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MATHEMATICAL PERSPECTIVE OF MODELLING FLUID FLOW INSIDE FRACTURED CARBONATE RESERVOIR PRODUCING UNDER STEAM FLOODING ENHANCED OIL RECOVERY

PRILOG MATEMATIČKOG MODELIRANJA PROTOKA FLUIDA U OŠTEĆENOM KREČNJAČKOM REZERVOARU PRI STIMULISANOM ISCRPKU NAFTE ISTISKIVANJEM PAROM

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 7.08.2020

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Keywords
- fractured carbonate reservoir
- partial differential equation
- correlations
- improved mathematical model
- implicit pressure and explicit saturation (IMPES)

Abstract

A total of around ~2 trillion barrels of viscous crude oil is held in a carbonate reservoir. The heavy oil held in a Fractured Carbonate Reservoir (FCR) encompasses an enormous potential to contribute to the world's oil needs. Continuous steam flooding Enhanced Oil Recovery (EOR) is used as a tertiary method to increase recovery from these complex classes of reservoirs. The mathematical perspective of modelling these reservoirs involves the inclusion of three flow equations (oil, water, and steam) and one heat balance equation. In recent studies, for modelling FCR, flow equations are defined for two different sub-domains, namely fracture and matrix. Both the grid blocks possess high porosity and permeability contrast, in which fracture has high transmissibility and the matrix has a high storage capacity of hydrocarbons. In this study, an attempt is made to derive flow equations (partial differential equation) for all phases in two dimensions, including gravity effect with the help of Darcy's law. The presence of pressure and saturation terms in flow equation for steam, oil, and water makes the modelling of steam flooding EOR in FCR more challenging. The derivation of fluid flow equations (oil, water, and steam) involves deriving of the continuity equation first, followed by combining it with Darcy's law. An under-saturated reservoir considered in this case, where the reservoir pressure is always higher than the bubble-point pressure, and hence, the fluid of interest is of aqueous phase only in the absence of any phase changes between steam and liquid. The present study is aimed at deducing an improved mathematical model by considering the reservoir fundamental parameters, namely, porosity, permeability, and compressibility, as a function of petrophysical parameters associated with a dual-continuum system. The correlations used in the derivation of the fluid flow equation incorporate separate equations defined mathematically for fracture and matrix porosity and permeability, respectively. Also, porosity is considered as a variable coefficient which changes with change in pressure. This work will help in modelling fractured reservoirs more efficiently by solving fluid flow equations with more ease.

Ključne reči
- oštećeni krečnjački rezervoar
- parcijalna diferencijalna jednačina
- korelacije
- poboljšani matematički model
- implicitni pritisak i eksplicitno zasićenje (IMPES)

Izvod

Ukupno oko ~2×10^{12} barela viskozne sirove nafte se nalazi u krečnjačkom rezervoaru. Sirova nafta u oštećenom krečnjačkom rezervoaru (FCR) predstavlja ogroman potencijal koji doprinosi potrebama za naftom u svetu. Tehnika povećanja iscrpka nafte (EOR) kontinualnim istiskivanjem parom se koristi kao tercijalna metoda u eksploataciji nafte iz ovih kompleksnih rezervoara. Matematičko modeliranje ovih rezervoara podrazumeva upotrebu tri jednačine protoka (nafta, voda, i para) i jednu jednačinu toplotnog balansa. U studijama rađenim nedavno, za modeliranje FCR, jednačine protoka se definišu za dva različita pod-domena, za oštećenje i za matricu. Obe ove sastavne celine imaju veliki kontrast poroznosti i permeabilnosti, gde oštećenje-lom ima veliku propusnost, a matrica ima veliki kapacitet za skladištenje ugljovodonika. U radu se izvode jednačine protoka (parcijalna diferencijalne jednačine) za sve faze u dve dimenzije, uključujući i uticaj gravitacije preko zakona Darsija. Uvođenje članova sa pritiskom i zasićenjem u jednačini protoka za paru, naftu i vodu predstavlja izazov za modeliranje istiskivanja nafte parom EOR u FCR. Izvođenje jednačina protoka (nafte, vode i pare) podrazumeva prvo izvođenje jednačine protoka, zatim smenom u zakon Darsija. Nezasićeni rezervoar u ovom slučaju, gde je pritisak u rezervoaru uvek veći od pritiska bubrenja, i stoga je fluid od interesa samo u tečnoj fazi sa odsustvom svake fazne promene pare i tečnosti. Cilj rada je u izvođenju poboljšanog matematičkog modela razmatranjem fundamentalnih parametara rezervoara, odnosno, poroznosti, permeabilnosti i stišljivosti, kao funkcije petrofizičkih parametara vezanih za kontinuum dva sistema. Korelacije primenjene u izvođenju jednačine protoka fluida sadrže posebne jednačine matematički definisane za poroznost i permeabilnost loma i matrice, respektivno. Takođe, poroznost se smatra promenljivim koeficijentom koji se menja sa pritiskom. Ovaj rad će pomoći u efikasnijem modeliranju oštećenih rezervoara, sa lakšim rešavanjem jednačine protoka fluida.

INTRODUCTION

Worldwide estimation of heavy oils in carbonate reservoirs is to be 1.6×10^{12} bbl in place, /1/. Heavy oils have high melting points due to large molecular structures, because of high oil viscosity from 100 to 10000 cP at reservoir temperature. Heavy oil deposits mostly occur in shallow depths (less than 900 m), having a thickness less than 150 m, having high porosity, high permeability, and high oil saturation, /2/. High viscosity is making the heavy oils more challenging to produce oil, which affects the mobility of crude oil.

Production from light and medium type oils are simple, but from heavy and extra heavy oils needs new technologies. However, the development of new technologies is crucial for the economic perspective.

Heterogeneities associated with FCR

A fractured reservoir fundamentally differs from that of a sandstone reservoir in the sense that it is characterized by a dual-continuum model, as explained, where the fluid storage and transmission takes place in two different units, namely low-permeable rock-matrix and high-permeable fracture, /3-12/. A fractured carbonate reservoir (FCR) becomes further complex as the geology is characterized by carbonates rather than by sandstone. Presence of high petrophysical contrast between these domains poses the biggest challenge to model reservoir efficiently, in turn, having severe impacts on reservoir optimisation. The sharp change in porosity and permeability in an FCR can clearly be understood in Fig. 1. Three different representative elementary volume (REV) are defined namely REV A, REV B and REV C. REV is the minimum volume which represents the reservoir at a microscopic scale to an upscaled macroscopic scale instead of the control volume, and it can state as a function of an average rock grain diameter of porous media, /13/. Since the flow equations defined in case of FCR producing under steam flooding EOR incorporates Darcy's law, the physics, and the governing flow equation (partial differential equation) gets bounded to be defined at a macroscopic scale. In other words, all the flow equations only hold valid to a REV and not to microscopic scale or pore scale. In REV A, the number of fractures present per unit area is abundant, which makes this REV more porous. Similarly, for REV B, the number of fractures per unit area is less than REV A but more than REV C, hence porosity in REV A is the highest followed by REV B, and least for REV C. With very less REV spacing, the heterogeneity associated with these reservoirs is very high. From the above discussion, the heterogeneity present in FCR can be clearly stated, and hence extreme care needs to be taken while handling the petrophysical terms like porosity and permeability.

Warren and Root /14/ developed an idealized model (dual porosity) to study the characteristic behaviour of porous media. They assumed that primary porosity or matrix porosity contributes significantly to the pore volume. However, secondary and fracture porosity contributes to fluid flow. They concluded that parameters such as the measure of the fluid capacitance of the secondary porosity and the scale of heterogeneity which is present in the reservoir system are sufficient to analyse the behaviour of deviation in flow in heterogeneous porous media, as compared to single continuum fluid flow system.

Zhongxiang et al. /15/ proposed a double porosity model for a transient pressure study, in which the flow of a slightly compressible fluid is considered along with an equipotential surface boundary condition. His results show that the ratio of matrix permeability and fracture permeability have an enormous impact on the pressure response in a double porosity media and it can also be valid for a two-layered reservoir because of the same system of equations used in a dual-porosity medium.

Bai et al. /16/ suggested transient fluid flow model in a naturally fractured reservoir incorporating dual-porosity in which no restrictions are made as far as assumptions considered for flow rate and permeability in a matrix subdomain. He concluded that along with permeability ratio (matrix to fracture permeability), the compressibility of fractures, fluid, and solid grains also controls the pressure profiles of transient fluid flow inside the matrix block. With a small increment in permeability ratio change, the fluid pressure got substantially delayed. It becomes necessary to consider the dual-porosity behaviour when the compressibility of the fractures decreases, compared to the increase in compressibility of the solid grains.

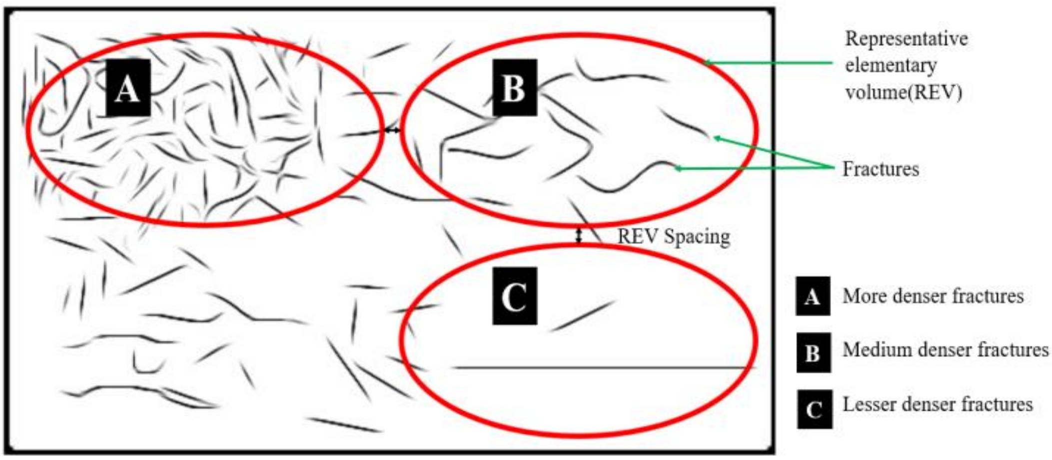

Figure 1. Petrophysical heterogeneities in an FCR.

Lu et al. /17/ developed an improved double permeability model in naturally fractured reservoirs when compared to the idealized dual-porosity model given by Warren and Root /14/. He incorporated the flow from the matrix subdomain to the well bore, which is mostly neglected in earlier studies. It concluded that a total of four stages exist in transient pressure behaviour in a naturally fractured reservoir. It is observed that the pressure profile is almost the same as the Warren and Root /14/ model, except for the fourth stage in which the pressure reaches steady state during the flow in the late-time region.

Here, a comprehensive study is carried out to model FCR containing viscous crude oil and produced under steam flooding EOR. An attempt is made to derive porosity, permeability and compressibility equations for fracture and matrix sub-domain. In addition to that, the ways of finding these essential parameters for modelling FCR are tabulated in the following sections. This study will help in optimising the well productivity by accurately modelling the fractured reservoir. The present study mathematically proves that the nature of pressure depletion and mechanism of fluid flow in an FCR is entirely different from the homogenous reservoir. The optimisation of the flow of fluid in an FCR needs extra care because there is a collective contribution of two subdomains, i.e. matrix and fracture and also an inter-porosity flow exists between both of them.

DERIVATION OF MULTI-PHASE FLUID FLOW EQUATIONS

In the earlier attempts, the author has focused on thermal EOR in a classical sandstone reservoir using multi-phase in-situ combustion process, /18-20/. The way the wettability gets altered has also been studied earlier in carbonate reservoirs /21-22/. In the present study, fluid flow equations for oil, water and steam are obtained by combining mass conservation equation with Darcy's momentum equation, /23/. The mass conservation principle can be stated for the representative elementary volume, defined in Fig. 2, as

$$m_{\text{into the REV}} - m_{\text{out from the REV}} = m_{\text{accumulated inside REV}} \quad (1)$$

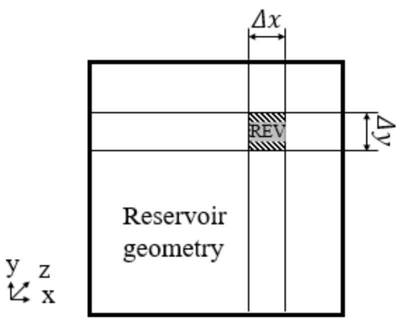

Figure 2. Representative Elementary Volume (REV) in reservoir geometry.

In the left-hand side of Eq.(1), m is the mass, defined as:

$$\text{Mass} = \text{mass flow rate} \times \text{time} = \dot{m} \times \Delta t \,,$$

where: mass flow rate = mass/time.

Mathematically it can be stated as:

$$\dot{m} = \frac{m}{t} = \frac{\rho \Delta V}{t} \,. \quad (2)$$

The volume of REV with unit thickness can be

$$\Delta V = \Delta x \Delta y = A_x \Delta x = A_y \Delta y \,. \quad (3)$$

The cross-sectional area of REV in the x-direction, $A_x = \Delta y$. The cross-sectional area of REV in the y-direction, $A_y = \Delta x$.

Substituting ΔV from Eq.(3) into Eq.(2), two different mass flow rates are obtained in the x and y-direction.

In the x-direction,

$$\dot{m}_x = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A_x \Delta x}{\Delta t} = \rho A_x u_x \,. \quad (4)$$

In the y-direction,

$$\dot{m}_y = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A_y \Delta y}{\Delta t} = \rho A_y u_y \,. \quad (5)$$

Considering the temporal variation of mass change inside the REV:

$$m_{\text{accumulated inside REV}} = (\rho \Delta V \phi S)_{t+\Delta t} - (\rho \Delta V \phi S)_t \,. \quad (6)$$

From mathematical relations in Eqs.(4), (5) and (6) and placing them in Eq.(1):

$$\left[(\rho A_x u)_x\right]\Delta t - \left[(\rho A_x u)_{x+\Delta x}\right]\Delta t + \left[(\rho A_y u)_y\right]\Delta t -$$
$$- \left[(\rho A_y u)_{y+\Delta y}\right]\Delta t = (\rho \Delta V \phi S)_{t+\Delta t} - (\rho \Delta V \phi S)_t \,. \quad (7)$$

Divide by $\Delta x \Delta y \Delta t$ on both sides of the above equation,

$$-\left[(\rho \Delta y u)_{x+\Delta x} - (\rho \Delta y u)_x\right]\Delta t \big/ (\Delta x \Delta y \Delta t) -$$
$$- \left[(\rho \Delta x u)_{y+\Delta y} - (\rho \Delta x u)_y\right]\Delta t \big/ (\Delta x \Delta y \Delta t) =$$
$$= \left[(\rho \phi S \Delta x \Delta y)_{t+\Delta t} - (\rho \phi S \Delta x \Delta y)_t\right] \big/ \Delta x \Delta y \Delta t \,. \quad (8)$$

From derivative first principle, the above equation can be written as:

$$-\frac{\partial}{\partial x}(\rho u)_x - \frac{\partial}{\partial y}(\rho u)_y = \frac{\partial}{\partial t}(\rho \phi S) \,. \quad (9)$$

For single-phase, the concept of saturation does not exist. The mass conservation equation for multi-phase fluid flow can be written as

$$-\frac{\partial}{\partial x}(\rho_i u_i)_x - \frac{\partial}{\partial y}(\rho_i u_i)_y = \frac{\partial}{\partial t}(\phi \rho_i S_i) \,, \quad (10)$$

where: operator i represents o, w and s for oil, water, and steam, respectively. In a multiphase fluid flow case, the concept of absolute permeability does not hold valid, and it needs to be replaced by relative permeability of the particular phase. Relative permeability is the ratio of effective permeability to absolute permeability,

$$k_{ri} = \frac{k_i}{k} \,. \quad (11)$$

The fluid velocity equation given by Darcy's equation can be given as:

- in the x-direction,

$$(u_i)_x = -\left(\frac{k_{ri} k}{\mu_i} \frac{\partial}{\partial x}(P_i - \rho_i g H)\right)_x \,, \quad (12)$$

- in the y-direction,

$$(u_i)_y = -\left(\frac{k_{ri} k}{\mu_i} \frac{\partial}{\partial y}(P_i - \rho_i g H)\right)_y \,. \quad (13)$$

Combining the mass conservation Eq.(10) and fluid velocity equation given by Darcy's law in Eqs.(12) and (13), gives the partial differential equation governing fluid flow in steam flooding application. Since in the case of steam flooding EOR in a fractured reservoir, a total three-phase will exist, and also in the field-scale scenario, the consideration of production becomes mandatory. If the rate of mass condensation exists between the steam and water phase, the equation for all the three phases changes slightly. Hence, the fluid flow equation comes as:

- for the oil phase,

$$\frac{\partial}{\partial x}\left(\rho_o \frac{k_{ro}k_x}{\mu_o}\frac{\partial}{\partial x}(P_o-\rho_o gH)\right)+\frac{\partial}{\partial y}\left(\rho_o \frac{k_{ro}k_y}{\mu_o}\frac{\partial}{\partial y}(P_o-\rho_o gH)\right)= =\phi\frac{\partial}{\partial t}(\rho_o S_o)+Q_o\,, \quad (14)$$

- for the water phase,

$$\frac{\partial}{\partial x}\left(\rho_w \frac{k_{rw}k_x}{\mu_w}\frac{\partial}{\partial x}(P_w-\rho_w gH)\right)+\frac{\partial}{\partial y}\left(\rho_w \frac{k_{rw}k_y}{\mu_w}\frac{\partial}{\partial y}(P_w-\rho_w gH)\right)= =\phi\frac{\partial}{\partial t}(\rho_w S_w)+Q_w+Q_c\,, \quad (15)$$

- for the steam phase,

$$\frac{\partial}{\partial x}\left(\rho_s \frac{k_{rs}k_x}{\mu_s}\frac{\partial}{\partial x}(P_s-\rho_s gH)\right)+\frac{\partial}{\partial y}\left(\rho_s \frac{k_{rs}k_y}{\mu_s}\frac{\partial}{\partial y}(P_s-\rho_s gH)\right)= =\phi\frac{\partial}{\partial t}(\rho_s S_s)+Q_s-Q_c\,. \quad (16)$$

We have three pressure and saturation equations in modelling the steam flooding mechanism in an FCR. Solving six variables, i.e. pressure and saturation for three phases (oil, water, and steam) is challenging with only three fluid flow equations, and there will be a requirement of three auxiliary equations. The first auxiliary equation comes from the fact that the sum of saturation of three-phase (oil, water, and steam) will be equal to 1. The other two equations can be obtained by defining capillary pressure at the oil/water and the oil/steam interface.

DERIVATION OF POROSITY AND PERMEABILITY FOR A FRACTURED RESERVOIR

Porosity developed during this stage is predominantly due to the deposition of different geological layers. Up to this stage, the hydrocarbon is stored in the intergranular pore space, however, which in turn causes less transmissibility of fluids due to very less developed porosity. Hence the flow up to this stage is negligible.

Secondary porosity is caused by a process like recrystallization, dolomitization and formation of vugs and pores during the post lithification process. This part of pore space contributes the most in fluid flow.

The role of fracture porosity, which constitutes a significant chunk of pore space in the secondary porosity formation, should be characterized comprehensively to better model the FCR. The volume fraction of different porosity and initial water saturation is depicted in Fig. 3.

From Fig. 3, the total porosity (ϕ_t) in the fractured oil reservoir can be written as:

$$\phi_t=\phi_1+\phi_2\,. \quad (17)$$

Mathematically, the primary and secondary porosity can be defined as: ϕ_1 = (matrix pore volume)/(total bulk volume); ϕ_2 = (fractured pore volume)/(total bulk volume); ϕ_m = (volume of pore space of the matrix)/(matrix bulk volume); ϕ_f = (fracture porosity)/(secondary porosity),

$$\phi_1=(1-\phi_2)\phi_m\,, \quad (18)$$

$$\phi_{1,eff}=(1-\phi_2)\phi_m(1-S_{wi})\,, \quad (19)$$

$$\phi_t=\phi_m+\phi_f-\phi_m\phi_f\,. \quad (20)$$

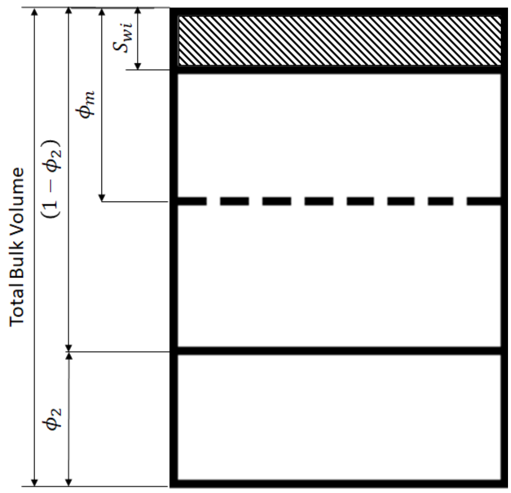

Figure 3. Depicting volume fraction of primary and secondary porosity in naturally fractured reservoir (NFR), /24/.

Being the major contributor of fluid flow in an FCR, this study has attempted to tabulate the different ways of finding the fracture porosity, refer Table 1. The different ways include empirical relationships, predetermined values, logging techniques, etc.

Table 1. The ways of obtaining fracture porosity.

Ways to find	Values of ϕ_f
Predetermined values of ϕ_f (fundamental of fracture reservoir)	Macro fracture network ϕ_f = (0.01-0.5)% Isolated fissures ϕ_f = (0.001-0.01)% Fissure network ϕ_f = (0.01-2)% Vugs ϕ_f = (0.1-3)%
Empirical relationship	$\phi_f < 0.1\phi_t$ when $\phi_t < 10\%$, and $\phi_f < 0.04\phi_t$ when $\phi_t > 10\%$
Using the FMI logging technique (logging source)	$\phi_f=(b)\times(l)\times\dfrac{400}{\pi r^2}$

A case study is taken from earlier published results, /26/:

$\phi_m = 5\times10^{-04}$, $\phi_f = 0.05$, $\phi_{eff} = \phi_f + \phi_m - \phi_f\,\phi_m$,

$\phi_{eff} = 0.05 + 5\times10^{-04} - 0.05\times5\times10^{-04} = 0.050475$,

$\phi_{total} = \phi_m + \phi_f = 0.05 + 5\times10^{-04} = 0.0505$,

error % = 0.0495 %.

Similar to the characterization of porosity for a fractured reservoir, the permeability is also defined for both subdomains (fracture and matrix). In this study, derivation of fracture permeability is performed for the inclined fracture with an angle of 'α'. The arrangement of the fracture is depicted in Fig. 4.

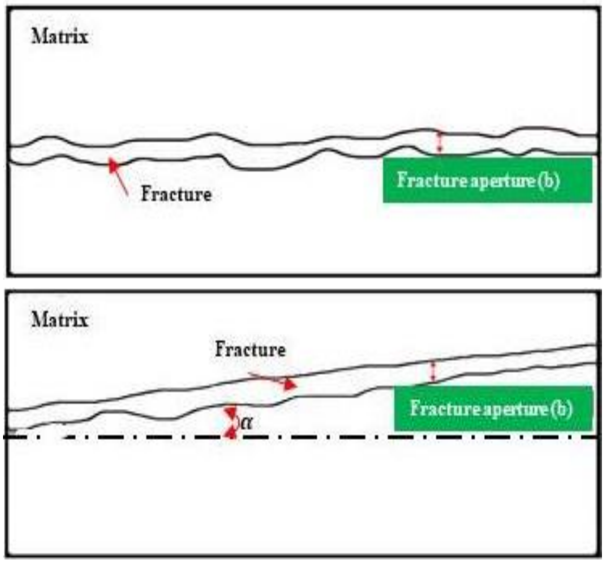

Figure 4. Depicting the arrangement of fracture with a matrix in the horizontal and inclined direction.

For horizontal fractures

$$q_f = a \times b \times \frac{b^2}{12\mu} \times \frac{\Delta P}{l}, \quad (21)$$

the flow rate 'q_f' through fracture is given by Darcy's law.

For inclined fractures

$$q_f = a \times b \times \frac{b^2 \cos\alpha}{12\mu} \times \frac{\Delta P}{l}. \quad (22)$$

For an inclined fracture, an additional term comprising of '$\cos\alpha$' is added to the horizontal fracture equation.

The entire area of cross-section is $M = a \times b$,

$$q_f = M \times \frac{b^2 \cos\alpha}{12\mu} \times \frac{\Delta P}{l}. \quad (23)$$

Also, we can write it as

$$q_f = M \times \frac{k_{ff}}{\mu} \times \frac{\Delta P}{l}. \quad (24)$$

Comparing Eq.(24) with Darcy's equation gives the relationship of fracture permeability with fracture aperture,

$$k_{ff} = \frac{b^2 \cos\alpha}{12}. \quad (25)$$

This equation can be used to determine fracture permeability for a series of fracture, making a fracture network.

The different ways to determine permeability are tabulated in Table 2. The different techniques include well testing, Kazemi /25/ model, and some empirical correlations.

AN APPROACH TOWARDS CONVENTIONAL AND FRACTURED RESERVOIR COMPRESSIBILITY

Total compressibility of the reservoir can be stated as a function of saturation and compressibility of fluids, and compressibility of formation. In case of an FCR, in addition to the compressibility of the matrix, the compressibility of vugs, fractures, need to be considered. Earlier experimental work reveals that the compressibility of vugs is approximately three times that of matrix compressibility.

Table 2. The ways of obtaining permeability, /26/.

Ways to find	The formula for finding permeability
From well testing techniques	$k_t = \frac{Q\mu[\ln(r_e/r_w)] + S}{2\pi h \Delta P}$
Using Kazemi model, /25/	$k_t = k_m + k_{ff} + \left(\frac{t \times b}{h}\right)$
Using idealized fracture reservoir model	for single fracture $k_f = \frac{b^2 \phi_f}{12}$ for multiple fractures $k_f = \frac{L_{FD} b^3}{12}$
Using empirical evaluation procedure (Teodoroc's method)	$k = A \times B \times C \times D$ where: *A* depends on rock type, *B* depends on the porosity and has a value from 25-30, if the rock has porosity > 25 %, *C* depends on pore size and has a coefficient equal to 16, if the maximum pore size is > 2.00 mm, *D* depends on the shape of pore and has a coefficient of 2, if elongated pores are present in the reservoir

For a three-phase (oil, water, and steam), the total compressibility is the summation of oil, water and steam phase, and the compressibility of rock,

$$C_T = S_o C_o + S_w C_w + S_s C_s + C_f. \quad (26)$$

For a conventional reservoir, the reservoir is considered homogenous, and all the phases are considered to be present in the single continuum. The effective oil compressibility in a homogenous reservoir is mathematically stated as

$$C_{eo} = C_o + \frac{(S_w C_w + S_s C_s + C_f)}{S_o}. \quad (27)$$

For a fractured reservoir

The main challenge that comes while approaching the compressibility in FCR is that in these reservoirs, two subdomains are present, namely matrix and fracture. In both, different fluids may be present. In case of enhanced oil recovery like steam flooding in FCR, the type of fluid that can be present in the fracture domain is the fluid that is injected as an EOR agent (like chemical in case of chemical flooding, or microbes in case of microbial flooding). Hence, in the present study, steam is considered to be present along with oil. So,

- in the matrix domain, both oil and water are present,
- in the fracture domain, less volume of oil, and a significant chunk of steam will be present,

which means, in a matrix domain

$$S_o + S_{wi} = 1. \quad (28)$$

Oil saturation in fracture

$$S_{of} \neq 1, \quad S_{wif} = 1, \quad (29)$$

$$S_{oim} = 1 - S_{wim}. \quad (30)$$

Using the same approach as in Eq.(27),

$$C_{eo} \neq C_o + C_w \frac{S_{wm}\phi_m}{\phi_m(1-S_{wim})+\phi_f} + C_{pm}\frac{\phi_m}{\phi_m(1-S_{wim})+\phi_f} + C_{pf}\frac{\phi_f}{\phi_m(1-S_{wim})+\phi_f}. \quad (31)$$

RESULTS AND DISCUSSION

The present model results are presented and discussed in this section. The input values are presented in Table 3. Initially, to validate and verify our model, the current model is compared with both experimental and numerical data of Jensen et al. /26/, respectively. Jensen et al. /26/ carried out experiments to study the hot water flood and steam flood in both fractured and non-fractured samples and has compared them with his numerical studies. However, at high injection rates and near residual oil saturations to steam, the Jensen et al. /26/ numerical simulation were unstable, because when steam condenses, the volume of steam decreases tremendously.

Table 3. Input values for performing numerical studies, /26/.

Variable	Value
length (cm)	30.39
width (cm)	03.86
porosity	0.186
initial oil saturation	0.500
matrix permeability (md)	100
S_{wirr} at room temp	0.270
S_{orw} at room temp	0.230
k_{rwo} (at S_{or})	0.052
k_{rgo} (at S_{wi})	0.950
S_{org} at room temp	0.230
S_{gc} at room temp	0.000
k_{rgo} (at S_{wi})	0.700
fracture permeability (md)	0.100
fracture height	0.025
k_{rog0} (at S_{wi})	0.400

In this current study, in Fig. 5a and 5b are the variations of average oil saturation in FCR, having different overburden pressure of 13.8 bar and 6.9 bar, respectively, and they also compare the present study with an experimental and numerical study of Jensen et al. /26/. Both plots in Fig. 5a and 5b show that average oil saturation is decreasing with increasing steam injections, which means increment in oil recovery. Furthermore, the decrement of oil saturation is more at a small volume of steam injection in Fig. 5a when compared to Fig. 5b, which signifies that overburden pressure plays a significant role in recovery in the initial stage; however, at a higher volume of steam injected, role of overburden pressure becomes insignificant. Early breakthrough is observed when high injection pressure rates are employed in the experimental and field applications of steam flooding.

The matrix permeability is playing a crucial role in oil production. The matrix permeability governs the rate of oil coming from the matrix to the fracture. In FCR, the oil must move from the matrix to fracture towards production. From Darcy's law, the mass flow rate of oil is directly proportional to permeability. It clearly shows that the matrix permeability is directly proportional to the mass flow rate of oil production.

(a)

(b)

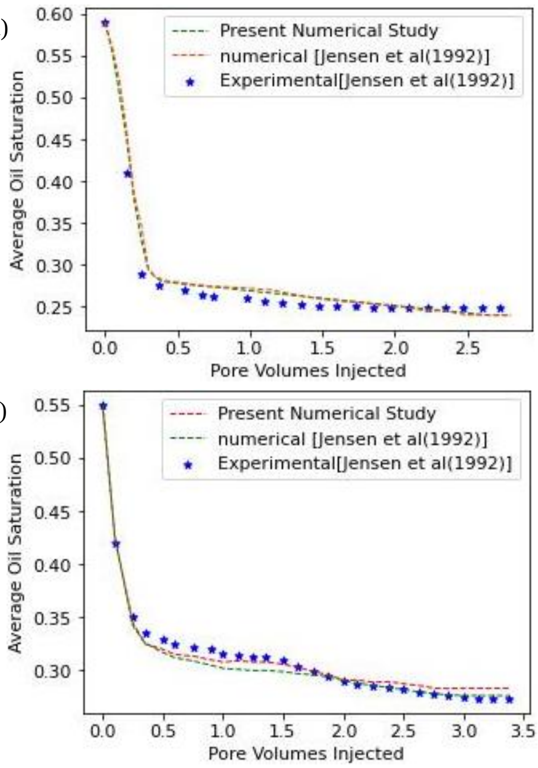

Figure 5. Validation and numerical verification of steam flooding in FCR with Jensen et al. /26/: a) overburden pressure 13.8 bar; b) overburden pressure 6.9 bar.

CONCLUSIONS

In the present study, an attempt has been made in order to deduce a detailed mathematical model along with the development of a numerical model for investigating EOR processes associated with a fractured reservoir.

The following conclusions have been drawn from the present study:

– fracture porosity at the network scale (as against at the scale of a single fracture) is found to be extremely sensitive in deciding the resultant production of highly viscous crude oil from an FCR,
– the porosity value deduced using the present mathematical model for an FCR matches very closely with the real field - scale scenario,
– various approaches to estimating porosity of FCR have been deduced in detail,
– the correlation between porosity and permeability has been established using the developed mathematical models,
– an improved method used in the present study in order to take into account the compressibility of the reservoir fluid in the fracture and rock-matrix is found to be significantly different,
– an early breakthrough has been observed upon the injection of steam into the fractured reservoir, which is found to be similar to that of a typical gas flooding,
– a significant reduction in the residual oil has been achieved upon the injection of steam at a reduced flow rate, which is found to displace a significant amount of oil from the larger pores initially, due to its associated lesser capillary pressure.

Nomenclature

P	pressure	g	gravity
m	mass	t	time
ρ	density	r_w	radius of well
ϕ	porosity	r	fracture radius (cm)
S	saturation	r_e	radius of investigation
u	Darcy's velocity	H	elevation
A	cross-sectional area	b	fracture width (cm)
$\dot{m}$	mass flow rates	l	average fracture length (cm)
k_i	intrinsic permeability	L_{FD}	linear fracture density
k_{ri}	relative permeability	M	area of cross-section
k	absolute permeability	Q	flow rate
k_{ff}	permeability of fracture	h	height of payzone
k_m	permeability of matrix	C	compressibility
ϕ_t	the total porosity	m	matrix
ϕ_1	primary porosity	f	fracture
ϕ_2	secondary porosity	S_{wirr}	irreducible water saturation
ϕ_m	matrix porosity	S_{orw}	residual oil satur. in water
ϕ_f	fracture porosity	k_{rwo}	relative permeab. of water
ϕ_{eff}	fracture's effective porosity	S_{wi}	initial water saturation

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