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VIBRATION STUDY INDUCED BY INTERNAL FLOW IN PIPELINES UNDER DIFFERENT PARAMETERS WITH A NUMERICAL ASPECT

STUDIJA O VIBRACIJAMA NASTALIM USLED PROTOKA U CEVOVODIMA POD RAZLIČITIM PARAMETRIMA SA NUMERIČKIM ASPEKTOM

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- fluid conveying pipe
- foundation
- FEM
- Matlab
- instability

Ključne reči
- cevovod za transport tečnosti
- temelj
- MKE
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- nestabilnost

Abstract

In this paper, the natural frequencies of a fluid conveying pipe and critical velocities are obtained using standard finite element method (FEM). Finite element beam type with two degrees of freedom per node was used. The natural frequencies of our system are calculated by using a program developed in MATLAB. The results are compared with those predicted by the differential transformation method (DTM), and with other results listed in the literature, where several examples are studied, for pipes with different boundary conditions: pinned-pinned and clamped-pinned. We determine the influence of the physical and geometrical parameters of the proper frequencies and the critical velocity for fluid conveying pipe to study and analyse instability with its concepts.

Izvod

U ovom radu prirodne frekvencije cevovoda za transport tečnosti i kritične brzine dobijene su standardnom metodom konačnih elemenata (MKE). Korišćen je tip snopa konačnih elemenata s dva stepena slobode po čvoru. Prirodne frekvencije ovog sistema izračunavaju se pomoću programa razvijenog u MATLAB-u. Rezultati se upoređuju s onima predviđenim metodom diferencijalne transformacije (MDT) i sa ostalim rezultatima navedenim u literaturi, gde je proučavano nekoliko primera za cevi sa različitim graničnim uslovima: zakovane i zakovane-ukljestene. Utvrđen je uticaj fizičkih i geometrijskih parametara odgovarajućih frekvencija i kritične brzine protoka cevovoda za transport tečnosti u cilju proučavanja i analize koncepta nestabilnosti.

INTRODUCTION

The pressure pulsations and mechanical vibrations in pipe systems may cause excessive noise and may even lead to damage of piping or machinery. The excitation mechanism can be hydraulic or mechanical /1/. In fluid-filled pipe systems pulsations and vibrations will be strongly coupled. The elastic fluid coupling forces depend on the relative movement of the structure, it gives coupling effects from mass, stiffness, damping, the coupling can cause dynamic instability by negative damping, and one then has a fluid-elastic instability. We will be particularly interested in the case of a pipe with an internal flow, see /2-7/. The first works on the subject are however those of Bourrières /8/, who obtained the linear equations of motion and made experimental observation of the oscillations of a cantilevered pipe.

The effect of internal fluid on free vibration of a pipe was studied by /9/. Dahmane and al. /10/ have studied the effect of Coriolis force of the internal fluid of pipeline by analytical approach using Galerkin method. There are others who used analytical method to study dynamic of pipe with internal fluid under different parameters as, differential quadrature method /11/, differential transformation method /12/, and such a generalized integral transform technique /13/. Independently analytical methods, numerical methods are very effective and faster to treat a physical problem of vibration under internal flow, such as finite element method /14-17/.

All these studies did not address the issue of the system stability, except what we find in Doaré's studies /18-21/; they have calculated critical velocity of liquid under the effect of physical and geometrical parameters of the system, and then study the two aspects of instability. Nevertheless, all these works do not take into account the fluid whether Newtonian or not, the effect of the parameter different on dynamic instability with different boundary conditions, study the margin (range) of static and dynamic instability by the numerical approach.

In the present study calculation methods have been developed for the analysis of vibrations in fluid-filled pipe systems. The analytical model is based on the Newtonian approach. The practicability of the calculation model and the effects of fluid-structure interaction are illustrated by calculations for some simple systems, for pipes with different boundary conditions pinned-pinned and clamped-pinned. The numerical methods were developed, modelling of solid-fluid was conducted by the standard finite element method; finite element beam type with two degrees of freedom per node was used. The frequencies of the system are calculated using a program developed on MATLAB language. After studying the convergence and validated program with /12/, several examples were studied. The study of these examples enabled us to determine the influence of these physical and geometrical parameters of the natural frequencies, and consequently their stability.

DERIVATION OF GOVERNING DIFFERENTIAL EQUATION

The problem to be considered is the vibration analysis of a fluid conveying pipe system on an elastic foundation. The derivation of the equation is based on Euler–Bernoulli elementary beam theory. The physical model of conveying pipe carrying fluid is shown in Fig. 1a. Figure 1b shows forces on fluid element while, Fig. 1c shows forces and moment of pipe element.

The pipe is long and straight L conveying an incomepressible fluid with steady speed U; the motions are small δs.

The pipe rests on an elastic foundation Winkler-model soil of modulus KX, m_s and m_f the masses per unit length of the pipe and the fluid, respectively. The boundary conditions are,

(a) Pinned-Pinned Pipe

$$Y|_{X=0} = \frac{\partial^2 Y}{\partial X^2}|_{X=0} = Y|_{X=L} = \frac{\partial^2 Y}{\partial X^2}|_{X=L} = 0 \quad (1)$$

(b) Clamped-Pinned Pipe

$$Y|_{X=0} = \frac{\partial Y}{\partial X}|_{X=0} = Y|_{X=1} = \frac{\partial^2 Y}{\partial X^2}|_{X=1} = 0 \quad (2)$$

The equation for conveying pipe-carrying fluid on a Winkler elastic foundation is given as /20/,

$$EI\frac{\partial^4 Y}{\partial X^4} + m_f U^2 \frac{\partial^2 Y}{\partial X^2} + 2 m_f U \frac{\partial^2 Y}{\partial X \partial T} + \left(m_s + m_f\right)\frac{\partial^2 Y}{\partial T^2} + KY = 0 \quad (3)$$

FINITE ELEMENT DISCRETIZATION

The Eq.(3) is a fourth-order partial differential equation in two independent variables subject to various boundary conditions. It is not easy to get its analytical solution, but through the use of finite element method we get its numerical solution. The equation of element deflection for straight two dimensional beam elements could have the form /23/,

$$W\left(X,T\right) = \sum_{i=1}^{N} N_i\left(X\right) W_i\left(T\right), \quad (4)$$

where: $[N_i]$ represent the shape function; $W_i(T)$ is the function which represents the shape of the displacements and rotations at nodes (the generalized coordinates).

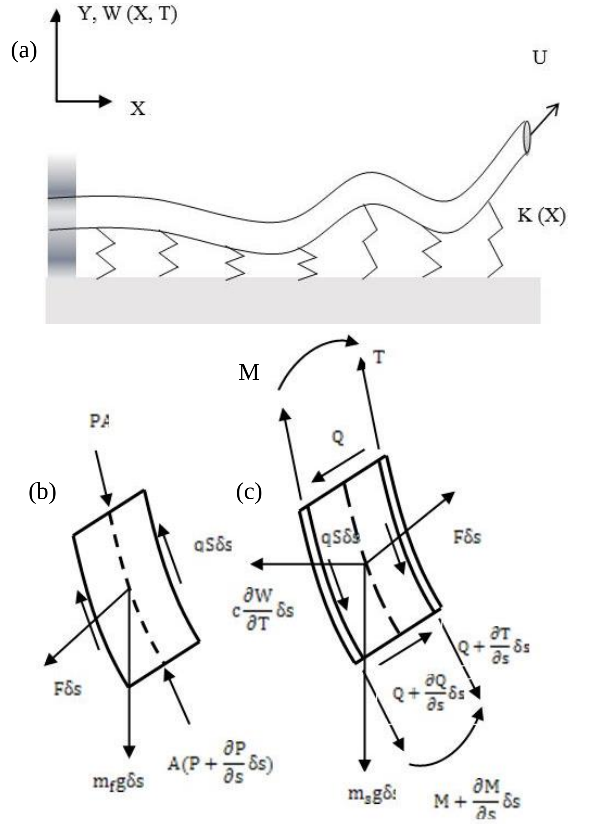


Figure 1. a) Representation of the pipe-conveying fluid resting on an elastic Winkler-type; b) forces on fluid element; c) forces and moments on pipe element δs /22/.

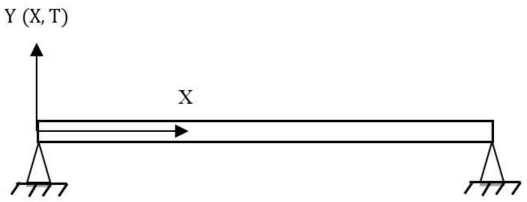


Figure 2. Pinned-pinned pipe.

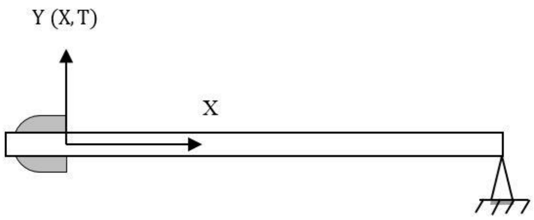


Figure 3. Clamped-Pinned Pipe

Figure 4. Beam element nodal displacements.

Therefore, Eq.(4) becomes

$$W(X,T) = N_1(X)W_1(T) + N_2(X)\theta_1(T) + \\ + N_3(X)W_2(T) + N_4(X)\theta_2(T) \qquad (5)$$

and

$$\theta(X,T) = N_1'(X)W_1(T) + N_2'(X)\theta_1(T) + \\ + N_3'(X)W_2(T) + N_4'(X)\theta_2(T) \qquad (6)$$

Determination of the element matrices

By using the energy principle. The potential (deformation) energy and the kinetic energy of the solid element can be expressed by /24-26/,

$$V_s = \frac{1}{2}\int_0^L EI\left(\frac{d^2W}{dX^2}\right)^2 dX\,, \qquad (7)$$

$$T_s = \frac{1}{2}\int_0^L m_s \frac{d^2W}{dT^2} dX\,. \qquad (8)$$

The kinetic energy of the fluid element can be expressed by /15/,

$$T_f = \frac{1}{2}\int m_f \left(U\frac{dW}{dX} + \frac{dW}{dT}\right)^2 dX\,. \qquad (9)$$

The potential energy over the length of elastic foundation can be expressed by /16/,

$$V' = \frac{1}{2}\int_0^L KW^2 dX\,. \qquad (10)$$

The different elementary matrices can be represented as follows,

$$[K_s] = \frac{m_f U^2}{30L}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \qquad (11)$$

$$[K_f] = \frac{m_f U^2}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & 3L^2 & -3L & 4L^2 \end{bmatrix}, \qquad (12)$$

$$[M] = \frac{(m_s + m_f)L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \qquad (13)$$

$$[C] = \frac{2m_f U}{30}\begin{bmatrix} -30 & 6L & 30 & -6L \\ -6L & 0 & 6L & -L^2 \\ -30 & -6L & 30 & 6L \\ 6L & L^2 & -6L & 0 \end{bmatrix}, \qquad (14)$$

$$[F] = \frac{KL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \qquad (15)$$

where: [K_s], [K_f], [M], [C] and [F], are respectively, the stiffness (structure and fluid), the masses, the damping, and the foundation matrices of the system, /17/.

Analysis of dynamic eigenvalues

Application of the Lagrange principle,

$$\frac{d}{dT}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial \dot{q}_i} + \frac{\partial V}{\partial q_i} = 0\,. \qquad (16)$$

The standard equation of motion in the finite element form is,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + ([K])\{q\} = 0\,, \qquad (17)$$

where: $[M] = [M_s] + [M_f]$, $[K] = [K_s] - [K_f]$.

The governing equation of the system (structure plus fluid) can be transformed into its state-space coordinates,

$$E\dot{z} + Gz = 0\,, \qquad (18)$$

where the state variable is,

$$z = \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix}. \qquad (19)$$

The matrices [E] and [G] are calculated through variable change as the following,

$$E = \begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}, \qquad (20)$$

$$G = \begin{bmatrix} C & K \\ -K & 0 \end{bmatrix}. \qquad (21)$$

Therefore, we can obtain the natural frequencies (eigenvalues) and mode shapes (eigenvectors) by solving the mathematically well-known characteristic equation of,

$$\lambda I - Hz = 0\,, \qquad (22)$$

where: λ is eigenvalues of the system and I is a unity matrix and,

$$H = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}. \qquad (23)$$

The solution of Eq.(22) can be written in the following form,

$$\{q\} = \{E\}.\exp(\lambda t)\,, \qquad (24)$$

$$z = \begin{Bmatrix} \lambda\{E\} \\ \{E\} \end{Bmatrix}\exp(\lambda t) = \{\tilde{E}\}\exp(\lambda t)\,. \qquad (25)$$

We obtain a homogeneous equation, which corresponds to a generalized eigenvalue problem of our system,

$$\left\{\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} - \lambda\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\right\}\begin{bmatrix} \lambda\{E\} \\ \{E\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \qquad (26)$$

We can compute the eigenvalues numerically from Eq.(26) and obtain the eigenfrequencies of conveying pipe carrying fluid for different various parameter values. The eigenvalues are complex,

$$\lambda^m = Re^m + j\omega^m\,, \qquad (27)$$

where: λ^m is the complex eigenvalue; Re is real part of eigenfrequencies; and the imaginary part of these roots represents the natural frequencies of damped system and m = 1, 2, ….N; j = √(-1). The critical flow velocity u_{cr} is characterized by max ($Re^m = 0$), The characteristic roots *m* are obtained here by using the (eigen) function of MATLAB. Using the non-dimensional parameters /22/, we obtain,

$$\beta = \frac{m_s}{m_f + m_s}, k = \frac{KL^4}{EI}, \Omega = \left(\frac{m_f + m_s}{EI}\right)^{1/2} \omega L^2,$$

$$u = \left(\frac{m_f}{EI}\right)^{1/2} UL^2$$

RESULTS AND DISCUSSION

In the current work, we rely on calculating the critical fluid velocity to study and analyse instability with its concepts. Results will be discussed for various values of β, length L, elastic foundation k (Winkler type) for pipes with different boundary conditions. Because the problem is very ramified, we use incompressible fluid, and the physical parameters as,

- Elastic modulus of pipe (211 GPa);
- Pipe length (1-2 m);
- Fluid density (1000 kg/m^3);
- Pipe density (7850 kg/m^3);
- Pipe thickness (β = 0.1- 0.5);
- Outer diameter of the pipe (0.03 m).

Pinned-pinned pipe with internal flow

The object of this section is the determination of proper frequencies for fluid conveying pipe without foundation. First, the validation of our program was made by doing a convergence study, convergence was performed for a velocity U = 100 m/s, see Fig. 5a, another study for critical velocity, where U = 175 m/s, the results obtained are shown in Fig. 5b.

Figure 5a shows that there is very fast convergence for the first two modes according to the number of elements and that for two different fluid velocities. Convergence is obtained for the third mode with 13 elements. On the other hand, the numerical results are given and compared with those obtained by DTM /12/ for pinned-pinned pipe with internal flow; the results obtained numerically are similar to those obtained by the analytical approach /12/.

Figures 7 and 8 represent the first eigen-modes of pipe on simple supports for different mass ratios: a) dimensionless frequencies; b) natural frequencies (Hz). It appears clearly on these figures that the mass ratio influences the first modes and consequently on the critical velocity and the stability of our system, these figures clearly show the distinction between the eigen-modes and the combined modes. We notice that the third critical speed is 9.44; this is not what we found in the previous literatures.

Figure 9 (physical results) and Fig. 10 (non-dimensional frequency) show the natural frequencies as a function of the fluid velocity for different length with two mass ratio. We observe in Fig. 9 that the critical speed is 170.27 m/s for a length of 1 m and 110 m/s for 2 m, we also note that the previous studies have not addressed the effects of these parameters; obviously, the length of the pipe has destabilizing effect on the vibration of the system.

Figure 10 shows the field of instability, where the flow velocity is critical or rather the pulsation of the system is zero, as we note that this instability margin changes according to length, which affects the stability of both types, we will explain in what follows this development in detail.

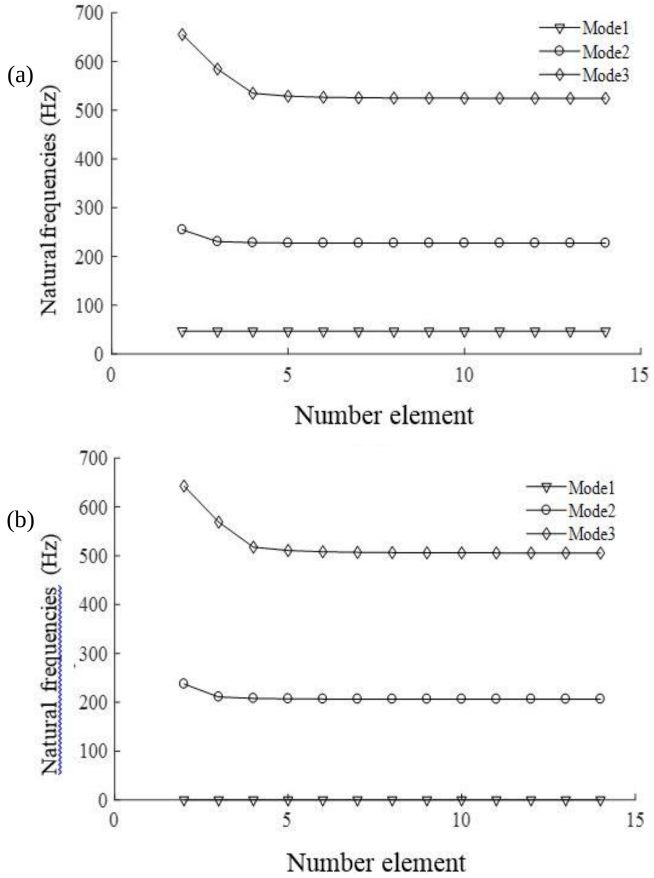


Figure 5. Convergence of the first three natural frequencies of pinned-pinned pipe: a) U = 100 m/s; b) U = 175 m/s, β = 0.5.

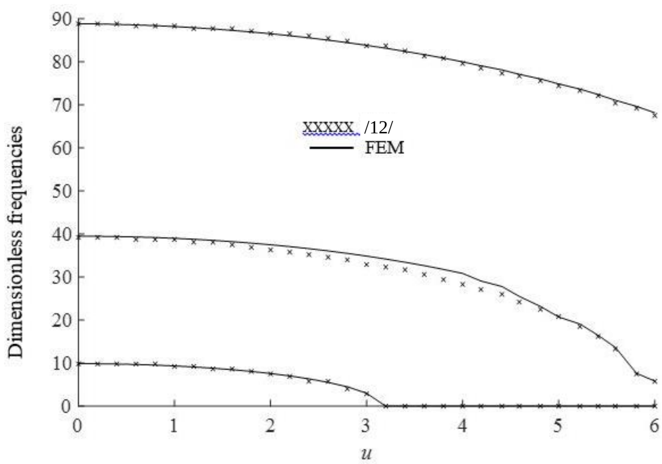


Figure 6. Dimensionless frequency for various values of u, for the lowest three modes of a pinned-pinned pipe conveying fluid, comparison DTM /12/ (xxx) and FEM (----), β = 0.1.

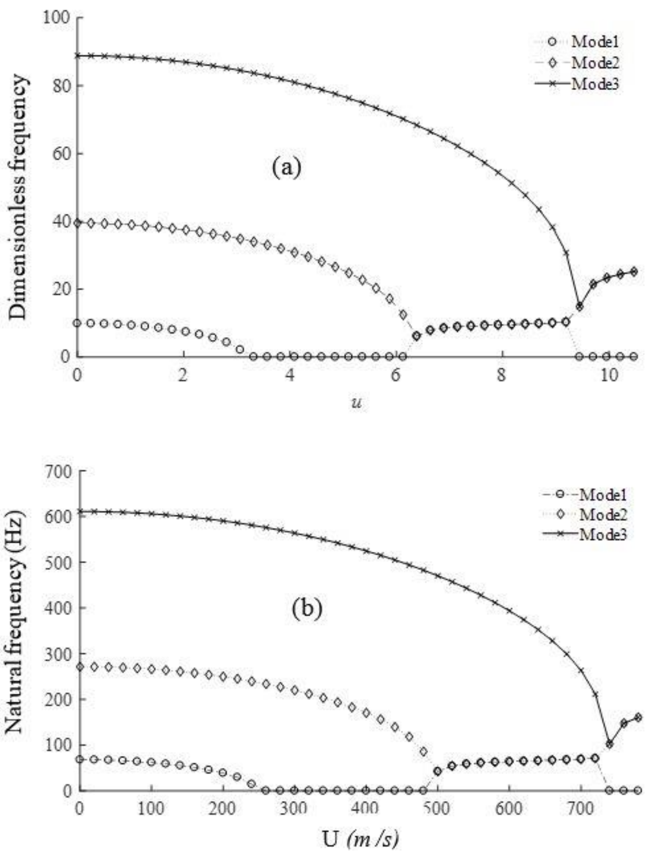


Figure 7. Three proper modes on fluid velocity function of pinned-pinned pipe conveying fluid, β = 0.3: a) dimensionless frequencies; b) naturel frequencies (Hz).

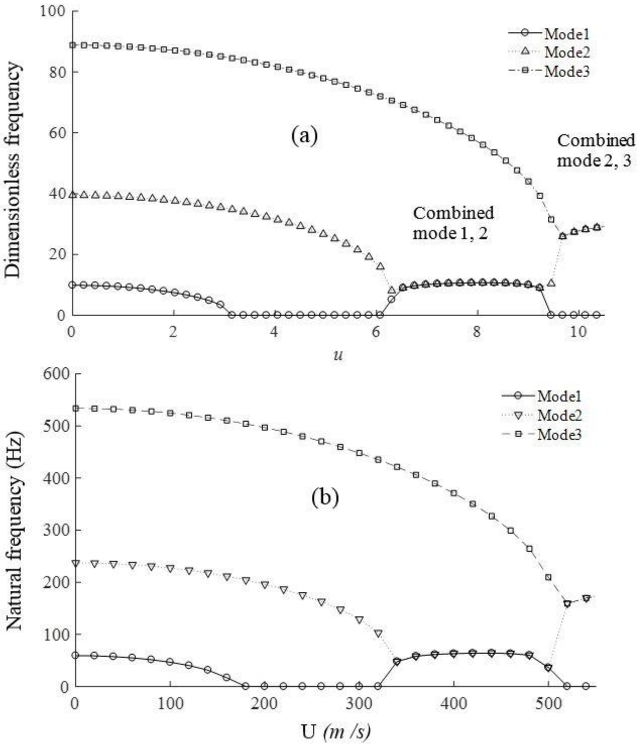


Figure 8. Three proper modes on fluid velocity function of pinned-pinned pipe conveying fluid, β = 0.5: a) dimensionless frequencies; b) naturel frequencies (Hz).

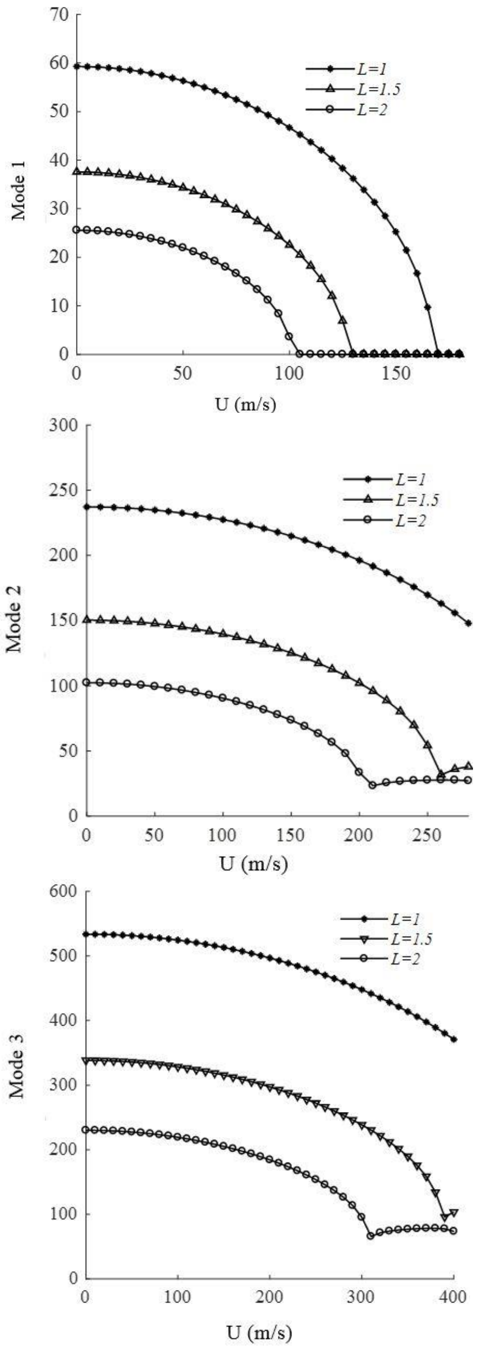
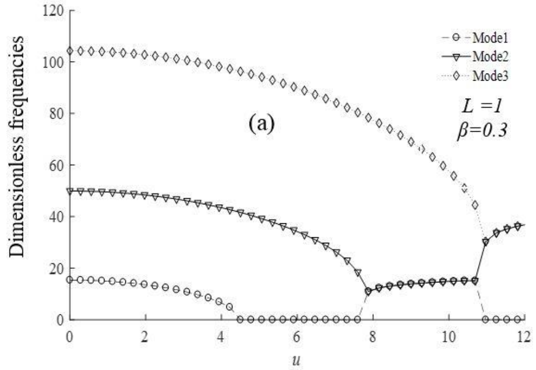


Figure 9. Effect of length on the natural frequency of the pinned-pinned pipe at different fluid velocities, β = 0.5



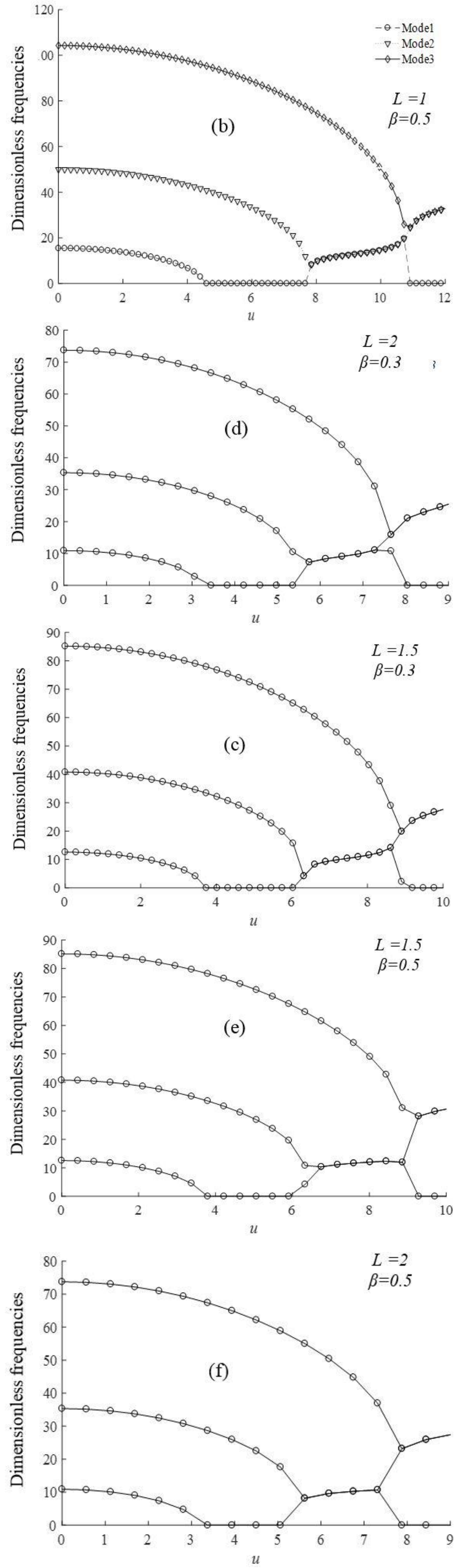


Figure 10. Effect of length on the natural frequency of the pinned-pinned pipe at different fluid velocities: a) for β = 0.3 and L = 1, b) for β = 0.5 and L = 1; c) for β = 0.3 and L = 1.5; d) for β = 0.3 and L = 2; e) for β = 0.5 and L = 1.5; f) for β = 0.5 and L = 2.

Clamped-pinned pipe with internal flow

In this section, the determinations of parameters frequencies for fluid conveying pipe, without and with foundation are calculated using the FEM. Beginning, the convergence was performed for a velocity U = 150 m/s, the results obtained are shown in Fig. 11, convergence is obtained for the three modes with 13 elements.

Figure 12 presents the natural frequency of the pipe at different fluid velocities for β = 0.3 and β = 0.5. Over an interval [0 12], we notice almost same result and same instability range.

Figure 13 shows the evaluation of these modes as a function of the speed of the fluid for different lengths L for two β. It appears in this figure that the increase in β implies a reduction in the thickness, that is to say a gain in the mass of the empty pipe, this increase has no great influence on the first mode while its influence the higher modes. For low velocity it is a gain, but at high velocity its effect is destabilizing.

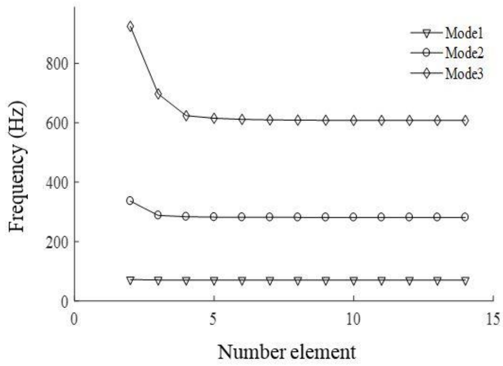


Figure 11. Convergence of the first three natural frequencies of clamped-pinned pipe, U = 150 m/s, β = 0.5.

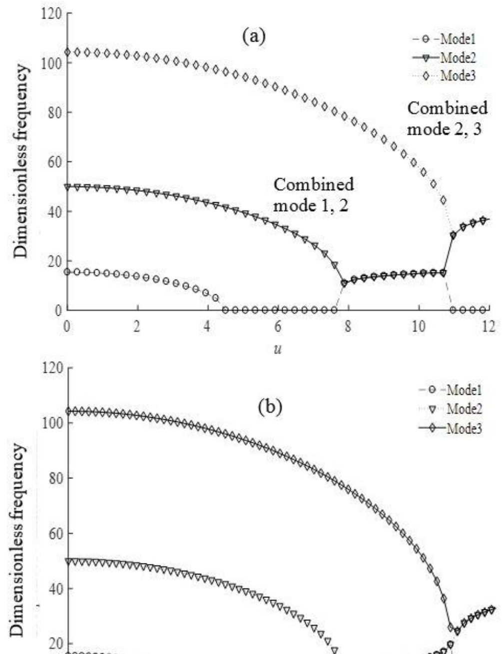
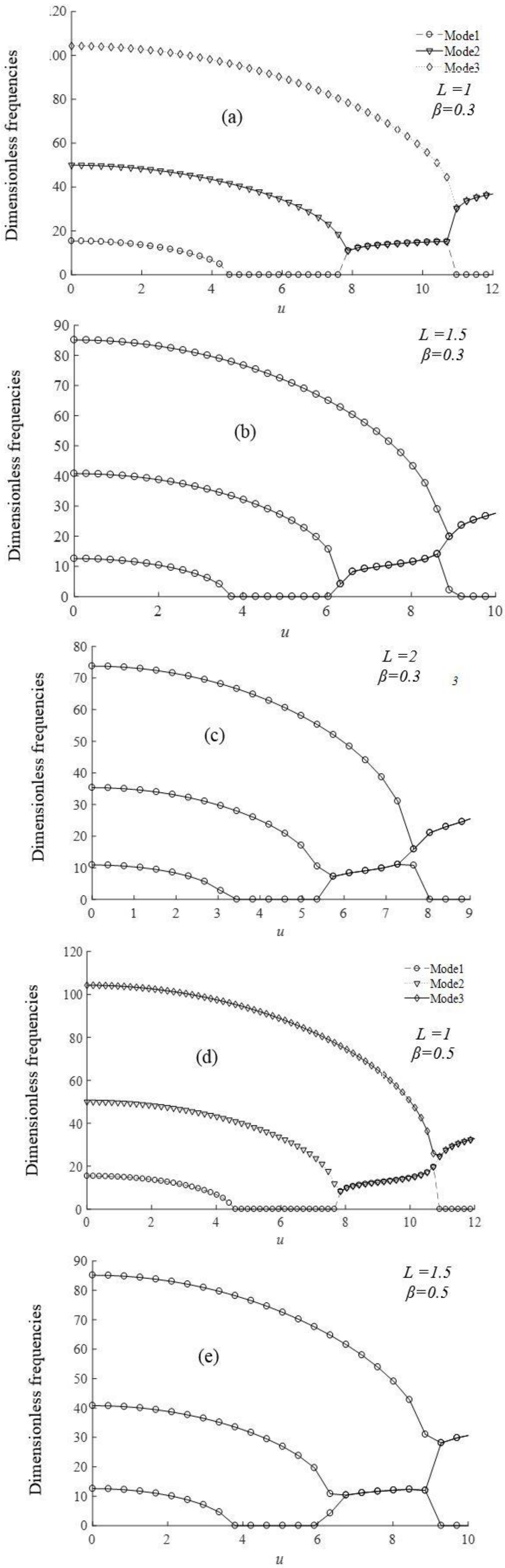


Figure 12. Three proper modes on fluid velocity function of clamped-pinned pipe conveying fluid: a) β = 0.3; b) β = 0.5.



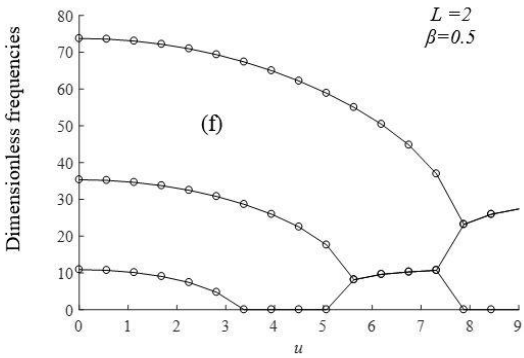
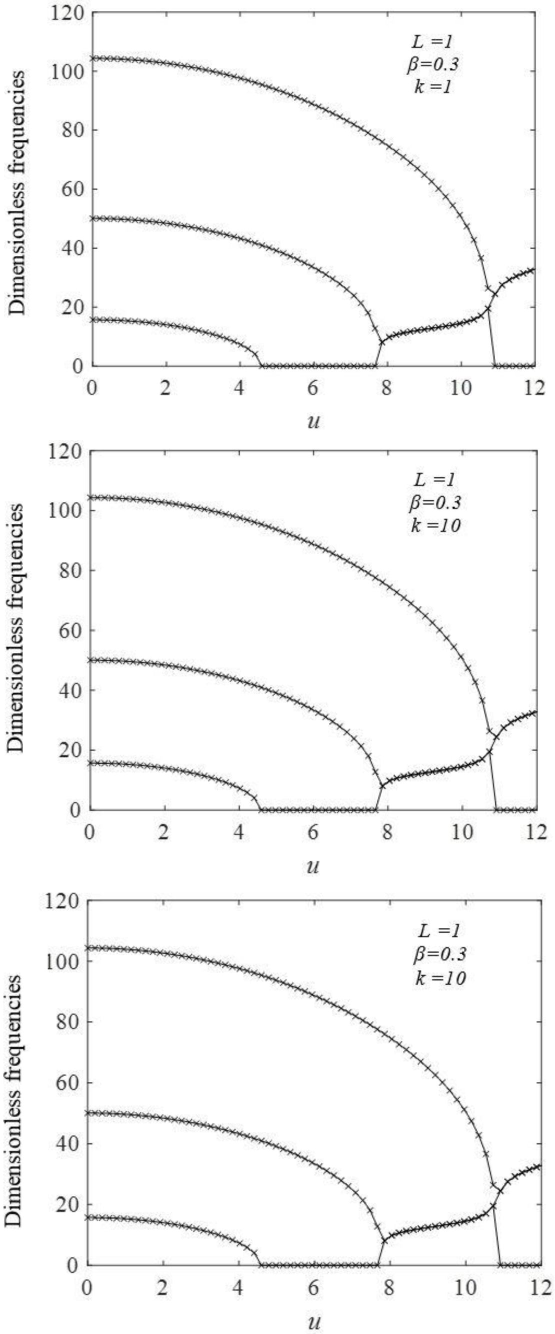
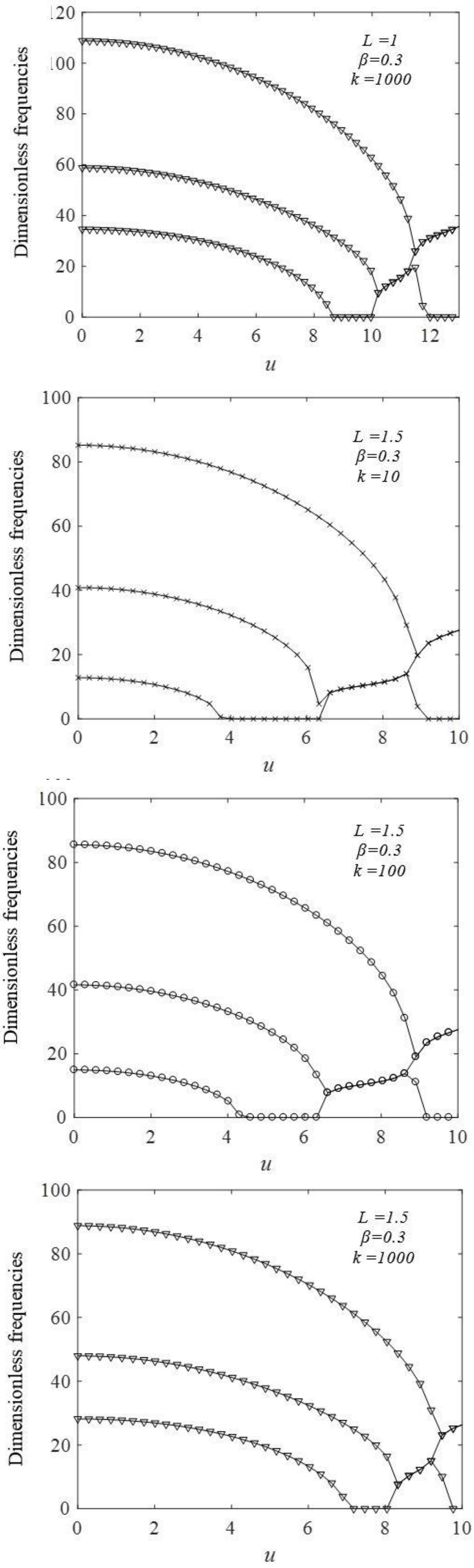


Figure 13. Effect of length on the natural frequency of clamped-pinned pipe at different fluid velocities: a) for β = 0.3 and L = 1; b) for β = 0.3 and L = 1.5; c) for β = 0.3 and L = 2; d) for β = 0.5 and L = 1; e) for β = 0.5 and L = 1.5; f) for β = 0.5 and L = 2.





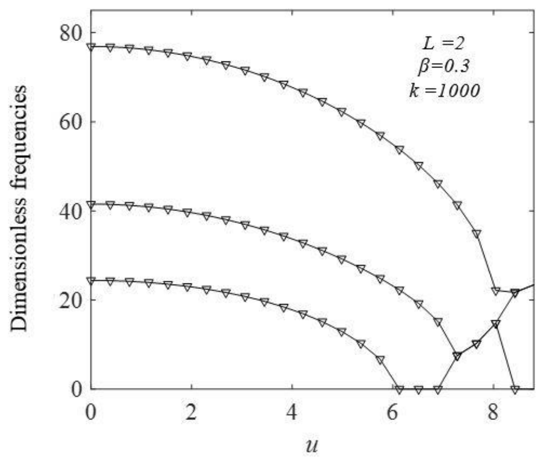


Figure 14. Effect of foundation stiffness on the natural frequency of the clamped-pinned pipe at different fluid velocities, $\beta = 0.3$.

The length has an effect of reducing the rigidity, which lowers the frequencies of the system according to the speed of fluid and consequently quickly reach the first critical velocity of static instability. For L = 1 m, the instability range is equal 3.55, is reduced to 1.96 for L = 2 m. The effect of the elastic is stabilizing for the system, as shown in Fig. 14, and the length weakens the rigidity of the system and therefore, has a destabilizing effect. In addition we note that the instability range of the first mode is reduced as a function of the stiffness. For the parameter k = 1, and L = 1, the instability range is equal to 3.11, the range is 1.33 for the parameter k = 1000, see Fig. 14.

CONCLUSIONS

We have studied in this work the free vibration of pipe transporting a fluid for different boundary conditions. The numerical aspect with the finite method gives solutions in a complex plane by determining the eigen modes, the numerical results are finally combined with the semi-analytic results to determine the different characteristics of instability of each system. The first observation that we can make that the natural frequencies of the system weight the velocity of the fluid. We observe that instability appears when the velocity exceeds a threshold called critical velocity of instability, when the first frequency is zero. According to the first two cases, we note the distinction between eigen modes and combined modes (first mode, combination between the first and second, second mode, combination between the second and third, third mode. We have noticed that increasing β slightly decreases the rigidity of the system (loss of rigidity) and the system consequently decreases their natural frequencies. The typical elastic foundation of Winkler increases the rigidity of the system and consequently the natural frequencies and the critical velocity. What distinguishes most of this research from others is its discussion of the axis of instability and what it means in this field that is why we did some analysis and calculation in this research, hoping to continue with other work in the same field.

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