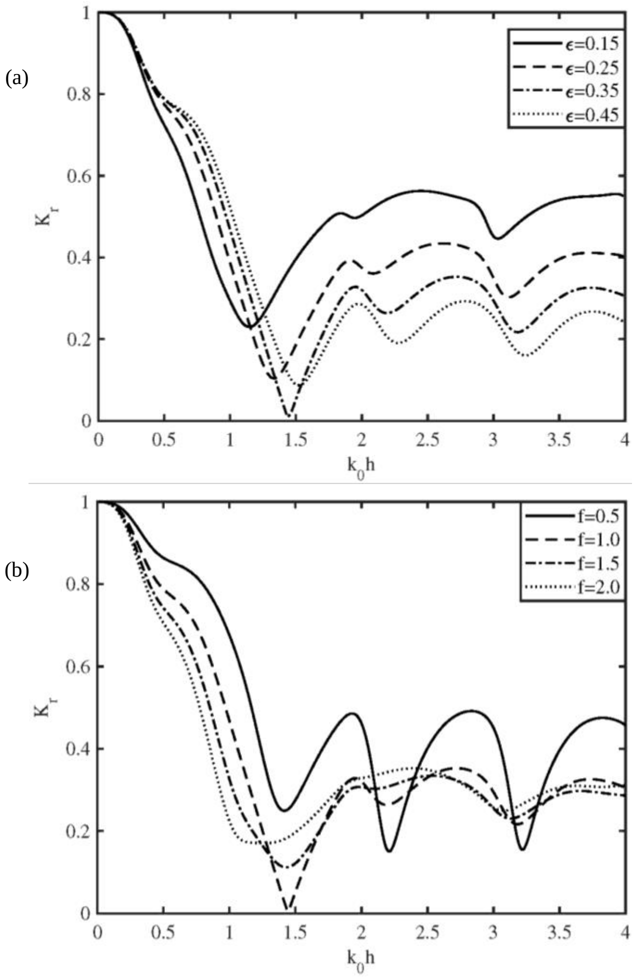

Figure 2. K_r vs. k_0h for distinct (a) porosity ε, (b) friction coefficient f.

Figures 3a and 3b show the effect of dimensionless breakwater width b/h and thickness b_2/h on K_r. Figure 3a depicts that K_r decreases with increase of b/h. The reason for the same is explained in Fig. 2a. Further, the minimal reflection i.e., zero reflection is observed for $b/h = 2.0$ at $k_0h \approx 1.5$. It is noticed that for intermediate wavelengths $1.5 < k_0h < 2.5$, the local minimum in K_r occurs owing to the interaction of incoming and reflected waves. In Fig. 3b it is clearly visible that the thickness of the breakwater has insignificant effects on K_r in the short-wave regime.

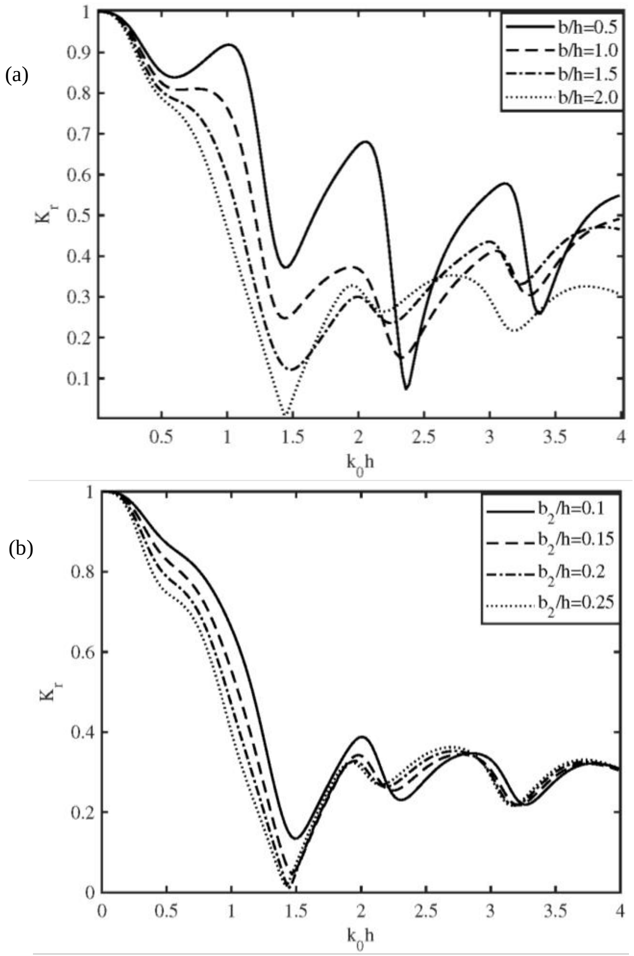

Figure 3. K_r vs. k_0h for distinct (a) b/h, (b) b_2/h.

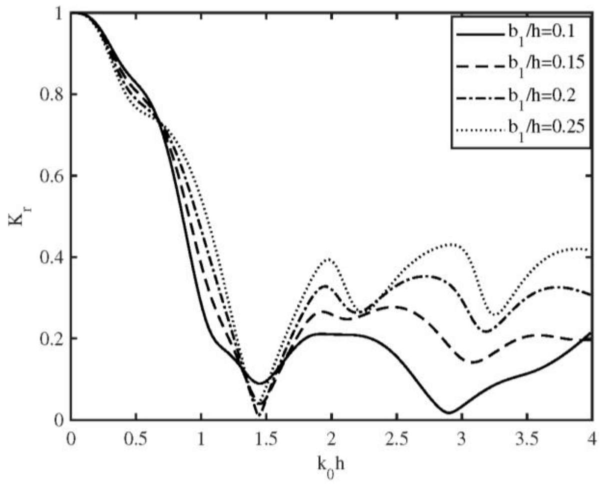

Figure 4. K_r vs. K_0h for distinct b_1/h.

The effect of b_1/h on K_r is plotted in Fig. 4. It is viewed that K_r drops steeply for $k_0h < 1.5$ and attains minimum around $k_0h \approx 1.5$. Thereafter, it increases in an oscillatory manner for larger k_0h. Further, K_r is more for the higher values of b_1/h except around $k_0/h = 1.5$, where a reverse trend is observed.

CONCLUSIONS

Water wave trapping by floating ⊥-shaped porous breakwater is studied with BEM using linear wave theory. The flow in the porous region is analysed by applying the Sollitt and Cross model. The effects of porosity, friction coefficient, width and thickness of the breakwater, and submergence depth of the breakwater, on wave reflection are analysed. The salient conclusions of the study:

- The reflection coefficient increases with an increase in structural porosity in the long-wave regions and reduces with an increase in porosity in short-wave regions. Further, also viewed is that the minimal K_r is obtained with 35% of the structural porosity.
- The reflection coefficient follows oscillatory in nature for lower values of the friction coefficient.
- The reflection from the barrier reduces with an increase in structural width except for intermediate wavelengths.
- The thickness of the breakwater has insignificant effect on wave reflection in short-wave regions. Moreover, the reflection coefficient is more for the higher submergence depth of the upper rectangular breakwater.

The present numerical method is also extendable to the porous structures of complex geometries.

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