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ANALYSIS OF CREEP STRESSES IN THIN ROTATING DISC COMPOSED OF PIEZOELECTRIC MATERIAL

ANALIZA NAPONA PUZANJA U TANKOM ROTIRAJUĆEM DISKU OD PIJEZOELEKTRIČNOG MATERIJALA

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- creep
- piezoelectric material
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- rotating disc

Abstract

Piezoelectricity is the characteristic of some materials due to which they generate electricity when subjected to mechanical load. There exist many materials (natural and man-made) that possess piezoelectric properties. For example, cane sugar, quartz, berlinite, Rochelle salt, topaz and bone are naturally occurring piezoelectric materials, and lead zirconate titanate, and barium titanate are man-made piezoelectric materials. In this paper we are discussing the analytic solution of creep stresses in thin rotating disc composed of piezoelectric material subjected to internal pressure. Creep stresses in the rotating disc are calculated by applying the concept of Seth's transition theory. A non-linear differential equation governing this physical problem is obtained by substituting the resultant relations into the equilibrium equation. The solution of nonlinear differential equation with applied boundary conditions gives the creep stresses and pressure. The obtained results are discussed numerically and presented graphically. With the help of mathematical calculations and numerical discussions we observed that creep stresses show significant increase with the increasing value of pressure and angular velocity.

Ključne reči
- puzanje
- pijezoelektrični materijal
- naponi
- unutrašnji pritisak
- rotirajući disk

Izvod

Pijezoeletrične karakteristike nekih materijala potiču od generisanja elektriciteta kada se mehanički opterete. Postoji više vrsta materijala (prirodni i veštački) koji poseduju pijezoelektrične osobine. Na primer, šećerna trska, kvarc, berlinit, Rošelova so, topaz i kost su prirodni pijezoelektrični materijali, a olovo cirkonat titanat, ili barijum titanat su veštački pijezoelektrični materijali. U radu je navedena diskusija analitičkog rešenja napona puzanja u tankom rotirajućem disku od pijezoelektričnog materijala, koji je opterećen unutrašnjim pritiskom. Naponi puzanja u rotirajućem disku se izračunavaju primenom koncepta Setove teorije prelaznih napona. Nelinearna diferencijalna jednačina, kojom se opisuje ovaj fizički problem, dobijena je smenom rezultujućih relacija u jednačinu ravnoteže. Rešenja nelinearne diferencijalne jednačine sa primenom graničnih uslova daju napone puzanja i pritisak. Diskusija obuhvata rezultate dobijene numerički i predstavljene grafički. Iz matematičkih proračuna i numeričke diskusije, primećujemo da naponi puzanja pokazuju značajan porast sa porastom pritiska i ugaone brzine.

INTRODUCTION

Creep deformation in materials is a time dependent deformation which occurs due to long term exposure of materials to high temperature and pressure. Failure of materials due to high temperature, creep, fatigue, and fracture, is an unavoidable issue in the safety production of structures in modern industries. During the past few years, a significant number of research works has been done in order to estimate the strength of structures working under high temperature. Piezoelectric materials generate an electric current when they are subjected to some external load. There exist a number of materials that possess piezoelectric properties, for example bone, quartz, proteins, and ceramics. These materials have numerous applications in sonar, generation of high-voltage and sound detection. Piezoelectric materials are also used in cigarette lighter and barbecue-grill igniters. Man-made piezoelectric materials are used in aviation and filters for radios and television. Due to electric and magnetic properties of piezoelectric materials, researchers are more interested in this area. Many authors have done significant work on creep deformations in piezoelectric materials.

Ali Ghorbanpour Arani et.al. /1/ applied the method of successive approximation to evaluate the stresses in functionally graded sphere of piezoelectric material and found that major part of electric potential is redistribution along the thickness. A. A. Mohammed et al. /2/ have discussed about the role of piezoelectric elements in finding the mechanical properties of solid structures used in industries. Their research has reviewed emerging technology and the role of piezoelectric elements in tests for various mechanical properties, such as creep, fracture toughness, hardness, and toughness, etc.

Successive approximation method for creep deformation of a piezoelectric cylinder made of functionally graded material has been applied by Ali Ghorbanpour Arani and Reza Kolahchi, /3/. R. Pramanik and A. Arockiarajan /4/ performed experimental and theoretical studies on creep deformations of piezo- composites, and found that experimental results are conceding with theoretical results. Jiayu Chena et al. /5/ studied piezoelectric materials for sustainable building structures and their industrial applications. They also discussed the latest techniques of using piezoelectric materials in energy harvesters, actuators and sensors for various building structures. Atrian et al. /6/ discussed the solution of functionally graded piezoelectric thick cylinder under the influence of electric field and mechanical loads by separation of variable method. Renato Caliò and others /7/ reviewed the state of art in harvesting of piezoelectric energy. Their work emphasizes on operating modes of material and configurations of devices. Elio /8/ described the phenomenon of forces in piezoelectric materials subjected to electric fields and showed that these materials have capability of producing nonlocal forces of induction. All the above mentioned authors applied the classical theory of deformation for solving these problems in the elastic region only. Borah /9/ has used the concept of transition theory by employing the concept of generalized finite strain measures. Many authors /10-25/ have applied this theory to solve the problems of different solid structures such as shells, cylinders and rotating disc, etc. In example, the stresses in circular cylinder composed of functionally graded material are evaluated by Aggarwal et al. /12/ and concluded that functionally graded material is better for constructing cylinders as compared to isotropic material. Sharma et al. /17/ evaluated the thermal shear stresses in torsion of functionally graded cylinder under pressure at inner and outer surface. Sharma and Panchal /18/, and Sharma et al. /19/, have evaluated the stresses in spherical shells and cylinder made of transversely isotropic material by applying the concept of transition theory. Sharma et al. /20/ calculated creep torsion in thick cylinder subjected to pressure at inner and outer surface and concluded that composite materials are better than isotropic materials for design.

In the present paper, creep stresses are evaluated in thin rotating disc composed of piezoelectric material under internal pressure by applying the concept of transition theory.

GOVERNING MATHEMATICAL EQUATIONS

A thin rotating disc having inner and outer radii, r_1 and r_2, respectively, is considered. The angular velocity of the disc is taken as ω. The thin disc considered here is effectively in a state of plane stress i.e. (T_{zz} = 0). The displacement coordinates in polar form are taken as

$$u = r(1-F);\quad v = 0 \quad\text{and}\quad w = dz\,, \tag{1}$$

where: F is a function of $r = \sqrt{(x^2 + y^2)}$; and d is constant.

By using generalized strain measure, the components of strains are given by

$$e_{rr} = \frac{1}{n}\left[1-(rF'+F)^n\right];\quad e_{\theta\theta} = \frac{1}{n}\left[1-F^n\right];$$
$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right];\quad e_{r\theta} = e_{\theta z} = e_{zr} = 0\,, \tag{2}$$

where: n is the strain measure; and $F' = dF/dr$.

Stress-strain relations for this problem are:

$$T_{rr} = (\lambda+2\mu)[e_{rr}+e_{\theta\theta}] + \lambda(e_{\theta\theta}+e_{zz}) - \epsilon_{11} E_r\,,$$
$$T_{\theta\theta} = (\lambda+2\mu)e_{\theta\theta} + \lambda(e_{rr}+e_{zz}) - \epsilon_{12} E_r\,,$$
$$T_{zz} = T_{zr} = T_{r\theta} = T_{\theta z} = 0\,. \tag{3}$$

The electric displacement equation is

$$D_r = \epsilon_{11} e_{rr} + \epsilon_{12} e_{\theta\theta} + \epsilon_{13} e_{zz} + \eta_{11}E_r\,,\quad D_\theta = D_z = 0\,, \tag{4}$$

where: λ and μ are Lame's constants; ϵ_{11}, ϵ_{12}, ϵ_{13} are piezoelectric coefficients; and η_{11} is dielectric constant.

From free charge equation we have

$$D_r = \frac{C_1}{r} = \frac{1}{r}\,,\quad E_r = \frac{1}{\eta_{11}}\left[\frac{1}{r} - \epsilon_{11} e_{rr} - \epsilon_{12} e_{\theta\theta}\right]. \tag{5}$$

By using Eqs.(3), (4) and (5), stresses are given as

$$T_{rr} = \frac{3}{(3-2C)Cn}(1-F^n(1+P)^n) + \frac{3(1-C)}{(3-2C)Cn}(1-F^n) +$$
$$+[(1-(1-d)^n)] + \frac{\epsilon_{11}}{n\eta_{11}}\left[\epsilon_{11}(1-F^n(1+P)^n) + \epsilon_{12}(1-F^n) - \frac{1}{r}\right],$$
$$T_{\theta\theta} = \frac{3(1-C)}{(3-2C)Cn}\left[(1-F^n(1+P)^n) + (1-(1-d)^n)\right] + \frac{3}{(3-2C)Cn}(1-F^n) +$$
$$+\frac{\epsilon_{12}}{n\eta_{11}}\left[\epsilon_{11}(1-F^n(1+P)^n) + \epsilon_{12}(1-F^n) - \frac{1}{r}\right], \tag{6}$$

where: $rF' = FP$; and $C = 2\mu/(\lambda + 1\mu)$.

The equation of equilibrium for rotating disc is given as

$$\frac{d}{dr}(T_{rr}) + \frac{T_{rr} - T_{\theta\theta}}{r} + \rho\omega^2 r = 0\,. \tag{7}$$

Using Eqs.(6) and (7), the governing differential equation is obtained as

$$-\left\{\left(\frac{3}{(3-2C)C}\right)F^{n+1}(1+P)^{n-1}P + \frac{\epsilon_{11}^2}{\eta_{11}}F^{n+1}(1+P)^{n-1}P\right\}\frac{dP}{dF} =$$
$$= \left(\frac{3}{(3-2C)C}\right)F^{n+1}(1+P)^{n-1}P + \left(\frac{3(1-C)}{(3-2C)C}\right)F^nP + \frac{\epsilon_{11}^2 F^nP}{\eta_{11}}(1+P)^n -$$
$$-\frac{3}{(3-2C)n}\left[F^n(1-(1+P^n))\right] - \frac{\epsilon_{11}(\epsilon_{11}-\epsilon_{12})}{n\eta_{11}}\left[1-F^n(1+P^n)\right] -$$
$$-\frac{\epsilon_{12}(\epsilon_{11}-\epsilon_{12})}{n\eta_{11}}(1-F^n) + \frac{\epsilon_{11}\epsilon_{12}F^nP}{\eta_{11}} - \frac{\epsilon_{12}}{rn\eta_{11}} - \rho r^2\omega^2\,. \tag{8}$$

The boundary conditions which are to be applied at inner and outer surfaces of the disc are taken as

$$T_{rr} = 0 \quad\text{at}\quad r = r_2 \quad\text{and}\quad T_{rr} = -p \quad\text{at}\quad r = r_1\,. \tag{9}$$

TRANSITION FROM ELASTIC TO CREEP

According to transition theory /10-25/, material in elastic state changes to creep at critical point $P \to -1$. For calculating the creep stresses, the transition function is taken as

$$R = T_{rr} - T_{\theta\theta},$$

$$R = \frac{3}{n(3-2C)}\left[F^n(1-(P+1)^n)\right] + \frac{\epsilon_1(\epsilon_1-\epsilon_2)}{\eta_{11}n}(1-F^n) + + \frac{\epsilon_2(\epsilon_1-\epsilon_2)}{\eta_{11}n}\left[1-F^n(P+1)^n\right] - \frac{(\epsilon_1-\epsilon_2)}{n\eta_{11}r}. \quad (10)$$

Taking logarithmic differentiation of Eq.(10) and using Eq.(8) and then applying $P \to -1$ we get,

$$R = AG,$$

where: $G = e^{\int g dr}$; and

$$g = \frac{\dfrac{\dfrac{3}{(3-2C)} + \dfrac{\epsilon_1(\epsilon_1-\epsilon_2)}{\eta_{11}}}{r\left[\dfrac{3}{C(3-2C)} + \dfrac{\epsilon_1^2}{\eta_{11}}\right]}\left\{\dfrac{3(C-1)F^n}{C(3-2C)} + \dfrac{3F^n}{n(3-2C)} - \dfrac{(\epsilon_1^2-\epsilon_2^2)}{n\eta_{11}} - \rho r^2\omega^2 - \dfrac{\epsilon_1^2 F^n}{n\eta_{11}} - \dfrac{\epsilon_2}{n\eta_{11}}\right\} - \dfrac{3F^n}{r(3-2C)} + \dfrac{\epsilon_2(\epsilon_1-\epsilon_2)F^n}{\eta_{11}r} + \dfrac{(\epsilon_1-\epsilon_2)}{n\eta_{11}r^2}}{\dfrac{3F^n}{n(3-2C)} + \dfrac{(\epsilon_1^2-\epsilon_2^2)}{n\eta_{11}} - \dfrac{\epsilon_2(\epsilon_1-\epsilon_2)F^n}{\eta_{11}n} - \dfrac{(\epsilon_1-\epsilon_2)}{n\eta_{11}r}} \quad (11)$$

With the help of Eqs.(11) and (7), creep stresses are obtained as follows

$$T_{rr} = B - \int \frac{AG}{r}dr - \frac{\rho r^2\omega^2}{2},$$

$$T_{\theta\theta} = B - \int \frac{AG}{r}dr - \frac{\rho r^2\omega^2}{2} - AG. \quad (12)$$

By using Eqs.(9) and (12) we have

$$A = \frac{-p - \dfrac{\rho\omega^2}{2}(r_2^2 - r_1^2)}{\int_{r_1}^{r_2}\dfrac{G}{r}dr},$$

$$B = \frac{-p - \dfrac{\rho\omega^2}{2}(r_2^2 - r_1^2)\left[\int \dfrac{G}{r}dr\right]_{r=r_2}}{\int_{r_1}^{r_2}\dfrac{G}{r}dr} + \frac{\rho r_2^2\omega^2}{2}. \quad (13)$$

On converting all the components in non-dimensional form we have

$$R = \frac{r}{r_2}; \quad R_0 = \frac{r_1}{r_2}; \quad \sigma_{rr} = \frac{T_{rr}}{E}; \quad \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{E};$$

$$p_1 = \frac{p}{E}; \quad \Omega^2 = \frac{\rho\omega^2 r_2^2}{E}. \quad (14)$$

From Eq.(12), transitional stresses are given as

$$\sigma_{rr} = B_1 - \int \frac{A_1 G^*}{R}dR - \frac{\Omega^2 R^2}{2},$$

$$\sigma_{\theta\theta} = B_1 - \int \frac{A_1 G^*}{R}dR - \frac{\Omega^2 R^2}{2} - A_1 G^*, \quad (15)$$

where: $G^* = e^{\int g^* r_2 dR}$; and

$$g^* = \frac{\dfrac{\dfrac{3}{(3-2C)} + \dfrac{\epsilon_1(\epsilon_1-\epsilon_2)}{\eta_{11}}}{r_2 R\left[\dfrac{3}{C(3-2C)} + \dfrac{\epsilon_1^2}{\eta_{11}}\right]}\left\{\dfrac{3(C-1)F^n}{(3-2C)} + \dfrac{3F^n}{n(3-2C)} + \dfrac{(\epsilon_1^2-\epsilon_2^2)}{n\eta_{11}} - \Omega^2 R^2 - \dfrac{\epsilon_1^2 F^n}{n\eta_{11}} - \dfrac{\epsilon_2}{n\eta_{11}r_2 R}\right\} - \dfrac{3F^n}{r_2 R(3-2C)} + \dfrac{\epsilon_2(\epsilon_1-\epsilon_2)F^n}{\eta_{11}r_2 R} + \dfrac{(\epsilon_1-\epsilon_2)}{n\eta_{11}r_2^2 R^2}}{\dfrac{3F^n}{n(3-2C)} + \dfrac{(\epsilon_1^2-\epsilon_2^2)}{n\eta_{11}} - \dfrac{\epsilon_2(\epsilon_1-\epsilon_2)F^n}{n\eta_{11}} - \dfrac{(\epsilon_1-\epsilon_2)}{n\eta_{11}r_2 R}}$$

$$A_1 = \frac{-p_1 - \Omega^2(1-R_0^2)/2}{\int_{R_0}^{1}\dfrac{G^*}{R}dR}, \quad B_1 = \frac{-p_1 - \dfrac{\Omega^2}{2}(1-R_0^2)\left[\int \dfrac{G^*}{R}dR\right]_{R=1}}{\int_{R_0}^{1}\dfrac{G^*}{R}dR} + \frac{\Omega^2}{2}.$$

NUMERICAL DISCUSSION

Figures 1a, 1b and 1c represent circumferential and radial creep stresses in thin rotating disc with angular velocity $\Omega = 10$ with internal pressure 5, 10 and 15 for piezoelectric material PZT 4 at different radii ratios. It can be seen in Fig. 1a that radial and circumferential creep stresses are tensile in nature. It is also observed that radial stress shows significant increase with increasing radii ratios and attains its maximal value at outer surface of the disc. Circumferential creep stresses reach maximum at inner surface and decrease with the increase in ratios. It is noticed from Fig. 1b and 1c that as we increase the value of internal pressure, the creep stresses increase significantly, but the behaviour of stresses is the same as in Fig. 1a.

With the increase in angular velocity ($\Omega = 20$) of the disc, values of circumferential and radial stresses increase significantly with internal pressure 5, 10 and 15, respectively, as can be observed from Figs. 2a, 2b and 2c. In Fig. 3a, 3b and

3c, it is observed that circumferential and radial creep stresses show significant increase with the incremented value of angular velocity (Ω = 30) and internal pressure of the disc.

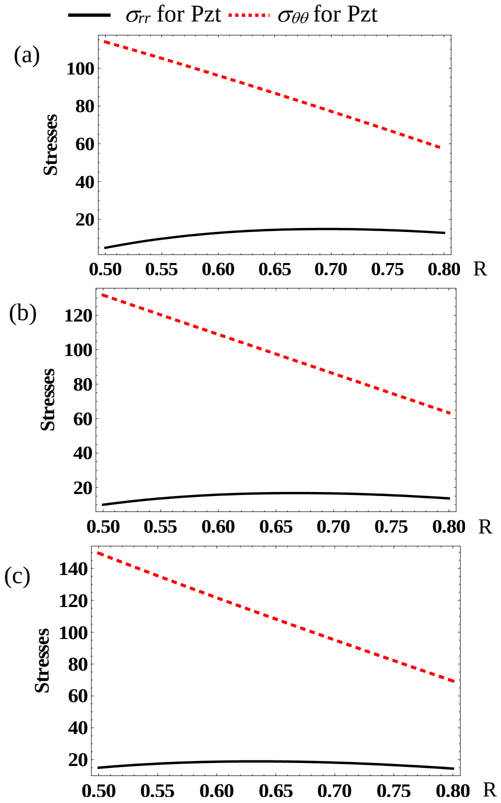

Figure 1. Circumfer. and radial creep stresses for piezoelectric material with angular velocity 10 and inter. pressure 5, 10, 15 in respect.

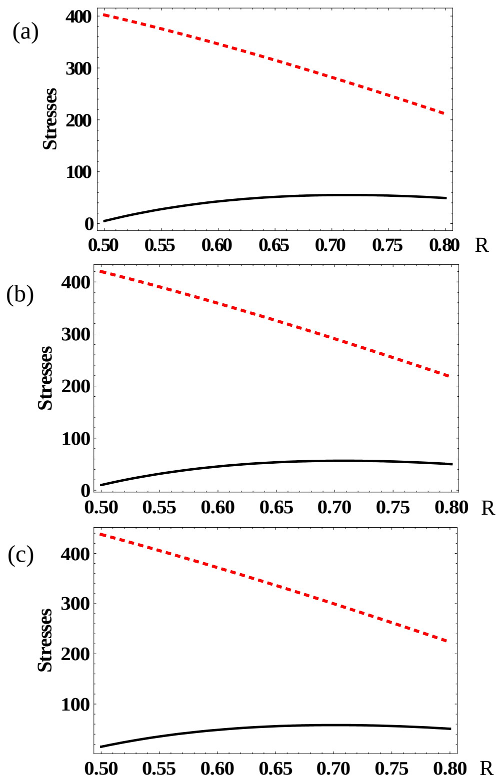

Figure 2. Circumfer. and radial creep stresses for piezoelectric material with angular velocity 20 and internal pressure 5, 10, 15 in respect.

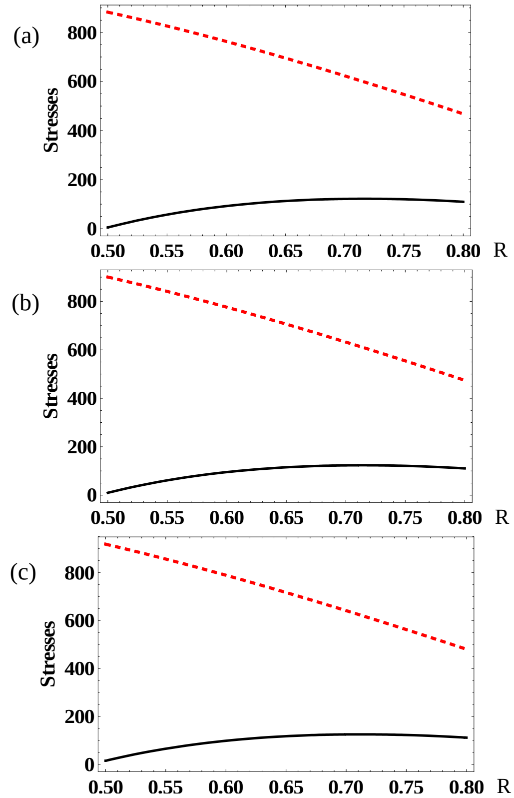

Figure 3. Circumfer. and radial creep stresses for piezoelectric material with angular velocity 30 and internal pressure 5, 10, 15 in respect.

CONCLUSION

The analytical solution is obtained for creep stresses in piezoelectric material using transition theory for different angular speeds and pressure at internal surface. With the help of mathematical calculations and numerical discussions we observed that creep stresses show significant increase with the increasing value of pressure and angular velocity.

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