Nitish Kumar Guru, Saral Kumar Jain*

FREE AXISYMMETRIC VIBRATION OF THICK CIRCULAR SANDWICH PLATES USING A HIGHER-ORDER THEORY

SLOBODNE OSNOSIMETRIČNE VIBRACIJE DEBELIH KRUŽNIH SENDVIČ PLOČA PRIMENOM TEORIJE VIŠEG REDA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 6.4.2021

Adresa autora / Author's address:
Department of Mathematics, Birla Institute of Technology, Mesra, Ranchi-835215, India
email: nitishkguru@gmail.com

Keywords
- circular sandwich plates
- axisymmetric vibrations
- Hamilton's principle
- Chebyshev collocation technique
- Reddy's Theory
- mode shapes

Abstract

Reddy's third-order theory is adopted to analyse the free axisymmetric vibrations of thick circular sandwich plates with a relatively stiff uniform core and membrane facings. Hamilton's principle is used to develop the equations of motion and natural boundary conditions. Numerical solution for frequency equations of simply-supported, clamped and free edged plates are obtained using Chebyshev collocation method. The least three roots obtained are reported as the natural frequency parameters for the first three vibration modes. Validation of the results presented in the paper is done by making comparison with their counterparts accessible in available published works. Results are exhibited numerically and graphically for studying the influence of thicknesses of the core and facings on the natural frequencies. The significance of the proposed model is established by showing that for the estimation of natural frequencies of thick cored circular sandwich plates, previously published models based on first-order shear deformation theory are not sufficient. Mode shapes of the initial three modes for each boundary condition have been plotted.

Ključne reči
- kružne sendvič ploče
- osnosimetrične vibracije
- Hamiltonov princip
- metoda kolokacije Čebiševa
- Redijeva teorija
- oblici modova

Izvod

Redijeva teorija trećeg reda je usvojena za analizu slobodnih osnosimetričnih vibracija debelih kružnih sendvič ploča s relativno krutim uniformnim jezgrom i membranskim stranama. Hamiltonov princip je upotrebljen za izvođenje jednačina kretanja i sopstvenih graničnih uslova. Dobijena su numerička rešenja za frekvencijske jednačine prosto oslonjenih i uklještenih ploča sa slobodnim krajem, dobijena metodom kolokacije Čebiševa. Bar tri izračunata korena su dobijeni kao parametri sopstvenih frekvencija za prva tri moda vibracija. Provera rezultata prikazanih u radu izvedena je poređenjem sa rezultatima dostupnim u objavljenim radovima. Rezultati su prikazani numerički i grafički radi proučavanja uticaja debljina jezgra i strana na sopstvene frekvencije. Značaj predloženog modela opravdava se time što za procenu sopstvenih frekvencija kružnih sendvič ploča s debelim jezgrom, prethodni objavljeni modeli na osnovu teorije deformacije smicanja prvog reda nisu zadovoljavajući. Dati su dijagrami oblika modova za početna tri moda, za svaki granični uslov.

INTRODUCTION

The analysis of free vibration of circular/annular sandwich structures are of great importance in civil, mechanical, marine, aerospace and other various dimensions of engineering as they have become widely adopted constructional elements due to their outstanding qualities such as good energy absorption, lightweight, high bending rigidity and cost effectiveness. A sandwich plate consists of two thin facings adhered to a core. Face sheets are usually stiff and strong enough to counter all the longitudinal loads while the core sustains the transverse shear. Efficient use of sandwich structures requires fairly exact knowledge of their vibrational characteristics which has made the dynamic analysis of these structures essential. The recent trends of work in the field of vibration analysis of sandwich structures are given in the surveys articulated by Sayyad and Ghugal /1/ and Birman and Kardomateas /2/.

For studying the vibrational characteristics of sandwich plates, numerous theories have appeared /3, 4/ for the past few decades. Various studies on dynamic analyses of these plates in the framework of classical plate theory (CPT) /5/ and first order-shear deformation theories (FSDT) /6-9/ are well documented by Habip /10, 11/ and Bert /12, 13/ in their review articles. Kao and Ross /14/ employed a variational theorem to study the fundamental natural frequencies of simply-supported and clamped circular sandwich plates. Mirza and Singh /15/ discussed the axisymmetric vibration of circular sandwich plate with honeycomb type core and isotropic facings. Prasad and Gupta /16/ solved the equations formed due to asymmetric vibration of sandwich plate of circular geometry by using Bessel functions. Jain et al. studied the radially symmetric vibration of sandwich circular plates with stiff core of linearly /17, 18/ and parabolically /18/ varying thickness using Chebyshev collocation method. Using the same method, results are computed for axisymmetric vibration of honeycomb cored sandwich plates of circular type with thickness varying linearly /18/. Guojun and Yingjie /19/ presented axisymmetric large amplitude vibration of circular sandwich plates by considering the flexural rigidity of the facings. Differential equations for axisymmetric large amplitude free vibration of circular sandwich plate under static loading has been analytically solved in /20/. Starovoitov et al. /21/ discussed the analytical solution for axisymmetric transverse vibration of circular sandwich plates under impulsive surface and linear loads by using Heaviside functions and Dirac delta function. Zhou and Stronge /22/ investigated axisymmetric and non-axisymmetric modes of vibration of circular sandwich panels using Mindlin-Reissner plate theory and solved by applying numerical finite element method. Using two stress functions and two displacement functions, /23/ worked on free vibration of transversely isotropic laminated annular, circular and sectoral plates. Shariyat and Alipour /24/ analysed the free bending vibration of functionally graded viscoelastic circular sandwich plates by employing a global-local zigzag theory. The radially symmetric vibration of sandwich plate of circular geometry with parabolically /25/ and linearly /26/ varying core thicknesses are studied by Lal and Rani assuming facings as membranes.

The above discussed works are either based on CPT or FSDT. We know that the effects of normal strains and transverse shear deformation is neglected by CPT due to which its adoptability is constrained only for the thin plates whereas FSDT assumes a constant transverse shear stress throughout the thickness of the plate which limits their acceptability only up to moderately thick plates. Moreover, in case of FSDT, the shear stresses do not vanish along the top and bottom surface of the plate which leads to inclusion of a shear correction factor.

To remove these constraints and make them valid for thick plates, various higher-order shear deformation theories (HSDT) /27-33/ are developed. Among all the HSDT, the theories of Reddy /29, 30/ are most common as they are simple. Moreover, Rohwer in his work /34/ has stated that the Reddy's theory /30/ for the laminated, composite and sandwich plates is best among all the theories in context of many parameters.

According to surveys /1, 2/ and the literature discussed above, it is found that the works available for analysis of vibrational behaviour of annular or circular plates in the framework of HSDTs are very few. The analysis of radially symmetric bending and buckling of functionally graded circular plates is studied by Ma and Wang /35/ on the basis of Reddy's HSDT /29/. In the paper they have also presented a relationship between the results obtained for isotropic circular plates and those obtained by CPT. The closed form solution for free vibration analysis of thick circular plate is presented by Hosseini-Hashemi et al. /36/ by employing Reddy's HSDT /29/. The unconstrained third-order theory /37/ is used in ref. /38/ to analyse the radially symmetric bending and buckling of functionally graded plates of circular dimension. A paper by Saharee and Saidi /39/ is a study on radially symmetric stretching and bending of a functionally graded circular plate loaded uniformly by a plate theory of fourth order. On the basis of the theory of Reddy /30/. Najafizadeh and Heydari /40/ solved the critical buckling problem of a circular functionally graded plate. Bisadi et al. /41/ have analytically solved the problem of free vibration of annular plate. In order to find the static, buckling and free vibration of composite sandwich plates, Nguyen-Xuan et al. /42/ used an isogeometric finite element method consistent with a fifth order theory. The large amplitude free vibration analysis of annular sandwich plate resting on elastic foundation is presented in /43/. The face sheets considered in this paper are made up of functionally graded composite reinforced with carbon nanotube. The discussions mentioned above show that the analysis of free vibration of circular sandwich plates based on HSDT has not been attempted so far in the literature.

The work discussed in this paper is an attempt to analyse the free radially symmetric vibrations of a sandwich plate of circular geometry and uniform thickness by adopting Reddy's HSDT /30/. The core is considered to be thick, and facings are treated as membranes. Derivation for equations of motion are on the basis of Hamilton's principle. Frequency equations for free, simply-supported and clamped edged plates are obtained using Chebyshev collocation method. The initial three zeroes are obtained numerically and addressed as dimensionless parameter of frequency for initial three vibrational modes. The obtained results for various values of facing and core thicknesses are given along with graphical and numerical comparisons with the works available.

MATHEMATICAL FORMULATION

A uniform circular sandwich plate is considered here with radius a and total thickness ($h_c + 2h_f$), where h_c is the core thickness and the thickness of membrane facings is denoted by h_f (see Fig. 1). Let us consider the cylindrical coordinates (r,θ,z) into account to define the geometry of the plate. The origin is assumed to be at the centre of the horizontal mid-plane $z = 0$ with $r = 0$ taken as the axis of rotation.

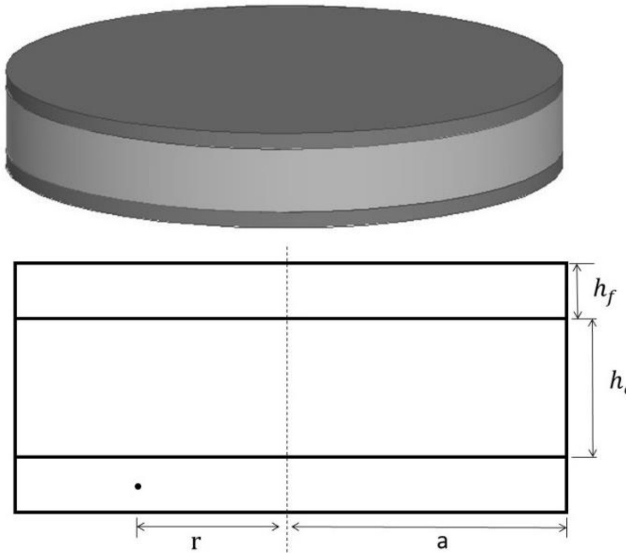

Figure 1. The mid-vertical cross section of a circular sandwich plate.

Strain- displacement relation

Here we consider $w(r,t)$ for the plate's displacement in the vertical direction, whereas the angle of rotation in radial direction is denoted by the symbol $\phi_r(r,t)$. According to the theory of Reddy, the axisymmetric deformation in the core and lower face sheet of the plate are:

$$u_c(r,z,t) = -\frac{4}{3}\frac{z^3}{h_c^2}\frac{\partial w}{\partial r} + z\left(1-\frac{4}{3}\frac{z^2}{h_c^2}\right)\phi_r ,$$

and $$u_f(r,z,t) = u_c\left(r,\frac{h_c}{2},t\right) = -\frac{h_c}{6}\frac{\partial w}{\partial r} + \frac{h_c}{3}\phi_r ,$$

where the subscripts f and c are used for facings and core, respectively. Resulting strains in terms of displacement are

$$\varepsilon_{c_{rr}} = \frac{\partial u_c}{\partial r} ,\ \varepsilon_{c\theta\theta} = \frac{u_c}{r} ,\ \text{and}\ \varepsilon_{c_{rz}} = \frac{\partial u_c}{\partial z} + \frac{\partial w}{\partial r}$$

in the core of the plate, and

$$\varepsilon_{f_{rr}} = \frac{\partial u_f}{\partial r} ,\ \varepsilon_{f\theta\theta} = \frac{u_f}{r}$$

in the lower face sheet. The values of strain corresponding to the upper facing are $-\varepsilon_{frr}$ and $-\varepsilon_{f\theta\theta}$, respectively.

Stress-strain relations

The stress values in terms of strain for homogeneous and isotropic core and facing may be written as

$$\sigma_{c_{rr}} = \lambda_c[\varepsilon_{c_{rr}} + \nu_c\varepsilon_{c\theta\theta}] ,\ \sigma_{c\theta\theta} = \lambda_c[\varepsilon_{c\theta\theta} + \nu_c\varepsilon_{c_{rr}}] ,$$

$$\sigma_{c_{rz}} = \mu_c\varepsilon_{c_{rz}} ,\ \sigma_{f_{rr}} = \lambda_f[\varepsilon_{f_{rr}} + \nu_f\varepsilon_{f\theta\theta}] ,\ \text{and}$$

$$\sigma_{f\theta\theta} = \lambda_f[\varepsilon_{f\theta\theta} + \nu_f\varepsilon_{f_{rr}}] ,$$

where: $\lambda_c = \dfrac{E_c}{1-\nu_c^2}$; $\mu_c = \dfrac{E_c}{2(1+\nu_c)}$; and $\lambda_f = \dfrac{E_f}{1-\nu_f^2}$; E_c and E_f are Young's moduli; and ν_c and ν_f are Poisson's ratios for the core and face sheets, respectively.

Expressions for energies and equations of motion

The kinetic energy corresponding to the core and lower face sheet of the plate are expressed as

$$T_c = \pi\rho_c\int_0^a\int_{-\frac{h_c}{2}}^{\frac{h_c}{2}}\left[\dot{u}_c^2 + \dot{w}^2\right]rdzdr , \tag{1}$$

and $$T_f = \pi\rho_f h_f\int_0^a\left[\dot{u}_f^2 + \dot{w}^2\right]rdr , \tag{2}$$

where: dot $(^{\cdot})$ over u_c, u_f and w denotes their corresponding time derivatives; ρ_c and ρ_f are the density corresponding to materials of core and facings, respectively. Similarly, the total strain energy in the form of stored potential associated with the core and lower face sheet is given by:

$$U_c = \pi\int_0^a\Bigg[rM_{c_r}\frac{\partial^2 w}{\partial r^2} + rN_{c_r}\frac{\partial\phi_r}{\partial r} + M_{c_\theta}\frac{\partial w}{\partial r} + N_{c_\theta}\phi_r + $$
$$+ rP_{c_{rz}}\frac{\partial w}{\partial r} + rQ_{c_{rz}}\phi_r\Bigg]dr , \tag{3}$$

and

$$U_f = \pi h_f\int_0^a\Bigg[-\frac{h_c}{6}r\sigma_{f_{rr}}\frac{\partial^2 w}{\partial r^2} + \frac{h_c}{3}r\sigma_{f_{rr}}\frac{\partial\phi_r}{\partial r} - \frac{h_c}{6}\sigma_{f\theta\theta}\frac{\partial w}{\partial r} + $$
$$+ \frac{h_c}{3}\sigma_{f\theta\theta}\phi_r\Bigg]dr , \tag{4}$$

where: $\{M_{c_r}, M_{c_\theta}\} = \int_{-h_c/2}^{h_c/2} -\frac{4}{3}\frac{z^3}{h_c^2}\{\sigma_{c_{rr}}, \sigma_{c_{\theta\theta}}\}dz$; $\{N_{c_r}, N_{c_\theta}\} = \int_{-h_c/2}^{h_c/2} z\left(1-\frac{4}{3}\frac{z^2}{h_c^2}\right)\{\sigma_{c_{rr}}, \sigma_{c_{\theta\theta}}\}dz$; $P_{c_{rz}} = Q_{c_{rz}} = \int_{-h_c/2}^{h_c/2}\left(1-\frac{4z^2}{h_c^2}\right)\sigma_{c_{rr}}dz$ are the moments.

According to Hamilton's principle:

$$\delta\int_0^t[(T_c - U_c) + 2(T_f - U_f)]dt = 0 .$$

Putting the energy values in the given equation gives us equations of motion in dynamic form for the considered plate

$$r\left(\rho_c I_2^c - \frac{1}{9}\rho_f h_c^2 h_f\right)\frac{\partial\ddot{w}}{\partial r} + r\left(\rho_c I_3^c + \frac{2}{9}\rho_f h_c^2 h_f\right)\ddot{\phi}_r - $$
$$-\left[\left(N_{c_r} + \frac{2}{3}h_c h_f\sigma_{f_{rr}}\right) - \left(N_{c_\theta} + \frac{2}{3}h_c h_f\sigma_{f\theta\theta}\right)\right] - $$
$$-r\left(\frac{\partial N_{c_r}}{\partial r} + \frac{2}{3}h_c h_f\frac{\partial\sigma_{f_{rr}}}{\partial r}\right) + rQ_{c_{rz}} = 0 , \tag{5}$$

and

$$-\left(\frac{\partial M_{c_\theta}}{\partial r} - \frac{1}{3}h_c h_f\frac{\partial\sigma_{f\theta\theta}}{\partial r}\right) + 2\left(\frac{\partial M_{c_r}}{\partial r} - \frac{1}{3}h_c h_f\frac{\partial\sigma_{f_{rr}}}{\partial r}\right) - P_{c_{rz}} - r\frac{\partial P_{c_{rz}}}{\partial r} + $$
$$+r\left(\frac{\partial^2 M_{c_r}}{\partial r^2} - \frac{1}{3}h_c h_f\frac{\partial^2\sigma_{f_{rr}}}{\partial r^2}\right) - \left(\rho_c I_1^c + \frac{1}{18}\rho_f h_c^2 h_f\right)\frac{\partial\ddot{w}}{\partial r} - $$
$$-r\left(\rho_c I_1^c + \frac{1}{18}\rho_f h_c^2 h_f\right)\frac{\partial^2\ddot{w}}{\partial r^2} - \left(\rho_c I_2^c - \frac{1}{9}\rho_f h_c^2 h_f\right)\ddot{\phi}_r - $$
$$-r\left(\rho_c I_2^c - \frac{1}{9}\rho_f h_c^2 h_f\right)\frac{\partial\ddot{\phi}_r}{\partial r} + r(\rho_c I_0^c + 2\rho_f h_f)\ddot{w} = 0 , \tag{6}$$

where: I_0^c , I_1^c , I_2^c , and I_3^c are given by $\{I_0^c, I_1^c, I_2^c, I_3^c\} =$

$$= \int_{-h_c/2}^{h_c/2}\left\{1, \frac{16}{9}\frac{z^6}{h_c^4}, -\frac{4}{3}\frac{z^4}{h_c^2}\left(1-\frac{4}{3}\frac{z^4}{h_c^2}\right), z^2\left(1-\frac{4}{3}\frac{z^4}{h_c^2}\right)^2\right\}dz .$$

Equations of motion in terms of displacement

Equations (5) and (6) are reconstructed by putting stress values in terms of displacement. Resulting equations are:

$$P_1\ddot{\phi}_r + P_2\frac{\partial\ddot{w}}{\partial r} + u_1\frac{\partial^2\phi_r}{\partial r^2} + u_2\frac{\partial\phi_r}{\partial r} + u_3\phi_r + u_4\frac{\partial^3 w}{\partial r^3} + u_5\frac{\partial^2 w}{\partial r^2} + $$
$$+u_6\frac{\partial w}{\partial r} = 0 , \tag{7}$$

and

$$P_3\frac{\partial\ddot{\phi}_r}{\partial r} + P_4\ddot{\phi}_r + P_5\frac{\partial^2\ddot{w}}{\partial r^2} + P_6\frac{\partial\ddot{w}}{\partial r} + P_7\ddot{w} + u_7\frac{\partial^3\phi_r}{\partial r^3} + u_8\frac{\partial^2\phi_r}{\partial r^2} + $$

$$+u_9\frac{\partial\phi_r}{\partial r}+u_{10}\phi_r+u_{11}\frac{\partial^4 w}{\partial r^4}+u_{12}\frac{\partial^3 w}{\partial r^3}+u_{13}\frac{\partial^2 w}{\partial r^2}+u_{14}\frac{\partial w}{\partial r}=0 \quad (8)$$

where: $P_1=r^2\left(\rho_c I_3^c+\frac{2}{9}\rho_f h_c^2 h_f\right)$; $P_2=r^2\left(\rho_c I_2^c-\frac{1}{9}\rho_f h_c^2 h_f\right)$; $P_3=-rP_2$; $P_4=-P_2$; $P_5=-r^3\left(\rho_c I_1^c+\frac{1}{18}\rho_f h_c^2 h_f\right)$; $P_6=-r^2\left(\rho_c I_1^c+\frac{1}{18}\rho_f h_c^2 h_f\right)$; $P_7=r^3\left(\rho_c I_0^c+2\rho_f h_f\right)$; $u_1=r^2\left(-\lambda_c I_3^c-\frac{2}{9}\lambda_f h_c^2 h_f\right)$; $u_2=r\left(-\lambda_c I_3^c-\frac{2}{9}\lambda_f h_c^2 h_f\right)$; $u_3=\left(\lambda_c I_3^c+\frac{2}{9}\lambda_f h_c^2 h_f+\mu_c r^2 H\right)$; $u_4=r^2\left(-\lambda_c I_2^c+\frac{1}{9}\lambda_f h_c^2 h_f\right)$; $u_5=r\left(-\lambda_c I_2^c+\frac{1}{9}\lambda_f h_c^2 h_f\right)$; $u_6=\left(\lambda_c I_2^c-\frac{1}{9}\lambda_f h_c^2 h_f+\mu_c r^2 H\right)$; $u_7=-ru_4$; $u_8=-2u_4$; $u_9=-ru_6$; $u_{10}=\left(\lambda_c I_2^c-\frac{1}{9}\lambda_f h_c^2 h_f-\mu_c r^2 H\right)$; $u_{11}=r^3\left(\lambda_c I_1^c+\frac{1}{18}\lambda_f h_c^2 h_f\right)$; $u_{12}=2r^2\left(\lambda_c I_1^c+\frac{1}{18}\lambda_f h_c^2 h_f\right)$; $u_{13}=r\left(-\lambda_c I_1^c-\frac{1}{18}\lambda_f h_c^2 h_f-\mu_c r^2 H\right)$; $u_{14}=\left(\lambda_c I_1^c+\frac{1}{18}\lambda_f h_c^2 h_f-\mu_c r^2 H\right)$, where: $H=\int_{-h_c/2}^{h_c/2}\left(1-4\frac{z^2}{h_c^2}\right)dz$.

Now, to obtain the non-dimensional equations, the following dimensionless parameters are introduced: $\xi = r/a$, $\bar z = z/a$, $H_c = h_c/a$, $H_f = h_f/a$, $\bar w = w/a$, $R_\rho = \rho_f/\rho_r$, $R_f = \lambda_f/\mu_c$, $R_c = \lambda_c/\mu_c$, and assuming $\bar w(\xi,t) = \bar w(\xi)\sin(\omega t)$ and $\psi_r(\xi,t) = \psi_r(\xi)\sin(\omega t)$, where the symbol '$\omega$' is used for the angular frequency of harmonic motion, Eqs.(7) and (8) get reduced to

$$\bar u_1\frac{d^2\phi_r}{d\xi^2}+\bar u_2\frac{d\phi_r}{d\xi}+\bar u_3\phi_r+\bar u_4\frac{d^3\bar w}{d\xi^3}+\bar u_5\frac{d^2\ddot w}{d\xi^2}+\bar u_6\frac{d\bar w}{d\xi}-\Omega^2\bar S_1\phi_r-\Omega^2\bar S_2\frac{d\bar w}{d\xi}=0\,, \quad (9)$$

and

$$\bar u_7\frac{d^3\phi_r}{d\xi^3}+\bar u_8\frac{d^2\phi_r}{d\xi^2}+\bar u_9\frac{d\phi_r}{d\xi}+\bar u_{10}\phi_r+\bar u_{11}\frac{d^4\bar w}{d\xi^4}+\bar u_{12}\frac{d^3\bar w}{d\xi^3}+\bar u_{13}\frac{d^2\bar w}{d\xi^2}+\bar u_{14}\frac{d\bar w}{d\xi}-\Omega^2\bar S_3\frac{d\phi_r}{d\xi}-\Omega^2\bar S_4\phi_r-\Omega^2\bar S_5\frac{d^2\bar w}{d\xi^2}-\Omega^2\bar S_6\frac{d\bar w}{d\xi}-\Omega^2\bar S_7\bar w=0\,, \quad (10)$$

where: $\bar S_1=\xi^2\left(\bar I_3^c+\frac{2}{9}R_\rho H_c^2 H_f\right)$, $\bar S_2=\xi^2\left(\bar I_2^c-\frac{1}{9}R_\rho H_c^2 H_f\right)$, $\bar S_3=-\xi\bar S_2$, $\bar S_4=-\bar S_2$, $\bar S_5=-\xi^3\left(\bar I_1^c+\frac{1}{18}R_\rho H_c^2 H_f\right)$, $\bar S_6=-\xi^2\left(\bar I_1^c+\frac{1}{18}R_\rho H_c^2 H_f\right)$, $\bar S_7=\xi^3\left(\bar I_0^c+2R_\rho H_f\right)$, $\bar u_1=\xi^3\left(-R_c\bar I_3^c-\frac{2}{9}R_f H_c^2 H_f\right)$, $\bar u_2=\xi^2\left(-R_c\bar I_3^c-\frac{2}{9}R_f H_c^2 H_f\right)$, $\bar u_3=\left(R_c\bar I_3^c+\frac{2}{9}R_f H_c^2 H_f+\xi^2\bar H\right)$, $\bar u_4=\xi^2\left(-R_c\bar I_2^c+\frac{1}{9}R_f H_c^2 H_f\right)$, $\bar u_5=\xi\left(-R_c\bar I_2^c+\frac{1}{9}R_f H_c^2 H_f\right)$, $\bar u_6=\left(R_c\bar I_2^c-\frac{1}{9}R_f H_c^2 H_f+\xi^2\bar H\right)$, $\bar u_7=-\xi\bar u_4$, $\bar u_8=-2\bar u_4$, $\bar u_9=-\xi\bar u_6$, $\bar u_{10}=\left(R_c\bar I_2^c-\frac{1}{9}R_f H_c^2 H_f-\xi^2\bar H\right)$, $\bar u_{11}=\xi^3\left(R_c\bar I_1^c+\frac{1}{18}R_f H_c^2 H_f\right)$, $\bar u_{12}=\xi^2\left(R_c\bar I_1^c+\frac{1}{18}R_f H_c^2 H_f\right)$, $\bar u_{13}=\xi\left(-R_c\bar I_1^c-\frac{1}{18}R_f H_c^2 H_f-\xi^2\bar H\right)$, and $\bar u_{14}=\left(R_c\bar I_1^c+\frac{1}{18}R_f H_c^2 H_f-\xi^2\bar H\right)$,

where: $\{\bar I_0^c,\bar I_1^c,\bar I_2^c,\bar I_3^c\}=$
$$=\int_{-H_c/2}^{H_c/2}\left\{1,\frac{16}{9}\frac{\bar z^6}{H_c^4},-\frac{4}{3}\frac{\bar z^4}{H_c^2}\left(1-\frac{4}{3}\frac{\bar z^2}{H_c^2}\right),\bar z^2\left(1-\frac{4}{3}\frac{\bar z^2}{H_c^2}\right)^2\right\}d\bar z\,;$$
$\bar H=\int_{-H_c/2}^{H_c/2}\left(1-4\frac{\bar z^2}{H_c^2}\right)d\bar z$; and $\Omega^2=\frac{\rho_c a^2\omega^2}{\mu_c}$, where Ω is the dimensionless frequency parameter.

METHOD OF SOLUTION

Chebyshev collocation technique is applied here for solving the obtained differential equation with variable coefficients. Since the range of applicability of this method is [-1, 1], we introduce a new variable x such that $x = [2\xi - 1]$.

Transforming Eqs.(9) and (10) within the applicability range and taking

$$\frac{d^4\psi_r}{dx^4}=\sum_0^{m-5}a_{k+5}T_k \quad\text{and}\quad \frac{d^4\bar w}{dx^4}=\sum_0^{n-5}b_{k+5}T_k\,,$$

where: m and n are chosen in such a way that one is even and the other is odd. Their successive integration gives us

$$\phi_r=a_1+a_2x+a_3\frac{x^2}{2}+a_4\frac{x^3}{6}+\sum_0^{m-5}a_{k+5}T_k^4\,,$$

and
$$\bar w=b_1+b_2x+b_3\frac{x^2}{2}+b_4\frac{x^3}{6}+\sum_0^{n-5}b_{k+5}T_k^4\,,$$

and equations of motion finally get reduced to

$$(V_3)a_1+(V_3x+V_2)a_2+\left(V_3\frac{x^2}{2}+V_2x+V_1\right)a_3+\left(V_3\frac{x^3}{6}+V_2\frac{x^2}{2}+V_1x\right)a_4+\sum_0^{m-5}(V_3T_k^4+V_2T_k^3+V_1T_k^2)a_{k+5}+(V_6)b_2+(V_6x+V_5)b_3+$$
$$+\left(V_6\frac{x^2}{2}+V_5x+V_4\right)b_4+\sum_0^{n-5}(V_6T_k^3+V_5T_k^2+V_4T_k^1)b_{k+5}-\Omega^2\left[(S_1)a_1+(S_1x)a_2+\left(S_1\frac{x^2}{2}\right)a_3+\left(S_1\frac{x^3}{6}\right)a_4+\sum_0^{m-5}(S_1T_k^4)a_{k+5}+(S_2)b_2+\right.$$

$$+(S_2x)b_3+\left(S_2\frac{x^2}{2}\right)b_4+\sum_{0}^{n-5}(S_2T_k^3)b_{k+5}\Bigg]=0\,, \quad (11)$$

and

$$(V_{10})a_1+(V_{10}x+V_9)a_2+\left(V_{10}\frac{x^2}{2}+V_9x+V_8\right)a_3+\left(V_{10}\frac{x^3}{6}+V_9\frac{x^2}{2}+V_8x+V_7\right)a_4+\sum_{0}^{m-5}(V_{10}T_k^4+V_9T_k^3+V_8T_k^2+V_7T_k^1)a_{k+5}+V_{14}b_2+$$

$$+(V_{14}x+V_{13})b_3+\left(V_{14}\frac{x^2}{2}+V_{13}x+V_{12}\right)b_4+\sum_{0}^{n-5}(V_{14}T_k^3+V_{13}T_k^2+V_{12}T_k^1+V_{11}T_k)b_{k+5}-\Omega^2\Bigg[S_4a_1+(S_4x+S_3)a_2+\left(S_4\frac{x^2}{2}+S_3x\right)a_3+$$

$$+\left(S_4\frac{x^3}{6}+S_3\frac{x^2}{2}\right)a_4+\sum_{0}^{m-5}(S_4T_k^4+S_3T_k^3)a_{k+5}+S_7b_1+(S_7x+S_6)b_2+\left(S_7\frac{x^2}{2}+S_6x+S_5\right)b_3+\left(S_7\frac{x^3}{6}+S_6\frac{x^2}{2}+S_5x\right)b_4+$$

$$+\sum_{0}^{n-5}(S_7T_k^4+S_6T_k^3+S_5T_k^2)b_{k+5}\Bigg]=0\,, \quad (12)$$

where: T_k is k-th Chebyshev's polynomial and T_k^j; $1\le j\le 4$ denotes the j-th integral of k-th Chebyshev polynomial; and $V_{11}=16\bar{u}_{11}$, $\{V_4,V_7,V_{12}\}=8\{\bar{u}_4,\bar{u}_7,\bar{u}_{12}\}$, $\{S_5,V_1,V_5,V_8,V_{13}\}=4\{\bar{S}_5,\bar{u}_1,\bar{u}_5,\bar{u}_8,\bar{u}_{13}\}$, $\{S_2,S_3,S_6,V_2,V_9,V_{14}\}=2\{\bar{S}_2,\bar{S}_3,\bar{S}_6,\bar{u}_2,\bar{u}_6,\bar{u}_9,\bar{u}_{14}\}$, $\{S_1,S_4,S_7,V_3,V_{10}\}=\{\bar{S}_1,\bar{S}_4,\bar{S}_7,\bar{u}_3,\bar{u}_{10}\}$.

Equations (11) and (12) satisfy $(m+n-3)/2$ collocation points given by

$$x_i=\cos\left(\frac{\pi(2i+1)}{m+n-3}\right);\quad 1\le i\le\frac{m+n-3}{2}\,,$$

where: x_i are the roots of T_k for $k=(m+n-3)/2$.

Putting the values of x_i in Eqs.(11) and (12), we obtain a set of $(m+n-3)$ equations in $(m+n)$ unknowns, denoted by a_j and b_k, where j ranges from 1 to m and k varies from 1 to n. The obtained system can also be expressed as:

$$[G-\Omega^2H][X]=0\,, \quad (13)$$

where: matrices H and G are of size $(m+n-3)\times(m+n)$ and X is of order $(m+n)\times 1$.

CONDITIONS AT BOUNDARIES AND FREQUENCY EQUATIONS

The edge conditions can be expressed mathematically as:

1. $w=0$, $\phi_r=0$ and $\partial w/\partial r=0$, for clamped edge
2. $w=0$, $\left(N_{c_r}+\frac{2}{3}h_ch_f\sigma_{f_{rr}}\right)=0$ and $\left(M_{c_r}-\frac{1}{3}h_ch_f\sigma_{f_{rr}}\right)=0$, for simply-supported edge
3. $\left(M_{c_\theta}-\frac{1}{3}h_ch_f\sigma_{f_{\theta\theta}}\right)+rP_{c_{rz}}-\left(M_{c_r}-\frac{1}{3}h_ch_f\sigma_{f_{rr}}\right)-$
$-r\left(\frac{\partial M_{c_r}}{\partial r}-\frac{1}{3}h_ch_f\frac{\partial\sigma_{f_{rr}}}{\partial r}\right)+r\left(\rho_cI_2^c-\frac{1}{9}\rho_fh_c^2h_f\right)\ddot{\psi}_r+$
$+r\left(\rho_cI_1^c+\frac{1}{18}\rho_fh_c^2h_f\right)\frac{\partial\ddot{w}}{\partial r}=0$, $\left(N_{c_r}+\frac{2}{3}h_ch_f\sigma_{f_{rr}}\right)=0$,
$\left(M_{c_r}-\frac{1}{3}h_ch_f\sigma_{f_{rr}}\right)=0$, for free edge.

Here, each boundary conditions are applied at the edge of the plate, i.e. at $x=1$. We have three equations for each case itemized above, which together with Eq.(13) creates a full set of $(m+n)$ equations in terms of $(m+n)$ unknowns. The matrix equation for a clamped circular sandwich plate can be expressed as

$$[G^c-\Omega^2H^c]X=0\,, \quad (14)$$

with $G^c=\begin{bmatrix}G\\B_G^c\end{bmatrix}$ and $H^c=\begin{bmatrix}H\\B_H^c\end{bmatrix}$, where B_G^c and B_H^c are matrices of order $3\times(m+n)$ which come from the boundary condition for the clamped edge.

For the solution to be non-trivial, we must have

$$\left|G^c-\Omega^2H^c\right|=0\,. \quad (15)$$

Similarly, for simply-supported and clamped sandwich plates, we have

$$\left|G^s-\Omega^2H^s\right|=0\,, \quad (16)$$

and

$$\left|G^f-\Omega^2H^f\right|=0\,, \quad (17)$$

where: $G^s=\begin{bmatrix}G\\B_G^s\end{bmatrix}$, $G^f=\begin{bmatrix}G\\B_G^f\end{bmatrix}$, $H^s=\begin{bmatrix}H\\B_H^s\end{bmatrix}$ and $H^f=\begin{bmatrix}H\\B_H^f\end{bmatrix}$.

NUMERICAL RESULTS AND DISCUSSION

Solutions for Ω are obtained from Eqs.(15), (16) and (17) for different plate parameter values by applying hybrid bisection-secant method. In order to analyse the effect of thickness of core thickness H_c and facing thickness H_f for all three boundary conditions, the first three zeroes of the system are obtained and recorded as least three natural frequencies. Material of the core is taken as polyvinyl chloride while facings are considered to be made up of aluminium. The various elastic constants are: $R_p=2.76$, $R_f=1232.21$, $R_c=2.85$, $\nu_f=0.30$ and $\nu_c=0.30$.

Table 1 exhibits the percentage error of the least three modal frequencies for each boundary case considered with $H_c=0.1$ and $H_f=0.005$. In the table, it is found that the percentage errors involved in the least three modal frequencies diminish with increasing m and n values and become almost zero after $m=37$ and $n=38$. So, we fix $m=37$ and $n=38$ for evaluations involved in this work.

In Table 2, the numerical results of discussed work with $H_f=0$ are compared with the analytically obtained results available in literature for thick circular plates based on Reddy's HSDT. The frequency parameter considered here is $\Omega = \omega a\sqrt{(\rho_c H_c/D)}$, where $D = E_c a^3 H_c^3/12(1-\nu_c^2)$. An excellent agreement of frequencies is found in all the cases which validates and verifies the obtained results.

Table 3 compares the results obtained by the proposed approach with the available works based on FSDT. Percentage difference is calculated using formula % diff. = $[(\Omega_{HSDT} - \Omega_{FSDT})/\Omega_{HSDT}]\times 100$.

Considerable improvement is observed in results obtained for HSDT which validates the significance of the proposed work. Results of the present HSDT based formulation for varying values of H_c and H_f are given in Tables 4 and 5.

Figure 2 exhibits the effect of core thickness H_c on the percentage difference between the values of Ω obtained for HSDT (present) and FSDT for the initial three modes of vibrations of each considered boundary case. The graph's pattern reveals that with the rising value of H_c, the percentage difference increases monotonically. It is noted that with an increase in the mode number, the percentage difference in all the three plates increases.

Figure 3 shows the effect of core thickness H_c on the Ω value for H_f = 0.0025, 0.0075. The range of H_c is taken from 0.1 to 0.5. Results for both FSDT and HSDT are plotted for a better visual comparison. In the figure, Ω is found to rise monotonically with an increase in the H_c value. In all the three modes of vibration of each boundary case, the pattern is observed to be the same. The value of Ω for HSDT is also noted to be greater than its counterpart obtained for FSDT and the discrepancy between their values increases with the value of H_c increasing. As we shift into higher vibration modes in each boundary case, the effect appears to be more pronounced. It is observed that values of Ω for H_f = 0.0025 are more than the equivalent value measured for H_f = 0.0075.

The influence of H_f on Ω is shown in Fig. 4. Curves of both FSDT and HSDT are plotted for the least three vibration modes taking H_c = 0.2, 0.4. Values of H_f is taken from 0.0025 to 0.02. It is observed that H_f inversely affects the frequency parameter Ω in all the three boundary cases. It is also found that the curves for FSDT are lower in position as compared to that of HSDT. It is clear from the observation that for a particular mode of vibration, the discrepancy in Ω values obtained for HSDT and FSDT remains almost the same with increasing value of H_f and hence the curves plotted for them are equidistant at each point. This gap in between the corresponding curves of HSDT and FSDT increases when we move towards the higher modes of vibration in all the plates. The value of Ω for H_c = 0.4 is found to be more than that obtained for H_c = 0.2.

Figure 5 is a three-dimensional surface plot of clamped, simply-supported and free plates for the first three modes of vibration for H_c = 0.2 and H_f = 0.005.

Table 1. Percentage error under varying boundary conditions for first three modes of vibration. H_c = 0.1 and H_f = 0.005. Percentage error = $[|\Omega_i^{(j,k)} - \Omega_i^{(37,38)}|/\Omega_i^{(37,38)}]\times 100$, where $\Omega_i^{(j,k)}$ is the i-th mode of frequency for $j = m$ and $k = n$.

		Percentage error*								
		Clamped			Simply supported			Free		
m	n	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
28	29	0.2687	0.1838	0.1477	0.0000	0.0185	0.0208	0.0109	0.0148	0.0287
29	30	0.1598	0.1115	0.0878	0.0739	0.0618	0.1023	0.0054	0.0074	0.0159
30	31	0.0944	0.0663	0.0523	0.0000	0.0062	0.0076	0.0054	0.0049	0.0096
31	32	0.0581	0.0392	0.0299	0.0277	0.0185	0.0265	0.0000	0.0025	0.0064
32	33	0.0291	0.0211	0.0168	0.0000	0.0000	0.0019	0.0000	0.0025	0.0032
33	34	0.0218	0.0120	0.0093	0.0092	0.0062	0.0076	0.0000	0.0000	0.0032
34	35	0.0073	0.0060	0.0056	0.0000	0.0000	0.0000	0.0000	0.0025	0.0000
35	36	0.0073	0.0030	0.0019	0.0000	0.0031	0.0019	0.0000	0.0000	0.0000
36	37	0.0000	0.0000	0.0019	0.0000	0.0000	0.0000	0.0000	0.0025	0.0000
37	38	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
38	39	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
39	40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2. Comparison with Hosseini-Hashemi et al. /36/ . $\Omega = \omega a\sqrt{(\rho_c H_c/D)}$ where $D = E_c a^3 H_c^3/12(1-\nu_c^2)$.

Boundary condition	H_c	Ω_1		Ω_2	
		Present	Benchmark	Present	Benchmark
Clamped	0.2	9.2650	9.26503	30.4750	30.4749
	0.25	8.8464	8.84637	27.6223	27.6223
	0.3	8.4113	8.41130	25.1011	25.1011
	0.35	7.9783	7.97828	22.9183	22.9183
Simply-Supported	0.2	4.7787	4.77871	25.0414	25.0414
	0.25	4.6985	4.69853	23.3190	23.3190
	0.3	4.6070	4.60704	21.6757	21.6757
	0.35	5.5070	4.50697	20.1569	20.1569
Free	0.2	8.5084	8.50842	31.1748	31.1748
	0.25	8.2723	8.27233	28.6931	28.6931
	0.3	8.0151	8.01507	26.3883	26.3883
	0.35	7.7467	7.74669	24.2979	24.2979

Table 3. Percentage difference with respect to FSDT /18, 26/. H_f = 0.005. Asterisk (*) denotes the present result. % difference = $[(\Omega_{HSDT} - \Omega_{FSDT})/\Omega_{HSDT}]\times 100$.

H_c		Ω_1	% diff.	Ω_2	% diff.	Ω_3	% diff.
				Clamped plate			
0.2	HSDT *	1.7876	9.56	4.2309	10.58	6.7764	11.44
	FSDT	1.6166		3.7831		6.0013	
0.4	HSDT*	2.1677	11.83	5.1300	13.66	8.3079	15.91
	FSDT	1.9112		4.4291		6.9861	
				Simply - supported plate			
0.2	HSDT*	1.5250	6.09	4.1095	8.46	6.6208	9.44
	FSDT	1.4322		3.7619		5.9957	
0.4	HSDT*	1.9251	7.12	4.8901	9.67	7.9456	12.15
	FSDT	1.7881		4.4172		6.9830	
				Free plate			
0.2	HSDT*	2.5342	6.67	5.1769	8.62	7.7444	9.71
	FSDT	2.3651		4.7309		6.9925	
0.4	HSDT*	3.1582	7.99	6.2235	10.45	9.3730	12.94
	FSDT	2.9060		5.5730		8.1600	

Table 4. Influence of the thickness of core H_c on frequency parameter Ω.

H_c	Frequency parameter (Ω)					
	H_f = 0.0025			H_f = 0.0075		
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
			Clamped plate			
0.10	1.5589	3.9133	6.4260	1.2275	2.9155	4.6637
0.15	1.8100	4.4352	7.2102	1.4570	3.4311	5.4720
0.20	1.9735	4.7786	7.7365	1.6239	3.8122	6.0822
0.25	2.0904	5.0298	8.1343	1.7538	4.1143	6.5787
0.30	2.1796	5.2276	8.4606	1.8596	4.3656	7.0037
0.35	2.2511	5.3921	8.7439	1.9484	4.5820	7.3811
0.40	2.3106	5.5347	9.0003	2.0251	4.7736	7.7255
0.45	2.3616	5.6623	9.2391	2.0926	4.9471	8.0463
0.50	2.4065	5.7796	9.4659	2.1531	5.1070	8.3494
			Simply - supported plate			
0.10	1.0954	3.7564	6.3187	1.0212	2.8563	4.6052
0.15	1.3577	4.2782	7.0664	1.2638	3.3539	5.3768
0.20	1.5429	4.6080	7.5486	1.4394	3.7117	5.9452
0.25	1.6808	4.8385	7.8984	1.5735	3.9874	6.3958
0.30	1.7874	5.0114	8.1738	1.6799	4.2097	6.7719
0.35	1.8724	5.1480	8.4044	1.7665	4.3954	7.0979
0.40	1.9418	5.2608	8.6065	1.8387	4.5549	7.3887
0.45	1.9996	5.3571	8.7901	1.8999	4.6948	7.6540
0.50	2.0485	5.4418	8.9609	1.9526	4.8201	7.9004
			Free plate			
0.10	1.8998	4.6941	7.3180	1.7124	3.5879	5.3701
0.15	2.3221	5.3486	8.1991	2.0942	4.2266	6.2899
0.20	2.6129	5.7700	8.7747	2.3689	4.6910	6.9744
0.25	2.8260	6.0708	9.1959	2.5787	5.0528	7.5228
0.30	2.9896	6.3014	9.5274	2.7458	5.3482	7.9841
0.35	3.1197	6.4877	9.7992	2.8829	5.5978	8.3859
0.40	3.2263	6.6446	10.0217	2.9983	5.8142	8.7445
0.45	3.3156	6.7809	10.1809	3.0971	6.0057	9.0701
0.50	3.3920	6.9022	10.1984	3.1832	6.1779	9.3692

Table 5. Influence of the thicknesses of face sheets H_f on frequency parameter Ω.

H_f	Frequency parameter Ω					
	H_c = 0.2			H_c = 0.4		
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
			Clamped plate			
0.0025	1.9735	4.7786	7.7365	2.3106	5.5347	9.0003
0.0050	1.7876	4.2309	6.7764	2.1677	5.1300	8.3079
0.0075	1.6239	3.8122	6.0822	2.0251	4.7736	7.7255
0.0100	1.4939	3.4925	5.5612	1.9025	4.4763	7.2447
0.0125	1.3895	3.2402	5.1534	1.7980	4.2263	6.8417
0.0150	1.3037	3.0350	4.8233	1.7084	4.0131	6.4984
0.0175	1.2318	2.8641	4.5492	1.6305	3.8288	6.2017
0.0200	1.1705	2.7190	4.3170	1.5622	3.6675	5.9420
			Simply - supported plate			
0.0025	1.5429	4.6080	7.5486	1.9418	5.2608	8.6065
0.0050	1.5250	4.1095	6.6208	1.9251	4.8901	7.9456
0.0075	1.4394	3.7117	5.9452	1.8387	4.5549	7.3887
0.0100	1.3531	3.4046	5.4371	1.7480	4.2733	6.9287
0.0125	1.2760	3.1609	5.0390	1.6644	4.0359	6.5431
0.0150	1.2088	2.9621	4.7167	1.5895	3.8330	6.2147
0.0175	1.1503	2.7963	4.4489	1.5227	3.6575	5.9308
0.0200	1.0990	2.6553	4.2220	1.4631	3.5038	5.6823
			Free plate			
0.0025	2.6129	5.7700	8.7747	3.2263	6.6446	10.0217
0.0050	2.5342	5.1769	7.7444	3.1582	6.2235	9.3730
0.0075	2.3689	4.6910	6.9744	2.9983	5.8142	8.7445
0.0100	2.2136	4.3112	6.3891	2.8407	5.4640	8.2140
0.0125	2.0792	4.0078	5.9276	2.6989	5.1661	7.7656
0.0150	1.9641	3.7593	5.5526	2.5735	4.9104	7.3818
0.0175	1.8650	3.5514	5.2403	2.4626	4.6884	7.0490
0.0200	1.7788	3.3741	4.9751	2.3642	4.4936	6.7571

CONCLUSIONS

Free axisymmetric vibrations of thick sandwich plates of circular geometry with clamped, simply-supported and free edges are studied in this article. The plate's core is assumed to be solid as well as of uniform thickness. Face sheets are taken as membranes. Reddy's HSDT is used to define the displacement field. Chebyshev collocation method is adopted to get the frequency equations. From the results obtained in the previous section, the following conclusions are drawn:

- frequency parameter increases monotonically with increasing value of H_c for all boundary conditions keeping H_f constant,
- with an increase in H_f keeping H_c fixed, the frequency parameter Ω decreases monotonically,
- the frequency value computed for HSDT exceeds its corresponding value computed for FSDT in all the three plates keeping the plate parameters same,
- the difference between frequency values computed for FSDT and HSDT increase while moving towards higher modes of vibration keeping the values of H_c and H_f constant,

- increasing the value of H_c keeping H_f constant increases the difference between the numerical value of Ω computed for HSDT and FSDT. This effect is more pronounced when we move towards higher modes of vibration,
- the difference in frequency values obtained for FSDT and HSDT for a particular mode of vibration remains almost the same with increase in H_f for a fixed H_c,
- the analyses of references /18, 26/ are not suitable for the analysis of axisymmetric vibration of thick sandwich plates of circular geometry.

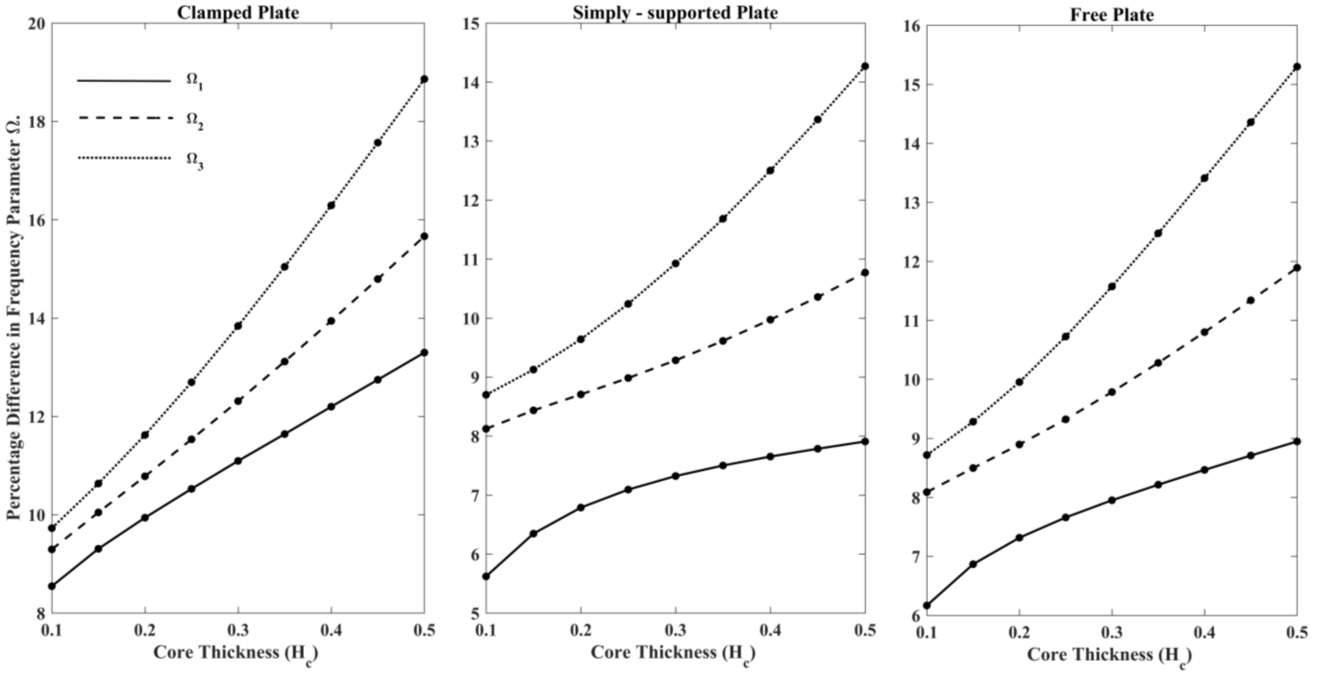

Figure 2. Effect of core thickness H_c on percentage difference and frequency parameter Ω, H_f = 0.0075. % diff. = $[(\Omega_{HSDT}-\Omega_{FSDT})/\Omega_{HSDT}]\times 100$.

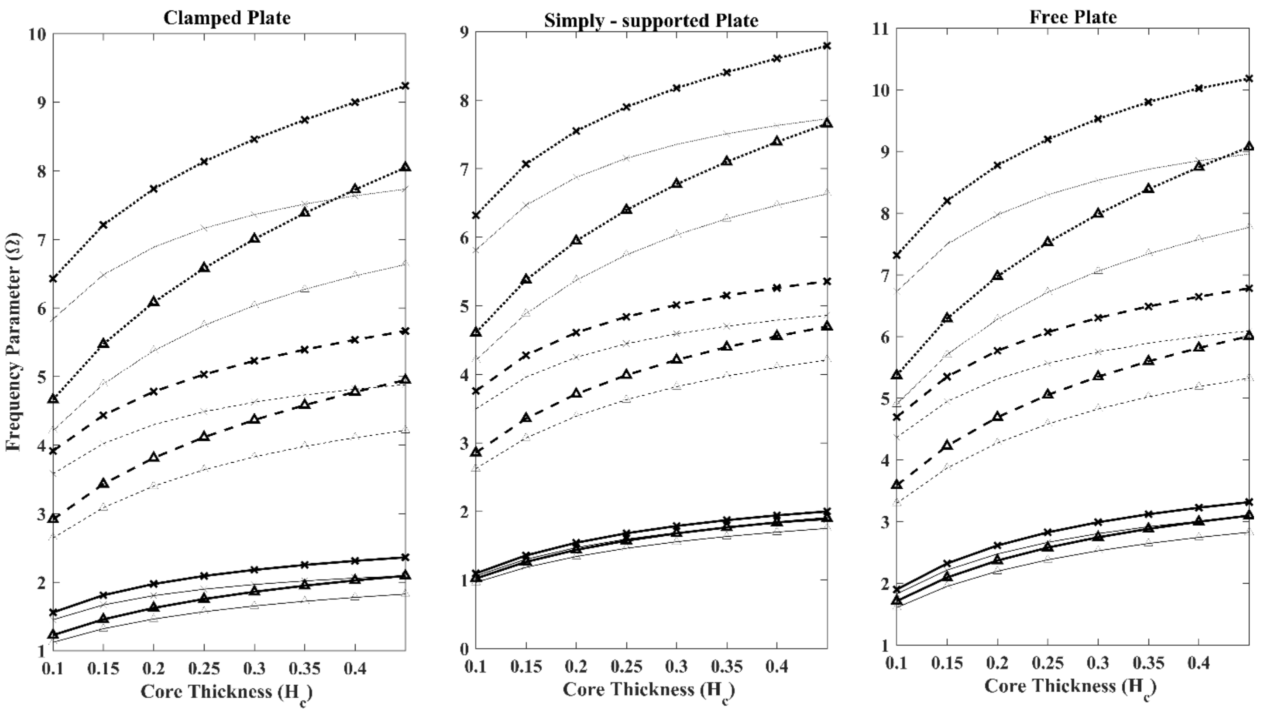

Figure 3. Effect of varying core thickness H_c on frequency parameter Ω. Darker lines: HSDT (present), faded lines: FSDT /18, 26/. H_f = 0.0025 (×), 0.0075 (Δ). — First mode, - - - Second mode, ... Third mode

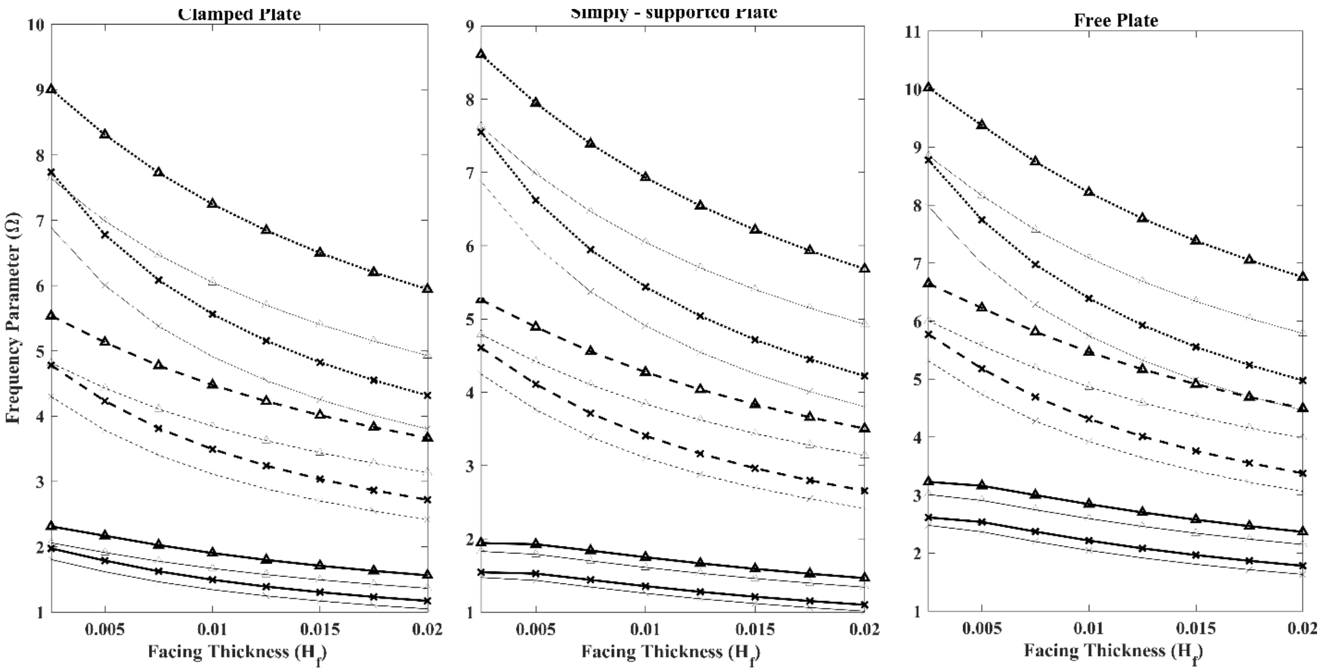

Figure 4. Effect of varying facing thickness H_f on frequency parameter Ω. Darker lines: HSDT (present), Faded lines: FSDT /18, 26/. H_c = 0.2 (×), 0.4 (Δ). —— First mode, - - - Second mode, ... Third mode

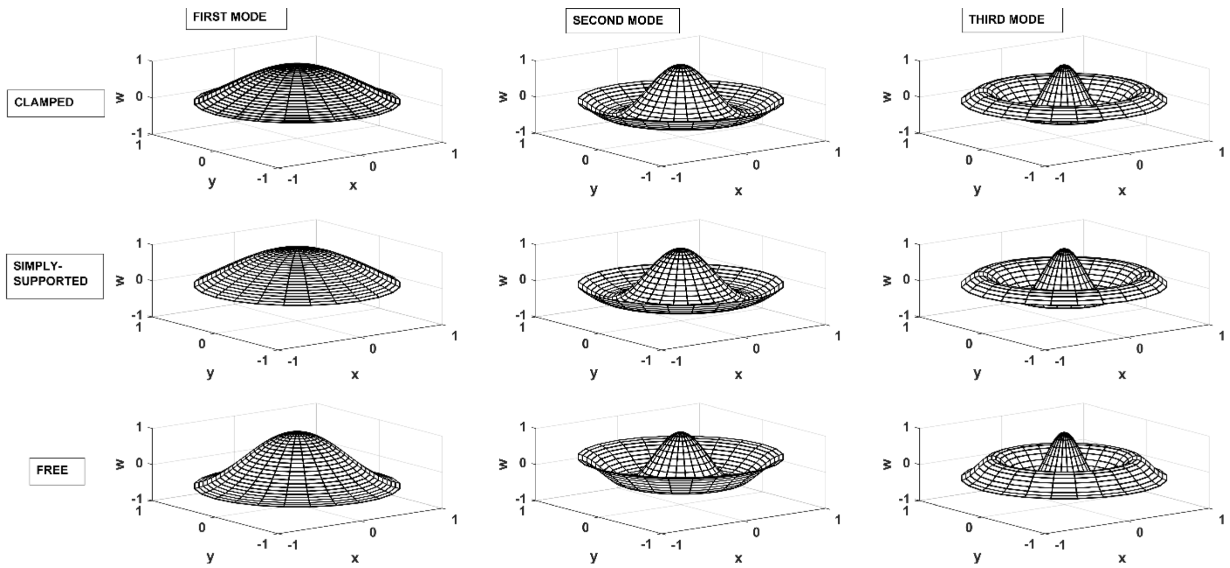

Figure 5. Three-dimensional mode shapes for initial three vibrational modes. H_c = 0.2, H_f = 0.005.

REFERENCES

1. Sayyad, A.S., Ghugal, Y.M. (2015), *On the free vibration analysis of laminated composite and sandwich plates: A review of recent literature with some numerical results*, Compos. Struct. 129: 177-201. doi: 10.1016/j.compstruct.2015.04.007
2. Birman, V., Kardomateas, G.A. (2018), *Review of current trends in research and applications of sandwich structures*, Compos. Part B: Eng. 142: 221-240. doi: 10.1016/j.compositesb.2018.01.027
3. Altenbach, H. (1998), *Theories for laminated and sandwich plates*, Mech. Compos. Mater. 34(3): 243-252. doi: 10.1007/BF02256043
4. Carrera, E., Brischetto, S. (2009), *A survey with numerical assessment of classical and refined theories for the analysis of sandwich plates*, Appl. Mech. Rev. 62(1): 010803. doi: 10.1115/1.3013824
5. Love, A.E.H. (1934), A Treatise on the Mathematical Theory of Elasticity, Cambridge University Press.
6. Reissner, E. (1945), *The effect of transverse shear deformation on the bending of elastic plates*, J Appl. Mech. 12(2): A69-A77. doi: 10.1115/1.4009435
7. Mindlin, R. (1951), *Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates*, J Appl. Mech. 18: 31-38.
8. Reissner, E. (1985), *Reflections on the theory of elastic plates*, Appl. Mech. Rev. 38(11): 1453-1464. doi: 10.1115/1.3143699
9. Touratier, M. (1991), *An efficient standard plate theory*, Int. J Eng. Sci. 29(8): 901-916. doi: 10.1016/0020-7225(91)90165-Y
10. Habip, L.M. (1964), *A review of recent Russian work on sandwich structures*, Int. J Mech. Sci. 6(6): 483-487. doi: 10.1016/S0020-7403(64)80010-2
11. Habip, L.M. (1965), *A survey of modern developments in the analysis of sandwich structures*, Appl. Mech. Rev. 18: 93-98.

12. Bert, C.W. (1978), *Recent research in composite and sandwich plate dynamics*, Shock Vibr. Digest, 11(10): 13-23.
13. Bert, C.W., Research on dynamics of composite and sandwich plates, 1979-81, Tech. Report No.27, School of Aerospace, Mech. and Nucl. Eng., University of Oklahoma, 1982.
14. Kao, J.S., Ross, R.J. (1969), *Fundamental natural frequencies of circular sandwich plates*, AIAA Journal, 7(12): 2353-2355.
15. Mirza, S., Singh, A.V. (1974), *Axisymmetric vibration of circular sandwich plates*, AIAA J, 12(10): 1418-1420. doi: 10.2514/3.49501
16. Prasad, C., Gupta, A.P. (1979), *Asymmetric vibration of circular sandwich plates*, Ind. J Pure Appl. Math. 10(8): 1002-1008.
17. Jain, S.K., Gupta, U.S., Lal, R. (2001), *Axisymmetric vibrations of circular sandwich plates with core of linearly varying thickness*, In: Indo-US workshop on 'Advances in elastic vibrations and smart structures', pp.43-50, UOR, Roorkee, Phoenix Ltd.
18. Jain, S.K., Vibration of non-uniform composite plates. PhD Thesis, University of Roorkee, 1993.
19. Guojun, D., Yingjie, C. (1996), *Further study on large amplitude vibration of circular sandwich plates*, Appl. Math. Mech. 17(11): 1087-1094. doi: 10.1007/BF00119957
20. Guojun, D., Huijian, L. (2000), *Nonlinear vibration of circular sandwich plate under the uniformed load*, Appl. Math. Mech. 21(2): 217-226. doi: 10.1007/BF02458523
21. Starovoitov, E., Leonenko, D., Yarovaya, A. (2003), *Circular sandwich plates under local impulsive loads*, Int. Appl. Mech. 39(8): 945-952. doi: 10.1023/A:1027464715958
22. Zhou, D., Stronge, W.J. (2006), *Modal frequencies of circular sandwich panels*, J Sandwich Struct. Mater. 8(4): 343-357. doi: 10.1177/1099636206063501
23. Xu, R.Q. (2008), *Three-dimensional exact solutions for the free vibration of laminated transversely isotropic circular, annular and sectorial plates with unusual boundary conditions*, Archive Appl. Mech. 78(7): 543-558. doi: 10.1007/s00419-007-0177-2
24. Shariyat, M., Alipour, M.M. (2012), *A zigzag theory with local shear correction factors for semi-analytical bending modal analysis of functionally graded viscoelastic circular sandwich plates*, J Solid Mech. 4(1): 84-105.
25. Lal, R., Rani, R. (2015), *On radially symmetric vibrations of circular sandwich plates of non-uniform thickness*, Int. J Mech. Sci. 99: 29-39.
26. Lal, R., Rani, R. (2016), *On the use of differential quadrature method in the study of free axisymmetric vibrations of circular sandwich plates of linearly varying thickness*, J Vibr. Control, 22(7): 1729-1748. doi: 10.1177/1077546314544695
27. Whitney, J.M., Sun, C.T. (1973), *A higher order theory for extensional motion of laminated composites*, J Sound Vibr. 30(1): 85-97. doi: 10.1016/S0022-460X(73)80052-5
28. Lo, K.H., Christensen, R.M., Wu, E.M. (1977), *A high-order theory of plate deformation - Part 1: Homogeneous plates*, J Appl. Mech. 44(4): 663-668. doi: 10.1115/1.3424154
29. Reddy, J.N. (1984), *A refined nonlinear theory of plates with transverse shear deformation*, Int. J Solids Struct. 20(9-10): 881-896. doi: 10.1016/0020-7683(84)90056-8
30. Reddy, J.N. (1984), *A simple higher-order theory for laminated composite plates*, J Appl. Mech. 51(4): 745-752. doi: 10.1115/1.3167719
31. Hanna, N.F., Leissa, A.W. (1994), *A higher order shear deformation theory for the vibration of thick plates*, J Sound Vibr. 170(4): 545-555. doi: 10.1006/jsvi.1994.1083
32. Shi, G. (2007), *A new simple third-order shear deformation theory of plates*, Int. J Solids Struct. 44(13): 4399-4417. doi: 10.1016/j.ijsolstr.2006.11.031
33. Mantari, J.L., Oktem, A.S., Soares, C.G. (2012), *A new higher order shear deformation theory for sandwich and composite laminated plates*, Compos. Part B: Eng. 43(3): 1489-1499. doi: 10.1016/j.compositesb.2011.07.017
34. Rohwer, K. (1992), *Application of higher order theories to the bending analysis of layered composite plates*, Int. J Solids Struct. 29(1): 105-119. doi: 10.1016/0020-7683(92)90099-F
35. Ma, L.S., Wang, T.J. (2004), *Relationships between axisymmetric bending and buckling solutions of FGM circular plates based on third-order plate theory and classical plate theory*, Int. J Solids Struct. 41(1): 85-101. doi: 10.1016/j.ijsolstr.2003.09.008
36. Hosseini-Hashemi, S., Es'haghi, M., Taher, H.R.D., Fadaie, M. (2010), *Exact closed-form frequency equations for thick circular plates using a third-order shear deformation theory*, J Sound Vibr. 329(16): 3382-3396. doi: 10.1016/j.jsv.2010.02.024
37. Leung, A.Y.T., Niu, J., Lim, C.W., Song, K. (2003), *A new unconstrained third-order plate theory for Navier solutions of symmetrically laminated plates*, Comp. Struct. 81(26-27): 2539-2548. doi: 10.1016/S0045-7949(03)00290-6
38. Saidi, A.R., Rasouli, A., Sahraee, S. (2009), *Axisymmetric bending and buckling analysis of thick functionally graded circular plates using unconstrained third-order shear deformation plate theory*, Comp. Struct. 89(1): 110-119. doi: 10.1016/j.compstruct.2008.07.003
39. Sahraee, S., Saidi, A.R. (2009), *Axisymmetric bending analysis of thick functionally graded circular plates using fourth-order shear deformation theory*, Euro. J Mech.-A/Solids, 28(5): 974-984. doi: 10.1016/j.euromechsol.2009.03.009
40. Najafizadeh, M.M., Heydari, H.R. (2008), *An exact solution for buckling of functionally graded circular plates based on higher order shear deformation plate theory under uniform radial compression*, Int. J Mech. Sci. 50(3): 603-612. doi: 10.1016/j.ijmecsci.2007.07.010
41. Bisadi, H., Es'haghi, M., Rokni, H., Ilkhani, M. (2012), *Benchmark solution for transverse vibration of annular Reddy plates*, Int. J Mech. Sci. 56(1): 35-49. doi: 10.1016/j.ijmecsci.2011.12.007
42. Nguyen-Xuan, H., Thai, C.H., Nguyen-Thoi, T. (2013), *Isogeometric finite element analysis of composite sandwich plates using a higher order shear deformation theory*, Compos. Part B: Eng. 55: 558-574. doi: 10.1016/j.compositesb.2013.06.044
43. Ansari, R., Torabi, J., Hasrati, E. (2018), *Axisymmetric nonlinear vibration analysis of sandwich annular plates with FG-CNTRC face sheets based on the higher-order shear deformation plate theory*, Aerosp. Sci. Technol. 77: 306-319. doi: 10.1016/j.ast.2018.01.010