Gaurav Verma[1], Devinder S. Pathania[2]

THERMAL CREEP STRESS ANALYSIS OF FUNCTIONALLY GRADED SPHERICAL SHELL UNDER INTERNAL AND EXTERNAL PRESSURE

ANALIZA TERMIČKIH NAPONA PUZANJA KOD SFERNE LJUSKE OD FUNKCIONALNOG KOMPOZITA PRI UNUTRAŠNJEM I SPOLJAŠNJEM PRITISKU

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 17.3.2020

Adresa autora / Author's address:
[1] Hans Raj Mahila Maha Vidyalaya, Jalandhar, Punjab, India email: gk_gaurav35@yahoo.com
[2] Guru Nanak Dev Eng. College (GNDEC), Ludhiana, India

Keywords
- creep
- stresses
- thermal
- spherical shell
- pressure

Ključne reči
- puzanje
- naponi
- termički
- sferna ljuska
- pritisak

Abstract

Thermal creep analysis of spherical shell made up of functionally graded material is done under influence of internal and external pressure. The strain measures are used in generalized form to solve complex situation of creep in shell. Creep stresses are examined along the internal and external part of shell by using generalized strain measure. The influence of linear to nonlinear measure on shell is shown. The results are numerically derived and shown graphically.

Izvod

Izvedena je analiza termičkih napona puzanja kod sferne ljuske od funkcionalnog kompozitnog materijala, opterećena unutrašnjim i spoljašnjim pritiscima. Primenjene su mere deformacija u generalisanom obliku radi rešavanja složenog stanja puzanja u ljusci. Naponi puzanja su proračunati duž unutrašnjeg i spoljašnjeg dela ljuske, primenom generalisane mere deformacija. Prikazan je uticaj linearne i nelinearne mere deformacija na ljusku. Rezultati su dobijeni numeričkom obradom i predstavljeni su grafički.

INTRODUCTION

Shells are the most difficult form of structure to analyse and the form with the most complex behaviour. As shell is the most efficient way of using the material, it can be very useful in case of storage of fluids and solids. The various techniques of producing spherical shell with different materials with less expenditure and better life have been developed. These structures formed of compressible and incompressible materials are expected to find various industrial applications. For example, spherical shell can be used as inertial confinement fusion targets; as a principal component in lightweight structural materials for nuclear space stations; as containers for phase-change heat-storage chemical media; as containers for hazardous materials; employed as catalytic surface agents. In various situations, spherical shells have to work under highly warmed and pressurized environments causing significant creep and thus reducing its service life. Therefore, the analysis of thermal creep stresses and strains in shells under influence of pressure is very important due to its various applications. In order to work under severe mechanical conditions, functionally graded materials are widely used in the engineering field. The FGMs have smooth grading of components that gives better thermal properties, higher fracture toughness, improved residual stress distribution and reduced stress intensity factors. These properties of FGMs allow shells to withstand high pressure and thermal environment, /1, 2/.

Zhang et al. /3/ have done elastic analysis of functionally graded spherical shell under influence of internal pressure. Circumferential stresses are obtained in pressurized vessel made up of composite material. Moosaie et al. /4/ performed nonlinear thermal analysis of functionally graded spherical shell. The perturbation technique is used to analytically solve heat conduction equation. Thakur et al. /5/ worked on thermal creep problem of non-homogeneous shell. Shariyat et al. /6/ worked on the rates of creep strains and stresses of FGM shell under effect of the hygrothermal degradation of material properties. Norton's creep law is used to solve time dependent governing equations of pressurized shell. Gupta et al. /7/ described steady state creep behaviour of shell made up of composite material. Governing equations are solved by establishing well known Von-Mises yield criterion. The tangential, radial stresses, as well as corresponding strain rates are derived in shell. Pathania et al. /8/ explained the problem of elastic-plastic transition in spherical shell under internal and external pressure. The nature of the stresses is depicted for different cases of internal and external pressure. Stresses are found to be tensile as well as compressive. Tian et al. /9/ explained the electromagnetic behaviour of shells made up of graded material along the thickness of shell. The effect of FGMs is seen on piezoelectric shell over homogeneous shells. In order to solve creep complexity in shell under influence of pressure and heat, authors assumed various semi-empirical laws based on Norton's creep law and Von Mises yield criteria. These assumptions are taken from classical theory of continuum mechanics which are based on classical measures of deformation. As creep the phenomenon occurring in the spherical shell is nonlinear in nature, so there exists a need of generalization of measures of deformation. Seth /10, 11/ has developed concept of strain measures in generalized form and transition theory of elastic-plastic and creep. In series of papers, Sharma et al. /12, 15/ worked on various problems of functionally graded thick-walled cylinder under internal and external pressure. The thermal creep stresses are examined in thick-walled cylinder subjected to pressure by using Seth transition theory. The recognition of transition state or mid-zone as a separate state is the main feature of transition theory. In this research paper, the concept of creep transition and generalized strain measure is used to compute thermal stresses in functionally graded spherical shell under combination of internal and external pressure.

Seth /11/ has defined the concept of generalized strain measures as

$$e_{ii}=\int_0^{e_{ii}^A}[1-2e_{ii}^A]^{\frac{n-2}{2}}\,de_{ii}^A=\frac{1}{n}\left[1-(1-2e_{ii}^A)\right]^{n/2},\ i=1,2,3 \quad (1)$$

where: n is the measure; and e_{ii}^A are Almansi finite strain components.

Non-homogeneity in the spherical shell has been taken as the compressibility of the material as

$$C=C_0r^{-k}, \quad (2)$$

where: $a\le r\le b$; C_0 and k are real positive constants.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

We consider a spherical shell of internal and external radii a and b, respectively, subjected to uniform internal and external pressure p_1 and p_2, correspondingly, with gradually increasing temperature Θ_0, applied to the internal surface $r = a$ of the spherical shell. The components of displacement in spherical coordinates (r, θ, ϕ) are given as

$$u=r(1-g),\quad v=0,\quad w=0, \quad (3)$$

where: u, v, w (displacement components); and $g = g(r)$.

Generalized components of strain are given by Seth /11/ as

$$e_{rr}=\frac{1}{n^m}\left[1-(rg'+g)^n\right]^m,\ e_{\theta\theta}=\frac{1}{n^m}\left[1-g^n\right]=e_{\phi\phi},$$

$$e_{r\theta}=e_{\theta\phi}=e_{\phi r}=0, \quad (4)$$

where: $g' = dg/dr$. (5)

Stress-strain relation: stress-strain relations for thermo-elastic isotropic material are given by Parkus, /13/:

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij}-\xi\Theta\delta_{ij},\ (i,j=1,2,3), \quad (6)$$

where: T_{ij} are stress components; λ and μ are Lame's constants; $I_1 = e_{kk}$ is the first strain invariant; δ_{ij} is Kronecker's delta; $\xi = \alpha(3\lambda + 2\mu)$; α being the coefficient of thermal expansion; and Θ is temperature. Further, Θ has to satisfy:

$$\nabla^2\Theta=0. \quad (7)$$

Substituting the strain components from Eq.(4) in Eq.(6), the stresses are obtained by taking $m = 1$ as:

$$T_{rr}=\frac{2\mu}{n}\left[1-(rg'+g)^n\right]+\frac{\lambda}{n}\left[3-(rg'+g)^n-2g^n\right]-\xi\Theta,$$

$$T_{\theta\theta}=T_{\phi\phi}=\frac{2\mu}{n}\left[1-g^n\right]+\frac{\lambda}{n}\left[3-(rg'+g)^n-2g^n\right]-\xi\Theta,$$

$$T_{r\theta}=T_{\theta\phi}=T_{\phi r}=0, \quad (8)$$

$$\frac{dT_{rr}}{dr}+\frac{2}{r}(T_{rr}-T_{\theta\theta})=0, \quad (9)$$

where: T_{rr} and $T_{\theta\theta}$ are the radial and hoop stress, respectively.

Boundary conditions: the temperature satisfying Laplace Eq.(7) with boundary conditions:

$$\Theta=\Theta_0\ \text{ and }\ T_{rr}=-p_1\ \text{ at }\ r=a$$
$$\Theta=0\ \text{ and }\ T_{rr}=-p_2\ \text{ at }\ r=b, \quad (10)$$

where: Θ_0 is constant, given by Parkus, /13/:

$$\Theta=\frac{\Theta_0\ln(r/b)}{\ln(a/b)}. \quad (11)$$

Critical points or turning points: using Eq.(8) in Eq.(9), we get a nonlinear differential equation in g as:

$$nQ(Q+1)^{n-1}g^{n+1}\frac{dQ}{dg}=r\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\Big[(3-2C)-g^n\{2(1-C)+$$
$$+(1+Q)^n\}\Big]-2C'r(1-g^n)-ng^nQ\{1(1-C)+(1+Q)^n\}+$$
$$+2Cg^n\left[1-(1+Q)^n\right]-\frac{nc\bar{\Theta}_0}{2\mu g^n}\left[\xi+r\xi'\log\frac{r}{b}\right], \quad (12)$$

where: $\bar{\Theta}_0=\Theta_0/\log(a/b)$; $C = 1\mu/(\lambda + 2\mu)$; and $rg' = gQ$.

Transition points or turning points of g in Eq.(12) are $Q \to -1$ and $Q \to \pm\infty$. Hereby, we are interested in finding creep stresses corresponding to $Q \to -1$.

ANALYTICAL SOLUTION THROUGH STRESS DIFFERENCE

It has been shown that thermal creep stresses and strain rates can be evaluated by taking the transition function through principal stress difference /14-18/ at the transition point $Q \to -1$. We define the transition function Ψ as:

$$\Psi=T_{rr}-T_{\theta\theta}=\frac{2\mu g^n}{n}\left[1-(Q+1)^n\right], \quad (13)$$

where: Ψ is a function of r only.

Taking the logarithmic differentiating of Eq.(13) with respect to r and substituting the value of dQ/dg from Eq.(12) and taking asymptotic value $Q \to -1$, after integration we get

$$\frac{d}{dr}(\log\Psi)=\frac{nQ}{r}+\frac{\mu'}{\mu}-\frac{r\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\Big[(3-2C)-g^n\{2(1-C)+}{rg^n\left[1-(1+Q)^n\right]}$$
$$\frac{+(1+Q)^n\}\Big]-2rC'(1-g^n)-ng^nQ\left[2(1-C)+(1+Q)^n\right]+}{rg^n\left[1-(1+Q)^n\right]}$$
$$\frac{+2Cg^n\{1-(1+Q)^n\}-\frac{nc\bar{\Theta}_0}{2\mu}\left[\xi+r\xi'\log\frac{r}{b}\right]}{rg^n\left[1-(1+Q)^n\right]}. \quad (14)$$

Taking the asymptotic value of Eq.(14) at $Q = -1$,

$$\frac{d}{dr}\log\Psi=\frac{3\mu'}{\mu}-\frac{2C'}{C}-\frac{3n}{r}+X, \quad (15)$$

where:

$$X=\frac{2(n-1)C}{r}-\frac{2C\mu'}{\mu}+\frac{2C'}{g^n}-\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\frac{(3-2C)}{g^n}+$$
$$+\frac{nC\bar{\Theta}_0}{2\mu rg^n}\left[\xi+r\xi'\log\frac{r}{b}\right].$$

On integrating Eq.(15), we get

$$\Psi=A_1\frac{\mu^3}{C^2r^{3n}}\exp m\,, \tag{16}$$

where: $m=\int Xdr$; and A_1 is constant of integration.

From Eqs.(13) and (16), we have

$$T_{rr}-T_{\theta\theta}=A_1\frac{2r\mu^3}{2C^2r^{3n+1}}\exp m=\frac{ArM}{2}\,, \tag{17}$$

where: $M=\dfrac{2\mu^3}{C^2r^{3n+1}}\exp m$.

Substituting the value of $T_{rr}-T_{\theta\theta}$ from Eq.(17) in Eq.(9) and integrating, we get

$$T_{rr}=A_2-A_1\int Mdr\,, \tag{18}$$

where: A_2 is a constant of integration and asymptotic value of g as $Q\to-1$ is D/r; D being a constant. The constants A_1 and A_2 can be obtained by using boundary conditions Eq.(10) in Eq.(18) as

$$A_1=\frac{p_2-p_1}{\int_a^b Mdr}\,,\quad A_2=-p_2+A_1\left[\int Mdr\right]\quad\text{at}\quad r=b. \tag{19}$$

Using the value of A_2 in Eq.(18), we get

$$T_{rr}=-p_2+\frac{(p_2-p_1)\int_r^b Mdr}{\int_a^b Mdr}\,. \tag{20}$$

The value of stress $T_{\theta\theta}$ is obtained from Eq.(17) by using Eq.(20),

$$T_{\theta\theta}=T_{\phi\phi}=T_{rr}-\frac{(p_2-p_1)rM}{\int_a^b Mdr}\,. \tag{21}$$

These are expressions for radial and hoop stresses in spherical shell under influence of internal and external pressure. Now we introduce the non-homogeneity in spherical shell due to variable compressibility as given in Eq.(2), the Eqs.(20) and (21) become

$$T_{rr}=-p_2+\frac{(p_2-p_1)\int_r^b M_1dr}{\int_a^b M_1dr}\,, \tag{22}$$

$$T_{\theta\theta}=T_{\phi\phi}=T_{rr}-\frac{(p_2-p_1)rM_1}{\int_a^b M_1dr}\,, \tag{23}$$

where

$$M_1=\frac{2\mu^3}{C^2r^{3n+1}}\exp m_1=\frac{r^{-(3n+k+1)}C_0\lambda^3}{4(1-C_0r^{-k})}\exp m_1\,,$$

$$m_1=-\frac{2(n-1)}{k}C_0r^{-k}-\frac{2kC_0r^{n-k}}{D^n(n-k)}+\frac{kC_0}{D^n}\int\frac{r^{n-k-1}(3-2C_0r^{-k})}{1-C_0r^{-k}}dr+$$
$$+2\log(1-C_0r^{-k})+\frac{\alpha n\bar{\Theta}_0}{D^n}\int(1-C_0r^{-k})\left[3+\frac{C_0r^{-k}}{1-C_0r^{-k}}-\right.$$
$$\left.-\frac{kC_0r^{-k}\log(r/b)}{1-C_0r^{-k}}\right]r^{n-1}dr\,,$$

$$\frac{\mu'}{\mu}=\frac{C'}{C(1-C)}\,,\quad C'=-kC_0r^{-k-1}\,.$$

Equations (22) and (23) show thermal creep stresses for a spherical shell made up of functionally graded material under internal and external pressure. We introduce the following non-dimensional components as: $R=r/b$, $R_0=a/b$, $S_{rr}=T_{rr}/E$, $S_{\theta\theta}=T_{\theta\theta}/E$, $P_2-P_1=(p_2-p_1)/E$, where: $E=2\mu(3-2C)/(2-C)$. Now, Eqs. (22) and (23) in non-dimensional form become

$$S_{rr}=-P_2+\frac{(P_2-P_1)\int_R^1 M_2dR}{\int_{R_0}^1 M_2dR}\,, \tag{24}$$

$$S_{\theta\theta}=S_{\phi\phi}=S_{rr}-\frac{(P_2-P_1)RM_2}{\int_{R_0}^1 M_2dR}\,, \tag{25}$$

where
$$M_2=\frac{(bR)^{-(3n+k+1)}C_0\lambda^3}{4\left[1-C_0(bR)^{-k}\right]^3}\exp m_2\,, \tag{26}$$

$$m_2=-\frac{2(n-1)}{k}C_0(bR)^{-k}-\frac{2kC_0(bR)^{n-k}}{D^n(n-k)}+\frac{kC_0b^{n-k}}{D^n}\times$$
$$\times\int\frac{R^{n-k-1}[(3-2C_0(bR)^{-k}]}{1-C_0(bR)^{-k}}dr+2\log[1-C_0(bR)^{-k}]+\frac{n\alpha\bar{\Theta}_0b^n}{D^n}\times$$
$$\times\int[1-C_0(bR)^{-k}]\left[3+\frac{C_0(bR)^{-k}-kC_0(bR)^{-k}\log R}{1-C_0(bR)^{-k}}\right]R^{n-1}dR\,. \tag{27}$$

NUMERICAL RESULTS AND DISCUSSION

For calculating creep stresses in spherical shell made up of functionally graded material, the following values have been taken: $\nu=0.3$; compressibility factor $C_0=0.5$, non-homogeneity parameter $k=0$, 0.5, -0.5, 1.5; strain measure $n=1$, 1/3, 1/5 (i.e. $N=1$, 3, 5); $\alpha=15.0\cdot10^{-5}\,\text{degF}^{-1}$ (for methyl methacrylate, /20/); $\Theta_1=\alpha\Theta_0=0.0$ and 1.5; and $a=1$, $b=2$, $D=1$. In classical theory, measure N is equal to $1/n$. The definite integrals in Eqs.(24)-(25) are evaluated by using Simpson's rule.

Influence on creep stresses when (internal pressure) $P_1>P_2$ (external pressure)

In Fig. 1, curves for creep stresses are produced along radii ratio $R=r/b$ by taking into consideration non-homogeneity parameter k. For linear measure, $n=1$, creep stresses have maximum effect on the internal surface of the body as compared to external part. It is seen that when compressibility of material increases along the radius for $k=-1$, -1.5, then creep stresses have maximum effect at internal surface as compared to homogeneous material and low functionally graded material. But as we go from linear behaviour of strain measure to nonlinear behaviour, i.e. from $n=1$ to 1/3, 1/5, creep stresses have maximum effect on external part of shell for highly functionally graded material. It is observed that circumferential stresses are tensile in nature as compared to radial stresses for all values of non-homogeneity parameter *k*. In Fig. 2, the creep stresses are drawn along the combined effect of thermal and nonlinear measure. It is seen that the values of creep stresses are increased for linear measure at the internal surface of spherical shell as compared to nonlinear measure with the introduction of temperature condition.

Influence on creep stresses when (external pressure) P_2 > P_1 (internal pressure)

It can be seen from Fig. 3 that creep stresses obtained are compressible in nature due to high external pressure. For linear measure $n = 1$, the creep stresses are maximum at interior part of spherical shell but for nonlinear measure, circumferential stresses are maximum at external surface. It is observed that creep stresses attain maximum value for low graded material at $n = 1$, but these values get maxima for $n = 1/3$, 1/5 for highly graded materials. In Fig. 4, the effect of temperature factor is seen on creep stresses when external pressure is higher than internal pressure. Creep stresses are found to be increased at internal surface for homogenous material and low graded material for linear measure, whereas, for nonlinear measure, these values of creep stresses get lowered for highly graded material.

CONCLUSION

The paper gives behaviour of creep stresses in functionally graded spherical shell under combination of temperature and pressure. Creep stresses have observed tensile, as well as compressive, nature due to the high internal- and high external pressure, respectively. Creep stresses are found to be very effective at the internal, as well as the external surface of shell, in case of spherical shell made up of low graded material and homogenous material that cause severe damage of the shell. The effect of temperature component is seen in the case of linear measure, as compared to nonlinear measure. Thus it can be concluded that spherical shell made up of highly graded material are more useful from a safety point of view.

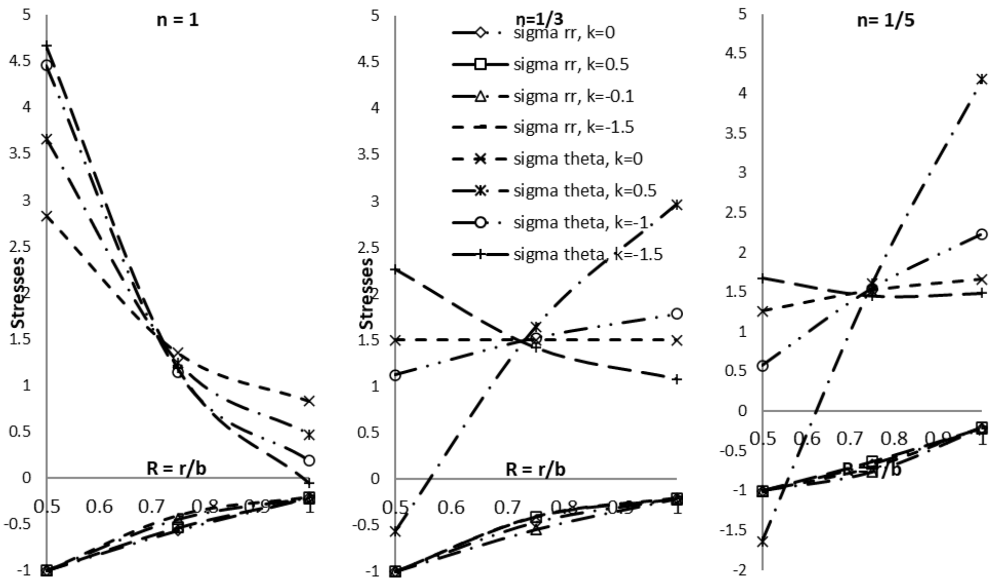

Figure 1. Creep stresses in functionally graded spherical shell when internal pressure > external pressure.

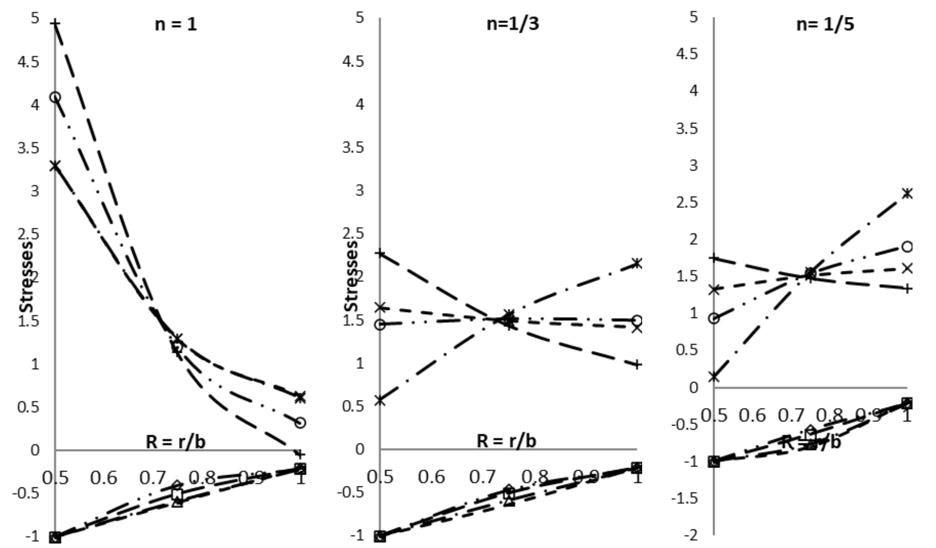

Figure 2. Creep stresses in funct. graded spherical shell under steady state temperature $\Theta_1 = 1.5$ when internal pressure > external pressure.

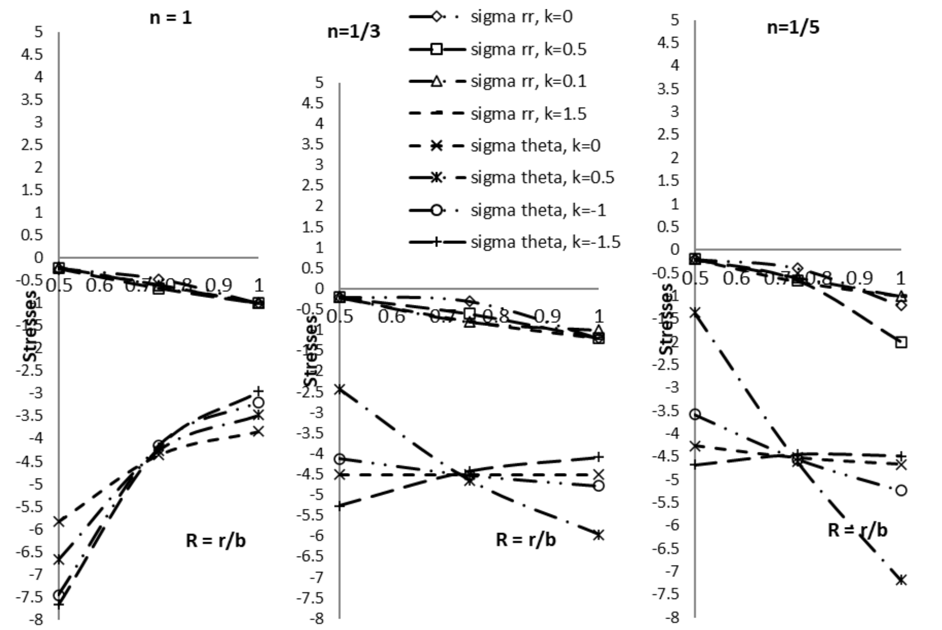

Figure 3. Creep stresses in functionally graded spherical shell when external pressure > internal pressure.

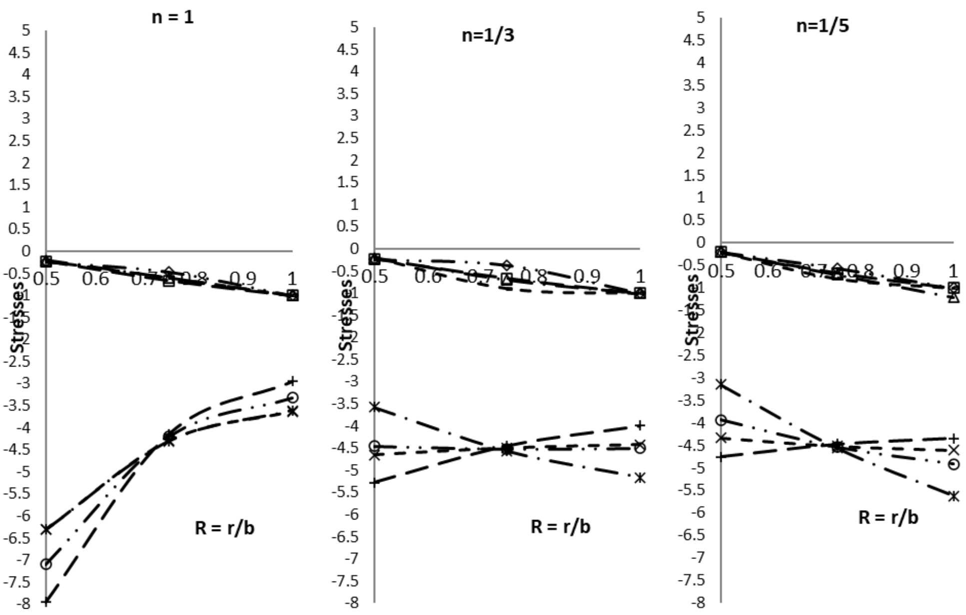

Figure 4. Creep stresses in funct. graded spherical shell under steady state temperature $\Theta_1 = 1.5$ when external pressure > internal pressure.

REFERENCES

1. Hamed, E., Bradford, M.A., Gilbert, R.I. (2010), *Nonlinear long-term behaviour of spherical shallow thin-walled concrete shells of revolution*, Int. J Solids Struct. 47(2): 204-215. doi: 10.1016/j.ijsolstr.2009.09.027
2. Kashkoli, M.D., Nejad, M.Z. (2014), *Effect of heat flux on creep stresses of thick-walled cylindrical pressure vessels*, J Appl. Res. Technol. 12(3): 585-597.
3. You, L.H., Zhang, J.J., You, X.Y. (2005), *Elastic analysis of internally pressurized thick-walled spherical pressure vessels of functionally graded materials*, Int. J Pres. Ves. Piping, 82(5): 347-354. doi: 10.1016/j.ijpvp.2004.11.001
4. Moosaie, A., Panahi-Kalus, H. (2017), *Thermal stresses in an incompressible FGM spherical shell with temperature-dependent material properties*, Thin-Walled Structures, 120: 215-224. doi: 10.1016/j.tws.2017.09.005
5. Thakur, P., Verma, G., Pathania, D.S., Singh, S.B. (2017), *Thermal creep analysis in non-homogeneous spherical shell*, Struct. Integ. and Life, 17(2): 89-95.
6. Shariyat, M., Ghafourinam, M. (2019), *Hygrothermomechanical creep and stress redistribution analysis of thick-walled FGM spheres with temperature and moisture dependent material properties and inelastic radius changes*, Int. J Pres. Ves. Piping, 169: 94-114. doi: 10.1016/j.ijpvp.2018.11.011
7. Sandhu, S.S., Singh, T., Gupta, V.K. (2019), *Analysis of steady state creep behaviour in spherical vessels made of composite material*, Mater. Today: Proc. 18(7): 3401-3408. doi: 10.1016/j.matpr.2019.07.267
8. Verma, G., Pathania, D.S. (2019), *Elastic-plastic stress analysis of spherical shell under internal and external pressure*, Struct. Integ. and Life, 19(1): 3-7.
9. Wu, X.-H., Chen, C., Shen, Y.-P., Tian, X.-G. (2002), *A high order theory for functionally graded piezoelectric shells*, Int. J Solids Struct. 39(20): 5325-5344.

10. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi: 10.1038/195896a0
11. Seth, B.R. (1966), *Measure concept in Mechanics*, Int. J Non-Linear Mech. 1(1): 35-40. doi: 10.1016/0020-7462(66)90016-3
12. Sharma, S., Aggarwal, A., Sharma, R. (2013), *Safety analysis of thermal creep non-homogeneous thick-walled circular cylinder under internal and external pressure using Lebesgue strain measure*, Multidisc. Model. Mater. Struct. 9(4): 499-513. doi: 10.1108/MMMS-09-2012-0013
13. Parkus, H., Thermoelasticity, Springer-Verlag Wien, New York, 1976. doi: 10.1007/978-3-7091-8447-9
14. Thakur, P. (2015), *Analysis of thermal creep stresses in transversely thick-walled cylinder subjected to pressure*, Struct. Integ. and Life, 15(1): 19-26.
15. Gupta, V. (2019), *Creep analysis in composite disc in the presence of thermal gradients*, Struct. Integ. and Life, 19(3): 167-174.
16. Mehta, P.D., Mishra, L., Sahni, M. (2019), *Thermomechanical stress analysis of thick-walled cylinder with inner FGM layer*, Struct. Integ. and Life, 19(3): 211-223.
17. Sharma R., Radaković, Z. (2020), *Analytical solution of elastic-plastic stresses in thin rotating disc made up of piezoelectric material*, Struct. Integ. and Life, 20(1): 21-25.
18. Thakur, P., Sethi, M. (2020), *Creep deformation and stress analysis in a transversely material disk subjected to rigid shaft*, Math. Mech. Solids, 25(1): 17-25. doi: 10.1177/1081286519857109