Gaurav Verma

CREEP MODELLING OF SPHERICAL SHELL UNDER INFLUENCE OF INTERNAL AND EXTERNAL PRESSURE

MODELIRANJE PUZANJA SFERNE LJUSKE POD UTICAJEM UNUTRAŠNJEG I SPOLJAŠNJEG PRITISKA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 22.11.2019

Adresa autora / Author's address:
PG Department of Mathematics, Hans Raj Mahila Maha Vidyalaya, Jalandhar, Punjab, India
email: gkdon85@gmail.com

Keywords
- creep
- pressure
- spherical shell
- stress

Abstract

In the present paper, creep analysis of spherical shell has been done under the influence of internal and external pressure. Creep stresses in a spherical shell are derived by using principal strain measure and Seth's transition theory. A novel feature of the transition theory is its ability to give a uniform treatment of plastic and creep deformation without making ad hoc assumptions like yield criterion or creep laws. The expressions obtained are valid for compressible as well as incompressible materials.

Ključne reči
- puzanje
- pritisak
- sferna ljuska
- napon

Izvod

U radu je data analiza puzanja sferne ljuske pod uticajem unutrašnjeg i spoljašnjeg pritiska. Naponi puzanja u sfernoj ljuski su izvedeni primenom mere glavne deformacije i teorije prelaznih napona Seta. Novost u teoriji prelaznih napona jeste mogućnost uniformnog pristupa plastične deformacije i deformacije puzanja bez uvođenja ad hoc pretpostavki, kao što su kriterijum puzanja ili zakoni puzanja. Dobijeni izrazi važe za stišljiv kao i za nestišljiv materijal.

INTRODUCTION

The proposed creep modelling can be used for the simulation of creep processes occurring in spherical shell structures under influence of internal and external pressure. It is clear that the use of the generalized creep laws is connected with great experimental efforts, so in many practical cases we have to work with creep equations containing a reduced number of parameters. On the other hand, the proposed creep-damage model is, in some situations, incomplete and does not reflect the real damage behaviour. The reason for this inaccuracy is that we deal only with one damage parameter. Many researchers have done creep modelling in spherical and cylindrical shell under various conditions. Miller /1/ derived a solution for stresses and strains in a thick circular shell exposed to internal and external loads. In addition to plastic behaviour, the shell material is accepted to experience both wet creep and dimensional changes as the shell is pressurized. Zhang *et al.* /2/ presented an precise method to find out stresses in thick-walled spherical pressure vessels under influence of uniform internal pressure. The impacts of Young's modulus of the external layer on the distortions and stresses in the vessels comprising of the three unique layers are inspected. A technique to get practically consistent circumferential stresses in the vessels comprising of the practically evaluated material just are researched. Gilbert *et al.* /3/ investigated nonlinear creep behaviour of spherical shells of revolution including domes subjected to sustained loads. A nonlinear axisymmetric hypothetical model, which represents the impacts of creep and shrinkage, is developed. The governing field equations are derived using variational principles, equilibrium requirements, and integral-type constitutive relations. Hansen *et al.* /4/ presented the new finite volume method for solving three-dimensional thermal convection in a spherical shell under strong temperature and pressure-dependent viscosity in which the spherical shell is divided into six cubes. The performing model is validated by taking parameters of steady-state cubic and tetrahedral convection with other published spherical models and a detailed convergence test on successively refined grids. Kashkoli *et al.* /5/ assumed that the thermo-creep response of the material is governed by Norton's law, and an analytical solution of the problem is presented for determining time-dependent creep stresses and displacements of homogeneous thick-walled pressure vessels. The nonlinear behaviour of the material arises by taking into consideration creep behaviour at high temperature under quasi-static conditions. All the authors mentioned above have determined the solutions of the problems by considering assumptions of creep-strain laws like Norton and incompressibility condition, etc. These conditions are based on classical assumptions of creep transition. These conditions are no longer valid at transition state of the solid and this state is nonlinear in nature. Seth /6-9/ has developed the transition theory which is helpful to solve various problems related to plastic and creep deformations in solids. Seth's transition theory can be applied to various problems of creep transition. Neither the yield criterion, nor the associated flow rule is assumed here. Sharma *et al.* /10-12/ investigated behaviour of transversely isotropic cylinder under internal and external pressure. The results are derived for plastic and creep stresses in cylinder by using the concept of generalized strain measures and transition theory. Pathania *et al.* /13, 14/ worked on the problem of elastic-plastic and thermal creep stresses under combined effect of internal and external pressure. In this paper, I shall derive the results for creep stresses under combined effect of internal and external pressure in a spherical shell without using semi-empirical laws. The results are derived and shown graphically.

FORMULATION OF THE MATHEMATICAL PROBLEM

Consider a thick-walled spherical shell, whose inward and outer radii are a and b respectively, subjected to uniform internal pressure p_1 and external pressure p_2. The components of displacement in spherical co-ordinates (r, θ, ϕ) are given as

$$u = r(1-g), \quad v = 0, \quad w = 0, \tag{1}$$

where: u, v, w (displacement components); and $g = g(r)$.

Generalized components of strain are given by Seth /15-16/ as

$$e_{rr} = \frac{1}{n^m}\left[1-(rg'+g)^n\right]^m,$$
$$e_{\theta\theta} = \frac{1}{n^m}\left[1-g^n\right] = e_{\phi\phi}, \tag{2}$$
$$e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0,$$

where: $g' = dg/dr$.

Stress-strain relation: the constitutive equation for stress-strain relations for an isotropic material is given in /17/,

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij}, \quad (i,\ j = 1, 2, 3), \tag{3}$$

where: T_{ij} are the stress components; λ and μ are Lame's constants; $I_1 = \Sigma e_{kk}$ is the first strain invariant; δ_{ij} is the Kronecker delta.

By using Eqs.(2) in Eq.(3), the stresses are obtained as:

$$T_{rr} = \frac{\lambda}{n^m}\left[2(1-g^n)^m + \{1-(rg'+g)^n\}^m\right] + \frac{2\mu}{n^m}\left[1-(rg'+g)^n\right]^m,$$
$$T_{\theta\theta} = \frac{\lambda}{n^m}\left[2(1-g^n)^m + \{1-(rg'+g)^n\}^m\right] + \frac{2\mu}{n^m}\left[1-g^n\right]^m = T_{\phi\phi},$$
$$T_{r\theta} = T_{\theta\phi} = T_{\phi r} = 0. \tag{4}$$

Equation of equilibrium: the radial equilibrium of an element of the spherical shell requires:

$$\frac{dT_{rr}}{dr} - \frac{2(T_{\theta\theta}-T_{rr})}{r} = 0, \tag{5}$$

where: T_{rr} and $T_{\theta\theta}$ are the radial and circumferential stresses.

Boundary conditions: boundary conditions of the problem are written as

$$T_{rr} = -p_1 \ \text{ at } \ r = a \quad \text{and} \quad T_{rr} = -p_2 \ \text{ at } \ r = b. \tag{6}$$

Critical points or turning points: using Eqs.(4) in Eq.(5), we get a nonlinear differential equation in g as:

$$\left\{1-(rg'+g)^n\right\}^{m-1}(rg'+g)^{n-1}(rg''+2g') + 2(1-c)(1-g^n)^{m-1}\times$$
$$\times g^{n-1}g' - \frac{2c}{mnr}\left[\left\{1-(rg'+g)^n\right\}^m - (1-g^n)^m\right] = 0 \tag{7}$$

where: $c = 2\mu/\lambda + 2\mu$; and putting $rg' = gQ$ (Q is function of g, and g is function of r), we get

$$\left\{1-g^n(Q+1)^n\right\}^{m-1}Q(Q+1)^{n-1}g\frac{dQ}{dg} + Q(Q+1)^n\times$$
$$\times\left\{1-g^n(Q+1)^n\right\}^{m-1}2Q(1-c)(1-g^n)^{m-1} - \frac{2c}{mng^n}\times$$
$$\times\left[\left\{1-g^n(Q+1)^n\right\}^m - (1-g^n)^m\right] = 0. \tag{8}$$

Transition points of g in Eq.(8) are $Q \to -1$ and $Q \to \pm\infty$. Here, we are only interested in finding creep stresses corresponding to $Q \to -1$.

TRANSITION FUNCTION AND CREEP LAW

We define the transition function R through principal stress difference (see Thakur /18-21/, Sharma /22/, Gupta /23/, Verma /24-25/) at the transition point $Q \to -1$. The transition function R is given as:

$$R = (T_{\theta\theta} - T_{rr}) = \frac{2\mu}{n^m}\left[(1-g^n)^m - \left\{1-g^n(Q+1)^n\right\}^m\right]. \tag{9}$$

Taking the logarithmic differentiation of Eq.(9) with respect to g and substituting the value of dQ/dg from Eq.(8) and taking asymptotic value $Q \to -1$, we get:

$$\frac{d\log R}{d\log g} = -2c\frac{d\log r}{dg} - \frac{(3-2c)mng^n(1-g^n)^{m-1}}{\left\{1-g^n(Q+1)^n\right\}^m - (1-g^n)^m}. \tag{10}$$

This gives

$$\log\left(\frac{R}{A_0}\right) = -2c\log r + (3-2c)\log\left\{1-(1-g^n)^m\right\}, \tag{11}$$

as $Q \to -1$, where A_0 is a constant of integration. Therefore, we have

$$T_{\theta\theta} - T_{rr} = A_0 r^{-2c}\left\{1-(1-g^n)^m\right\}^{3-2c}. \tag{12}$$

Since $g \to g_0/r$ as $Q \to -1$, where g_0 is constant, we get

$$T_{\theta\theta} - T_{rr} = A_0 r^{-2c}\left\{1-(1-g_o r^{-n})^m\right\}^{3-2c}. \tag{13}$$

Equation (13) will lead to a very general form of solutions in creep. For this problem, I shall take $m = 1$. Then Eq.(13) gives

$$T_{\theta\theta} - T_{rr} = Ar^{-3n+2c(n-1)}, \tag{14}$$

where: A is constant. Combining Eqs.(14) and (5), we have

$$T_{rr} = \frac{2A}{-3n+2c(n-1)}r^{-3n+2c(n-1)} + B, \tag{15}$$

where: B is an integration constant.

From Eqs.(14) and (15),

$$T_{\theta\theta} = \frac{A\left\{-3n+2+2c(n-1)\right\}}{-3n+2c(n-1)}r^{-3n+2c(n-1)} + B. \tag{16}$$

By applying boundary conditions from Eq.(6) into Eqs. (15) and (16), we have

$$A=\frac{-(p_1-p_2)\{-3n+2c(n-1)\}}{2\left[a^{-3n+2c(n-1)}-b^{-3n+2c(n-1)}\right]}, \quad B=-p_1+\frac{(p_1-p_2)a^{-3n+2c(n-1)}}{\left[a^{-3n+2c(n-1)}-b^{-3n+2c(n-1)}\right]}. \tag{17}$$

By using the values of constants A and B, we have creep stresses in the spherical shell as

$$T_{rr}=\frac{(p_1-p_2)\left[\left(\frac{a}{b}\right)^t-\left(\frac{r}{b}\right)^t\right]}{\left[\left(\frac{a}{b}\right)^t-1\right]}-p_1, \quad T_{\theta\theta}=T_{rr}-\frac{(p_1-p_2)\left[t\left(\frac{r}{b}\right)^t\right]}{2\left[\left(\frac{a}{b}\right)^t-1\right]}, \tag{18}$$

where: $t = -3n + 2c(n-1)$.

Equations (18) give creep stresses for the spherical shell under the combined load of internal and external pressure. We introduce the following non-dimensional components: $R = r/b$, $R_0 = a/b$, $\sigma_{rr} = T_{rr}/E$, $\sigma_{\theta\theta} = T_{\theta\theta}/E$, $P_1 - P_2 = (p_1 - p_2)/E$, where $E = 2\mu(3-2c)/(2-c)$. Equations (18) in a non-dimensional form become:

$$\sigma_{rr}=\frac{(P_1-P_2)\left[(R_0)^t-(R)^t\right]}{\left[(R_0)^t-1\right]}-p_1, \quad \sigma_{\theta\theta}=\sigma_{rr}-\frac{(P_1-P_2)\left[t(R)^t\right]}{2\left[(R_0)^t-1\right]}, \tag{19}$$

where: $t = -3n + 2c(n-1)$.

NUMERICAL DISCUSSION ON CREEP STRESSES AND STRAIN RATES

For calculating creep stresses based on the above analysis, the following values have been taken: incompressible material $C = 0$, and compressible material $C = 0.25$, 0.50, measure $n = 1/7$, $1/5$, 1 (*i.e.* $N = 7$, 5, 1). In the classical theory, measure N is equal to $1/n$. The creep stresses derived from Eqs.(19) are plotted in graphs along radii ratio R under different cases of internal and external pressure. In Fig. 1, creep curves are drawn by taking into consideration that internal pressure is higher than external pressure. Curves for radial, as well as circumferential stresses, are produced against radii ratio R for different value of measures. It is found that radial stresses have more impact on the internal surface of the sphere as compared to the external surface of the shell for different materials with compressibilities $c = 0$, 0.25, 0.50. The values of creep stresses are negative due to compressive nature of pressure. It is also seen that values of creep stresses get lowered for $n = 1/5$ as compared to $n = 1/7$. For linear measure $n = 1$, it is seen that radial stresses, as well as circumferential stresses, show same values for all types of compressible materials. Radial stresses are more effective at internal surface as compared to circumferential stresses. In Fig. 2, creep curves are drawn by taking into consideration that external pressure is higher than internal pressure. The effect of stresses gets reversed in this case. The circumferential stresses have more effect on outer boundary of shell as compared to the radial stresses. It is further seen that with increase in compressibility of the material, creep stresses have more influence on interior as well as the exterior part of the shell as compared to incompressible material. It means that under effect of higher external pressure, the spherical shell made up of high compressible material will face more damage. In Fig. 3, creep curves are drawn under effect of external pressure only. It is found that for nonlinear measure $n = 1/7$ and $1/5$ creep stresses have more influence on exterior part of spherical shell in absence of internal pressure. The stress values are more in case of $n = 1/7$ as compared to $n = 1/5$. For linear measure $n = 1$, circumferential stresses are maximum, irrespective of compressible material nature. In Fig. 4, creep curves are drawn under effect of internal pressure only. It is observed that circumferential stresses are tensile in nature that will

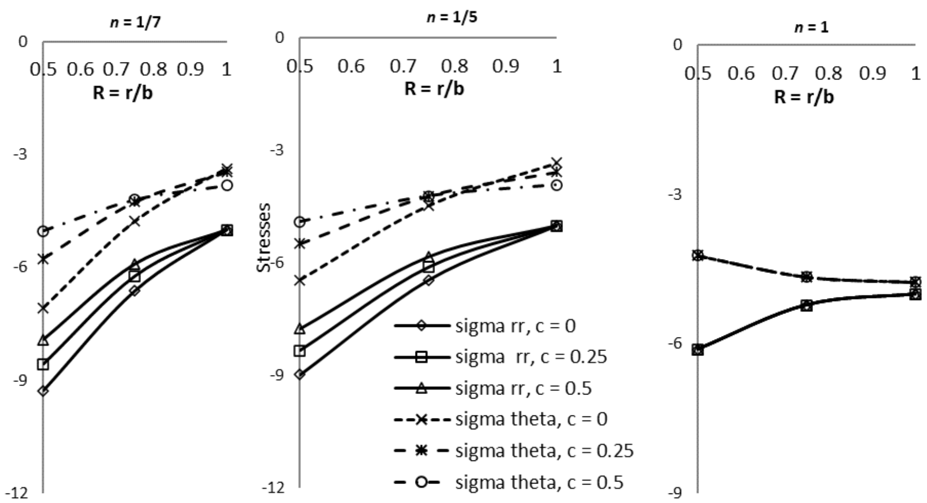

Figure 1. Creep stresses along radii ratio at $P_1 = 15$ and $P_2 = 5$.

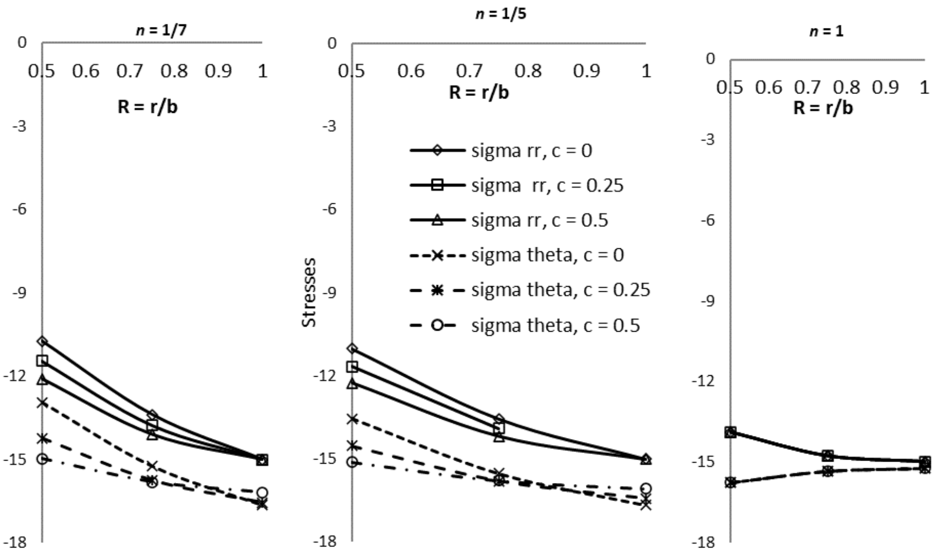

Figure 2. Creep stresses along radii ratio at $P_1 = 5$ and $P_2 = 15$.

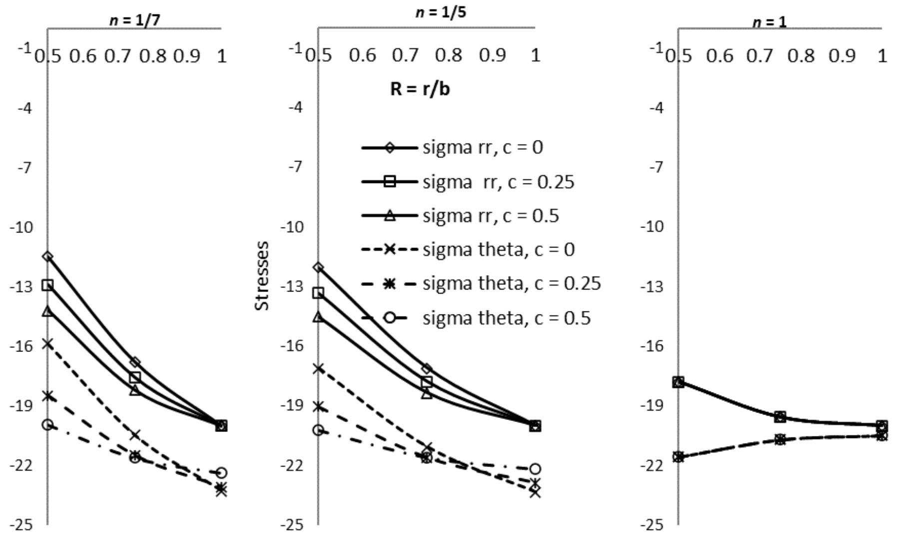

Figure 3. Creep stresses along radii ratio at $P_1 = 0$ and $P_2 = 20$.

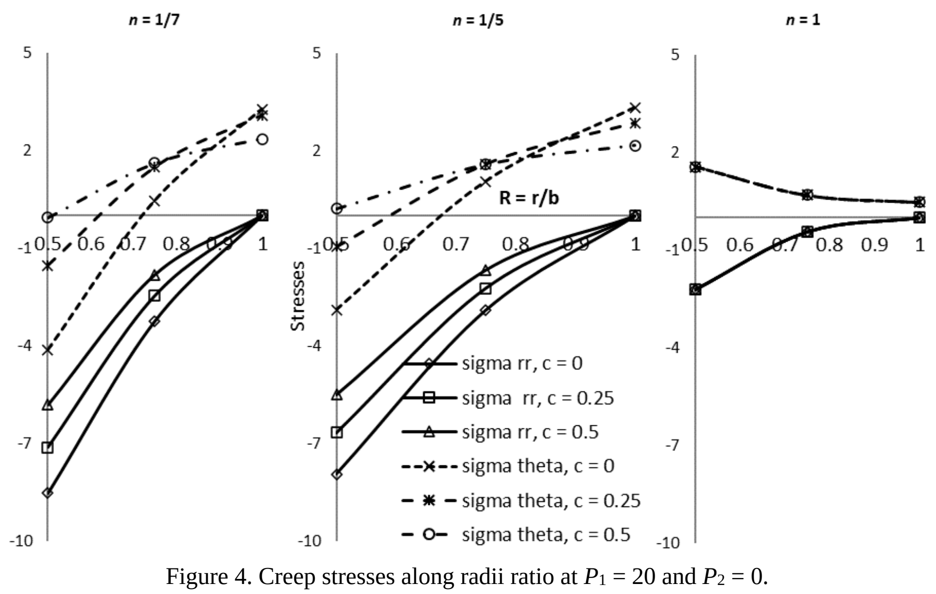

Figure 4. Creep stresses along radii ratio at $P_1 = 20$ and $P_2 = 0$.

lead to expansion of spherical shell. Radial stresses are more effective inside the shell and they have no influence on the boundary of the shell for linear as well as nonlinear measure values. Incompressible material is under less influence of stresses as compared to the compressible material.

CONCLUSION

It can be concluded from above discussed results that spherical shell made up of incompressible material has a longer life as compared to the shell made up of compressible material, under internal and external pressure environments. The influence of nonlinear measure n =1/7 on creep stresses is maximum in the spherical shell, as compared to other values which leads to more damage in the shell.

REFERENCES

1. Miller, G.K. (1995), *Stresses in a spherical pressure vessel undergoing creep and dimensional changes*, Int. J Solids Struct. 32(14): 2077-2093. doi: 10.1016/0020-7683(94)00197-5
2. You, L.H., Zhang, J.J., You, X.Y. (2005), *Elastic analysis of internally pressurized thick-walled spherical pressure vessels of functionally graded materials*, Int. J Pres. Ves. Piping, 82(5), 347-354. doi: 10.1016/j.ijpvp.2004.11.001
3. Hamed, E., Bradford, M.A., Gilbert, R.I. (2010), *Nonlinear long-term behaviour of spherical shallow thin-walled concrete shells of revolution*, Int. J Solids Struct. 47(2): 204-215. doi: 10 .1016/j.ijsolstr.2009.09.027
4. Stemmer, K., Harder, H., Hansen, U. (2006), *A new method to simulate convection with strongly temperature- and pressure-dependent viscosity in a spherical shell: Applications to the Earth's mantle*, Phys. Earth Planet. Interiors, 157(3-4): 223-249. doi: 10.1016/j.pepi.2006.04.007
5. Kashkoli, M.D., Nejad, M.Z. (2014), *Effect of heat flux on creep stresses of thick-walled cylindrical pressure vessels*, J Appl. Res. Technol. 12(3): 585-597, (2014). doi: 10.1016/S166 5-6423(14)71637-2
6. Seth, B.R., *Transition theory of creep rupture*, MRC Tech. Summary Report no. 321, 1962.
7. Seth, B.R. (1964), *Generalized strain and transition concepts for elastic-plastic deformation-creep and relaxation*, Appl. Mech., Proc. 11th Int. Congress Applied Mechanics, Munich, Germany, 1964: 383-389.
8. Seth, B.R. (1970), *Transition conditions: The yield condition*, Int. J Non-Linear Mech. 5(2): 279-285. doi: 10.1016/0020-746 2(70)90025-9
9. Seth, B.R. (1972), *Creep transition in cylinders*, Courses and Lectures No.149, Int. Centre for Mech. Science, Udine, 1972: 1-29.
10. Aggarwal, A.K., Sharma, R., Sharma, S. (2013), *Safety analysis using Lebesgue strain measure of thick-walled cylinder for functionally graded material under internal and external pressure*, Scient. World J. 2013: 676190: 1-10. doi: 10.1155/2013/ 676190
11. Aggarwal, A.K., Sharma, R., Sharma, S. (2014), *Collapse pressure analysis of transversely isotropic thick-walled cylinder using Lebesgue strain measure and transition theory*, The Scient. World J. 2014: 1-10. doi: 10.1155/2014/240954
12. Sharma, S., Yadav, S., Sharma, R. (2018), *Creep torsion in thick-walled circular cylinder under internal and external pressure*, Struct. Integ. and Life, 18(2): 89-97.
13. Verma, G., Pathania, D.S. (2019), *Elastic-plastic stress analysis of spherical shell under internal and external pressure*, Struct. Integ. and Life, 19(1), 3-7.
14. Thakur, P., Singh, S.B., Pathania, D., Verma, G. (2017), *Thermal creep stress and strain analysis in non-homogeneous spherical shell*, J Theor. Appl. Mech. 55(4): 1155-1165. doi: 10.156 32/jtam-pl.55.4.1155
15. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi: 10.10 38/195896a0
16. Seth, B.R. (1966), *Measure-concept in mechanics*, Int. J Non-Linear Mech. 1(1): 35-40. doi: 10.1016/0020-7462(66)90016-3
17. Sokolnikoff, I.S., Mathematical Theory of Elasticity, McGraw Hill, 1946.
18. Thakur, P., Kaur, J., Singh, S.B. (2016), *Thermal creep transition stresses and strain rates in a circular disc with shaft having variable density*, Eng. Comput. 33(3). doi: 10.1108/EC-05-201 5-0110
19. Thakur, P. (2010), *Creep transition stresses in a thin rotating disc with shaft by finite deformation under steady state temperature*, Thermal Science, 14(2): 425-436. doi: 10.2298/TSCI100 2425P
20. Thakur, P. (2011), *Creep transition stresses of a thick isotropic spherical shell by finitesimal deformation under steady-state of temperature and internal pressure*, Thermal Science, 15(2): 157-165. doi: 10.2298/TSCI101004083P
21. Thakur, P., Sethi, M. (2020), *Creep deformation and stress analysis in a transversely material disk subjected to rigid shaft*, Math. Mech. Solids, 25(1): 17-25. doi: 10.1177/108128651985 7109
22. Sharma, R. (2019), *Elastic-plastic stresses in a thin rotating disk of functionally graded material with exponentially varying thickness under internal pressure*, Struct. Integ. and Life, 19 (3): 185-194.
23. Gupta, V. (2019), *Creep analysis in composite disc in presence of thermal gradients*, Struct. Integ. and Life, 19(3): 167-174.
24. Verma, G., Rana, P., Pathania, D.S., Thakur, P. (2017), *Creep transition in the rotating spherical shell under the effect of density variable by Seth's transition theory*, AIP Conference proceeding 1802(1), 020020 (2017). doi: 10.1063/1.4973270
25. Pathania, D.S., Verma, G. (2019), *Temperature and pressure dependent creep stress analysis of spherical shell*, Int. J Appl. Mech. Eng. 24(1): 105-115. doi: 10.2478/ijame-2019-0007