Avanish Mishra, Suresh Kumar Govindarajan

MATHEMATICAL MODEL FOR CBM (COAL BED METHANE) PRODUCTION UNDER THE COUPLED EFFECT OF MULTI-MECHANISTIC METHANE FLOW AND COAL GEOMECHANICS

MATEMATIČKI MODEL PROIZVODNJE METANA IZ LEŽIŠTA UGLJA (CBM) KAO SPREGNUTI MULTI-MEHANIČKI UTICAJ PROTOKA METANA I GEOMEHANIKE UGLJA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 7.08.2020

Adresa autora / Author's address:
Reservoir Simulation Laboratory, Indian Institute of Technology Madras, Chennai, India
email: avanishra@gmail.com

Keywords
- methane flow
- dual porosity
- multi-mechanistic
- coal deformation
- Klinkenberg effect

Ključne reči
- protok metana
- dvojna poroznost
- multi-mehanički
- deformacija uglja
- Klinkenbergov efekat

Abstract

The dry CBM (coal bed methane) reservoir contains methane gas and irreducible water, contrary to wet reservoir. Studies have shown economical production can be achieved in dry CBM reservoir. The coal seam consists of cleat and matrix arrangement where cleat have more permeability and less porosity, while matrix accounts for storage i.e. high porosity. The variation of porosity and permeability in cleat and matrix have order of difference due to which it requires to be characterized as a dual porosity model. The migration of dry methane (after desorption) in coal beds follows three-stage gas migration process i.e. desorption in matrix, diffusion and seepage in cleat. Earlier published models consider either equilibrium/non-equilibrium sorption, Klinkenberg effect, a dynamic porosity and permeability model, Darcy flow or geo-mechanical model, but only fewer studies are performed considering all these effects together. The proposed mathematical model differs by encapsulating multi-mechanistic flow with non-equilibrium sorption and geo-mechanical aspect. On this framework, a dual-porosity, single-phase, non-equilibrium adsorption, geo-mechanical, multi-mechanistic (Darcy and slip velocity), nonlinear coupled mathematical model has been developed. The proposed mathematical model approaches dry coalbed methane reservoir to investigate methane gas production performance with coupled effect of geo-mechanics and dynamic petrophysical parameter in the reservoir.

Izvod

Suvi tip ležišta uglja sa metanom (CBM) sadrži metan gas i vodu, koja ne otiče, suprotno mokrom ležištu. Studije pokazuju da se ekonomičnost proizvodnje postiže u suvom ležištu CBM. Ugljeni sloj sačinjava ispucalost i matričnost, gde proslojna ispucalost ima veću permeabilnost i manju poroznost, dok matrica ima veću poroznost. Razlike u poroznosti i permeabilnosti ispucalog prosloja i matrice su reda veličine, usled čega se mogu okarakterisati modelom dvojne poroznosti. Prostiranje suvog metana (posle desorpcije) u ugljenom sloju teče prema procesu migracije gasa u tri faze, na pr. desorpcija u matrici, difuzija i procurivanje u ispucalom prosloju. Stariji objavljeni modeli uzimaju u obzir ravnotežnu/neravnotežnu sorpciju, Klinkenbergov efekat, model dinamičke poroznosti i permeabilnosti, Darsijev zakon, ili geomehanički model, ali samo manji broj radova razmatra sve ove uticaje istovremeno. Predloženi matematički model se razlikuje, jer sadrži multi-mehanički protok sa neravnotežnom sorpcijom i geomehaničkim aspektom. Stoga je razvijen nelinearni spregnuti matematički model, koji obuhvata dvojnu poroznost, jednofaznu neravnotežnu adsorpciju, geomehaniku i multi-mehaniku (Darsijeva brzina i brzina na granici fluid-čvrsto). Predloženi model je primenjen na suvi tip ležišta uglja sa metanom za istraživanje mogućnosti proizvodnje metana, pri istovremenom geomehaničkom i dinamičkom petrofizičkom uticaju, unutar datog nalazišta.

INTRODUCTION

At the outset, in order to meet the enhanced energy requirements Coal-Bed-Methane (CBM) becomes a very valuable and demanding natural energy resource. The CBM reservoir is very similar to a fractured reservoir and it behaves both as a source rock as well as a reservoir rock. Fractured reservoirs are characterized based on multi-continuum concept and it can be modelled using a dual porosity approach with fracture and matrix as fundamental entities /1-6/. In a CBM reservoir, most of the methane stored in coal-beds are adsorbed on the coal-matrix surfaces, while the high permeability cleat or fractures contain the free gases. Thus, we have two distinct pressure drawdown regimes: one within the high permeable fracture; and other within the low-permeability rock-matrix; with a fluid mass transfer exchange at the fracture-matrix interface /7-9/. The methane migration in single phase coal-seams gets initiated from gas desorption followed by free molecular diffusion from coal-matrix to cleats /10/; and then seepage through the cleats towards the production well as shown in Fig. 1.

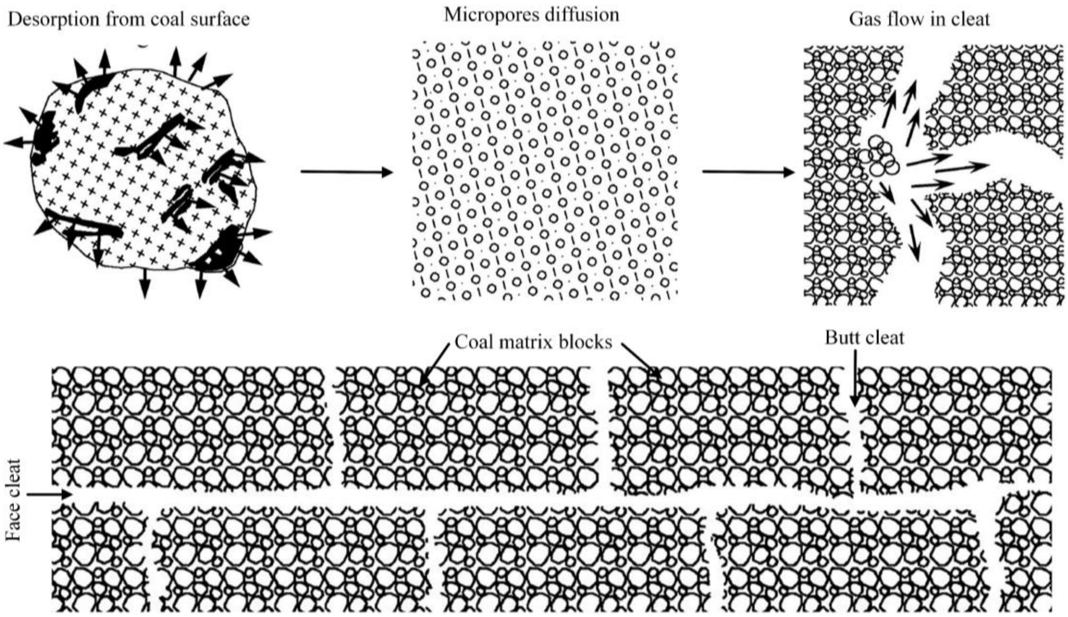

Figure 1. Methane gas migration in coalbed /11/.

During this methane-gas migration, reservoir involves a coupled effect of gas flow and geo-mechanical deformation. This coupled effect in a CBM reservoir makes the petro-physical parameters as dynamic quantity which subsequently leads to nonlinearity in the respective mathematical model of describing methane migration.

The geo-mechanical effect in the reservoir can be expressed in terms of variation in porosity and permeability of coal seams due to matrix swelling and shrinkage effect with stress/strain generated during the process.

In order to develop a fully coupled mathematical model of methane migration in the coal seams, various studies have been performed by researchers. These studies have considered various aspect of gas flow dynamics in coal seams, like equilibrium/non-equilibrium sorption, Klinkenberg effect, dynamic porosity and permeability model, Darcy flow or geo-mechanics of reservoir.

Turgay Ertekin et al. /12/ have proposed a mathematical formulation for gas migration in gas reservoir considering the effect of Klinkenberg and slippage velocity for single and multi-phase gas flow, and they have concluded that the compositional effects of the slippage factor tended to increase the difference between the dynamic and constant slippage approaches; differences increased with decreasing permeability. Valliappan and Zang /13/ further developed the mathematical model for gas flow in coal seams by considering the geo-mechanical aspect of reservoir under anisotropic medium and stated that during the gas migration, the pressure and concentration gets effected by non-linear behaviour of adsorption. Palmer and Mansoori /14/ proposed mathematical model for the effective stress law in coal seams for single porosity medium by considering matrix shrinkage, not as an average but as a function of reservoir pressure during drawdown. Gilman and Beckie /15/ presented mathematical model for diffusion-controlled gas migration rather than permeability controlled in gas migration and concluded that steeper the sorption isotherm, the slower will be the methane production, when other properties remain constant. Yu Wu et al. /16/ developed anisotropic permeability model for nonlinear coupled effect of gas migration and geo-mechanics with fractures spacing, and concluded that the properties of components in coal are governed by BC. Tong-qiang Xia et al. /17/ proposed inertia and slip effect in the CBM migration and concluded that the inertial effect gives significant effect in coal-seam with high permeability and high gas pressure, whereas the slip effect is primarily dominant in coal-seam with low permeability and low gas pressure. Chaolin Zhang et al. /18/ investigated CBM production in parallel boreholes under dynamically evolved coal seam parameters. Studies shows permeability variation in coal seam based on location and production stages. Shouqing Lu et al. /19/ studied adsorbed and free gas effect on mechanical properties of CBM reservoir. The model illustrates the variation in the intensity of pressure and temperature. Xinfu Liu et al. /20/ developed a mathematical model for pressure drop and stimulated flow pressure and analysed its effect on CBM reservoir. Gang Wang et al. /21/ proposed permeability models based on double strain spring and effective stress principle for cleat-spacing. The model demonstrated pore pressure variation with Klingenberg effect under the effective stress.

Given this background, most of the proposed mathematical models are based on the study of specific flow and mechanical property under the restricted medium but fewer studies have been proposed to incorporate all the possible flow and cross coupling option together. In this manuscript, an attempt has been made in order to model a CBM reservoir system that covers most of the flow and mechanical processes involved in the methane migration in coal seams with their coupled effect on dual porosity CBM reservoir. This cross-coupling makes mathematical models highly nonlinear and rigorous to solve.

The primary objective of the proposed work is to develop a multi-mechanistic fully coupled geo-mechanical model that describes the methane migration through a fractured CBM reservoir while considering the stress sensitivity, geomechanics, nonlinear non-equilibrium sorption, Klinkenberg effect, and dynamic porosity and permeability values for the coupled fracture-matrix system.

MATHEMATICAL MODEL

To establish the mathematical model for methane flow in a dual porosity coal seam, proposed work considers two-phase methane migration. In study, two separate pressure values are defined: one for matrix and another for cleat/fracture. The following assumptions are made during the mathematical model establishment.

1. Gas saturated coalbed reservoir.
2. Single phase methane gas migration with ignoring water effect.
3. The reservoir is at isothermal condition.
4. Methane gas viscosity is constant at the isothermal condition.
5. Coalbed reservoir is a homogenous, isotropic and poro-elastic medium.

The assumptions are to reduce the capillary, phase permeability and wettability effect. While assumption validates the viscosity variation no more than 15%.

The mass conservation equation for methane gas can be expressed as:

$$\frac{\partial m}{\partial t}+\nabla(\rho_g v_g)=Q\,, \tag{1}$$

where: m is gas mass content; t is time; ρ_g is density of gas; v_g is gas velocity; and Q_S is the source/sink term.

Gas mass content and gas velocity in cleat can be expressed as

$$m_f=\rho_g\phi_f\,, \tag{2}$$

$$v_{fg}=\frac{-k_f}{\mu}\nabla p_f\,, \tag{3}$$

where: m_f is gas mass in the fracture; ϕ_f is the porosity of matrix; p_f is the pore pressure; k_f is the fracture permeability and v_{fg} is gas velocity in the matrix.

Gas mass content (free and adsorbed) and gas velocity in matrix can be expressed as

$$m_m=\rho_g\phi_m+\rho_{ga}\rho_c m_b\,, \tag{4}$$

$$v_{mg}=v_{mg}^D+v_{mg}^S\,, \tag{5}$$

$$v_{mg}=\frac{-k_m}{\mu}\nabla p_m-D\nabla C\,, \tag{6}$$

where: m_m, ϕ_m, ρ_{ga}, ρ_c, v_{mg}, $v_{mg}{}^D$, $v_{mg}{}^S$, k_m, D, C, M_g, Z, R, T, ω and p_m denote: gas mass content in the matrix; matrix porosity; absolute gas density; density of coal; velocity in the matrix; Darcy velocity; Fickian velocity component; matrix permeability; diffusion coefficient; molar concentration; molecular weight; compressibility factor; universal gas constant; gas temperature; fracture-matrix coupling factor; and matrix pore pressure, respectively.

The real gas density term can be calculated by

$$\rho_g=\frac{M_g p}{ZRT}\,. \tag{7}$$

The Klinkenberg effect, i.e. apparent permeability on gas pressure will be

$$k_e=k_\infty\left(1+\frac{b}{P}\right). \tag{8}$$

The source and sink term are given by

$$Q_s=\frac{\omega M_g}{ZRT}(p_f-p_m)\,. \tag{9}$$

Methane flow in the matrix

The mass conservation equation for methane gas through matrix can be expressed as

$$\frac{\partial m_m}{\partial t}+\nabla(\rho_g v_{mg})=Q_s\,. \tag{10}$$

Now putting Eq.(4) in Eq.(10) we get

$$\frac{\partial}{\partial t}(\rho_g\phi_m+\rho_{ga}\rho_c m_b)+\nabla(\rho_g v_{mg})=Q_s\,. \tag{11}$$

Using the real density law from Eq.(7) and source/sink term from Eq.(9) leads to

$$\frac{\partial}{\partial t}(p_m\phi_m+p_m\rho_c m_b)+\nabla(p_m v_{mg})=\omega(p_f-p_m)\,. \tag{12}$$

By applying chain rule in temporal term and substituting in Eq.(6) leads to

$$\phi_m\frac{\partial p_m}{\partial t}+\rho_c p_{ga}\frac{\partial m_b}{\partial t}+\nabla\left[p_m\left(\frac{-k_m}{\mu}\nabla p_m-D\nabla C\right)\right]==\omega(p_f-p_m)\,, \tag{13}$$

$$\phi_m\frac{\partial p_m}{\partial t}+\rho_c p_{ga}\frac{\partial m_b}{\partial t}+\nabla\left(\frac{-k_m}{\mu}p_m\nabla p_m-Dp_m\nabla C\right)==\omega(p_f-p_m)\,. \tag{14}$$

The molar concentration from gas law can be obtained as

$$C=\frac{\rho_g}{M_g}\,. \tag{15}$$

By using Eq.(15) in Eq.(14), we get

$$\phi_m\frac{\partial p_m}{\partial t}+\rho_c p_{ga}\frac{\partial m_b}{\partial t}+\nabla\left[\frac{-k_m}{\mu}p_m\nabla p_m-Dp_m\nabla\left(\frac{M_g p_m}{ZRTM_g}\right)\right]==\omega(p_f-p_m)\,. \tag{16}$$

After rearrangements, Eq.(16) can be expressed as

$$\phi_m\frac{\partial p_m}{\partial t}+\rho_c p_{ga}\frac{\partial m_b}{\partial t}+\nabla\left(\frac{-k_m}{\mu}p_m\nabla p_m-\alpha Dp_m\nabla p_m\right)==\omega(p_f-p_m)\,. \tag{17}$$

Here: $\frac{\partial m_b}{\partial t}=\frac{-1}{\tau}[m_b-m_e(p_m)]$; $\tau=\frac{1}{\alpha D}$; $m_e=\frac{V_{Lm}P_m}{P_L+P_m}$

where: m_b, m_e, τ, V_{Lm} and P_L denote average volume, gas in matrix element, desorption time, gas volume in the surface, Langmuir volume of gas in matrix, and Langmuir pressure, respectively.

Methane flow in the fracture

The mass conservation equation for methane gas through fracture can be expressed as

$$\frac{\partial}{\partial t}(\rho_g\phi_f)+\nabla(\rho_g v_{fg})=Q_s\,. \tag{18}$$

Using the real density law from Eq.(7) and source/sink term from Eq.(9) leads to

$$\frac{\partial}{\partial t}(p_f \phi_f)+\nabla(p_f v_{fg})=-\omega(p_f-p_m)\,. \tag{19}$$

By applying chain rule in temporal term and substituting Eq.(3) leads to

$$p_f\frac{\partial}{\partial t}(\phi_f)+\phi_f\frac{\partial}{\partial t}(p_f)+\nabla\left(\frac{-k_f}{\mu}p_f\nabla p_f\right)= =-\omega(p_f-p_m)\,. \tag{20}$$

By substituting Klinkenberg effect from Eq.(8) we get

$$p_f\frac{\partial}{\partial t}(\phi_f)+\phi_f\frac{\partial}{\partial t}(p_f)+\nabla\left(\frac{-k_e}{\mu}p_f\nabla p_f\right)= =-\omega(p_f-p_m)\,, \tag{21}$$

Here,

$$\frac{\partial\phi_f}{\partial t}=\frac{1}{M}\left(\beta_f\frac{\partial p_f}{\partial t}+\beta_m\frac{\partial p_m}{\partial t}\right)+\frac{\varepsilon_L P_L}{(P_L+p_m)^2}\left(\frac{K}{M}-1\right)\frac{\partial p_m}{\partial t}\,.$$

Geomechanical deformation model

The general geomechanical deformation equation for the coalbed can be presented as

$$Gu_{ijj}+\frac{G}{1-2\nu}u_{jji}-\beta_f p_{fi}-\beta_m p_{mi}+F_i=0\,. \tag{22}$$

Here, $\beta_m=\frac{K}{K_m}-\frac{K}{K_s}$; $\beta_f=1-\frac{K}{K_m}$; $K=\frac{E}{3(1-2\nu)}$;

$$K_m=\frac{E_m}{3(1-2\nu)};\; K_s=\frac{K_m}{\{1-3\phi_m(1-\nu)/[2(1-2\nu)]\}}\,,$$

where: G, u, ν, β_f, β_m, F, K, K_m, K_s and E denote shear module of coal; displacement on the boundary; Poisson ratio of the coal; effective stress coefficient for fracture; effective stress coefficient for matrix; body force; bulk modulus of coal; bulk modulus of coal grains; bulk modulus of coal skeleton; Young's modulus of coal, respectively.

Dynamic porosity and dynamic permeability model

The dynamic porosity and dynamic permeability model for coalbed can be expressed as

$$\frac{\phi_f}{\phi_{f0}}=1+\frac{1}{M\phi_{f0}}\left[B_f(p_f-p_{f0})+B_m(p_m-p_{m0})\right]+ +\frac{\varepsilon_L}{\phi_{f0}}\left(\frac{K}{M}-1\right)\left(\frac{p_m}{P_L+p_m}-\frac{p_m}{P_L+p_m}\right). \tag{23}$$

From the conventional cubic law $\frac{k_\infty}{k_{\infty 0}}=\left(\frac{\phi_f}{\phi_{f0}}\right)^3$,

$$\frac{k_\infty}{k_{\infty 0}}=\left\{1+\frac{1}{M\phi_{f0}}\left[B_f(p_f-p_{f0})+B_m(p_m-p_{m0})\right]+ +\frac{\varepsilon_L}{\phi_{f0}}\left(\frac{K}{M}-1\right)\left(\frac{p_m}{P_L+p_m}-\frac{p_m}{P_L+p_m}\right)\right\}^3. \tag{24}$$

By using Eq.(24), the Klinkenberg effect from Eq.(8) can be rewritten as

$$k_e=k_{\infty 0}\left(1+\frac{b}{p_f}\right)\left\{1+\frac{1}{M\phi_{f0}}\left[B_f(P_f-P_{f0})+B_m(p_m-p_{m0})\right]+ +\frac{\varepsilon_L}{\phi_{f0}}\left(\frac{K}{M}-1\right)\left(\frac{p_m}{P_L+p_m}-\frac{p_m}{P_L+p_m}\right)\right\}^3, \tag{25}$$

where: ε_L, M denote lateral strain and constrained axial modulus, respectively.

Initial and boundary conditions

Initial conditions in Ω for the governing equations are defined as

$$u_i(0)=u_0\,, \tag{26}$$

$$\sigma_{ij}(0)=\sigma_0\,, \tag{27}$$

$$p_m(0)=p_f(0)=p_0\,. \tag{28}$$

Dirichlet and Neumann boundary conditions on Ω for the governing equations are defined as

$$p_m=p_f=\tilde{p}\,, \tag{29}$$

$$\left(\frac{k_m}{\mu}\nabla p_m-\alpha D\nabla p_m\right)\cdot\vec{n}=Q_m(t)\,, \tag{30}$$

$$\frac{k_f}{\mu}\nabla p_f\cdot\vec{n}=Q_f(t)\,. \tag{31}$$

MODEL DISCUSSION

The derived mathematical model covers the various aspect of methane migration in coal seams. The conventional mass conservation laws are applied in fracture and matrix system explicitly. The matrix in coal seams responsible for storage module and is considered as with constant porosity and insignificant permeability, the matrix equation incorporates the nonlinear non-equilibrium sorption front. The considered nonlinear Langmuir sorption shows time dependency in sorption phenomenon. Multi-mechanistic flow phenomenon is comprised of Darcy and Fickien velocities considered together in the matrix. The fracture in coal seams are characterized as dynamically evolved porosity and permeability from the Palmer and Mansoori model. This dynamic model represents the resultant effect of effective stress and matrix shrinkage in the CBM reservoir with the assumption of considering uniaxial strain conditions and constant overburden stress. The standard cubic laws have been considered for calculating permeability from porosity. To couple fracture matrix system, the pressure-dependent source/sink term is introduced. The freely moving fluid in the porous media impacts the geomechanical response and changes the sedimentation, /16/. To capture the geomechanical effect, the Navier equation has been taken. The presented model can accommodate the above discussed flow and geomechanical process along with their coupled effect on the system. The characterization of reservoir parameters in the governing equation is shown in Table 1, given below.

The cross coupling of the multi-mechanistic and geomechanical mathematical equations can be expressed in Fig. 2.

The present mathematical model consists of highly nonlinear equations; in the absence of any direct analytical solution, numerical technique can be adapted. One of the numerical techniques like finite difference method, finite volume method and finite element method can be used with respective initial and boundary conditions to solve governing equations.

Table 1. Characteristics of parameters used in governing equation.

Type of PDE	coupled parabolic
Nature of PDE	nonlinear PDE
Nature of fluid	incompressible
Number of phases	single-phase
Spatial distribution of constants and variables	homogeneous and isotropic
Temporal evolution	transient
Number of continuums	dual-continuum
Nature of Reynold's fluid regime	laminar
Momentum description	non-Darcy-based
Number of partial differential equations	two
Number of unknowns	two
Number of dependent variables in PDEs	P_f and P_m
Number of independent variables in PDEs	x, y and t
Number of constant coefficients	k_m, ϕ_μ, μ, ρ_c, D, β_f, β_m
Number of variable coefficients	ϕ_f, k_f, V_l, P_l, m_b and α
Number of initial conditions for PDEs	two
Number of boundary conditions for PDEs	four

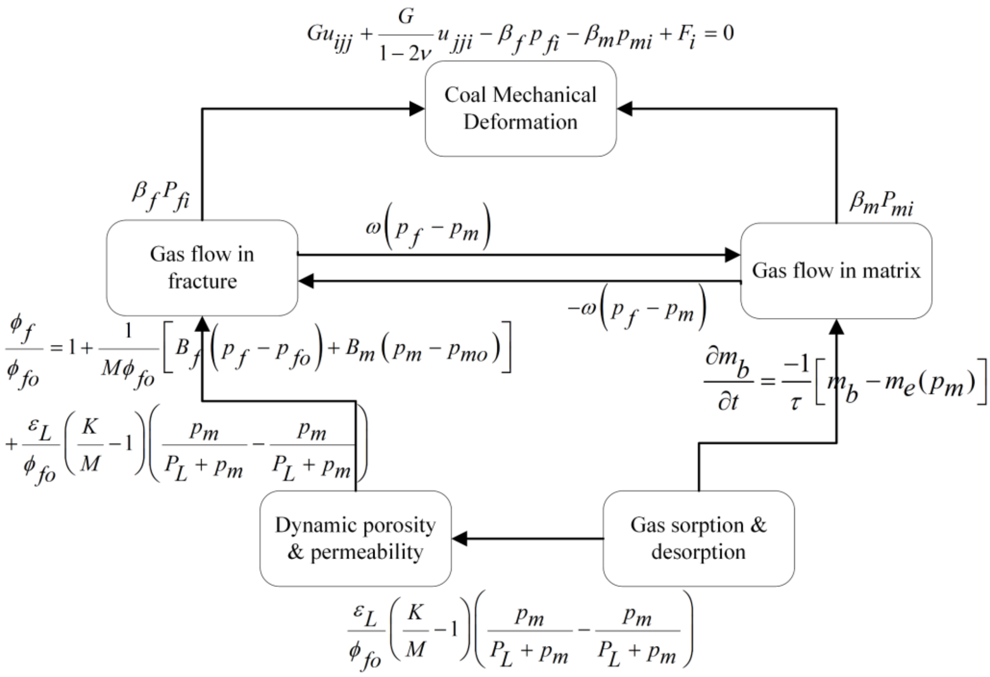

Figure 2. Cross-coupling between multi-physics process in CBM reservoir.

CONCLUSIONS

A mathematical model is presented to describe the coupled effect of methane flow and geomechanics of the system under the stress dependence dynamic porosity and permeability, nonlinear non-equilibrium sorption, Klinkenberg effect, and multi-mechanistic model.

Based on the proposed model, the following conclusions can be drawn:

– the present model considers methane migration as multi-mechanistic flow (Darcy and Fickian). Parallelly gas flow in matrix considers nonlinear time dependent sorption effect,
– this stress dependent dynamic porosity and permeability Palmer and Mansoori model have been adopted to capture matrix shrinkage and stress effect together,
– cross coupling has been done in the model considering the effect of fracture matrix interaction, gas flow and dynamically evolved petrophysical parameters with geomechanics.

REFERENCES

1. Kumar, Suresh G., Sekhar, M. (2005), *Spatial moment analysis for transport of nonreactive solutes in a fracture-matrix system*, J Hydrol. Eng. 10(3): 192-199.
2. Kumar, Suresh G., Ghassemi, A. (2005), *Numerical modeling of non-isothermal quartz dissolution/precipitation in a coupled fracture-matrix system*, Geothermics, 34(4): 411-439. doi: 10.1016/j.geothermics.2005.04.003
3. Ghassemi, A., Kumar, Suresh G. (2007), *Changes in fracture aperture and fluid pressure due to thermal stress and silica dissolution/precipitation induced by heat extraction from subsurface rocks*, Geothermics, 36(2): 115-140. doi: 10.1016/J.GEOTHERMICS.2006.10.001
4. Kumar, Suresh G. (2009), *Influence of sorption intensity on solute mobility in a fractured formation*, J Environ. Eng. 135(1): 1-7. doi: 10.1061/(ASCE)0733-9372(2009)135:1(1)
5. Kumar, Suresh G., Sekhar, M., Mishra, D. (2008), *Time-dependent dispersivity of linearly sorbing solutes in a single fracture with matrix diffusion*, J Hydrol. Eng. 13(4): 250-257. doi: 10.1061/(ASCE)1084-0699(2008)13:4(250)
6. Kumar, Suresh G. (2008), *Effect of sorption intensities on dispersivity and macro-dispersion coefficient in a single fracture with matrix diffusion*, Hydrogeol. J. 16(2): 235-249. doi: 10.1007/s10040-007-0234-5
7. Kumar, Suresh G. (2014), *Mathematical modelling on transport of petroleum hydrocarbons in saturated fractured rocks*, Sadhana, 39(5): 1119-1139. doi: 10.1007/s12046-014-0284-z
8. Kumar, Suresh G. (2014), *Mathematical modeling of groundwater flow and solute transport in saturated fractured rock using a dual-porosity approach*, J Hydrol. Eng. 19(12): 04014033-1-04014033-8. doi: 10.1061/(ASCE)HE.1943-5584.0000986

9. Kumar, Suresh G. (2016), *Modeling fluid flow through fractured reservoirs: Is it different from conventional classical porous medium?*, Current Sci. (Current Sci. Ass. & Indian Acad. Sci.), 110(4): 695-701. doi: 10.18520/cs/v110/i4/695-701
10. Kumar, Suresh G., Rakesh, T.V. (2018), *Numerical modeling of hyperbolic dominant transient fluid flow in saturated fractured rocks using Darcian approach*, Ground. Sustain. Devel. 7(1): 56-72. doi: 10.1016/j.gsd.2018.03.006
11. Liu, Q., et al. (2015), *A mathematical model of coupled gas flow and coal deformation with gas diffusion and Klinkenberg effects*, Rock Mech. Rock Eng., 48(3): 1163-1180. doi: 10.1007/s00603-014-0594-9
12. Ertekin, T., King, G.A., Schwerer, F.C. (1986), *Dynamic gas slippage: A unique dual-mechanism approach to the flow of gas in tight formations*, SPE Form. Eval., 1(1): 43-52. doi: 10.2118/12045-PA
13. Valliappan, S., Zhang, W. (1996), *Numerical modelling of methane gas migration in dry coal systems*, Int. J Numer. Anal. Methods Geomech. 20(8): 571-593. doi: 10.1002/(SICI)1096-9853(199608)20:8<571::AID-NAG840>3.0.CO;2-0
14. Palmer, I., Mansoori, J. (1996), *How permeability depends on stress and pore pressure in coalbeds: A new model*, SPE Reserv. Eng. 1(6): 539-543. doi: 10.2118/36737-MS
15. Gilman, A., Beckie, R. (2000), *Flow of coal-bed methane to a gallery*, Transp. Porous Media, 41(1): 1-16. doi: 10.1023/A:1006754108197
16. Wu, Y., et al. (2010), *Development of anisotropic permeability during coalbed methane production*, J Nat. Gas Sci. Eng. 2(4): 197-210. doi: 10.1016/j.jngse.2010.06.002
17. Xia, T.Q., Gao, F., Kang, J.H., Wang, X.X. (2016), *A fully coupling coal-gas model associated with inertia and slip effects for CBM migration*, Environ. Earth Sci. 75(7): 1-10. doi: 10.1007/s12665-016-5378-y
18. Zhang, C., et al. (2018), *Dynamic evolution of coal reservoir parameters in CBM extraction by parallel boreholes along coal seam*, Transp. Porous Media, 124(1): 325-343. doi: 10.1007/s11242-018-1067-5
19. Lu, S., Zhang, Y., Sa, Z., Si, S. (2019), *Evaluation of the effect of adsorbed gas and free gas on mechanical properties of coal*, Environ. Earth Sci., 78(6): 1-15. doi: 10.1007/s12665-019-8222-3
20. Liu, X., Liu, C., Liu, G. (2019), *Dynamic behavior of coalbed methane flow along the annulus of single-phase production*, Int. J Coal Sci. Technol. 6(4): 547-555. doi: 10.1007/s40789-019-00276-1
21. Wang, G., et al. (2020), *An improved coal permeability model with variable cleat width and Klinkenberg coefficient*, Geotech. Geol. Eng. 38: 3041-3051. doi: 10.1007/s10706-020-01205-9