CREEP MODELLING OF SPHERICAL SHELL UNDER INFLUENCE OF INTERNAL AND EXTERNAL PRESSURE

INTRODUCTION

The proposed creep modelling can be used for the simulation of creep processes occurring in spherical shell structures under influence of internal and external pressure. It is clear that the use of the generalized creep laws is connected with great experimental efforts, so in many practical cases we have to work with creep equations containing a reduced number of parameters. On the other hand, the proposed creep-damage model is, in some situations, incomplete and does not reflect the real damage behaviour. The reason for this inaccuracy is that we deal only with one damage parameter. Many researchers have done creep modelling in spherical and cylindrical shell under various conditions. Miller /1/ derived a solution for stresses and strains in a thick circular shell exposed to internal and external loads. In addition to plastic behaviour, the shell material is accepted to experience both wet creep and dimensional changes as the shell is pressurized. Zhang et al. /2/ presented an precise method to find out stresses in thick-walled spherical pressure vessels under influence of uniform internal pressure. The impacts of Young’s modulus of the external layer on the distortions and stresses in the vessels comprising of the three unique layers are inspected. A technique to get practically consistent circumferential stresses in the vessels comprising of the practically evaluated material just are researched. Gilbert et al. /3/ investigated nonlinear creep behaviour of spherical shells of revolution including domes subjected to sustained loads. A nonlinear axisymmetric hypothetical model, which represents the impacts of creep and shrinkage, is developed. The governing field equations are derived using variational principles, equilibrium requirements, and integral-type constitutive relations. Hansen et al. /4/ presented the new finite volume method for solving three-dimensional thermal convection in a spherical shell under strong temperature and pressure-dependent viscosity in which the spherical shell is divided into six cubes. The performing model is validated by taking parameters of steady-state cubic and tetrahedral convection with other published spherical models and a detailed convergence test on successively refined grids. Kashkoli et al. /5/ assumed that the thermo-creep response of the material is governed by Norton’s law, and an analytical solution of the problem is presented for determining time-dependent creep stresses and displacements of homogeneous thick-walled pressure vessels. The nonlinear behaviour of the material arises by taking into consideration creep behaviour at high temperature under quasi-static conditions. All the authors mentioned above have determined the solutions of the problems by considering assumptions of creep-strain laws like Norton and incompressibility condition, etc. These conditions are based on classical assumptions of creep transition. These conditions are no longer valid at transition state of the solid and this state is nonlinear in nature. Seth /6-9/ has developed the transition theory which is helpful to solve various problems related to plastic and creep deformations in solids. Seth’s transition theory can be applied to various problems.
of creep transition. Neither the yield criterion, nor the associated flow rule is assumed here. Sharma et al. /10-12/ investigated behaviour of transversely isotropic cylinder under internal and external pressure. The results are derived for plastic and creep stresses in cylinder by using the concept of generalized strain measures and transition theory. Pathania et al. /13, 14/ worked on the problem of elastic-plastic and thermal creep stresses under combined effect of internal and external pressure. In this paper, I shall derive the results for creep stresses under combined effect of internal and external pressure in a spherical shell without using semi-empirical laws. The results are derived and shown graphically.

FORMULATION OF THE MATHEMATICAL PROBLEM

Consider a thick-walled spherical shell, whose inward and outer radii are a and b respectively, subjected to uniform internal pressure p₁ and external pressure p₂. The components of displacement in spherical co-ordinates (r, θ, φ) are given as

\[ u = r(1 - g), \quad v = 0, \quad w = 0, \quad (1) \]

where: \( u, v, w \) (displacement components); and \( g = g(r) \).

Generalized components of strain are given by Seth /15-16/ as

\[ e_{rr} = \frac{1}{r^m} \left[ 1 - (r'g' + g)^n \right]^m, \]
\[ e_{θθ} = \frac{1}{r^m} \left[ 1 - g^n \right] = e_{φφ}, \]
\[ e_{rθ} = e_{θφ} = e_{φr} = 0, \quad (2) \]

where: \( g' = dg/dr \).

Stress-strain relation: the constitutive equation for stress-strain relations for an isotropic material is given in /17/,

\[ T_{ij} = \frac{1}{n} \left[ 2(1 - g)^m + (1 - (r'g' + g)^n)^m \right] + 2\mu \left[ 1 - (r'g' + g)^n \right]^m, \]
\[ T_{θθ} = \frac{1}{n} \left[ 2(1 - g)^m + (1 - (r'g' + g)^n)^m \right] + 2\mu \left[ 1 - (1 - g)^m \right]^m = T_{φφ}, \]
\[ T_{θφ} = T_{φθ} = T_{rφ} = T_{φr} = 0. \quad (3) \]

Equation of equilibrium: the radial equilibrium of an element of the spherical shell requires:

\[ \frac{dT_{rr}}{dr} = - \frac{2(T_{θθ} - T_{rr})}{r} = 0, \quad (5) \]

where: \( T_{rr} \) and \( T_{θθ} \) are the radial and circumferential stresses.

Boundary conditions: boundary conditions of the problem are written as

\[ T_{rr} = -p_1 \quad \text{at} \quad r = a \quad \text{and} \quad T_{rr} = -p_2 \quad \text{at} \quad r = b. \quad (6) \]

Critical points or turning points: using Eqs.(4) in Eq.(5), we get a nonlinear differential equation in g as:

\[ \left[ 1 - (r'g' + g)^n \right]^m - (r'g' + g)^n - (1 - g^n)^m \times \left[ 1 - (r'g' + g)^n \right]^m - (1 - g^n)^m = 0 \quad (7) \]

where: \( c = 2\mu(\lambda + 2\mu) \) and putting \( g' = gQ \) (Q is function of g, and \( g \) is function of \( r \)), we get

\[ \left[ 1 - g^n(Q+1)^n \right]^{-m} - 2Q(C(1) - g^n m)^{-1} - 2c \times \left[ 1 - g^n(Q+1)^n \right]^{-m} - (1 - g^n)^m = 0. \quad (8) \]

Transition points of \( g \) in Eq.(8) are \( Q \rightarrow -1 \) and \( Q \rightarrow ±∞ \). Here, we are only interested in finding creep stresses corresponding to \( Q \rightarrow -1 \).

TRANSITION FUNCTION AND CREEP LAW

We define the transition function \( R \) through principal stress difference (see Thakur /18-21/, Sharma /22/, Gupta /23/, Verma /24-25/ at the transition point \( Q \rightarrow -1 \). The transition function \( R \) is given as:

\[ R = (T_{θθ} - T_{rr}) = \frac{2\mu}{n} \left[ (1 - g^n)^m - (1 - g^n(Q+1)^n)^m \right]. \quad (9) \]

Taking the logarithmic differentiation of Eq.(9) with respect to \( g \) and substituting the value of \( dQ/dg \) from Eq.(8) and taking asymptotic value \( Q \rightarrow -1 \), we get:

\[ \frac{d\log R}{dg} = - 2\mu \frac{d\log r}{dg} - \left[ 1 - (1 - g^n)^m \right] \frac{1 - (1 - g^n)^m}{1 - (1 - g^n)^m}. \quad (10) \]

This gives

\[ \log \left( \frac{R}{A_0} \right) = - 2\mu \log r + (3 - 2c) \log \left[ 1 - (1 - g^n)^m \right], \quad (11) \]

as \( Q \rightarrow -1 \), where \( A_0 \) is a constant of integration. Therefore, we have

\[ T_{θθ} - T_{rr} = A_0r^{-2c} \left[ 1 - (1 - g^n)^m \right]^{3-2c}. \quad (12) \]

Since \( g \rightarrow g_0/r \) as \( Q \rightarrow -1 \), where \( g_0 \) is constant, we get

\[ T_{θθ} - T_{rr} = A_0r^{-2c} \left[ 1 - (1 - g_0^n)^m \right]^{3-2c}. \quad (13) \]

Equation (13) will lead to a very general form of solutions in creep. For this problem, I shall take \( m = 1 \). Then Eq.(13) gives

\[ T_{θθ} = T_{rr} = Ar^{-3n+2c(n-1)}, \quad (14) \]

where: \( A \) is constant. Combining Eqs.(14) and (5), we have

\[ T_{rr} = \frac{2A}{-3n+2c(n-1)} r^{-3n+2c(n-1)} + B, \quad (15) \]

where: \( B \) is an integration constant.

From Eqs.(14) and (15),

\[ T_{θθ} = \frac{A}{-3n+2c(n-1)} r^{-3n+2c(n-1)} + B. \quad (16) \]
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By applying boundary conditions from Eq.(6) into Eqs. (15) and (16), we have

$$A = \frac{-(p_1 - p_2)[a^{3n+2c(n-1)}]}{2b^{3n+2c(n-1)}}, \quad B = \frac{-(p_1 - p_2)[a^{3n+2c(n-1)}]}{b^{3n+2c(n-1)}}.$$ \hspace{1cm} (17)

By using the values of constants $A$ and $B$, we have creep stresses in the spherical shell as

$$T_{rr} = \frac{(p_1 - p_2)\left[\left(\frac{a}{b}\right)^r - \left(\frac{r}{b}\right)^r\right]}{\left(\frac{a}{b}\right)^r - 1} - p_1, \quad T_{\theta\theta} = T_{rr} = \frac{(p_1 - p_2)\left[\left(\frac{r}{b}\right)^r\right]}{2\left(\frac{r}{b}\right)^r - 1}.$$ \hspace{1cm} (18)

where: $t = -3n + 2c(n-1)$.

Equations (18) give creep stresses for the spherical shell under the combined load of internal and external pressure. We introduce the following non-dimensional components: $R = r/b$, $R_0 = a/b$, $\sigma_0 = T_{rr}/E$, $\sigma_{\theta\theta} = T_{\theta\theta}/E$, $p_1 - P_2 = (p_1 - p_2)/E$, where $E = 2\pi(3 - 2c)/(2 - c)$. Equations (18) in a non-dimensional form become:

$$\sigma_{rr} = \frac{(p_1 - P_2)\left[\left(\frac{R_0}{R}\right)^r - \left(\frac{R}{R_0}\right)^r\right]}{\left(\frac{R_0}{R}\right)^r - 1} - p_1,$$

$$\sigma_{\theta\theta} = \sigma_{rr} = \frac{(p_1 - P_2)\left[\left(\frac{R}{R_0}\right)^r\right]}{2\left(\frac{R}{R_0}\right)^r - 1},$$ \hspace{1cm} (19)

where: $t = -3n + 2c(n-1)$.

**NUMERICAL DISCUSSION ON CREEP STRESSES AND STRAIN RATES**

For calculating creep stresses based on the above analysis, the following values have been taken: incompressible material $C = 0$, and compressible material $C = 0.25, 0.50, 0.75$; measure $n = 1/7, 1/5, 1$ (i.e. $N = 7, 5, 1$). In the classical theory, measure $N$ is equal to $1/n$. The creep stresses derived from Eqs.(19) are plotted in graphs along radii ratio $R$ under different cases of internal and external pressure. In Fig. 1, creep curves are drawn by taking into consideration that internal pressure is higher than external pressure. Curves for radial, as well as circumferential stresses, are produced against radii ratio $R$ for different value of measures. It is found that radial stresses have more impact on the internal surface of the sphere as compared to the external surface of the shell for different materials with compressibilities $c = 0, 0.25, 0.50$. The values of creep stresses are negative due to compressive nature of pressure. It is also seen that values of creep stresses get lowered for $n = 1/5$ as compared to $n = 1/7$. For linear measure $n = 1$, it is seen that radial stresses, as well as circumferential stresses, show same values for all types of compressible materials. Radial stresses are more effective at internal surface as compared to circumferential stresses. In Fig. 2, creep curves are drawn by taking into consideration that external pressure is higher than internal pressure. The effect of stresses gets reversed in this case. The circumferential stresses have more effect on outer boundary of shell as compared to the radial stresses. It is further seen that with increase in compressibility of the material, creep stresses have more influence on interior as well as the exterior part of the shell as compared to incompressible material. It means that under effect of higher external pressure, the spherical shell made up of high compressible material will face more damage. In Fig. 3, creep curves are drawn under effect of external pressure only. It is found that for nonlinear measure $n = 1/7$ and $1/5$ creep stresses have more influence on exterior part of spherical shell in absence of internal pressure. The stress values are more in case of $n = 1/7$ as compared to $n = 1/5$. For linear measure $n = 1$, circumferential stresses are maximum, irrespective of compressible material nature. In Fig. 4, creep curves are drawn under effect of internal pressure only. It is observed that circumferential stresses are tensile in nature that will

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**Figure 1. Creep stresses along radii ratio at $P_1 = 15$ and $P_2 = 5$.**
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Figure 2. Creep stresses along radii ratio at $P_1 = 5$ and $P_2 = 15$.

Figure 3. Creep stresses along radii ratio at $P_1 = 0$ and $P_2 = 20$.

Figure 4. Creep stresses along radii ratio at $P_1 = 20$ and $P_2 = 0$. 

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lead to expansion of spherical shell. Radial stresses are more effective inside the shell and they have no influence on the boundary of the shell for linear as well as nonlinear measure values. Incompressible material is under less influence of stresses as compared to the compressible material.

CONCLUSION

It can be concluded from above discussed results that spherical shell made up of incompressible material has a longer life as compared to the shell made up of compressible material, under internal and external pressure environments. The influence of nonlinear measure $n = 1/7$ on creep stresses is maximum in the spherical shell, as compared to other values which leads to more damage in the shell.

REFERENCES


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