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THERMOMECHANICAL ANALYSIS OF MAGNETO-MICROPOLAR SOLID SUBJECTED TO THERMAL LASER HEAT SOURCE

TERMOMEHANIČKA ANALIZA MAGNETNO-MIKROPOLARNOG ČVRSTOG TELA POD LASERSKIM IZVOROM TOPLOTE

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 6.08.2020

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Keywords
- micropolar thermoelastic
- Hall current
- thermal laser heat source
- rotation
- temperature dependence

Abstract

The model of the equations of generalized magneto-micropolar thermoelasticity with two relaxation times in an isotropic medium with temperature-dependent mechanical properties under the effect of Hall current is established. Elastodynamic interactions in magneto-micropolar thermo-elastic half-space consider the effect of Hall current and rotation subjected to laser heating. The micropolar theory of thermoelasticity by Eringen (1966) has been used to investigate the problem. Normal mode analysis technique is used to solve the resulting non-dimensional coupled field equations to obtain displacement, stress components and temperature distribution. Numerical computed results of all the considered variables are shown graphically to depict the combined effect of the Hall current, laser heat source and rotation. Some particular cases of interest are also deduced from the present study.

Ključne reči
- mikropolarni i termoelastični
- Holov efekat
- laserski izvor toplote
- rotacija
- temperaturska zavisnost

Izvod

Izveden je model jednačina generalisane magnetno-mikropolarne termoelastičnosti sa dva vremena relaksacije u izotropnoj sredini sa mehaničkim osobinama zavisnim od temperature, pod uticajem Holovog efekta. Elastodinamičke interakcije u magnetno-mikropolarnom termoelastičnom poluprostoru podrazumevaju Holov efekat i rotaciju, pod uticajem zagrevanja laserom. Za istraživanje problema je primenjena mikropolarna teorija termoelastičnosti Eringena (1966). Metoda analize normalnih modova je upotrebljena za rešavanje spregnutih jednačina rezultujućeg bezdimenzionog polja, radi dobijanja pomeranja, komponenata napona i raspodele temperature. Rezultati numeričkog proračuna svih razmotrenih veličina su prikazani grafički radi uočavanja kombinovanog uticaja Holovog efekta, laserskog izvora toplote i rotacije. U radu su takođe razmotreni neki posebni slučajevi od značaja.

INTRODUCTION

The elastic modulus is an important physical property of materials reflecting the elastic deformation capacity of the material when subjected to an applied external load. Most of the investigations are done under the assumption of the temperature independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature, e.g. at high temperature the material characteristic such as the modulus of elasticity. The researches concerning the investigation of the effect of magnetic field (that may be Earth's magnetic field or other human generated high intensity magnetic field) and thermal loading by lasers on various type of materials are of great importance in seismological research and in engineering applications. The linear theory of micropolar elasticity was developed by Eringen /1/. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motion. Rigid chopped fibers, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals are examples of such materials.

The theory of magneto thermoelasticity has a wide range of applications and possibilities of research in the field of geology, earth sciences, plasma physics and engineering. When a particle is stationary under the effect of magnetic field, the field has no effect on this particle. Also, consider a particle moving in parallel direction of the magnetic field, the particle will move undeflected. Now in case a particle is moving within a path, having a component normal to the magnetic field, the particle will be deflected due to a force acting on it. In addition to this deflected motion this particle will experience the electric field. The combined force is the Lorentz force. There are a consideration that mechanical and electromagnetic field interactions take place due to Lorentz forces. Conductivity perpendicular to the direction of the magnetic field decreases due to the free spiralling of negatively charged electrons and other ions about the magnetic field lines before colliding, and a current is induced perpendicular to both the electric and magnetic field. This phenomenon is called the Hall Effect. When the magnetic field intensity is very high, the Hall effect cannot be neglected. Zakaria /2/ investigated the effects of Hall current and rotation on magneto-micropolar generalized thermoelasticity, including the boundary condition with a source of ramp type heating.

A thermal shock induces very rapid movement in the structural elements, giving rise to very significant inertial forces, and a rise to oscillations. The ultrashort lasers have pulse durations ranging from nano- to femtoseconds. Also, in an ultrashort laser pulse, the high energy flux and short duration result in a very large thermal gradient. So, the Fourier law of heat conduction is no longer valid. For one-point laser heat input, Rose /4/ provided an accurate mathematical basis. Scruby et al. /3/ investigated a mathematical model of point source to study the ultrasonic evolution by lasers. He studied the physics of heated plate by laser heat loading in the thermoelastic system as a surface centre of expansion (SCOE). Later McDonald /5/ and Spicer /6/ gave a mathematical model known as laser-generated ultrasound model by introducing the thermo-diffusion concept. Dubois /7/ verified by experimental results that penetration depth plays an important role in the generation of laser-ultrasound. Kim et al. /10/ and Chen et al. /9/ investigated some other such type of research. Aouadi /15/ explained the real behaviour of materials, i.e. the temperature dependence of elastic moduli in a micropolar generalized medium. Abo-Dahab and Abbas /8/ investigated the LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Thermoelastic behaviour of laser heating context of different theories of thermoelasticity was presented by Youssef and Al-Bary /11/. A 2-dimensional problem in generalised thermoelastic medium with thermo-diffusion was investigated by Elhagary /12/. Kumar et al. /13/ studied the elastodynamical interactions of input heat source with microstretch thermoelastic medium. Kumar et al. /17/ and Kumar and Kumar /16/ discussed some dynamical problems in micropolar thermoelasticity. Ailawalia and Sachdeva /18/ presented a problem involving temperature dependent thermoelastic half space with micro temperatures subjected to the effect of internal heat source.

The aim of the present study is to investigate the interaction in magneto micropolar thermoelastic medium, taking into consideration the effect of Hall current, laser heat source and rotation. Components of displacement, stress, current density and temperature distribution are obtained by using normal mode analysis. The problem has become more interesting with the inclusion of thermal laser heat source, normal and tangential forces. The resulting quantities are computed numerically and depicted graphically.

BASIC EQUATIONS

We consider a micropolar thermoelastic medium permeated by an initial strong magnetic field $\mathbf{H} = (0, H_0, 0)$. The considered medium is rotating with constant angular velocity assumed to be equal to Ω. For magneto-micropolar thermoelastic medium the basic equations and constitutive relation in absence of body forces, body couples and stretch forces, following Eringen /1/, Al Qahtani and Dutta /14/, and Zakaria /2/, are given by:

$$(\lambda+\mu)\nabla(\nabla.\mathbf{u})+(\mu+K)\nabla^2\mathbf{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T+$$

$$+\mu_0\varepsilon_{rji}J_r H_j=\rho\left[\ddot{\mathbf{u}}+\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})+2\mathbf{\Omega}\times\frac{\partial u}{\partial t}\right], \qquad (1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\mathbf{u}=\rho j\left(\ddot{\boldsymbol{\phi}}+\mathbf{\Omega}\times\frac{\partial\boldsymbol{\phi}}{\partial t}\right), \qquad (2)$$

$$K^*\nabla^2 T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\beta_1 T_0(\nabla.\dot{\mathbf{u}}-Q)+$$

$$+\nu_1 T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^* \qquad (3)$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijk}\phi_k)-$$

$$-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T\,, \qquad (4)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}\,, \qquad (5)$$

where: λ, μ, α, β, γ, and K are constants depending on the nature of material; ρ is density of the medium; $\mathbf{u} = (u_1, u_2, u_3)$ and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ are displacement and microrotation vectors respectively; T is temperature; T_0 is the reference temperature; K^* is the coefficient of thermal conductivity; c^* is the specific heat at constant strain; j is the microinertia; $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$, $\nu_1 = (3\lambda + 2\mu + K)\alpha_{t2}$; α_{t1} and α_{t2} are coefficients of linear thermal expansion; t_{ij} are components of stress; m_{ij} are components of coupled stress; δ_{ij} is the Kronecker delta function; τ_0 and τ_1 are thermal relaxation times with $\tau_0 \geq \tau_1 \geq 0$.

The micropolar thermoelastic medium is supposed to be rotating with angular velocity $\mathbf{\Omega}$. In such type of medium, the equations of motion have two additional terms,

(i) the centripetal acceleration $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ due to time varying motion,

(ii) the Coriolis acceleration $2(\mathbf{\Omega}\times\dot{\mathbf{u}})$.

The current density vector $\mathbf{J}$ can be expressed as:

$$\mathbf{J}=\frac{\sigma_0}{1+m^2}\left[\mathbf{E}+\mu_0(\mathbf{u}\times\mathbf{H})-\frac{\mu_0}{en_e}(\mathbf{J}\times\mathbf{H})\right]. \qquad (6)$$

Here, $\mathbf{F} = \mu_0(\mathbf{J}\times\mathbf{H})$ is the Lorentz force; $\mathbf{H}$ is the magnetic field vector; $\mathbf{E}$ is the intensity of the electric field; m is Hall parameter; σ_0 is the electrical conductivity; e is the charge of an electron; n_e is the number density of electrons. Further, the plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0 f(t)g(x_1)h(x_3)\,, \qquad (7)$$

$$f(t)=\frac{t}{t_0^2}e^{-(t/t_0)}\,, \qquad (8)$$

$$g(x_1)=\frac{1}{2\pi r^2}e^{-(x_1^2/r^2)}\,, \qquad (9)$$

$$h(x_3)=\gamma^* e^{-\gamma^* x_3}\,, \tag{10}$$

where: I_0 is absorbed energy; t_0 is the pulse rising time; r is the beam radius. Figures 1-3 show profiles of $f(t)$, $g(x_1)$, $h(x_3)$.

Equation (7) with substitution of Eqs.(8-10) takes the form

$$Q=\frac{I_0\gamma^*}{2\pi r^2 t_0^2}\, t e^{-(t/t_0)} e^{-(x_1^2/r^2)} e^{-\gamma^* x_3}\,. \tag{11}$$

In the above equations symbol (,) followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (˙) denotes the derivative with respect to time, respectively.

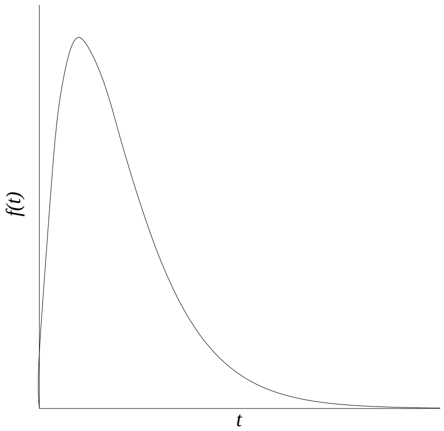

Figure 1. Temporal profile of *f(t)*.

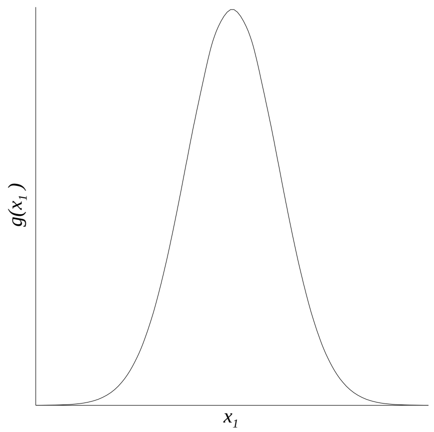

Figure 2. Profile of $g(x_1)$.

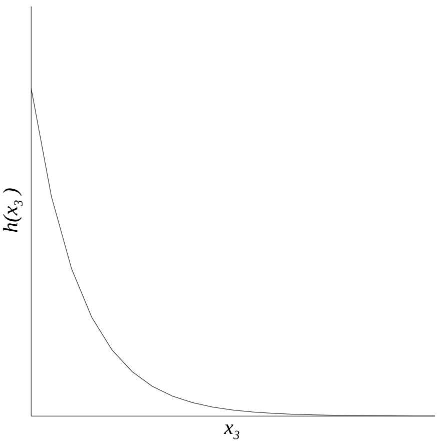

Figure 3. Profile of $h(x_3)$.

FORMULATION OF THE PROBLEM

We consider a temperature dependent magneto-micropolar thermoelastic medium with rectangular Cartesian coordinate system $0x_1x_2x_3$ having the x_2 axis pointing vertically downward the medium. A normal force/tangential force and ultrashort laser pulse are assumed to be acting at the origin of the rectangular Cartesian co-ordinate system. A component of Hall current H_0 is in the x_2 direction.

We consider a plane strain problem with all the field variables depending on (x_1, x_2, t). For two-dimensional problems, we take

$$\mathbf{u}=(u_1,0,u_2)\,,\quad \boldsymbol{\phi}=(0,\phi_2,0)\,. \tag{12}$$

Also, we are interested to investigate the effect of dependence of reference temperature on all elastic and thermal parameters. Therefore, we may assume:

$$\lambda=\lambda_0(1-\alpha^* T_0)\,,\quad \mu=\mu_0(1-\alpha^* T_0)\,,\quad K=K_0(1-\alpha^* T_0)\,,$$
$$\nu=\nu_0(1-\alpha^* T_0)\,,\quad \alpha=\alpha_0(1-\alpha^* T_0)\,,\quad \beta=\beta_0(1-\alpha^* T_0)\,,$$
$$\gamma=\gamma_0(1-\alpha^* T_0)\,,\quad K^*=K_0^*(1-\alpha^* T_0)\,,\quad c^*=c_0^*(1-\alpha^* T_0)\,,$$

where: λ_0, μ_0, K_0, ν_0, α_0, β_0, γ_0, K_0^*, j_0 are considered constants; α^* is called empirical material constant, in case the system is independent of reference temperature, $\alpha^* = 0$. For further consideration, it is convenient to introduce into Eqs.(1)-(3) the dimensionless quantities defined as:

$$x_i'=\frac{\omega^*}{c_1}x_i\,,\ u_i'=\frac{\rho\omega^* c_1}{\beta_1 T_0}u_i\,,\ \phi_i'=\frac{\rho c_1^2}{\beta_1 T_0}\phi_i\,,\ T'=\frac{T}{T_0}\,,\ t'=\omega^* t\,,$$
$$\tau_1'=\omega^*\tau_1\,,\ t_{ij}'=\frac{1}{\beta_1 T_0}t_{ij}\,,\ \omega^*=\frac{\rho c^* c_1^2}{K^*}\,,\ c_1^2=\frac{\lambda+2\mu+k}{\rho}\,,$$
$$m_{ij}^*=\frac{\omega^*}{c\beta_1 T_0}m_{ij}\,,\ \Omega^1=\frac{\Omega}{\omega^*}\,,\ M=\frac{\sigma_0\mu_0^2 H_0^2}{\rho\omega^*}\,,\ Q'=\frac{\beta_1^2}{\rho c_1^2}Q\,. \tag{13}$$

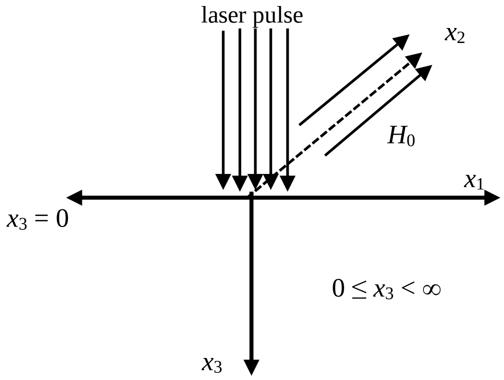

Figure 4. Geometry of the problem.

Making use of Eqs.(12)-(13) the system of Eqs.(1)-(3) reduces to:

$$\zeta_1\frac{\partial e}{\partial x_1}+\zeta_2\nabla^2 u_1-\zeta_3\frac{\partial\phi_2}{\partial x_3}+\Omega_0^2 u_1-2\Omega_0\frac{\partial u_3}{\partial t}+$$
$$+\frac{M}{1+m^2}\left(\frac{\partial u_1}{\partial t}+m\frac{\partial u_3}{\partial t}\right)-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\ddot{u}_1\,, \tag{14}$$

$$\zeta_1\frac{\partial e}{\partial x_3}+\zeta_2\nabla^2 u_3+\zeta_3\frac{\partial\phi_2}{\partial x_1}+\Omega_0^2 u_3+2\Omega_0\frac{\partial u_1}{\partial t}-$$

$$-\frac{M}{1+m^2}\left(m\frac{\partial u_1}{\partial t}-\frac{\partial u_3}{\partial t}\right)-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\ddot{u}_3, \quad (15)$$

$$\nabla^2\phi_2-2\zeta_4\phi_2+\zeta_4\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=\zeta_5\ddot{\phi}_2, \quad (16)$$

$$-\nabla^2 T+\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\zeta_6\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)(\dot{e}-Q)=$$

$$=Q_0 f^*(x_1,t)e^{-\gamma^* x_3}, \quad (17)$$

$$M_1=2\Omega_0+\frac{M}{1+m^2};\ \zeta_1=\frac{\lambda_0+\mu_0}{\rho c_1^2};\ \zeta_2=\frac{\mu_0+K_0}{\rho c_1^2};$$

$$\zeta_3=\frac{K_0}{\rho c_1^2};\ \zeta_4=\frac{K_0 c_1^2}{\gamma_0\omega^{*2}};\ \zeta_5=\frac{\rho j_0 c_1^2}{\gamma_0};\ \zeta_6=\frac{\beta_1 T_0^2}{\rho\omega^* K_0^*}.$$

Using Helmholtz's decomposition theorem, the displacement components u_1 and u_2 are related to the non-dimensional potential functions ϕ and ψ by the relation mentioned below:

$$u_1=\frac{\partial\phi}{\partial x_1}+\frac{\partial\psi}{\partial x_3},\ u_3=\frac{\partial\phi}{\partial x_3}-\frac{\partial\psi}{\partial x_1}. \quad (18)$$

Substituting the values of u_1 and u_2 from Eq.(18) into Eqs.(14)-(17), we obtain:

$$\left(\nabla^2+\Omega^2+\frac{M}{1+m^2}\frac{\partial}{\partial t}-\frac{\partial^2}{\partial t^2}\right)\phi+\left(2\Omega-\frac{mM}{1+m^2}\right)\frac{\partial\psi}{\partial t}-\left(1+\tau_1\frac{\partial}{\partial t}\right)T=0 \quad (19)$$

$$\left(a_3\nabla^2+\frac{M}{1+m^2}\frac{\partial}{\partial t}-\frac{\partial^2}{\partial t^2}\right)\psi-\left(2\Omega-\frac{mM}{1+m^2}\right)\frac{\partial\phi}{\partial t}-a_4\phi_2=0 \quad (20)$$

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+a_5\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi-\nabla^2 T=Q_0 f^*(x_1,t)e^{-\gamma^* x_3} \quad (21)$$

$$\left(\nabla^2-2a_1+a_2\frac{\partial^2}{\partial t^2}\right)\phi_2+a_1\nabla^2\psi=0, \quad (22)$$

where:

$$a_1=\frac{K_0 c_1^2}{\gamma_0\omega^{*2}},\ a_2=-\frac{\rho j_0 c_1^2}{\gamma_0},\ a_3=\frac{\mu_0+K_0}{\rho c_1^2},\ a_4=\frac{K_0}{\rho c_1^2},$$

$$a_5=\frac{\beta_1^2 T_0}{\rho K_0^*\omega^*},\ a_6=\frac{\lambda_0+\mu_0}{\rho c_1^2},\ Q_0=\frac{A^* a_{13} I_0\gamma^*}{2\pi r^2 t_0^2},\ A^*=\frac{1}{1-a^* T_0},$$

$$f(x_1,t)=\left[t+\varepsilon\tau_0\left(1-\frac{t}{t_0}\right)\right]e^{-\left(\frac{x_1^2}{r^2}+\frac{t}{t_0}\right)}.$$

SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi,\psi,T,\phi^*\}(x_1,x_3,t)=\{\bar{\phi},\bar{\psi},\bar{T},\bar{\phi}^*\}(x_3)e^{i(kx_1-\omega t)}. \quad (23)$$

Here, ω is the angular frequency and k is wave number.

Making use of Eq.(23) in Eqs.(19)-(22) and after some simplifications, yields:

$$[A\mathrm{D}^8+B\mathrm{D}^6+C\mathrm{D}^4+E\mathrm{D}^2+F]\bar{\phi}=f_1(\gamma^*,x_1,t)e^{-\gamma^* x_3}, \quad (24)$$

$$[A\mathrm{D}^8+B\mathrm{D}^6+C\mathrm{D}^4+E\mathrm{D}^2+F]\bar{T}=f_2(\gamma^*,x_1,t)e^{-\gamma^* x_3}, \quad (25)$$

$$[A\mathrm{D}^8+B\mathrm{D}^6+C\mathrm{D}^4+E\mathrm{D}^2+F]\bar{\phi}_2=f_3(\gamma^*,x_1,t)e^{-\gamma^* x_3}, \quad (26)$$

$$[A\mathrm{D}^8+B\mathrm{D}^6+C\mathrm{D}^4+E\mathrm{D}^2+F]\bar{\psi}=f_4(\gamma^*,x_1,t)e^{-\gamma^* x_3}. \quad (27)$$

Here, $\mathrm{D}=\frac{\partial}{\partial x_3}$, $k_1=-k^2+\Omega^2-\frac{i\omega M}{1+m^2}+\omega^2$, $k_2=-i\omega\left[2\Omega-\frac{mM}{1+m^2}\right]$,

$$k_3=\omega^2+\Omega^2-i\omega\frac{M}{1+m^2}-a_3k^2,\ k_4=k^2+\omega^2 a_2+2a_1,$$

$$k_5=i\omega(1-i\omega\tau_0)-k^2,\ k_6=a_5(i\omega+\omega^2\varepsilon\tau_0),$$

$$k_7=a_3(k_4+k_5)+k_3+a_3k_4,\ k_8=a_1a_4k^2-k_3k_4-k_5(k_3-a_1a_4+a_3k_4),$$

$$k_9=k_5(a_1a_4k^2-k_3k_4),\ k_{10}=k_6(a_3k^2+k_3+a_3k_4-a_1a_4),$$

$$k_{11}=k_6(2a_1a_4k^2-k_3k_4-a_3k_4k^2-k_3k^2),\ k_{12}=k_6k^2(k_3k_4-a_1a_4k^2),$$

$$A=-a_3,\ B=k_7-a_3k_1-\tau_{11}a_3k_6,\ C=k_8+k_1k_7+\tau_{11}k_{10}-k_2^2,$$

$$E=k_1k_8-k_9+\tau_{11}k_{11}+k_2^2(k_4+k_5),\ F=\tau_{11}k_{12}-k_1k_9-k_2^2k_4k_5.$$

The solution of the above system of Eqs.(24)-(27) satisfying the radiation conditions, that $(\bar{\phi}, \bar{\psi}, \bar{T}, \bar{\phi}_2)\rightarrow 0$ as $x_3\rightarrow\infty$, are given as following:

$$\bar{\phi}=\sum_{i=1}^{4}c_i e^{-m_i x_3}+\frac{f_1}{f_5}e^{-\gamma^* x_3}, \quad (28)$$

$$\bar{T}=\sum_{i=1}^{4}\alpha_i c_i e^{-m_i x_3}+\frac{f_2}{f_5}e^{-\gamma^* x_3}, \quad (29)$$

$$\bar{\phi}_2=\sum_{i=1}^{4}\beta_i c_i e^{-m_i x_3}+\frac{f_3}{f_5}e^{-\gamma^* x_3}, \quad (30)$$

$$\bar{\psi}=\sum_{i=1}^{4}\delta_i c_i e^{-m_i x_3}+\frac{f_4}{f_5}e^{-\gamma^* x_3}, \quad (31)$$

Here m_i^2 (i = 1,2,3,4) are the roots of characteristic equation of Eq.(24), and

$$\alpha_i=-\frac{-k_6a_3m_i^6+k_{10}m_i^4+k_{11}m_i^2+k_{12}}{-a_3m_i^6+k_7m_i^4+k_8m_i^2-k_9},$$

$$\beta_i=\frac{a_1k_2m_i^4-a_1k_2(k_5+k^2)m_i^2+a_1k_2k_5k^2}{-a_3m_i^6+k_7m_i^4+k_8m_i^2-k_9},\ i=1,2,3,4,$$

$$f_1=Q_0 f(x_1,t)(-a_3\gamma^{*6}+k_7\gamma^{*4}+k_8\gamma^{*2}-k_9),$$

$$f_2=Q_0 f(x_1,t)(-k_6a_3\gamma^{*6}+k_{10}\gamma^{*4}+k_{11}\gamma^{*2}+k_{12}),$$

$$f_3=Q_0 f(x_1,t)(a_1k_2\gamma^{*4}-a_1k_2(k_5+k^2)\gamma^{*2}+a_1k_2k_5k^2),$$

$$f_4=Q_0 f(x_1,t)(k_2\gamma^{*4}-k_2(k_5+k_4)\gamma^{*2}+k_2k_4k_5),$$

$$f_5=(A\gamma^{*8}+B\gamma^{*6}+C\gamma^{*4}+E\gamma^{*2}+F).$$

Substituting the values of $\bar{\phi}$, $\bar{T}$, $\bar{\phi}_2$, $\bar{\psi}$, from Eqs.(28)-(31) into Eqs.(4)-(5), and using Eqs.(12), (13), (18) and solving the resulting equations, we obtain:

$$\bar{t}_{33}=\sum_{i=1}^{4}G_{1i}e^{-m_i x_3}-M_1e^{-\gamma^* x_3}, \quad (32)$$

$$\bar{t}_{31}=\sum_{i=1}^{4}G_{2i}e^{-m_i x_3}-M_2e^{-\gamma^* x_3}, \quad (33)$$

$$\bar{m}_{32}=\sum_{i=1}^{4}G_{3i}e^{-m_i x_3}-M_3e^{-\gamma^* x_3}\,, \tag{34}$$

$$\bar{T}=\sum_{i=1}^{4}G_{4i}e^{-m_i x_3}-M_4e^{-\gamma^* x_3}\,, \tag{35}$$

where: $G_{mi} = g_{mi}C_i$, $i = 1,2,\ldots 4$; g_{mi} and M_i are mentioned in the Appendix A.

BOUNDARY CONDITIONS

We consider normal and tangential forces acting at the surface $x_3 = 0$ along with the vanishing of coupled stress at $x_3 = 0$ and $I_0 = 0$. Mathematically this can be written as:

$$t_{33}=-F_1e^{-(kx_1-\omega t)},\; t_{31}=-F_2e^{-(kx_1-\omega t)},\; m_{32}=0\,,$$
$$\frac{\partial T}{\partial x_3}=-F_3e^{-(kx_1-\omega t)}\,, \tag{36}$$

where: F_1 and F_2 are the magnitude of the applied mechanical forces and F_3 is thermal source magnitude.

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following:

$$\sum_{i=1}^{4}(g_{1i},g_{2i},g_{3i},g_{4i})c_i=(-F_1,-F_2,0,-F_3)\,. \tag{37}$$

The system of Eqs.(37) are solved by using the matrix method as follows:

$$\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}=\begin{bmatrix}g_{11}&g_{12}&g_{13}&g_{14}\\g_{21}&g_{22}&g_{23}&g_{24}\\g_{31}&g_{32}&g_{33}&g_{34}\\g_{41}&g_{42}&g_{43}&g_{44}\end{bmatrix}^{-1}\begin{bmatrix}-F_1\\-F_2\\0\\-F_3\end{bmatrix}. \tag{38}$$

SPECIAL CASE

Micropolar thermoelastic solid

If we neglect the Hall current in Eqs.(37) and put $I_0 = 0$, we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic half space.

NUMERICAL RESULTS AND DISCUSSION

The analysis is conducted for a magneto-micropolar material. For numerical computations, following Eringen /19/, the values of physical constants are: $\lambda = 9.4\cdot10^{10}$ Nm^{-2}, $\mu = 4.0\cdot10^{10}$ Nm^{-2}, $K = 1.0\cdot10^{16}$ Nm^{-2}, $\rho = 1.74\cdot10^{3}$ kgm^{-3}, $j = 0.2\cdot10^{-19}$ m^2, $\gamma = 0.779\cdot10^{-9}$ N.

Following Dhaliwal and Singh /20/, thermal parameters are given by: $c^* = 1.04\cdot10^3$ Jkg^{-1}K^{-1}, $K^* = 1.7\cdot10^6$ Jm^{-1}s^{-1}K^{-1}, $\alpha_{t1} = 2.33\cdot10^{-5}$ K^{-1}, $\alpha_{c1} = 2.48\cdot10^{10}$ K^{-1}, $T_0 = 298$ K, $\tau_0 = 0.02$, $\tau_1 = 0.01$, $\alpha_{c1} = 2.65\cdot10^{-4}$ m^{3}kg^{-1}, $a = 2.9\cdot10^4$ m^{2}s^{-2}K^{-1}, $b = 32\cdot10^5$ kg^{-1}m^{5}s^{-2}, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $\alpha^* = 0.051$ K^{-1}.

A comparison of the dimensionless form of the field variables for the cases of micropolar thermoelastic with Hall current, rotation, and input laser heat source (MPHCLSR) and micropolar thermoelastic (MPTH) is presented in Figs. 4-9. The values of all physical quantities for all cases are shown in the range $0 \le x_3 \le 5$.

Solid lines and dash lines correspond to micropolar thermoelastic with Hall current and input laser heat source (MPHCLSR) and micropolar thermoelastic (MPTH), in respect, for $t = 0.1$.

The computations are carried out in the absence and presence of laser pulse ($I_0 = 10^5$ and $I_0 = 0$) and on the surface of plane $x_1 = 1$, $t = 0.1$.

Thermal source

Figure 5 shows the variation of normal stress t_{33} with the distance x_3. It is noticed that for MPHCLSR and MPTH, the normal stress t_{33} shows similar behaviour initially. The normal stress in MPHCLSR initially increases and then shows oscillatory trend. The value of t_{33} approaches that on boundary surface away from the source.

Figure 6 displays the variation of tangential stress t_{31} with distance x_3. It is noticed that initially the behaviour of t_{31} for MPHCLSR and MPTH is similar. Initially t_{31} increases monotonically for MPHCLSR and also for MPTH but approaches to the boundary surface away from the point of application of temperature source.

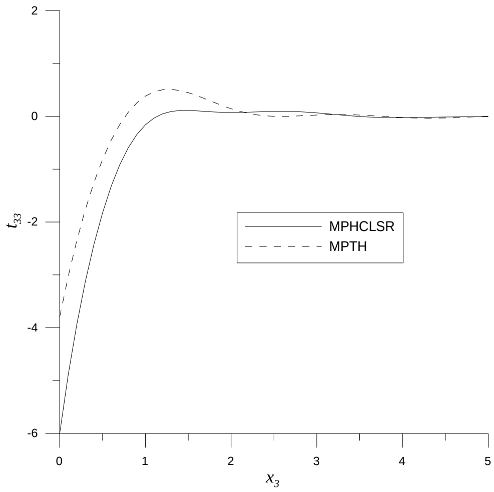

Figure 5. Variation of normal stress.

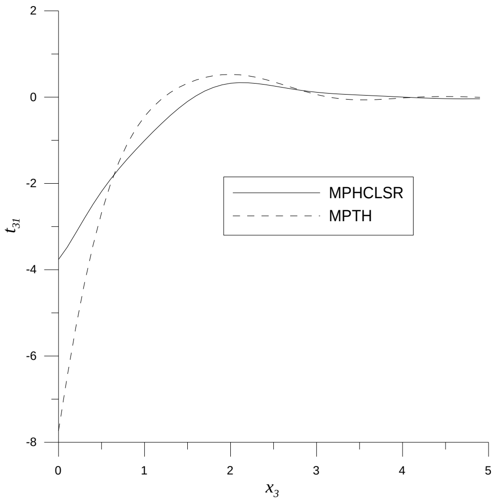

Figure 6. Variation of tangential stress.

Figure 7 clears the variation of coupled stress m_{32} with distance x_3 for MPHCLSR and MPTH. The variation of m_{32} for (MPHCLSR) is monotonically increasing in the region $0 \le x_3 \le 1$, whereas the trend of variation in MPTH is opposite. The m_{32} approaches zero away from the point of application of source. It is clear from Fig. 3 that Hall current has a significant effect on the value of m_{32} and causes significant oscillatory behaviour in MPHCLSR.

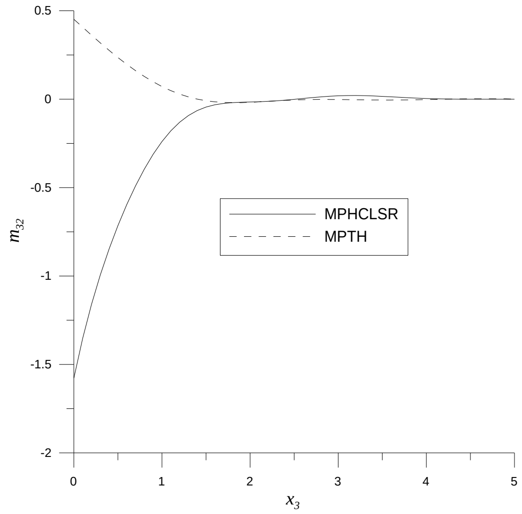

Figure 7. Variation of coupled tangential stress.

Figure 8 displays the variation of temperature T with distance x_3. The values of temperature change for MPTH show monotonically decreasing behaviour in the range $0 \leq x_3 \leq 5$. In case of MPHCLSR, the temperature decreases by exhibiting oscillatory trend due to the Hall effect and input laser heat source.

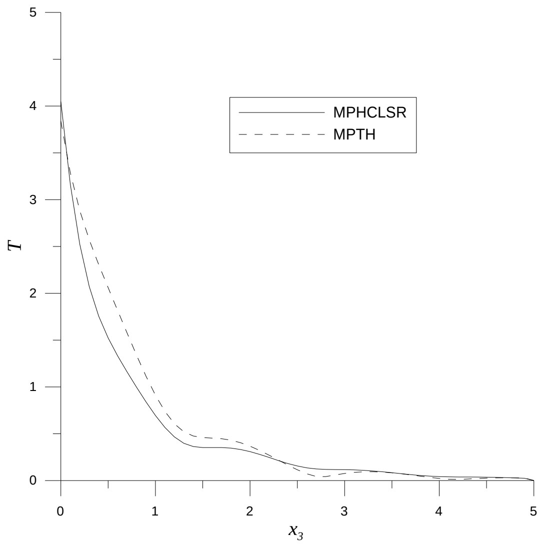

Figure 8. Variation of temperature.

Figures 9 and 10 exhibit the behaviour of displacement components u_1 and u_3 w.r.t. x_3. Both displacement components approach to boundary surface away from the application of normal force, which is in agreement to the generalized theory of thermoelasticity.

Particular case: for $\alpha^* = 0$, the results are obtained for temperature independent mechanical properties under the effect of Hall current when normal force is applied on the boundary surface. The graphical results in this case are represented in Figs. (11)-(16).

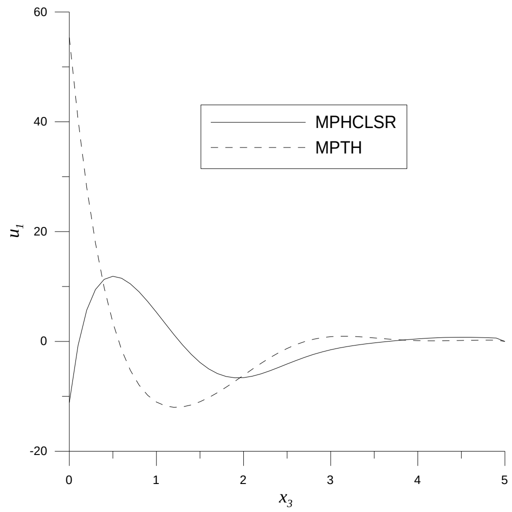

Figure 9. Variation of u_1 w.r.t. x_3.

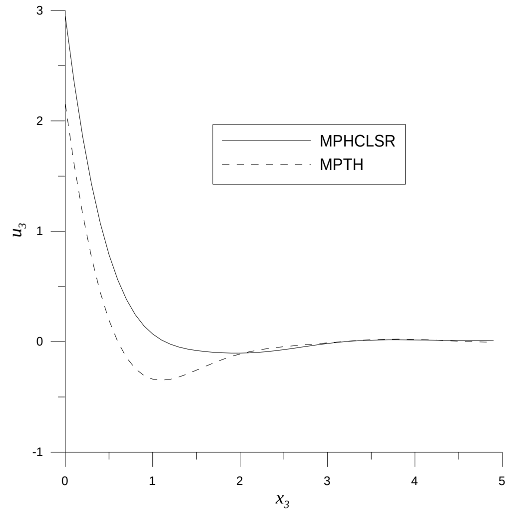

Figure 10. Variation of u_3 w.r.t. x_3.

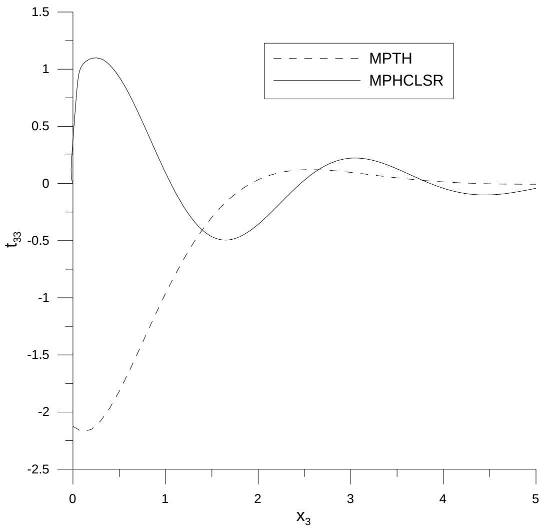

Figure 11. Variation of t_{33} w.r.t. x_3.

Figure 11 shows the variation of normal stress t_{33} with distance x_3. It is noticed that for MPHCLSR and MPTH, the normal stress t_{33} shows opposite behaviour initially. The normal stress in MPHCLSR initially increases and then shows oscillatory trend. The value of t_{33} approaches to boundary surface away from the source.

Figure 12 displays the variation of tangential stress t_{31} with distance x_3. It is noticed that initially the behaviour of t_{31} for MPHCLSR and MPTH is opposite. Initially t_{31} increases monotonically for MPHCLSR and decreases monotonically for MPTH but approaches to the boundary surface away from the point of application of normal force.

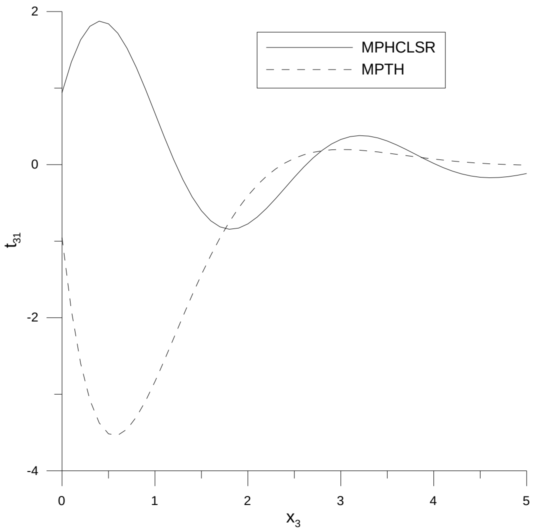

Figure. 12. Variation of t_{31} w.r.t. x_3.

Figure 13 clears the variation of coupled stress m_{32} with distance x_3 for MPHCLSR and MPTH. The variation of m_{32} for (MPHCLSR and MPTH) is monotonically increasing in the region $0 \leq x_3 \leq 1$ and is monotonically decreasing thereafter. The m_{32} approaches to zero away from the point of application of source. It is clear from Fig. 13 that the Hall current has a significant effect on the value of m_{32} and causes significant oscillatory behaviour in MPHCLSR.

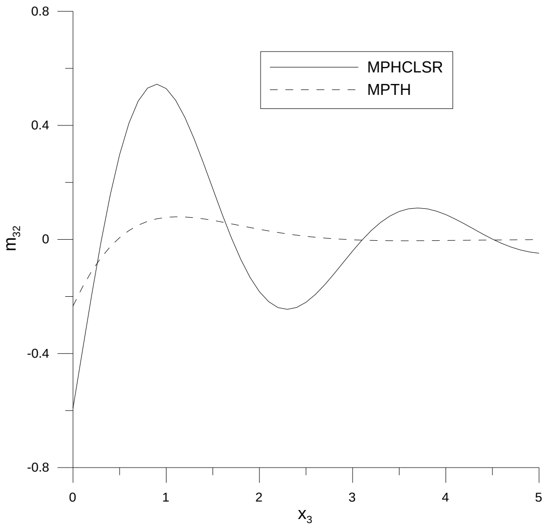

Figure. 13. Variation of m_{32} w.r.t. x_3.

Figure 14 displays the variation of temperature T with distance x_3. The values of temperature change for MPTH show monotonically decreasing behaviour in the range $0 \leq x_3 \leq 5$. In case of MPHCLSR the temperature decreases by exhibiting oscillatory trend due to the Hall effect and input laser heat source.

Figures 15 and 16 exhibit the behaviour of displacement components u_1 and u_3 w.r.t. x_3. Both displacement components approach to boundary surface away from the application of normal force which is in agreement to the generalized theory of thermoelasticity.

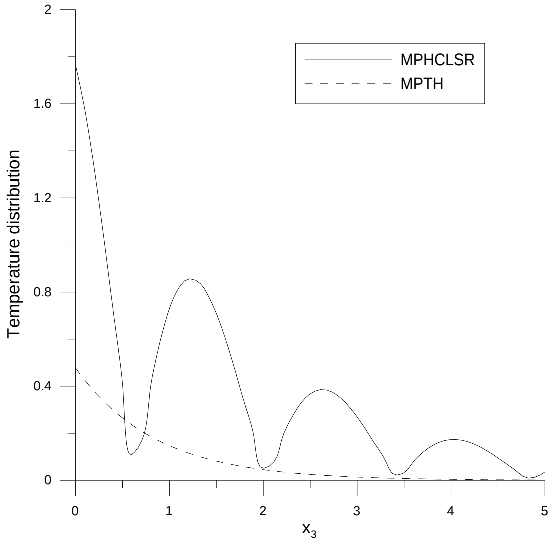

Figure. 14. Variation of temperature w.r.t. x_3

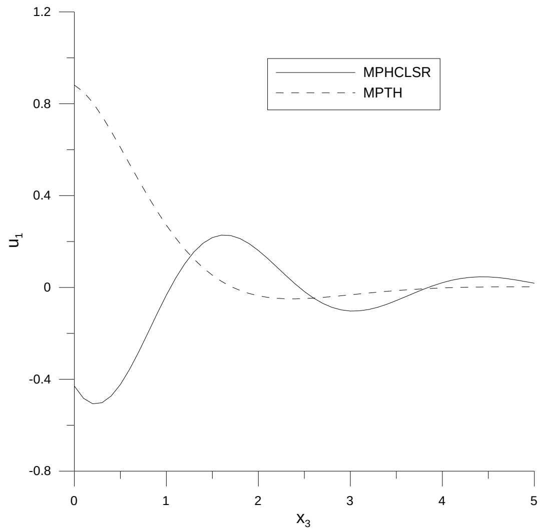

Figure. 15. Variation of u_1 w.r.t. x_3

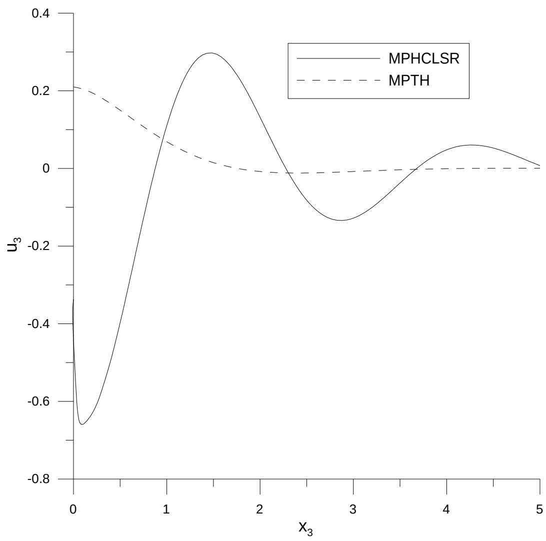

Figure. 16. Variation of u_3 w.r.t. x_3

CONCLUSIONS

The problem consists of investigating displacement components, temperature distribution, Hall current and stress components in a homogeneous isotropic micropolar thermoelastic temperature dependent half space due to various sources subjected to laser pulse. Normal mode analysis technique is employed to express the results mathematically.

The analysis of results permits some concluding remarks:
(1) It is clear from the figures that all the field variables have non-zero values only in the bounded region of space, indicating that all the results are in agreement with the generalized theory of thermoelasticity.
(2) The effect of the Hall current, rotation and ultra short laser is much pronounced in all the resulting quantities.

The new model is employed in magneto-micropolar thermoelastic medium as a new improvement in the field of thermoelasticity. The subject becomes more interesting due to Hall current involving rotation and irradiation of an ultra short laser pulse with an extensive short duration, or a very high heat flux. This type of problem has found numerous applications. The method used in this article is applicable to a wide range of problems in thermodynamics. Regarding the obtained results, it is expected that the present model of equations will serve as more realistic and will provide motivation to investigate micropolar thermoelasticity problems.

ACKNOWLEDGEMENT

The authors declare that there is no conflict of interest regarding the publication of this paper. Also, no funding from any agency is received for this research.

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APPENDIX A:

$$b_2 = \frac{\lambda_0}{\rho c_1^2},\ b_3 = \frac{2\mu_0 + K_0}{\rho c_1^2},\ b_5 = \frac{\mu_0 + K_0}{\rho c_1^2},\ b_6 = \frac{\mu_0}{\rho c_1^2},$$

$$b_7 = \frac{K_0}{\rho c_1^2},\ b_8 = \frac{\omega^{*2}\gamma}{\rho c_1^4},\ b_9 = \frac{\omega^{*2} b_0}{\rho c_1^4},\ b_{10} = \frac{\omega^{*2}}{\rho c_1^4},$$

$$g_{1i} = (m_i^2 - b_2 k^2) + \imath b_3 k m_i \alpha_{3i} - \tau_{11}\alpha_{1i},$$

$$g_{2i} = -\imath b_3 k m_i + (b_6 m_i^2 + b_5 k^2)\alpha_{3i} - b_7 \alpha_{2i},$$

$$g_{3i} = -b_8 \alpha_{2i} m_i,\ \ g_{4i} = -m_i \alpha_{1i},$$

$$M_1 = (b_1 f_2 + (\gamma^{*2} - b_2 k^2) f_1 - \tau_{11} f_3 + \imath b_3 k \gamma^* f_4) / f_5,$$

$$M_2 = (-\imath b_3 k \gamma^* f_1 + (b_6 \gamma^{*2} + b_5 k^2)) / f_5,$$

$$M_3 = -b_8 \gamma^* f_3 / f_5,\ M_4 = -\gamma^* f_2 / f_5.$$