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NUMERICAL MODEL OF FLUID-STRUCTURE INTERACTION FOR WATER TOWER ANALYSIS OF VIBRATIONS

NUMERIČKI MODEL INTERAKCIJE FLUID-KONSTRUKCIJA KOD ANALIZE VIBRACIJA VODOTORNJA

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Keywords
- water tower
- solid finite elements (FE)
- fluid-structure interaction (FSI)
- vibration analysis

Abstract

This paper deals with the aspects of vibration analysis of 3D numerical water tower models with the explicitly generated 3D finite element (FE) solid fluid model that simulates the fluid-structure interaction (FSI). The development and implementation of the 3D FE solid fluid model in modelling and analysing the system response is extremely important, both in terms of determining the impulsive and convective hydrodynamic pressure, and in terms of overall sizing of the water tower. Vibration of the water tower is calculated based on the analysis of its eigenvibrations, analysis of eigenvibrations occurring when charging and discharging the water tower reservoir (SFA - Staged Fluid Analysis), Steady-State Analysis (SSA) in the frequency domain and the analysis of the system response in the time domain (THA - Time History Analysis), followed by the transformation of the frequency domain response using the Fast Fourier Transform (FFT). The analysis of eigenvalues using the Ritz vectors (Ritz analysis) and the Power Spectral Density Analysis (PSDA) are carried out as the means of additional control and with the purpose of identifying vibration periods. The system's stiffness matrix and mass matrix are corrected using the 3D FE solid fluid model for analysing the water tower vibrations. Effects of the system's increasing vibration period are determined for the partially charged reinforced concrete (RC) water tower reservoir, given the strong action of the fluid mass that is excited for convective hydrodynamic action (sloshing) due to the water tower dimensions and the amount of fluid contained in the reservoir in this case.

Ključne reči
- vodotoranj
- konačni elementi čvrstog tela (FE)
- interakcija fluid-konstrukcija (FSI)
- analiza vibracija

Izvod

U radu su obrađeni aspekti analize vibracija primenom 3D numeričkih modela vodotornja, sa eksplicitno generisanim 3D modelom fluid-čvrsto telo konačnim elementima (FE), koji simulira interakciju fluid-konstrukcija (FSI). Od velikog je značaja razvoj i implementacija 3D modela fluid-čvrsto u modeliranju i analizi odziva sistema, kako u određivanju impulsnog i konvektivnog hidrodinamičkog pritiska, tako i pri projektovanju sklopa vodotornja. Proračun vibracija vodotornja se izvodi na osnovu analize sopstvenih vibracija, analize sopstvenih vibracija pri punjenju i pražnjenju rezervoara vodotornja (SFA - fazna analiza fluida), stacionarne analize (SSA) u frekventnom domenu i na osnovu analize odziva sistema u vremenskom domenu (THA), praćeno transformacijom odziva frekventnog domena primenom brze Furijeove transformacije (FFT). Analiza sopstvenih vrednosti primenom Ricovih vektora (Ritz analiza) i spektralnom analizom gustine snage (PSDA), izvedena je u smislu dopunske kontrole sa ciljem identifikacije perioda vibracija. Matrica krutosti sistema i matrica masa su korigovane primenom 3D FE modela čvrsto-fluid u analizi vibracija vodotornja. Uticaj povećanja perioda vibracija sistema je određen za delimično napunjen rezervoar vodotornja od ojačanog betona (RC), nastao usled jakog dejstva mase fluida, koja je pobuđena konvektivnim hidrodinamičkim talasanjem u datom slučaju dimenzija tornja i količini fluida sadržanog u rezervoaru.

INTRODUCTION

The main function of water towers is to store water at a specific altitude for the purpose of providing cities and towns with continuous water supply. One of the important steps when designing and analysing water tower structures is to determine the eigenmodes and corresponding periods of vibration. The development of software programs and implementation of *Finite Element Method* (FEM) has enabled modelling and analysing the system at a higher level as compared to the use of analytical method of treating the reservoir structure. In traditional engineering analysis, direct numerical modelling of fluid (water) in the water tower is not taken into account, except for water towers of large capacity or when providing expertise in case of accidents. The classic mathematical water tower model is being formed based on the geometry of high-level 3D models, and the approximation using beam and shell finite elements (FE), as well as on discretization of sub-domains with compatible FE mesh. Modelling of fluid is at an inferior level due to the complexity of the phenomena describing the static and dynamic state of fluid in the water tower reservoir. The static influence of fluid is approximated using a replacement model of pressure along the interior reservoir surface, whereas the dynamic influence is approximated using systems of lumped masses. The overall hydrodynamic effects of fluid that occur during the earthquake-excitation of the reservoir are modelled using these lumped masses.

Initial investigations of vibrations of the water tower, partially or completely filled with water, are presented in Housner /1, 2/, where analytical expressions for determining water tower vibrations are developed. These studies are still used for comparison with the results obtained by numerical procedures for their verification. The fluid-structure interaction is studied using FEM, i.e. the LUSAS software program, described in Algreane et al. /3, 4/; ANSYS software program presented in Sarokolayi et al. /5/, and SAP software, as described in Livaoglu and Dogangun /6/. Analytical and experimental investigation of the dynamic response of conical water tower reservoirs is described in Dammaty et al. /7/, while the analysis of resonant frequencies of spherical water tower reservoirs is presented in Curadelli et al. /8/. Modelling the mass of water in the reservoir with varying distributions along the walls of the 3D reservoir is presented in Algreane et al. /9/, while the equivalent mechanical model of masses and the water tower for the purpose of vibration analysis is considered in Joshi /10/. Modelling the mass of water in the tower using discrete numerical model with solid FE is discussed in Goudarzi and Sabbagh-Yazdi /11/. Effects of sloshing in the reservoir in the case of dynamic response and the analysis of response on the implemented numerical model using FEM are discussed in Mirzabozorg et al. /12/. Investigations regarding these effects are detailed in the book by Ibrahim /13/. Effects of sloshing in the reservoir are discussed in papers Wu et al. /14/, Jaiswal et al. /15/, Koli and Kulkarni /16/, Faltinsen and Timokha /17/.

The sophisticated model of fluid-structure interaction (FSI) explicitly simulates the hydrodynamic behaviour, the inertial effects, and effects of vibration with the goal of describing the actual behaviour of the water tower in service conditions. The ability of creating a model of a fluid filled water tower is based on the use of 3D fluid modelling based on solid FE, providing thereby the fluid with the property of spatial form of variations in the water tower. In this paper, a model is developed in order to analyse vibrations of the water tower with the inclusion of FSI interaction. The research is based on the discrete model of fluid in the water tower and its direct participation in vibration analysis. In addition to numerical modelling of the water tower and the fluid, the analysis of eigenvibrations of the water tower and the analysis of eigenvibrations occurring when charging and discharging the reservoir (SFA - *Staged Fluid Analysis*), as well as the *Steady-State Analysis* (SSA) in the frequency domain (FDA - *Frequency Domain Analysis*) were also undertaken. Prismatic and cylindrical forms of 3D FE solid fluid model are also considered, and the results obtained for the period of the first eigenmode, and compared with results obtained by experimental, analytical and numerical methods. The 3D analysis of vibrations using a 3D FE solid fluid model is illustrated on the example of two water towers; one of them is made of reinforced concrete (RC), and the other of steel.

GENERAL DYNAMIC WATER TOWER MODELS

In the process of static and dynamic analysis of the water tower, the basic starting point is to consider the horizontal and vertical pressures exercised by the fluid. Figure 1a shows the simplified water tower model and corresponding pressures arising from the water, i.e. the hydrostatic (Fig. 1b) and hydrodynamic (Fig. 1c) pressures. Hydrostatic pressures are taken into consideration for the steady-state of the water tower, while hydrodynamic pressures are taken into account for the condition when the water tower is exposed to earthquake.

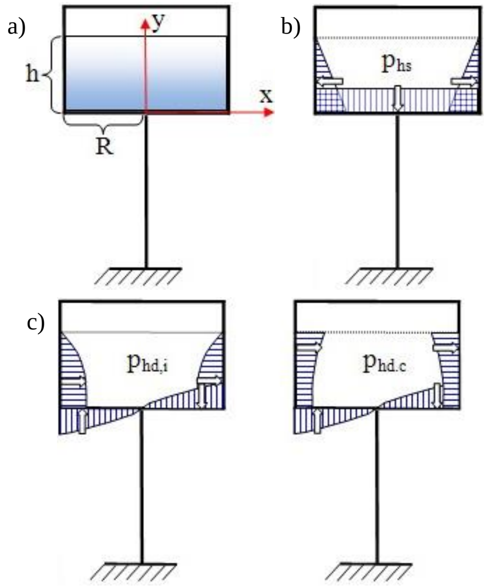

Figure 1. Simplified water tower model and corresponding pressures arising from the water: a) model with fluid; b) hydrostatic; and c) hydrodynamic water pressures.

When the water tower vibrates, the reservoir walls and bottom are influenced by two types of hydrodynamic pressures generated by water: impulsive and convective pressure /2/. Impulsive action implies that part of the fluid is in a stiff connection with the water tower (acting as quasi-static loading) while the convective action (sloshing) implies that there is an increase in force acting upon the water tower structure due to the rippling fluid. The convective pressure is significantly lower than impulsive pressure and it can cause rippling on the free fluid surface only in stiff reservoirs. The design of stiff reservoirs is significantly affected only by impulsive pressure, while the design of flexible reservoirs is strongly affected by convective pressure. The standard dynamic model for analysing the water tower consists of approximation of the system using lumped masses. The preliminary model requires the use of a system with a single degree of freedom (SDOF) with a lumped mass, Fig. 2a. This model is used to estimate the eigenmode and period of vibration. A more advanced level in the creation of dynamic water tower model consists of modelling the supporting structure with a concentrated mass of fluid (one-mass fluid model), Fig. 2b. This water tower model provides more degrees of freedom than the previous model.

The third and most advanced water tower model consists of structure with concentrated impulsive and convective masses of fluid (two- mass fluid model), Fig. 2c, /2/. Also, there is a possibility of modelling a fluid filled water tower with a larger number of masses (multi-mass fluid model), /18, 19/.

Position of the mass resulting from impulsive pressure in the cylindrical water tower reservoir is determined by the following equation:

$$h_i=\frac{3}{8}h\left\{1+1.33\left[\frac{m}{m_c}\left(\frac{R}{h}\right)^2-1\right]\right\}, \quad (1)$$

while mass resulting from the impulsive part of pressure is:

$$m_i=m\frac{\tanh(\sqrt{3}R/h)}{\sqrt{3}R/h}, \quad (2)$$

where: h is fluid level; and m is overall fluid mass in the water tower reservoir; while R is the radius of the reservoir.

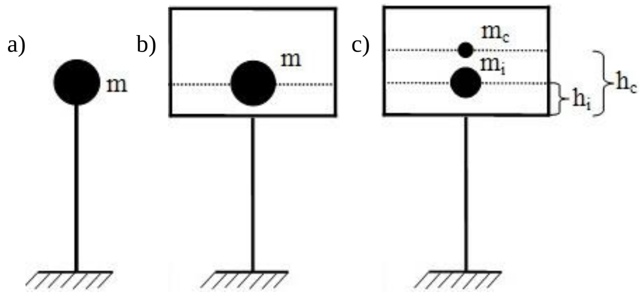

Figure 2. Water tower structural models for dynamic analysis with: a) single degree of freedom (SDOF); b) concentrated mass of fluid; c) concentrated masses of fluid (impulsive and convective).

The position of the mass resulting from convective pressure in the cylindrical water tower reservoir is determined by the following equation:

$$h_c=h\left[1-0.185\left(\frac{m}{m_c}\right)\left(\frac{R}{h}\right)^2-1.12\frac{R}{h}\sqrt{\left(\frac{mR}{3m_c h}\right)^2-1}\right], \quad (3)$$

while mass resulting from the convective part of pressure is:

$$m_c=0.6m\frac{\tanh(1.8h/R)}{1.8h/R}. \quad (4)$$

The mass of the impulsive part of pressure is associated with water tower through a stiff connection, while that of the convective part of pressure is associated through elastic connection, where the stiffness is equal to:

$$k_c=5.4\frac{m_c^2}{m}\frac{gh}{R^2}. \quad (5)$$

Periods of vibration of the fluid in the water tower are determined by:

$$T=2\pi\sqrt{\frac{m_c}{k_c}}. \quad (6)$$

Respective periods of vibration for impulsive and convective parts of pressure can be determined by:

$$T_i=2\pi\sqrt{\frac{m_i+m_s}{k_s}},\ \ T_c=2\pi\sqrt{\frac{m_c}{k_c}}, \quad (7)$$

where: m_s is the mass of the reservoir and the mass of one-third of water tower body height; k_s is the lateral stiffness of water tower body; while m_c and k_c are determined according to /20/.

In steady-state, the fluid surface (water mirror) is in a horizontal position, Fig. 3a, while when the water tower is excited for oscillations, the shape of this surface is much more complex due to the rippling fluid and slashing effects acting upon lateral sides. The simplified fluid model for the case when the water tower vibrates is shown in Fig. 3b: on one side, the reservoir fluid level is $h + d$, on the other $h - d$.

NUMERICAL MODELLING OF THE WATER TOWER AND FLUID

In relation to the analytical treatment of the water tower structure and the modelling of fluid, the problem of vibration can be analysed in a more efficient manner using FEM. When using the numerical analysis of the water tower, the focus is on modelling the fluid and the FSI interaction that can be implemented in the following ways:
– indirectly, by modelling the fluid using the replacement added mass method /21, 22/,
– by modelling the fluid using solid FE with embedded attributes of the fluid behaviour model according to Lagrange's formulation /23, 6/,
– by modelling the fluid using solid FE with embedded attributes of the fluid behaviour model according to Euler's formulation /24/,
– by modelling the fluid using solid FE with embedded attributes of the fluid behaviour model according to the Euler-Lagrange formulation, /25/.

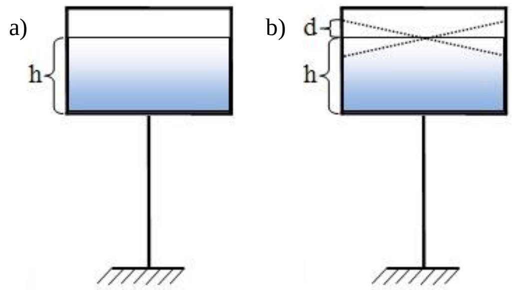

Figure 3. Fluid filled water tower model: a) steady-state; b) state of vibrations (sloshing effect).

The indirect fluid modelling procedure using replacement masses is analogous with the procedure applied for analytical treatment of the mass resulting from impulsive and convective pressure. The relation between mass, resulting from impulsive pressure, and the water tower structure is achieved by using rigid link elements, where m_{rle} = 0 and $k_{rle} \rightarrow \infty$. The relation between mass, resulting from convective pressure, and the water tower structure is achieved by using linear link elements, where m_{lle} = 0 and $k_{lle} = k_c$.

The procedure for defining the FSI interaction starts from the following differential equation for motion of the ideal and homogeneous fluid in the water tower reservoir:

$$0^2\theta=\frac{1}{C^2}\frac{\partial^2\theta}{\partial t^2}, \quad (8)$$

where: θ is the velocity potential; t is time; and C is the speed of sound. The research carried out in this paper is based on the discrete fluid model in the water tower and its direct participation in vibration analysis. For FSI interaction based on FEM and according to Lagrange's formulation, the fluid is assumed to be ideal, homogeneous, elastic, isotropic, compressible and non-viscous, /26/. The FE that describes the behaviour of fluid is based on the formulation where fluid dilatations are calculated from linear equations of displacements. If the consideration accounted with the fact that the fluid is non-compressible, then significant errors could occur in calculation, in the case of high frequencies. Using the 3D solid FE model with incompatible eigenmodes and very low shear modulus, the fluid behaviour in the reservoir can be fairly well described. The effect of viscosity in the dynamic analysis of the water tower is reduced with the increase of reservoir dimensions /27/. The stress/strain ratio (σ/ε) in fluids defined in this manner is:

$$\sigma = \lambda \varepsilon , \tag{9}$$

where: λ is the bulk modulus. The change of volume, $\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, while hydrostatic pressure, σ, indicates that stresses are equal in all directions. Based on stress-strain ratio for isotropic materials given in matrix form, the bulk modulus λ can be written as a function of the Young's elasticity modulus E and Poisson's coefficient ν,

$$\lambda = \frac{E}{3(1-2\nu)} . \tag{10}$$

In fluids, the bulk modulus λ is an independent constant value; for Poisson's coefficient ν its value is 0.5, with Young's elasticity modulus E and shear modulus G being negligibly small - these are taken to be equal to 0 (≈ 0). For isotropic materials, the bulk modulus λ and shear modulus G are known as Lamé's elastic constants, representing the fundamental physical properties of fluid, in this case water. The shear modulus G for isotropic materials is determined by:

$$G = \frac{E}{2(1+\nu)} , \tag{11}$$

while Poisson's coefficient ν is determined from:

$$\nu = \frac{3-2(G/\lambda)}{6+2(G/\lambda)} . \tag{12}$$

If the value of shear modulus G is extremely small compared to the bulk modulus λ, then the value of $\nu \approx 0.5$, while $E \approx 3G$. The main difference between the fluid and the deformable bodies is that the shear modulus G of the fluid is very low compared to the bulk modulus λ.

The main aspects in the stage of creating and pre-processing the 3D numerical water tower-fluid model are the approximation and discretization. In the process of approximation the underlying water tower-fluid domain is modelled by selecting the type of FE. The aspect of discretization refers to the formation of a FE mesh, i.e. to the selection of mesh quality and number of FE. The water tower structure is modelled using beam and shell FE, while the fluid domain is modelled using solid FE. In mathematical terms, these FE are 3D, as they are considered in a coordinate system defined by three axes. The number of nodes in the 3D solid FE is eight, each of them with three degrees of freedom for defining displacements located in the corners; for these nodes the 2×2×2 numerical integration is applicable via the Gaussian quadrature, /28/, Fig. 4.

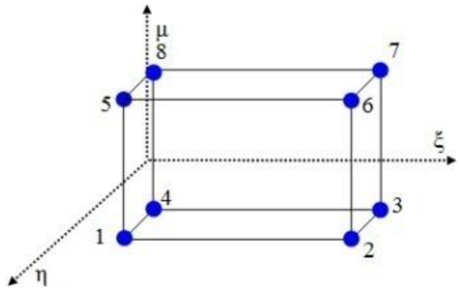

Figure 4. 3D solid FE for modelling the fluid.

The actual water tower model is shown in Fig. 5a, while the numerical water tower model along with the lumped mass fluid model is shown in Fig. 5b. The numerical water tower model is improved by introducing a discrete fluid model through FE mesh, Fig. 5c. The system's mass is modelled in a way that masses of the water tower structure and fluid are directly included in the analysis through appropriate masses of each FE. Using such an advanced numerical model enables realisation of more complex form of the water surface due to fluid ripping and its sloshing effects acting upon the lateral walls when the water tower is excited to vibrate.

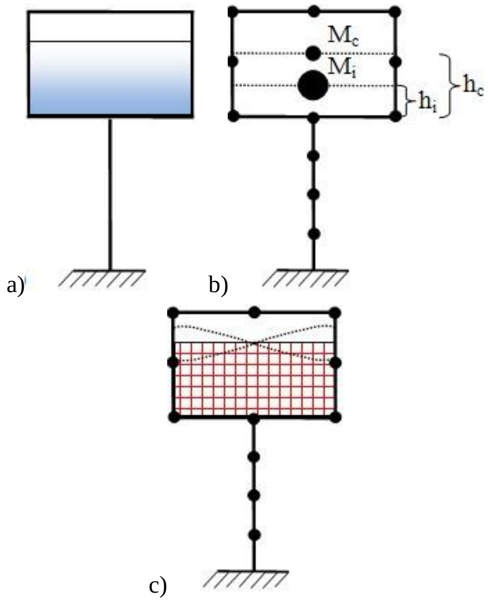

Figure 5. Water tower models: a) actual model. Numerical models: b) with lumped mass fluid; c) with discrete fluid.

MATHEMATICAL FORMULATION FOR VIBRATION ANALYSIS OF THE WATER TOWER

In this study three different types of analysis of water tower vibration are carried out: analysis of eigenvibrations of the water tower, analysis of eigenvibrations when charging and discharging the water tower reservoir (SFA) and SSA in the frequency domain. Determination of eigenfrequencies of the 3D water tower and fluid model consists of resolving the homogeneous system of equations without damping, /29/:

$$[M]\{\ddot{U}\}+[K]\{U\}=0 , \tag{13}$$

$$\{U\}=\{\bar{U}\}\sin(\omega t)\,,\ \{\ddot{U}\}=-\omega^2\{\bar{U}\}\sin(\omega t)\,, \qquad (14)$$

$$[K]\{\bar{U}\}\sin(\omega t)-\omega^2[M]\{\bar{U}\}\sin(\omega t)=0\,, \qquad (15)$$

$$\left([K_0]-\omega^2[M_0]\right)\{\bar{U}\}=0\,, \qquad (16)$$

where: $[M]$ is system's mass matrix; $[K]$ is system's stiffness matrix; $\{U\}$ is the vector of generalized displacements; and $\{\bar{U}\}$ is the amplitude vector of generalized displacements. Solutions for the homogeneous matrix Eq.(13) are the eigenfrequencies and eigenmodes of vibration.

Generally, the coupled problem of fluid-water tower can be represented via:

$$\begin{bmatrix} M_{ss} & 0 \\ 0 & M_{ff} \end{bmatrix}\begin{bmatrix} \ddot{U}_s \\ \ddot{U}_f \end{bmatrix}+\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad (17)$$

where: index s refers to the water tower structure; f refers to the fluid, while for the accelerations and displacements the following can be written:

$$\begin{bmatrix} \ddot{U}_s \\ \ddot{U}_f \end{bmatrix}\equiv\begin{bmatrix} \ddot{u}_s \\ \ddot{u}_f \end{bmatrix}+\begin{bmatrix} \ddot{v}_s \\ \ddot{v}_f \end{bmatrix},\quad \begin{bmatrix} U_s \\ U_f \end{bmatrix}\equiv\begin{bmatrix} u_s \\ u_f \end{bmatrix}+\begin{bmatrix} v_s \\ v_f \end{bmatrix}. \qquad (18)$$

When introducing Eq.(18) into Eq.(17), it follows:

$$\begin{bmatrix} M_{ss} & 0 \\ 0 & M_{ff} \end{bmatrix}\begin{bmatrix} \ddot{u}_s \\ \ddot{u}_f \end{bmatrix}+\begin{bmatrix} M_{ss} & 0 \\ 0 & M_{ff} \end{bmatrix}\begin{bmatrix} \ddot{v}_s \\ \ddot{v}_f \end{bmatrix}+\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix}\begin{bmatrix} u_s \\ u_f \end{bmatrix}+$$

$$+\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix}\begin{bmatrix} v_s \\ v_f \end{bmatrix}=0\,. \qquad (19)$$

The problem of eigenvibrations when charging and discharging the tower reservoir (SFA) is resolved through analysing the stages of charging/discharging. These analyses are carried out successively using the system's stiffness and mass matrix, and at the end of the previous analysis of eigenvibrations as the initial stiffness and mass matrix of the subsequent analysis of eigenvibrations. The mathematical formulation of the SFA is carried out based on Eq.(16):

$$y=0:\quad \left([K_0]-\omega^2[M_0]\right)=0\,, \qquad (20)$$

where: $[K_0]$ and $[M_0]$ are the stiffness and mass matrices of the empty reservoir, respectively. For the condition when the reservoir is filled to a specific initial level, the analysis is carried out according to:

$$y=1:\quad \left([K_1]-\omega^2[M_1]\right)=0\,,\ [K_1]=[K_0]+[K_1'],$$
$$[M_1]=[M_0]+[M_1'], \qquad (21)$$

where: $[K_1']$ and $[M_0']$ are stiffness and mass matrices of the 3D FE solid fluid that simulates the initial charge to a specific level, respectively. In the i-th stage of charging the reservoir, the analysis of eigenvibrations is determined by:

$$y=i:\quad \left([K_i]-\omega^2[M_i]\right)=0\,,\ [K_i]=[K_{i-1}]+[K_i']\,,$$
$$[M_i]=[M_{i-1}]+[M_i']\,, \qquad (22)$$

while the following applies to the final stage, when the reservoir is filled with fluid:

$$y=n:\quad \left([K_n]-\omega^2[M_n]\right)=0\,,\ [K_n]=[K_{n-1}]+[K_n'],$$
$$[M_n]=[M_{n-1}]+[M_n'], \qquad (23)$$

where: $[K_n]$ and $[M_n]$ are the stiffness and mass matrices of the 3D model of the water tower reservoir filled with fluid, respectively.

The SSA of the 3D water tower and fluid model is carried out by considering the harmonic load in the following format /26/:

$$\{r\}=\{f\}\sin(\omega t)\,. \qquad (24)$$

For the case when system damping is not considered, the system's equilibrium equations are as follows:

$$[M]\{\ddot{U}\}+[K]\{U\}=\{f\}\sin(\omega t)\,, \qquad (25)$$

where the solution of this expression is obtained from the condition that the displacement and acceleration are given according to Eq.(14). The response amplitude is obtained from the system of linear equations:

$$\left([K]-\omega^2[M]\right)\{\bar{U}\}=\{f\}\,. \qquad (26)$$

PRELIMINARY ANALYSES OF VIBRATIONS OF 3D FE SOLID FLUID MODEL

In order to increase the efficiency and accuracy of evaluating the effects of modelling the fluid as a 3D solid model, preliminary vibration analyses are carried out by using the SAP 2000 software program, /28/. The 3D solid fluid is assigned with properties previously defined in this paper. The fluid model is considered as an isolated model, simulating thereby the behaviour of fluid in the rigid reservoir with convective and impulsive actions (sloshing). The effect of a rigid reservoir is excluded from vibration analysis, its mass being neglected $m = 0$, while introducing the value of $k \to \infty$ for stiffness. The fluid-reservoir connection is realized through gap elements activated only by pressure. Preliminary analyses regarding the number and dimensions of FE in the 3D FE solid fluid model and a total of 1000 FE are obtained for each single model. The fluid (water) bulk modulus is λ = 2.1 GPa, while the density of water is ρ = 1000 kg/m³. The 3D FE solid fluid model is generated according to /15/, so that the comparison among the obtained solutions is also enabled. According to /1/, the vibration period of convective action (sloshing) of the fluid in the prismatic reservoir is as follows:

$$T_c=\left(\frac{1}{2\pi}\sqrt{\left(3.16g\tanh\left(3.16\frac{h}{L}\right)\right)/L}\right)^{-1}, \qquad (27)$$

while for the cylindrical reservoir the following applies:

$$T_c=\left(\frac{1}{2\pi}\sqrt{\left(3.68g\tanh\left(3.68\frac{h}{D}\right)\right)/D}\right)^{-1}, \qquad (28)$$

where: L is reservoir width; and D is reservoir diameter. Vibration periods obtained by the analysis of eigenvalues are verified using the *Ritz* vectors (*Ritz* analysis of vibration). Values for vibration periods of the first eigenmode of the 3D solid model of a prismatic and cylindrical fluid are presented in Table 1. The high level of agreement between the calculated values of vibration periods of the 3D FE solid fluid model and the experimentally observed values is evident.

Table 1. Vibration periods of the first eigenmode of 3D solid model of a prismatic and cylindrical fluid.

	T (s)			
model	experimental /15/	numerical /15/	analytical /1/	3D solid model
prismatic	0.599	0.602	0.594	0.597
cylindrical	0.429	0.450	0.431	0.444

Figure 6 shows the prismatic 3D FE solid fluid model for steady-state and first eigenmode of vibrations, while Fig. 7 shows the cylindrical 3D FE solid fluid model for steady-state and the first eigenmode of vibrations. Effects of convective action (sloshing) of fluid can be best seen through the elevation of fluid on one side and its drop on the other side, simulating the complex model of fluid vibrations in the water tower. The use of the 3D FE solid fluid model yields with better and more complex distribution of hydrodynamic pressures along the reservoir walls.

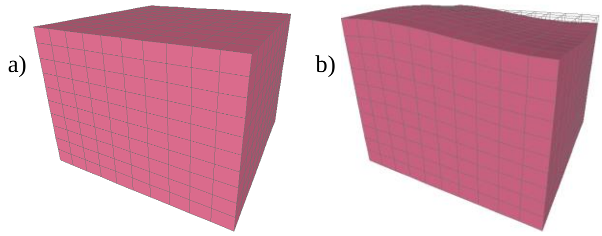

Figure 6. Prismatic 3D FE solid fluid model: a) steady-state; b) first eigenmode of vibrations.

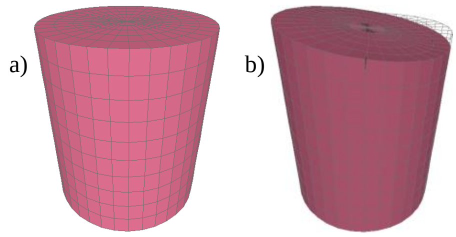

Figure 7. Cylindrical 3D FE solid fluid model: a) steady-state; b) first eigenmode of vibrations.

ANALYSIS OF VIBRATIONS OF THE 3D RC WATER TOWER WITH 3D FE SOLID FLUID MODEL

The reinforced concrete (RC) water tower is modelled using shell FE and 3D solid FE for the fluid. The influence of flexibility of the reservoir in the vibration analysis is introduced by modelling the shell elements. The level of accuracy and precision of this 3D water tower and fluid model is high and its geometry corresponds to the actual physical model. The contact between the reservoir and fluid is realized by applying nodal gap elements that are activated only by pressure, so that the compatibility of all nodes of FE is established. The 3D water tower model is generated according to /5/. Total height of the water tower is 39 m, the reservoir diameter is 14 m, while reservoir volume is 1260 m^3. Parameters of numerical water tower models are shown in Table 2, where cases of empty and full reservoir, as well as intermediate stages of charging are considered.

Figure 8 shows the 3D water tower model, the vertical cross-section of 3D water tower model, the vertical cross-section of the 3D water tower model with the 3D FE solid fluid model and 3D FE solid fluid.

Table 2. Parameters of RC water tower numerical models.

stage	shell FE	solid FE	no. of equations of the system equilibrium	no. of degrees of freedom for the system mass
empty water tower	832	-	4956	2454
first level	832	312	5829	3327
second level	832	624	6702	4200
full water tower	832	936	7284	4782

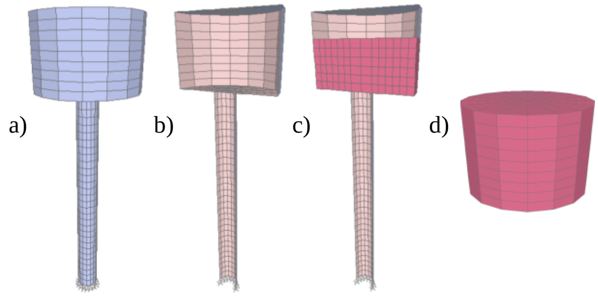

Figure 8. 3D RC water tower with 3D FE solid fluid: a) 3D water tower; b) vertical cross-section; c) vertical cross-section with 3D FE solid fluid; d) 3D FE solid fluid.

The 3D RC water tower the 3D FE solid fluid model is analysed regarding the eigenvibrations for simulating the stages of charging and discharging (SFA). The SFA consists of four static and four eigenvibration analyses which are conducted in a successive manner. Flowchart of the SFA vibration analysis of the RC water tower is shown in Fig. 9.

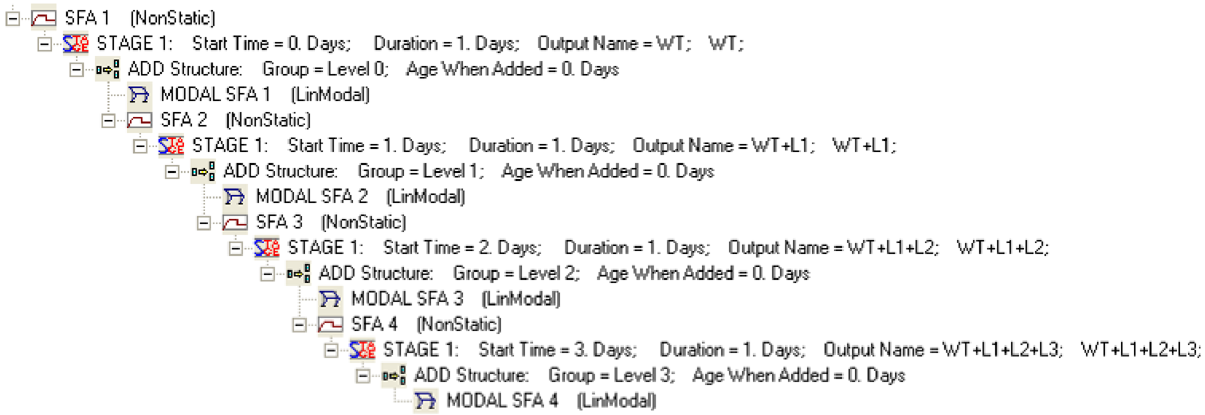

Figure 9. Flowchart of SFA vibration analysis of the RC water tower.

The normalized vibration periods T/T_e as the function of fluid level in the reservoir of RC water tower are shown in Fig. 10. The vibration period T is normalized in relation to the vibration period of empty reservoir T_e; the value of h for the empty reservoir is 0, while for the full reservoir, $h = 1$.

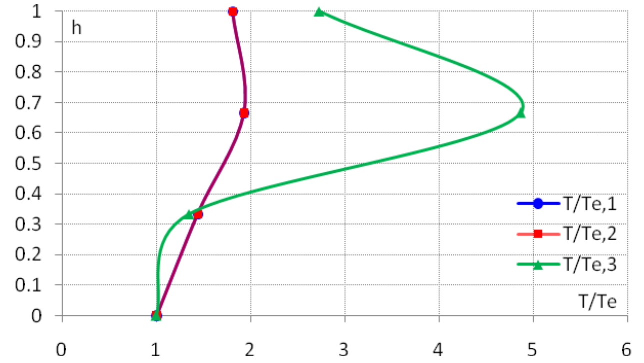

Figure 10. Normalized vibration periods T/T_e as a function of fluid level in the reservoir of RC water tower.

The first three eigenmodes (two translational and one torsional) of the 3D RC water tower, along with the 3D FE solid fluid model, are shown in Fig. 11. Larger periods of vibrations are realized for the first three eigenmodes when the reservoir is filled to the level of 2/3 than when it is completely full. This is due to the significant action of the convective hydrodynamic action (sloshing) in the 2/3 filled reservoir Fig. 11d, while the completely full reservoir is influenced only by hydrodynamic action.

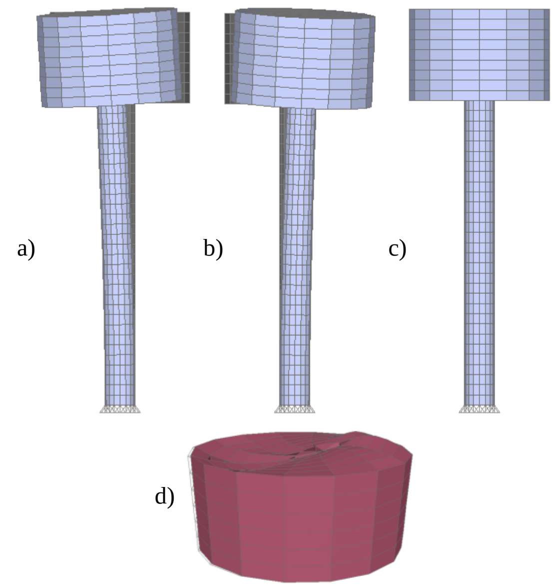

Figure 11. Eigenmodes of 3D RC water tower with 3D FE solid fluid model: a) I (translational); b) II (translational), c) III (torsion); d) eigenmode of vibrations of the 3D FE solid fluid model.

The effect of introducing the 3D FE solid fluid model and the correction of eigenmodes of the water tower are considered through modal load participation ratio and the modal participating mass ratio. The modal load participation ratio determines how well the calculated eigenmodes can represent the response to dynamic actions, /28/. For the n-th eigenmode, the modal load participation ratio is as follows:

$$r_n^D = \frac{f_n^2}{a^T p}, \tag{29}$$

where: a is the acceleration obtained through $Ma = p$, while f_n is determined according to:

$$f_n = \varphi_n^T p, \tag{30}$$

where: φ_n is the eigenvalue and p is the load vector. The cumulative sum of the modal load participation ratio for all calculated eigenmodes is determined according to:

$$R^D = \sum_{n=1}^{N} r_n^D = \frac{\sum_{n=1}^{N} (\varphi_n^T p)^2}{a^T p}, \tag{31}$$

where: N is the total number of calculated eigenmodes. The number of required eigenmodes is obtained iteratively and by tracking the R^D ratio, the minimum value of which should be at least 90 %. The modal participating mass ratio indicates the importance of the n-th eigenmode in the analysis of system response in all three orthogonal directions:

$$r_{xn} = \frac{f_{xn}^2}{M_x}, \quad r_{yn} = \frac{f_{yn}^2}{M_y}, \quad r_{zn} = \frac{f_{zn}^2}{M_z}, \tag{32}$$

where: f_{xn}, f_{yn}, and f_{zn} are the factors of modal participation for x, y, and z directions:

$$f_{xn} = \varphi_n^T m_x, \quad f_{yn} = \varphi_n^T m_y, \quad f_{zn} = \varphi_n^T m_z, \tag{33}$$

where: m_x, m_y and m_z are masses for orthogonal directions. The detailed consideration of eigenmodes of the 3D FE solid model of fluid in the reservoir is carried out by minimizing the dynamic effects of the water tower by reducing the domain of frequencies under consideration:

$$|f - \delta| \le s, \tag{34}$$

where: δ and s are the centre and radius of frequency domain, respectively. By using Eq.(34) frequencies specific only for the 3D solid fluid-model are separated. For the purpose of simplifying the presentation of eigenmodes of vibrations, only half of the water tower is considered. Figure 12 shows the first four eigenmodes of the 3D FE solid model of the fluid in the reservoir of RC water tower. The significance of hydrodynamic convective action (sloshing) of the fluid is evident.

The SSA of vibrations of the 3D RC water tower using 3D FE solid fluid model is conducted in the frequency domain (FDA). However, prior to the SSA, the analysis of system response in the time domain (THA - Time History Analysis) is carried out and the obtained response is transformed into the frequency domain by using Fast Fourier Transform (FFT), /30/. The frequency response realized by using the Fourier transform is represented through amplitude (FAS - Fourier Amplitude Spectrum). The input function for the THA is defined by:

$$y = f \sin(\omega t). \tag{35}$$

Responses of the 3D RC water tower with the 3D FE solid fluid model in the frequency domain of the THA for charging stages are shown in Fig. 13, while responses in the frequency domain of the SSA are shown in Fig. 14. Periods of vibration of the first, second and third eigenmode are identified based on peak amplitudes in the frequency domain.

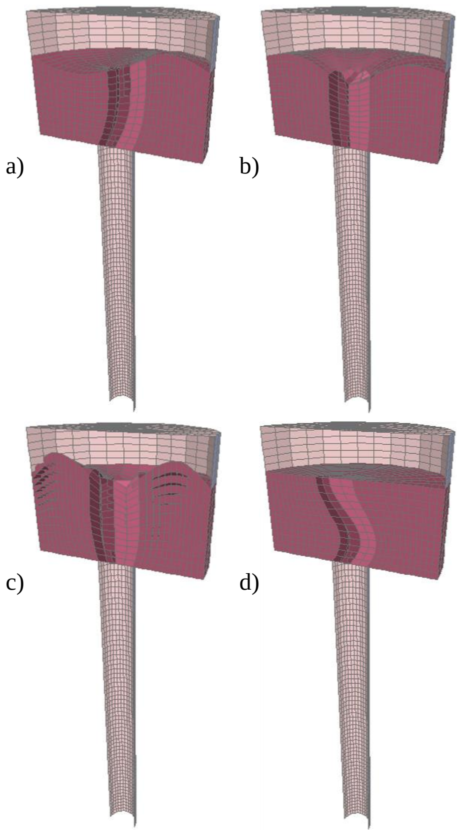

Figure 12. Eigenmodes of 3D FE solid model of the fluid of the RC water tower.

Solutions obtained by THA and SSA are compared with those obtained by analyses of eigenvibrations. Remarkably good agreements are obtained for vibration period of first and second eigenmode in all three analyses, while for vibration period of the third eigenmode small deviations are obtained for cases of second level charging and full reservoir.

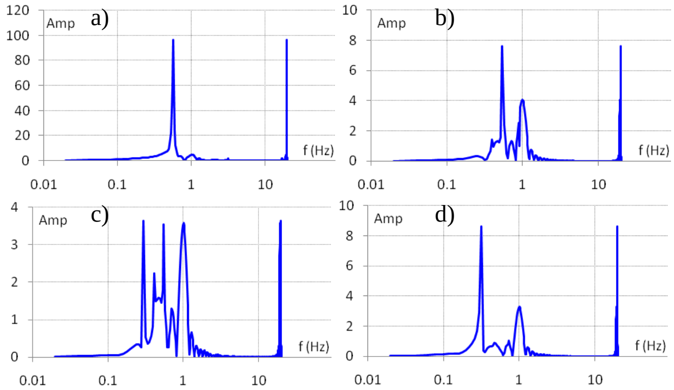

Figure 13. 3D RC water tower responses with 3D FE solid fluid model for THA in the frequency domain: a) empty; b) first level; c) second level; d) full.

ANALYSIS OF VIBRATIONS OF 3D STEEL WATER TOWER WITH 3D FE SOLID FLUID MODEL

The steel water tower is modelled using shell FE and 3D solid FE for the fluid, same as for the RC water tower (see previous chapter). The 3D water tower model is generated according to /31/. Total height of the water tower is 35.5 m, the reservoir diameter is 6.8 m, while the reservoir volume is 94 m^3. Parameters of numerical water tower models are shown in Table 3, where cases of empty and full reservoir, as well as intermediate stages of charging are considered. Figure 15 shows the 3D water tower model, the vertical cross-section of the 3D water tower model, the vertical cross-section of the 3D water tower model with 3D FE solid fluid model and 3D FE solid fluid.

Table 3. Parameters of 3D numerical steel water tower model.

stage	shell FE	solid FE	no. of equations of the system equilibrium	no. of degrees of freedom for the system mass
empty water tower	3552	-	20766	10335
first level	3552	1152	23646	13215
second level	3552	1728	25086	14655
third level	3552	2112	26046	15615
full water tower	3552	2304	27166	16225

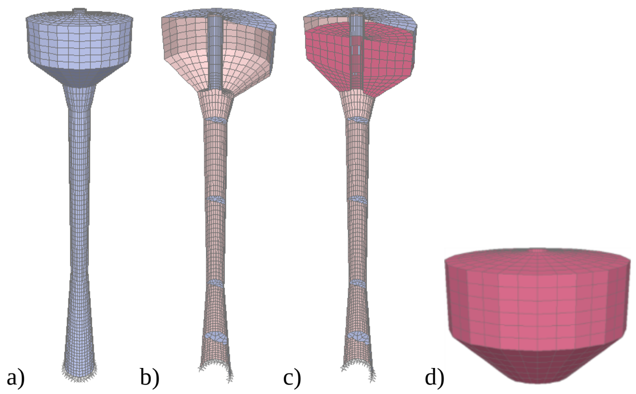

Figure 15. 3D steel water tower with 3D FE solid fluid: a) 3D water tower; b) vertical cross-section; c) vertical cross-section with 3D FE solid fluid; d) 3D FE solid fluid.

The 3D steel water tower with 3D FE solid fluid model is analysed regarding the eigenvibrations for simulating the stages of charging and discharging (SFA). The SFA consists of five analyses and five eigenvibration analyses which are conducted in a successive manner. The flowchart of the SFA vibration analysis of the steel water tower is shown in Fig. 16. The normalized periods of vibrations T/T_e of the steel water tower are shown in Fig. 17.

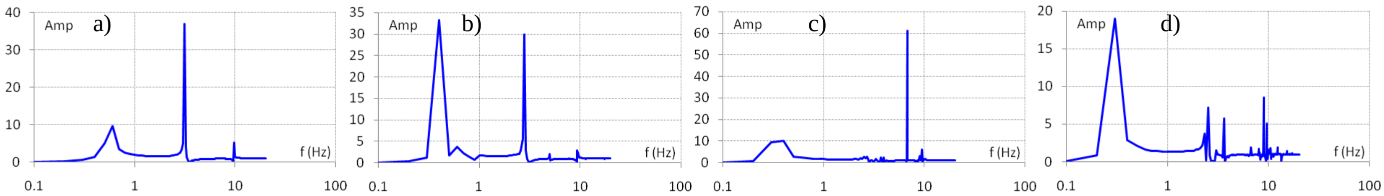

Figure 14. 3D RC water tower responses with 3D FE solid fluid model for SSA in the frequency domain: a) empty; b) first level; c) second level; d) full.

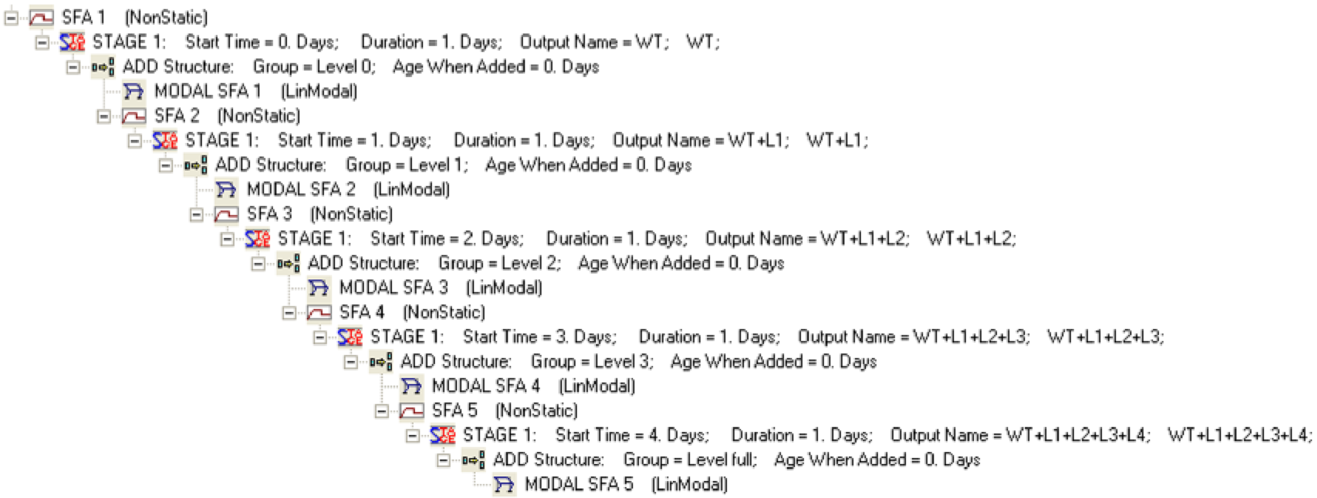

Figure 16. Flowchart of SFA vibration analysis of steel water tower.

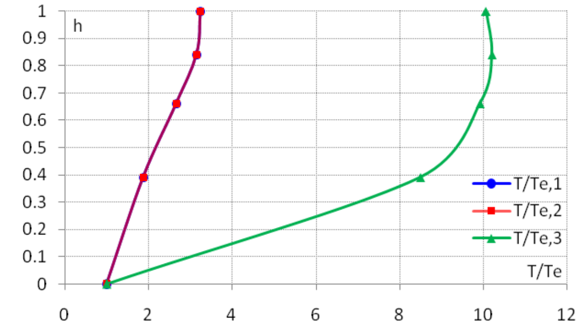

Figure 17. Normalized vibration periods T/T_e as a function of fluid level in the steel water tower reservoir.

The first three eigenmodes (two translational and one torsional) of the 3D steel water tower, along with the 3D FE solid fluid model, are shown in Fig. 18. As compared to the RC water tower, where larger periods of vibrations are realized when the reservoir is filled to the level of 2/3 than when it is completely full, in the case of steel water tower a gradual increment is observed while filling the reservoir. This is due to the fact that the increase of vibration periods has not been strongly influenced by convective hydrodynamic action (sloshing), with the prevailing influence of inductive hydrodynamic action.

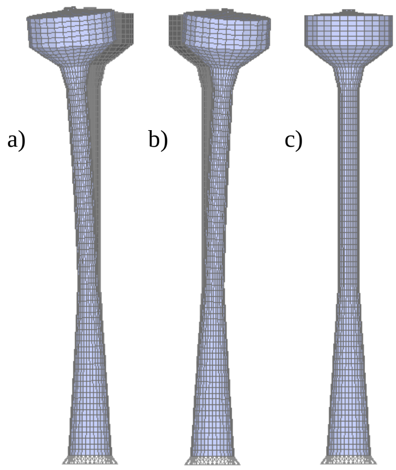

Figure 18. Eigenmodes of 3D steel water tower with 3D FE solid fluid model: a) I (translational); b) II (translational); c) III (torsion).

The maximum oscillation level of fluid in the reservoir is in direct correlation with the radius and the mass generated by the convective part of pressure. Figure 19 shows the eigenmodes of fluid in the reservoir for the stage when it is filled to the second and the third level. Given a vertical column existing in the steel water tower reservoir, which forms a toroidal body together with the reservoir, vibrations of the convective fluid mass are further reduced, making the influence generated by the convective part of the total pressure also lower.

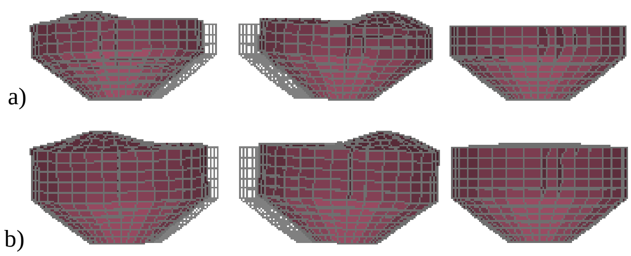

Figure 19. Eigenmodes of 3D FE solid fluid models: a) second level; b) third level.

Responses of the 3D steel water tower with the 3D FE solid fluid model in the frequency domain of THA for the charging stages are shown in Fig. 20, while responses in the frequency domain of SSA are shown in Fig. 21. Periods of vibration of the first, second and third eigenmode are identified based on peak amplitudes in frequency domain. Solutions obtained by THA and SSA are compared with solutions obtained by analysis of eigenvibrations. Remarkably good agreements of values are obtained for vibration period of the first, second and third eigenmode in THA. Some degree of deviation is observed for periods of vibration of the second and third eigenmode obtained by SSA in the case of filling the second and third level of the reservoir. These vibration periods are subsequently tested using the PSDA, and almost identical solutions are obtained as in SSA.

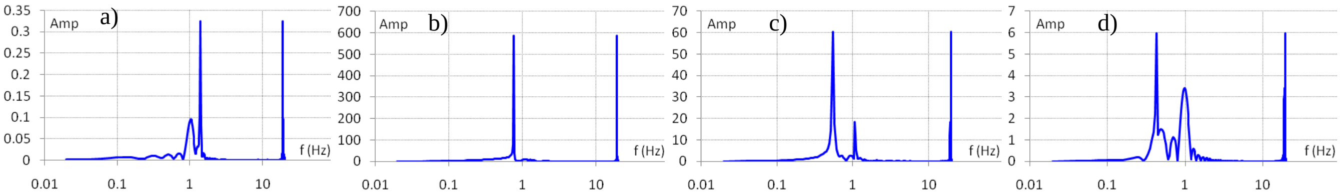

Figure 20. Responses of 3D steel water tower with 3D FE solid fluid model for THA in the frequency domain: a) empty; b) first level; c) second level; d) full.

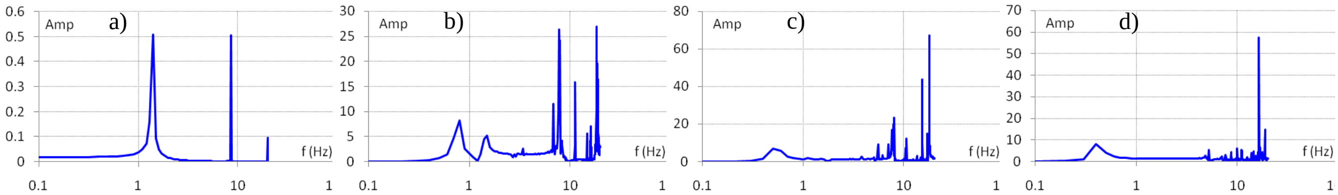

Figure 21. Responses of 3D steel water tower with 3D FE solid fluid model for SSA in the frequency domain. a) empty; b) first level; c) second level; d) full.

FINAL REMARKS AND CONCLUSIONS

The main concept of mathematical formulation and numerical research presented in this paper is to provide better approach and higher level of vibration analysis of the 3D water tower model. Development and implementation of 3D FE solid fluid models in modelling and analysing the system response is extremely important, both for determining the impulsive and convective hydrodynamic pressure, and sizing the water tower. Based on direct (explicit) fluid modelling using 3D solid FE, a discrete mathematical fluid model has been modelled for analysing the vibrations of the water tower. By using the 3D FE solid fluid model for analysing the vibrations of the water tower, the system's stiffness matrix and mass matrix are corrected. To determine the proper eigenmode, it is necessary to take into account the considerations regarding the modal load participation ratio and the modal participating mass ratio.

Vibrations of the water tower are calculated based on analysis of eigenvibrations, analysis of eigenvibrations occurring when charging and discharging the water tower reservoir (SFA), SSA in the frequency domain and analysis of the system response in time domain (THA), followed by transformation of frequency domain response using FFT. The PSDA was carried out as means of additional control and with the purpose of identifying the vibration periods.

Effects of the system's increasing vibration period are determined for the partially charged RC reservoir of the water tower, given the strong action of the fluid mass that is excited for convective hydrodynamic action (sloshing) due to the tower dimensions and the amount of fluid contained in the reservoir in this case. When comparing the solutions obtained by THA and SSA with the solutions obtained by analysis of eigenvibrations, remarkably good agreements are found between the values of vibration period of the first and second eignemodes, while for the vibration periods of the third eigenmode, small deviations are found in the case of partially filled reservoir.

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