Kabita Luitel[1], Dil Bahadur Gurung[2], Harihar Khanal[3], Kedar Nath Uprety[4]*

STABILITY AND CONVERGENCE OF IMPLICIT FINITE DIFFERENCE SCHEME FOR BIOHEAT TRANSFER EQUATION WITH CLOTHING EFFECT IN HUMAN THERMAL COMFORT

STABILNOST I KONVERGENCIJA IMPLICITNE SHEME KONAČNIH RAZLIKA ZA JEDNAČINU PRENOSA BIO-TOPLOTE SA UTICAJEM ODEĆE NA TERMIČKU UGODNOST

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 6.4.2021

Adresa autora / Author's address:
[1)] Bhaktapur Multiple Campus, Tribhuvan University, Nepal
[2)] School of Science, Kathmandu University, Nepal
[3)] Embry-Riddle Aeronautical University, Beach, Florida, USA
[4)] Central Department of Mathematics, Tribhuvan University, Nepal email: kabi123luitel@gmail.com

Keywords
- bioheat transfer
- clothing effect
- finite difference (FD) scheme
- effective clothing area factor

Ključne reči
- prenos bio-toplote
- uticaj odeće
- shema konačnih razlika (FD)
- faktor efikasnosti površine odeće

Abstract

This paper studies the stability and convergence of implicit Finite Difference (FD) scheme of the bioheat transfer model of Pennes' type with the clothing effect at the boundary node. Robin's boundary condition, in this study, incorporates the clothing insulation, effective clothing area factor in the combined heat transfer coefficient and observes their effects for the thermal comfort in the human body. Lemma and theorems for consistency, stability and convergence of FD scheme are established and the numerical results are graphically presented for validation of the model.

Izvod

U radu je proučena stabilnost i konvergencija implicitne sheme konačnih razlika (FD) prenosa bio-toplote prema modelu tipa Penesa, sa uticajem odeće u graničnim čvorovima. Ovde Robinov granični uslov sadrži izolaciju odeće i faktor efikasnosti površine odeće u okviru kombinovanog koeficijenta prenosa toplote, i posmatraju se njihovi uticaji na termičku ugodnost tela čoveka. Definisane su Leme i teoreme za konzistentnost, stabilnost i konvergenciju sheme FD, a numerički rezultati su predstavljeni grafički radi provere modela.

INTRODUCTION

The knowledge of temperature as well as heat transfer is essential for the treatment of cancer hyperthermia, cryosurgery, brain hypothermia and burn injury. A number of therapeutic and clinical applications of bioheat transfer models in the current century, including /1-3/, can be found in the field of biomathematics. The first and most popular bioheat transfer model based on the classical Fourier law was developed by H. Pennes /2/ in 1948 by incorporating the volumetry blood flow rate in tissue. Though many researchers /3-6/ have developed a bioheat transfer model, Pennes' model is still famous and widely used to study the human thermoregulatory system with the equilibrium human body temperature. Dai and Zhang /7/ developed and proved a three-level unconditionally stable and convergence of FD scheme for solving 1D Pennes' bioheat equation in a triple-layered skin structure by using discrete energy method. Zhao et al. /3/ developed two-level FD scheme for 1D Pennes' bioheat equation and proved unconditionally stable and convergence. Tuzikiewicz and Duda /8/ discussed on the stability of explicit scheme of bioheat transfer equation by Von Neumann approach. The physical and physiological factors along with clothing resistance are equally important phenomena for the thermoregulatory systems of the human body. Suitable management of clothes, on the other hand, provides better insulation and keeps a person in comfort position. Previously established models by Dai and Zhang /7/ and Zhao et al. /3/ have ignored the Robin's type boundary condition and also did not mention the effect of clothing resistance and insulation. These quantities cannot be neglected in the real-life situation. So, this paper focuses to study the mathematical model by incorporating the heat flux with clothing resistance, clothing area factor, and air insulation on the boundary. We construct the implicit finite difference scheme, establish and prove the theorems to show the FD scheme of our model is unconditionally stable and convergence having the same order of accuracy.

Role of clothes

The amount or rate of heat resists being transferred through clothes is defined as the clothing heat resistance. For thermal conductivity of clothes (W/m°C), / 9/,

$$k_{cl} = q\frac{L_{cl}}{\Delta T},$$

where: L_{cl} is the thickness of cloth (m); ΔT the temperature difference (°C); q is heat flow rate (W/m^2). The thermal resistance R_{cl} (m^2°C/W) is given by, /9/,

$$R_{cl} = \frac{\Delta T}{q} = \frac{L_{cl}}{k_{cl}}.$$

MATHEMATICAL MODEL

1D Pennes bioheat equation in cylindrical form is given by /2/ as

$$\rho c\frac{\partial T}{\partial t}=\frac{1}{r}\left(kr\frac{\partial T}{\partial r}\right)+w_b c_b(T_a-T)+q_m\ , \tag{1}$$

where: ρ is density (kg/m^3); c specific heat (J/kg°C); k thermal conductivity of tissue (W/m°C); w_b rate of blood perfusion (kg/m^{3}s); c_b specific heat of blood (J/kg°C); T_a arterial temperature (°C); q_m metabolic heat production (W/m^3).

The distance from the body core towards skin surface is denoted by r (m). The bioheat Eq.(1) with the clothing parameter P (W/m^3), /9/, is written as

$$\rho c\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+w_b c_b(T_a-T)+q_m+P\ , \tag{2}$$

where: $P = P_{cl}(T_{sk} - T_{cl})$; and $P_{cl} = k_{cl}/A_{cl}$ (W/m$^{3\circ}$C); A_{cl} is the surface area of clothed human body (m^2).

Left boundary condition at r = 0 (body core)

The boundary condition for the interior part of the living tissue is considered uniform as

$$\text{at}\quad r=0,\quad \frac{\partial T_t}{\partial t}=0\ . \tag{3}$$

Right boundary condition at the clothes surface

The right side is fitted with clothes and directly exposed to the atmospheric environment, so boundary condition of the Robin type is

$$\text{at}\quad r=R,\quad -k_{cl}\frac{\partial T_{cl}}{\partial r_{cl}}=h_A(T_{cl}-T_\infty)\ , \tag{4}$$

where: r_{cl} is thickness of clothes; h_c the convective heat transfer coefficient (W/m$^{2\circ}$C); h_r radiative heat transfer coefficient (W/m$^{2\circ}$C); T_{cl} clothes temperature (°C); T_∞ atmospheric temperature (°C). The clothing efficiency factor F_{cl} (dimensionless) is given by, /9-12/,

$$F_{cl}=\frac{I_a}{I_T}=\frac{I_a}{I_{cl}+I_a/f_{cl}}\ ,$$

where: I_{cl} is the clothing insulation (m$^{2\circ}$C/W); I_a (m$^{2\circ}$C/W) air insulation (m). The dimensionless clothing factor is the ratio of clothed surface and naked body surface area, $f_{cl} = A_{cl}/A_b$.

METHODOLOGY

Finite difference (FD) scheme is used as the numerical method. The main assumptions in this study are heat flows from body core toward the skin surface in radial direction, (R - 1)-th node is the interface between the skin surface and clothes. Clothing appears in the boundary, the R-th node is the clothes surface exposed to the environment. So, we use the boundary condition Eq.(4). The domain discretization can be seen in the circular limb of the body in Fig. 1.

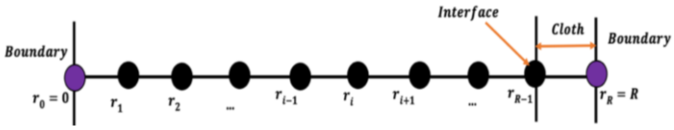


Figure 1. Discretization of mesh in radial direction.

Finite difference (FD) scheme

We consider the cylindrical coordinate system covered by the cylindrical differential mesh with step size Δr in radial direction. In this case we also take n, n + 1 as two successive time level with time step Δt. Taking the implicit FD scheme for the internal i-th node with i = 1, 2, 3,…, R-1 is given by

$$\rho c\frac{T_i^{n+1}-T_i^n}{\Delta t}=k_t\frac{T_{i-1}^{n+1}-2T_i^{n+1}+T_{i+1}^{n+1}}{\Delta r^2}+\frac{k_t}{2i\Delta r}\times$$
$$\times\left(T_{i+1}^{n+1}-T_{i-1}^{n+1}\right)+w_b\left(T_a-T_i^{n+1}\right)+q_m\ . \tag{5}$$

The construction of FD scheme in Eq.(5) has truncation error of order $0(\Delta r^2 + \Delta t)$ for (t^n, r_i) on each interior gride point $n \geq 1$, $0 < i < R$-1.

When, $D = k_t/\rho c$, $\alpha = D\Delta t/\Delta r^2$, $M = w_b c_b/\rho c$, $S = q_m/\rho c$, and $F = \Delta t(MT_a + S)$. Rewriting the above Eq.(5), we have for $0 < i < R$-1,

$$D_i T_{i-1}^{n+1}+E_i T_i^{n+1}+B_i T_1^{n+1}-F=T_i^n\ , \tag{6}$$

where: $D_i=\left(-\alpha+\dfrac{\alpha}{2i}\right)$; $E_i=(1+2\alpha+M\Delta t)$; $B_i=\left(-\alpha-\dfrac{\alpha}{2i}\right)$.

FD scheme for left boundary condition at i = 0

Due to the constant temperature at the core, when r = 0, then $\dfrac{1}{r}\dfrac{\partial T_t}{\partial r}\to 0$. In this situation we use L'Hospital rule to remove this indeterminate form at the left boundary node and get the equation at i = 0 as

$$\rho c\frac{\partial T_t}{\partial t}=2D\left(\frac{\partial^2 T_t}{\partial r^2}\right)+M(T_a-T_t)+S\ . \tag{7}$$

FD scheme at i = 0 with the help of boundary condition given in Eq.(3) is

$$E_0 T_0^{n+1}-4\alpha T_1^{n+1}-F=T_0^n\ . \tag{8}$$

FD scheme for skin surface at the interface at i = R - 1

$$D_{R-1}T_{R-2}^{n+1}+E_{R-1}T_{R-1}^{n+1}+B_{R-1}T_R^{n+1}-F=T_{R-1}^n\ . \tag{9}$$

Equation (9) is FD scheme at R - 1, the system is taken as tight fitting or negligible air gap between body and clothes at $i = R$-1, the contact point between skin and clothes.

The interface thermal conductivity in /13/ is

$$k_t=k_{cl}=\frac{k_t k_{cl}(\Delta r+\Delta r_{cl})}{k_{cl}\Delta r+k_t\Delta r_{cl}}\ . \tag{10}$$

The continuity and interface condition is given by /13/

at $i = R$-1, $k_t\dfrac{\partial T_{sk}}{\partial r}=k_{cl}\dfrac{\partial T_{cl}}{\partial r_{cl}}$, $k_t\dfrac{T_R^{n+1}-T_{R-1}^{n+1}}{\Delta r}=k_{cl}\dfrac{T_R^{n+1}-T_{R-2}^{n+1}}{\Delta r+\Delta r_{cl}}$.

From Eq.(10) we get, $k_t k_{cl}\dfrac{T_R^{n+1}-T_{R-2}^{n+1}}{k_T}=k_{cl}\dfrac{T_R^{n+1}-T_{R-1}^{n+1}}{\Delta r_{cl}}$,

where: Δr_{cl} is thickness of clothes; Δr is mesh size for body part; and $k_T = k_{cl}\Delta r + k_t\Delta r_{cl}$.

Now, suppose $\beta = k_t\Delta r_{cl}/k_T$, then

$$T_R^{n+1}=T_{R-1}^{n+1}+\beta\left(T_R^{n+1}-T_{R-2}^{n+1}\right)\ . \tag{11}$$

With the help of Eq.(11), Eq.(9) can be written as

$$D_{R-1}T_{R-2}^{n+1}+E_{R-1}T_{R-1}^{n+1}+B_{R-1}\Big[T_{R-1}^{n+1}+\beta\Big(T_{R}^{n+1}-T_{R-2}^{n+1}\Big)\Big]-F=T_{R-1}^{n},$$
$$(D_{R-1}-\beta B_{R-1})T_{R-2}^{n+1}-(E_{R-1}+E_{R-1})T_{R-1}^{n+1}+BT_{R-1}^{n+1}-F=T_{R-1}^{n}. \quad (12)$$

Now, at $r = R$, $M = 0$, $S = 0$, the Eq.(6) for clothing part is given in FD scheme as

$$\left(-\alpha_1+\frac{\alpha_1}{2R}-P_1\right)D_{R-1}T_{R-1}^{n+1}+(1+2\alpha_1+P_1)T_R^{n+1}+$$
$$+\left(-\alpha_1+\frac{\alpha_1}{2R}\right)T_{R+1}^{n+1}=T_R^n, \quad (13)$$

where: $\alpha_1=\dfrac{k_{cl}\Delta t}{\rho_{cl}c_{cl}(\Delta r_{cl})^2}$; $P_1=\dfrac{P\Delta t}{\rho_{cl}c_{cl}}$.

Boundary condition at the clothing surface at i = R

The right boundary condition at the clothing surface in Eq.(4) is

at $r=R \quad -k_{cl}\dfrac{\partial T_{cl}}{\partial r_{cl}}=h_A(T_{cl}-T_\infty)$.

FD scheme for boundary condition is

$$T_{R+1}^{n+1}=T_{R-1}^{n+1}-2h_A(T_{cl}-T_\infty). \quad (14)$$

From Eqs.(13) and (14), the FD scheme for the right boundary node is

$$(-2\alpha_1-P_1)T_{R-1}^{n+1}+\left[(1+2\alpha_1+P_1)+\alpha_1+\frac{\alpha_1}{2R}\right]T_R^{n+1}+$$
$$+2h_AR_{cl}T_\infty\left(-\alpha_1-\frac{\alpha_1}{2R}\right)=T_R^n,$$

$$D_{Rcl}T_{R-1}^{n+1}+[E_{Rcl}-2h_AR_{cl}T_\infty B_{Rcl}]T_R^{n+1}+$$
$$+2h_AR_{cl}\left(-\alpha_1-\frac{\alpha_1}{2R}\right)=T_R^n. \quad (15)$$

$$D_{Rcl}=(-2\alpha_1-P_1),\quad E_{Rcl}=(1+2\alpha_1+P_1),$$
$$B_{Rcl}=\left(-\alpha_1-\frac{\alpha_1}{2R}\right),\quad F_R=2h_AR_{cl}\left(-\alpha_1-\frac{\alpha_1}{2R}\right).$$

The systems Eqs.(8), (6) and (15) together can be written as the matrix equation of the form

$$AT^{n+1}=T^n+B, \quad (16)$$

where: A is $(R+1)\times(R+1)$ tridiagonal square matrix; B is the column vector of size $(R+1)\times1$ and

$$T^n=\begin{cases}T_t^n & \text{for } 1\le i\le R-1\\ T_{cl}^n & \text{for } i=R\end{cases} \quad (17)$$

ANALYSIS OF THE MODEL

To prove solvability and stability of Eq.(2), we introduce a column vector of size $(R+1)\times1$ representing the numerical solution at time step t^n as $T^n=[T_0^n, T_1^n, \dots, T_i^n, \dots, T_R^n]$.

Theorem 1 (solvability of the model): for each time step n, Eq.(16) is unconditionally solvable.

Proof. To show solvability of Eq.(16), it is sufficient to prove that the matrix A is invertible, i.e. the matrix A is diagonally dominant, where the absolute value of diagonal element of each row of the coefficient matrix A in Eq.(16) is greater than the sum of the absolute value of remaining elements in corresponding row of matrix A. Then we have

R_1 (1st row): $|a_{jj}|\ge\Sigma|a_{ij}|$ since $|1+4\alpha+M\Delta t|>|-4\alpha|$

R_i (i-th row): $|a_{jj}|\ge\sum\limits_{i=2,i\ne j}^{R-2}|a_{ij}|$.

Since $E_i=|1+2\alpha+M\Delta t|$, $|D_i|=\left|-\alpha+\dfrac{\alpha}{2i}\right|$, $|B_i|=\left|-\alpha-\dfrac{\alpha}{2i}\right|$,

So, $|1+2\alpha+M\Delta t|>\left|-\alpha+\dfrac{\alpha}{2i}\right|+\left|-\alpha-\dfrac{\alpha}{2i}\right|=|-2\alpha|$,

R_{R-1} (R-1)-th row: $|a_{jj}|\ge\Sigma|a_{ij}|$

Since $|E_{R-1}+B_{R-1}|=\left|(1+2\alpha+P_1)+\left(-\alpha-\dfrac{\alpha}{2(R-1)}\right)\right|$

$$|E_{R-1}+B_{R-1}|=\left|(1+P_1)+\left(\alpha-\frac{\alpha}{2(R-1)}\right)\right|$$

on the other hand, $|D_{R-1}-\beta B_{R-1}|+|\beta B_{R-1}|\ge|D_{R-1}|=$

$=\left|-\left(\alpha-\dfrac{\alpha}{2(R-1)}\right)\right|=\left(\alpha-\dfrac{\alpha}{2(R-1)}\right)$.

So $|E_{R-1}+B_{R-1}|>|D_{R-1}-\beta B_{R-1}|+|\beta B_{R-1}|$.

R_R R-th row: $|a_{jj}|\ge\Sigma|a_{ij}|$

Since $|E_{Rcl}-B_{Rcl}|>|D_{Rcl}|$

$\Rightarrow\left|(1+2\alpha+P_1)+2h_AR_{cl}\left(\alpha_1+\dfrac{\alpha_1}{2R)}\right)\right|>|-2\alpha_1-P_1|=|2\alpha_1+P_1|$.

Clearly, each row is also diagonally dominant. So, matrix A is invertible. Hence Eq.(16) is unconditionally solvable.

Lemma 1. If λ_i for $i = 0,1,2, \dots, R$ represents the eigenvalues of the square matrix A and '$\|\ \|_2$' represents the second matrix norm (i.e. $\|A\|_2=\max\limits_i|\lambda_i|$, then we have the following results

(i) $$|\lambda_i|=\begin{cases}1+M\Delta t & \text{for } 1\le i\le R-1\\ 1+P_1 & \text{for } i=R\end{cases}$$

(ii) $$\|A^{-1}\|_2\ge\begin{cases}\dfrac{1}{1+M\Delta t}\le1 & \text{for } 1\le i\le R-1\\ \dfrac{1}{1+P_1}\le1 & \text{for } i=R\end{cases}$$

Theorem 2 (consistency): a finite difference (FD) scheme of Eq.(2) with truncation error $\tau(\Delta r,\Delta t)$ is consistent if

$$\tau(\Delta r,\Delta t)\to0 \quad \text{as} \quad \Delta r, \Delta t\to0.$$

It can be easily said that we have approximated our model Eq.(2) by FD scheme in Eqs.(8), (6), (12) and (13), which has truncation error $\tau(\Delta r,\Delta t) = O((\Delta r)^2+\Delta t)$.

So, $\lim\limits_{\Delta r,\Delta t\to0}\tau(\Delta r,\Delta t)=0$.

Hence, the FD scheme of our model Eq.(2) is consistent.

Theorem 3 (stability): The FD scheme of Eq.(2) is unconditionally stable with respect to initial data if

$$\left\|E^{n+1}\right\|_2 \le \left\|E^0\right\|_2,$$

where: $E^{n+1} = T^{n+1} - T^{*(n+1)}$ is an error equation; $T^{*(n+1)}$ is the small perturb in T^{n+1}.

<u>*Proof.*</u> Operating Eq.(17) by A^{-1} we obtain

$$T^{n+1} = A^{-1}T^n + A^{-1}B. \quad \text{(I)}$$

Take $T^{*(n+1)}$ be small perturb in T^{n+1}, then

$$AT^{*(n+1)} = T^{*(n)} + B.$$

$$T^{*(n+1)} = A^{-1}T^{*(n)} + A^{-1}B \quad \text{(II)}$$

Assume that

$E^{n+1} = T^{n+1} - T^{*(n+1)}$ be the error equation, then from Eq.(I) and Eq.(II) we get

$$E^{n+1} = (A^{-1}T^n + A^{-1}B) - (A^{-1}T^{*(n)} + A^{-1}B)$$

$$E^{n+1} = A^{-1}(T^n - T^{*(n)}) = A^{-1}E^n = A^{-1}(A^{-1}E^{n-1}) =$$

$$= (A^{-1})^2 E^{n-1} = (A^{-1})^2(A^{-1}E^{n-2}) = (A^{-1})^3 E^{n-2} =$$

$$= \ldots = (A^{-1})^n E^0 \quad \text{for} \quad n = 1, 2, \ldots, n.$$

So, we have, $E^{n+1} = (A^{-1})^n E^0$ for $n = 1, 2, \ldots, n$.

With respect to second norm, $\left\|E^{n+1}\right\|_2 \le \left\|A^{-1}\right\|_2 \left\|E^0\right\|_2$.

From Lemma 1, we apply $\left\|A^{-1}\right\|_2 \le 1$, which implies

$$\left\|E^{n+1}\right\|_2 \le \left\|E^0\right\|_2.$$

Hence, by Lax-Ritchtmyer's Theorem the FD scheme of the extended bioheat equation with clothing system is unconditionally stable with respect to initial data.

<u>Theorem 4 (convergence)</u>: FD scheme of Eq.(2) is unconditionally convergent.

<u>Proof.</u> Let

$T^n = [T(r_0,t^n), T(r_1,t^n), \ldots, T(r_i,t^n), \ldots, T(r_R,t^n)]'$ of size $(R + 1)\times 1$ be the exact solution at time step t^n such that

$$AT^{n+1} = T^n + B + \tau^{n+1}, \quad (18)$$

where: τ^{n+1} be the truncation error vector at level t^n. We have the numerical solution in T in Eq.(16) based on the discretization schemes.

If $E^{n+1} = T^{n+1} - T^{n+1}$ is an error equation, then Eq.(16) and Eq.(18) yield

$$AE^{n+1} = E + \tau^{n+1}$$

$$E^{n+1} = A^{-1}E^n + A^{-1}\tau^{n+1} =$$

$$= A^{-1}(A^{-1}E^{n-1} + A^{-1}\tau^n) + A^{-1}\tau^{n+1} =$$

$$= (A^{-1})^2(A^{-1}E^{n-2} + A^{-1}\tau^{n-1}) + (A^{-1})^2\tau^n + A^{-1}\tau^{n+1} =$$

$$= (A^{-1})^3 E^{n-2} + (A^{-1})^3\tau^{n-1} + (A^{-1})^3\tau^n + A^{-1}\tau^{n+1} =$$

$$= \ldots = (A^{-1})^2 E^0 + \sum_k^n (A^{-1})^k \tau^{n-k}$$

$$E^{n+1} = (A^{-1})^2 E^0 + \sum_k^n (A^{-1})^k \tau^{n-k}. \quad (19)$$

Initially we take $T^0 = 0$, since initially no error occurs at $t = 0$. So, $E^0 = 0$, and taking norm on both sides of Eq.(19) gives the following result,

$$\left\|E^{n+1}\right\|_2 \le \left\|A^{-1}\right\|_2^2 \left\|E^0\right\|_2 + \left(\sum_{k=0}^n \left\|A^{-1}\right\|_2^k\right) \max_{1\le k\le R} \left\|\tau^k\right\|_2$$

Since $E^0 = 0$ and from Lemma 1 we have $\left\|A^{-1}\right\|_2 \le 1$,

$$\left\|E^{n+1}\right\|_2 \le \left\|E^0\right\|_2 + \left(\sum_{k=0}^n \left\|A^{-1}\right\|_2^k\right) \max_{1\le k\le R} \left\|\tau^k\right\|_2.$$

By the construction of $\tau(\Delta r + \Delta t)$, we have

$$\left\|\tau^k\right\|_2 \to 0 \quad \text{as} \quad \Delta r \to 0, \Delta t \to 0.$$

This implies

$$\lim_{\Delta r, \Delta t \to 0} \left\|E^{n+1}\right\|_2 = 0 \quad \text{for} \quad 1 \le k \le R.$$

Hence the system is unconditionally convergent.

NUMERICAL VERIFICATIONS

One dimensional Pennes bioheat equation given in Eq.(2) is numerically discretized in Eq.(6) with Backward Scheme in Time and Central Difference Scheme in Space (BTCS). The tissue thickness of the human cylindrical limb in this model is taken $R = 0.03$ m /5, 14/ from body core to skin surface. So far the clothes effect is concerned, the thickness of clothes 0.005 m /4/ is added to tissue thickness and the results are calculated with the new thickness $R = 0.03 + 0.005$. Other various physical and physiological parameters from Table 1 (for human body) and Table 2 (for clothing parameters) are chosen for the numerical experiments.

Table 1. Thermophysical parameters /6, 14, 15/.

Parameters		value	unit
Thermal conductivity	k_t	0.48	W/m°C
Specific heat of blood	c_b	3850	J/kg°C
Blood density	ρ_b	1000	kg/m^3
Blood perfusion rate	w_b	3	kg/s·m^3
Metabolism	q_m	1085	W/m^3
Arterial temperature	T_a	37	°C
Convectional heat transfer coefficient	h_c	10.023	W/m^2·°C
Environmental temperature	T_s	28	°C

Table 2. Thermophysical parameters /9, 10, 12, 16/.

Parameters		value	unit
Thermal conductivities of clothes	k_{cl}	2.0462	W/m°C
Thickness of clothes	L_{cl}	0.010	m
Density of clothes	ρ_{cl}	1550	kg/m^3
Specific heat of clothes	c_{cl}	1340	J/kg°C
Clothing Insulation	I_{cl}	1.34	m^2·°C/W
Air insulation	I_a	0.025	m^2·°C/W
Area of nude body	A_b	1.6	m^2
Clothing area factor	f_{cl}	1.75	-

GRAPHICAL REPRESENTATION

The stability of the developed FD scheme Eq.(16) for cylindrical shape of the body with protective clothing system at the boundary node and its effects has also been verified by considering different parameters mentioned in Tables 1 and 2.

Different mesh sizes 25, 40, and 55, as well as mesh sizes 100, 500 and 1000 in both non-clothing and clothing cases are taken for the validity and applicability of the numerical (implicit FD) scheme Eq.(16) at time $\Delta t = 0.01$ s.

This numerical verification has been performed in Fig. 2 and Fig. 3 with three different meshes of 25, 40 and 55 with the time increment Δt = 0.01 s for non-clothing and clothing system of the body. In Fig. 2, the temperature of nude body at skin is 28 °C. In clothing state, on the other hand, the temperature becomes 32 °C in Fig. 3 which is because of the effect of clothing insulation and other parameters related to clothes. Similar results can be seen in Figs. 4 and 5 with mesh sizes 100, 500 and 1000 at time step Δt = 0.01 s in 180 seconds. The temperatures obtained in Figs. 2 and 4 almost coincide on the same curve even though the mesh sizes are different for the non-clothing case (nude body).

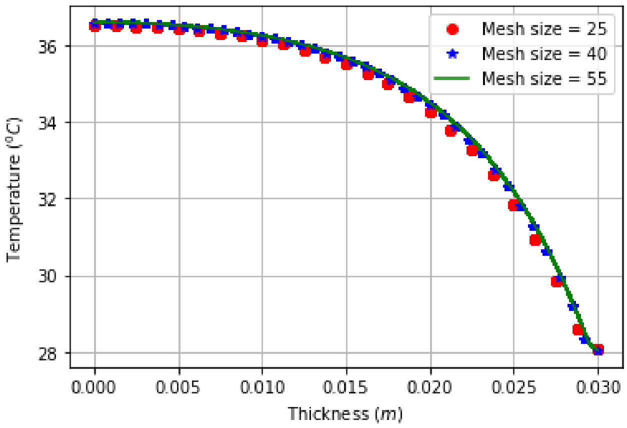


Figure 2. Radial temperature profile in 180 s.

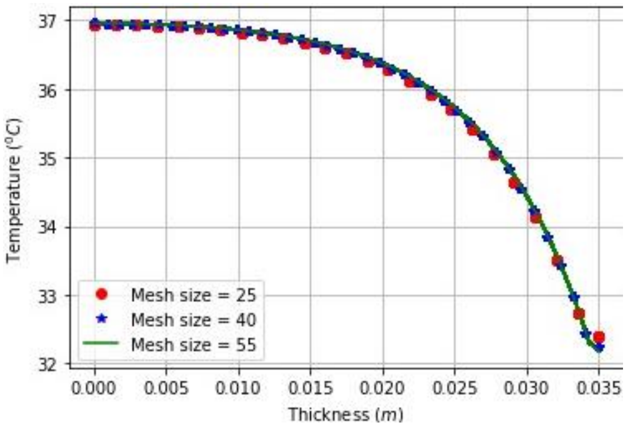


Figure 3. Radial temperature profile with clothing in 180 s.

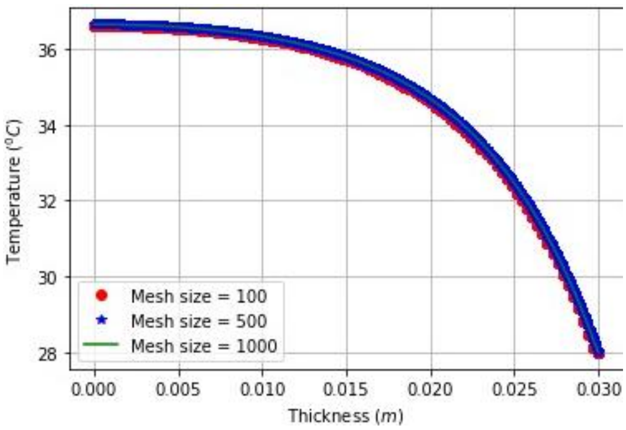


Figure 4. Radial temperature profile in 180 s.

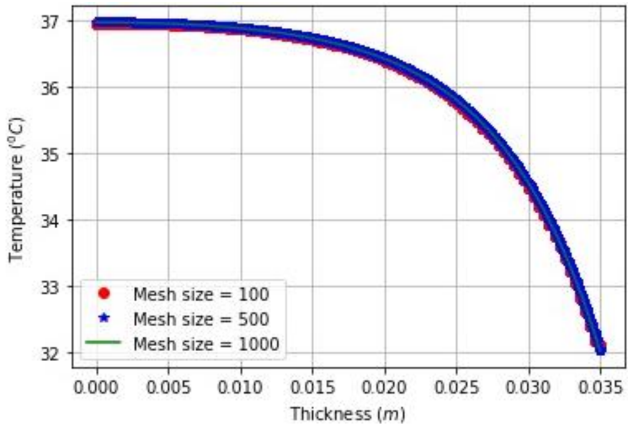


Figure 5. Radial temperature profile with clothing in 180 s.

The results in Figs. 3 and 5 in the clothing system are almost the same. In both cases, the figures are independent no matter what the mesh sizes are. These results verify the stability of implicit FD scheme for newly developed model.

CONCLUSION

In this study, an Implicit Finite Difference (FD) scheme, for one-dimensional transient bioheat transfer model with protective clothing at the boundary node of cylindrical body has been developed. The theorems are established and proven for analysing solvability, consistency, unconditionally stable and convergence of the model.

Also, the numerical verification of the model is represented graphically showing its stability and validity on one hand, and the presence of clothes of significant effect for thermal comfort of the person, on the other. The developed model fosters the advanced clothing system to achieve the comfort state of the human body.

ACKNOWLEDGEMENT

The first author expresses deep acknowledgement to the University Grant Commission (UGC), Nepal, for providing the Fellowship award no. PhD-75/76-S&T-10 for the PhD study.

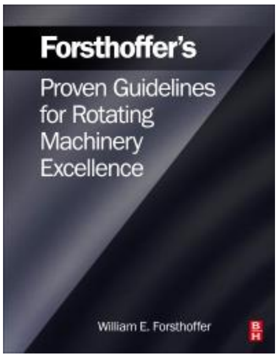
REFERENCES

1. Gurung, D.B., Luitel, K. (2012), *Development of bio-heat equation and its application*, In: Proc. Nepal Math. Soc. 2012, pp. 85-94.
2. Pennes, H.H. (1948), *Analysis of tissue and arterial blood temperatures in resting human forearm*, J Appl. Physiol. 1(2): 93-122. doi: 10.1152/jappl.1948.1.2.93
3. Zhao, J.J., Zhang, J., Kang, N., Yang, F. (2005), *A two level finite difference scheme for one dimensional Pennes' bioheat equation*, Appl. Math. Comp. 171(1): 320-331. doi: 10.1016/j.amc.2005.01.052
4. Gurung, D.B., Saxena, V.P. (2010), *Transient temperature distribution in human dermal part with protective layer at low atmospheric temperature*, Int. J Biomath. 3(4): 439-451. doi: 10.1142/S1793524510001070
5. Luitel, K. (2017), *Mathematical model for temperature distribution in cylindrical human body*, 2nd Int. Conf. on Man and Machine Interfacing (MAMI), Bhubaneswar, IEEE, 2017, pp. 1-5. doi: 10.1109/MAMI.2017.8307886
6. Luitel, K., Gurung, D.B., Uprety, K.N. (2018), *Effect of various parameters for temperature distribution in human body: An ana-*

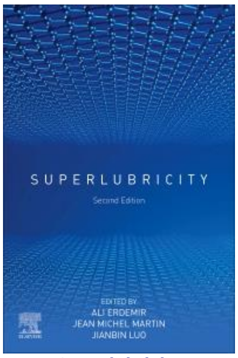
lytic approach, Spec. Issue Recent Adv. Eng. Syst. J (ASTESJ), 3(5): 421-426. doi: 10.25046/aj030548
7. Dai, W.Z., Zhang, J. (2002), *A three level finite difference scheme for solving the Pennes bioheat transfer in a triple-layered skin structure*, Tech. Rep. No.343-02, Department of Computer Science, University of Kentucky, 2002, pp.1-12.
8. Tuzikiewicz, W., Duda, M. (2015), *Bioheat transfer equation. The problem of FDM explicit scheme stability*, J Appl. Math. Comp. Mech. 14(4): 139-144. doi: 10.17512/jamcm.2015.4.14
9. Luitel, K., Gurung, D.B., Khanal, H., Uprety, K.N. (2021), *Bio-heat transfer equation with protective layer*, Math. Problems Eng., Vol.2021, Art. ID 6639550. doi: 10.1155/2021/6639550
10. Luitel, K., Gurung, D.B., Khanal, H., Uprety, K.N. (2020), *Role of clothing insulation for thermal comfort: A numerical study using bio-heat transfer model*, J Math. Statist. 16(1): 224-232. doi: 10.3844/jmssp.2020.224.232
11. Havenith, G., Holmer, I., Parsons, K. (2002), *Personal factors in thermal comfort assessment: clothing properties and metabolic heat production*, Energy Build. 34(6): 581-591. doi: 10.1016/S0378-7788(02)00008-7
12. Havenith, G., Kuklane, K., Fan, J., et al. (2015), *A database of static clothing thermal insulation and vapor permeability values of non-western ensembles for use in ASHRAE standard 55, ISO 7733 and ISO 9920*, ASHRAE Trans. 121(1): 197-215.
13. Patankar, S.V., Numerical Heat Transfer and Fluid Flow, CRC Press, Boca Raton, 1980. doi: 10.1201/9781482234213
14. Yue, K., Zhang, X., Yu, F. (2004), *An analytic solution of one-dimensional steady-state Pennes' bioheat transfer equation in cylindrical coordinates*, J Therm. Sci. 13(3): 255-258. doi: 10.1007/s11630-004-0039-y
15. Luitel, K., Gurung, D.B., Khanal, H., Uprety, K.N. (2019), *Numerical study of transient bio-heat transfer model with heat transfer coefficient and conduction effect in cylindrical living tissue*, The Nepali Math. Sci. Report, 36(1-2): 17-26. doi: 10.3126/nmsr.v36i1-2.29967
16. Voelker, C., Hoffmann, S., Kornadt, O., et al. (2009), *Heat and moisture transfer through clothing*, IPSA Building Simulation 2009, University of Strathclyde, Glasgow, Scotland, 2009. p.1-7. https://escholarship.org/uc/item/8qk6h840

New Elsevier Book Titles – Woodhead Publishing – Academic Press – Butterworth-Heinemann – …

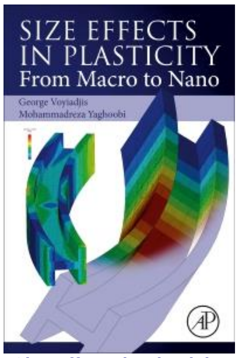
Fretting Wear and Fretting Fatigue, 1st Edition, Fundamental Principles and Applications
Tomasz Liskiewicz Daniele Dini (Eds.)
Elsevier, Jan 2022
ISBN: 9780128240960



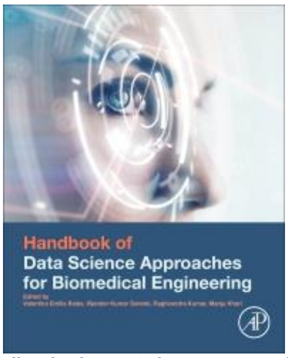
Forsthoffer's Proven Guidelines for Rotating Machinery Excellence 1st Edition
William Forsthoffer
Butterworth-Heinemann, Aug 2021
ISBN: 9780323854337



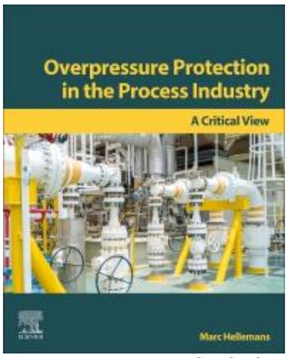
Superlubricity 2nd Edition.
Ali Erdemir Jean-Michel Martin Jianbin Luo Jianbin Luo (Eds.)
Elsevier, Sep 2020
ISBN: 9780444643131
EISBN: 9780444643148



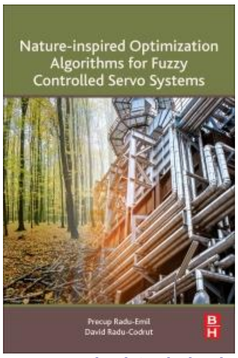
Size Effects in Plasticity 1st Edition, From Macro to Nano
George Voyiadjis Mohammadreza Yaghoobi
Academic Press, Aug 2019
ISBN: 9780128122365
EISBN: 9780128135136



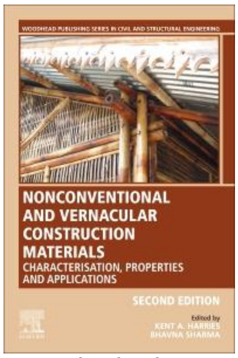
Handbook of Data Science Approaches for Biomedical Engineering 1st Edition
Valentina Balas Vijender Solanki Raghvendra Kumar Manju Khari (Eds.)
Academic Press, Nov 2019
ISBN: 9780128183182
EISBN: 9780128183199



Overpressure Protection in the Process Industry, 1st Edition, A Critical View
Marc Hellemans
Elsevier, Jan 2022
ISBN: 9780323909563



Nature-Inspired Optimization Algorithms for Fuzzy Controlled Servo Systems, 1st Edition
Radu-Emil Precup Radu-Codrut David
Butterworth-Heinemann, Apr 2019
ISBN: 9780128163580
EISBN: 9780128166062



Nonconventional and Vernacular Construction Materials, 2nd Edition, Characterisation, Properties and Applications
Kent Harries Bhavna Sharma (Eds.)
Woodhead Publishing, Nov 2019
ISBN: 9780081027042
EISBN: 9780081027059