Pankaj Thakur[1], Naresh Kumar[2], Sukhvinder[3]

ELASTO-PLASTIC DENSITY VARIATION IN A DEFORMABLE DISK

ELASTOPLASTIČNA PROMENA GUSTINE KOD DEFORMABILNOG DISKA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 27.09.2019

Adresa autora / Author's address:
[1)] Depart. of Mathematics, Faculty of Science and Techn., ICFAI University Baddi, Solan, India
email: pankaj_thakur15@yahoo.co.in
[2)] Research Scholar, Depart. of Mathematics, Faculty of Science and Techn., ICFAI University Baddi, Solan, India
[3)] Depart. of Math., Swami Premanand Mahavidyalaya, Hoshiarpur, Panjab University Chandigarh, India

Keywords
- disk
- density
- stresses
- speed
- yielding

Ključne reči
- disk
- gustina
- naponi
- brzina
- tečenje

Abstract

The purpose of this paper is to present a study of elasto-plastic density variation in a deformable disk by using Seth's transition theory. It has been observed that the effect of density variation parameter in a rotating disk requires lesser values of angular speed for compressible material as well as incompressible materials. The hoop stress has a maximum at the inner surface for incompressible materials as compared to compressible materials. With increased values of density parameter, the ratio of angular velocity for the fully-plastic state with respect to initial plasticity is increased to large values.

Izvod

Cilj ovog rada je predstavljanje istraživanja elastoplastične promene gustine kod deformabilnog diska, primenom teorije prelaznih napona Seta. Uočeno je da uticaj promene parametra gustine kod rotirajućeg diska ima manje vrednosti ugaone brzine za stišljiv materijal, kao i za nestišljiv materijal. Cirkularni napon ima maksimum na unutrašnjoj površini za nestišljiv materijal, u poređenju sa stišljivim materijalima. Sa porastom vrednosti parametra gustine, odnos ugaone brzine za stanje potpune plastičnosti prema vrednostima za početnu plastičnost raste do visokih vrednosti.

INTRODUCTION

Theoretical investigation of deformations in a disk induced by centrifugal forces is an important topic due to its various applications in engineering components such as gas turbine rotors, internal combustion engines, casting of ship propellers, turbojet engines, high-speed gears, flywheels, rotors, and compact disks, etc. Optimising the design of a rotating disk and also assessing the failure risk requires understanding its behaviour in the elasto-plastic regime. In this context, numerical investigations have been extensively used to predict the deformation, failure, and stress and strain fields in a uniform rotating disk under different loading conditions. Elasto-plastic analysis in a rotating disk has always attracted a lot of research interest because of their importance in engineering applications.

The stress field in a uniform rotating disk subjected to elasto-plastic loading condition is not uniform and maximal stress occurs at the centre of the disk. The non-uniform distribution of stresses is a key barrier in designing rotating disks with enhanced performance. The distribution of stresses and displacement in an elasto-plastic rotating disk is a classical problem in engineering design and can be found in many textbooks /1, 5, 6, 7/. Reddy and Srinath /8/ investigated the influence of material density on stresses and radial displacements in a rotating polar orthotropic disk. You et al. /9/ investigated elastic–plastic rotating disks with arbitrary variable thickness and density. Alexandrova et al. /10/ investigated the problems of elastic-plastic stress distribution in a rotating annular disk. Gupta et al. /11/ analysed elastic-plastic transition in a thin rotating disk with inclusion under thermal effect by using Seth's transition theory. Mohammad et al. /12/ investigated the problems of linear thermo-elastic analysis of a functionally graded (FG) rotating disk with different boundary conditions using Adomian decomposition method.

Methodology: Seth's transition theory /3/ includes classical macroscopic solving problems in creep and relaxation, plasticity, and assumes semi-empirical yielding conditions. The nonlinear transition regions through which yielding occurs are neglected. Apparently, transition theory is used to solve problems in a general way, employing the concept of generalised strain measure and asymptotic solution at the transition points of differential equations, defining the deformed field and has been successfully applied to a large number of problems /3, 4, 11, 13-21/. The abstract measure theory has been highly developed; it has not been suitably exploited in the domain of nonlinear mechanics. In classical mechanics, the ordinary measures have found sufficient extensions but none of them have been generally made. If a continuous phenomenon will be represented by a spectrum, nonlinearity exhibits itself at their end which corresponds to the transition states. In the current literature, such transition states require semi-empirical laws to match the solutions and thus a discontinuity arises where they do not exist. For the introduction of nonlinear measures, a continuum approach is necessary. Elastic-plastic deformation, creep, relaxation, fatigue and shocks are some of the well-known examples of such irreversible processes. Classical measures of deformation are totally inadequate to deal with such transitions and make constitutive equations of the medium very complicated. If for a very small interval, the number of fluctuations is very large, the Riemann integral concept for ordinary measure fails and the measures as those of Lebesgue have to be used. In the same manner, generalised measures given by weighted integral representations give very satisfactory results in problems like that of plasticity and creep.

Weighted integral measures representations: the ordinary uniaxial Cauchy measure is given by

$$\int_{l_0}^{l}\frac{dl}{l_0}=\frac{l-l_0}{l_0},$$

where l and l_0 are deformed and undeformed lengths. The first weighted measure called Hencky measure can be written as

$$\int_{l_0}^{l}\left(\frac{l_0}{l}\right)\frac{dl}{l_0}=\ln\frac{l}{l_0}$$

and is widely used in plasticity problems. But for the creep problems, it is found useful only in secondary or stationary creep, not in the transient or fracture stages. The second weighted measure used by /2/ is

$$\int_{l_0}^{l}\left(\frac{l_0}{l}\right)^2\frac{dl}{l_0}=\frac{l-l_0}{l}.$$

In finite elasticity, Almansi and Green measures, the deformed and undeformed states are taken as reference frameworks respectively, and are extensively used. The third weighted measures are

$$\int_{l_0}^{l}\left(\frac{l_0}{l}\right)^3\frac{dl}{l_0}=\frac{1}{2}\left[1-\left(\frac{l_0}{l}\right)^2\right],\quad \int_{l_0}^{l}\left(\frac{l}{l_0}\right)^3\frac{dl_0}{l}=\frac{1}{2}\left[\left(\frac{l}{l_0}\right)^2-1\right],$$

and in this case, the weighting functions are $(l_0/l)^3$ and $(l/l_0)^3$ respectively. An obvious generalization of these measures is

$$\int_{l_0}^{l}\left(\frac{l_0}{l}\right)^{n+1}\frac{dl}{l_0}=\frac{1}{2}\left[1-\left(\frac{l_0}{l}\right)^n\right]$$

in which the weighting function is $(l_0/l)^{n+1}$. For n = –2, –1, 0, 1, 2, it gives Green, Cauchy, Hencky, Swainger and Almansi measures, respectively. Thus, in the general case, if the principal Almansi and Green measures are denoted by $\overset{A}{\varepsilon}_{ii}$ and $\overset{G}{\varepsilon}_{ii}$, the generalized measures in Cartesian coordinates may be written in the form:

$$\varepsilon_{ii}^{M}=\int_0^{\overset{A}{\varepsilon}_{ii}}\left[1-2\varepsilon_{ii}^{A}\right]^{\frac{n}{2}-1}d\overset{A}{\varepsilon}_{ii}=\frac{1}{n}\left[1-(1-2\varepsilon_{ii}^{A})^{\frac{n}{2}}\right],\ i=1,2,3 \quad (1)$$

and $$\varepsilon_{ii}^{M}=\int_0^{\overset{G}{\varepsilon}_{ii}}\left[1+2\varepsilon_{ii}^{G}\right]^{\frac{n}{2}-1}d\varepsilon_{ii}^{G}=\frac{1}{n}\left[(1+2\varepsilon_{ii}^{G})^{\frac{n}{2}}-1\right].$$

The main objective of the present paper is to develop a consistent analytical model capable to resolve a class of control problems for rotating disk, due to the importance of the control problems in practical engineering design. On the other hand, the importance of material properties in the burst speed of disks is investigated. The novelty in the current research is to include three control factors such as rotating speed, density parameter and mechanical load in the consideration of the optimal performance of the disk. We assume that the density of disk varies along the radius in the form:

$$\rho(r)=\rho_0(r/r_0)^{-m}, \quad (2)$$

where: ρ_0 is constant density at $r = r_0$; and m is the density variation parameter. Results are obtained numerically and depicted graphically.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

We consider a thin annular disk of variable density with central bore of inner radius r_i and outer radius r_o. The disk is rotating with angular speed ω of gradually increasing magnitude about an axis perpendicular to its plane and passing through the centre. The thickness of the disk is assumed small so that the disk is effectively in a state of plane stress, i.e. the axial stress τ_{zz} is zero.

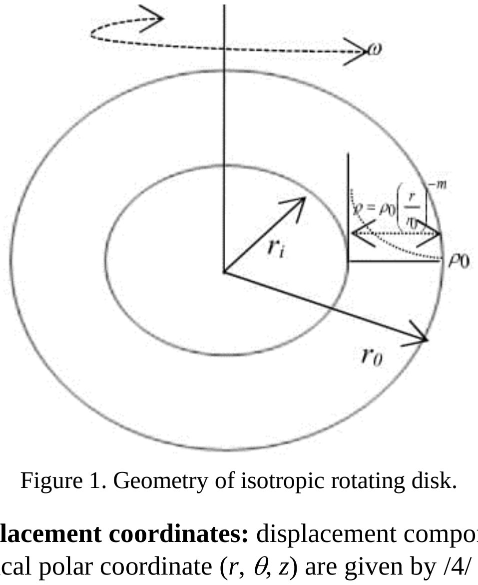

Figure 1. Geometry of isotropic rotating disk.

Displacement coordinates: displacement components in cylindrical polar coordinate (r, θ, z) are given by /4/ as:

$$u=r(1-\eta),\quad v=0,\quad w=dz, \quad (3)$$

where: η is position function, depending on $r = \sqrt{(x^2 + y^2)}$ only; and d is a constant.

Finite strain components: the finitesimal components of strain are given by /3, 4/ as:

$$\overset{A}{\varepsilon}_{rr} \equiv \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-(r\eta'+\eta)^2\right],$$

$$\overset{A}{\varepsilon}_{\theta\theta} \equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left[1-\eta^2\right],$$

$$\overset{A}{\varepsilon}_{zz} \equiv \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-(1-d)^2\right],$$

$$\overset{A}{\varepsilon}_{r\theta} = \overset{A}{\varepsilon}_{\theta z} = \overset{A}{\varepsilon}_{zr} = 0\,, \tag{4}$$

where: $\eta' = d\eta/dr$.

Generalized strain components: substituting Eq.(4) in Eq.(1), the components of generalized strain measure are given as:

$$\varepsilon_{rr} = \frac{1}{n}\left[1-(r\eta'+\eta)^n\right], \quad \varepsilon_{\theta\theta} = \frac{1}{n}\left[1-\eta^n\right],$$

$$\varepsilon_{zz} = \frac{1}{n}\left[1-(1-d)^n\right], \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0\,, \tag{5}$$

where: n is measure; ε_{rr}, $\varepsilon_{\theta\theta}$ and ε_{zz} are strain components.

Stress-strain relation: the stress-strain relations for an isotropic material are given /1/ as:

$$\tau_{ij} = \lambda\delta_{ij}I_1 + 2\mu\varepsilon_{ij} \quad i,j = 1,2,3\,, \tag{6}$$

where: τ_{ij} and ε_{ij} are stress and strain components respectively; also λ and μ are Lame's constants; $I_1 = \varepsilon_{kk}$ is the first strain invariant; and δ_{ij} is Kronecker's delta.

Equation (6) for this problem becomes

$$\tau_{rr} = \frac{2\lambda\mu}{\lambda+2\mu}[\varepsilon_{rr}+\varepsilon_{\theta\theta}] + 2\mu\varepsilon_{rr}\,,$$

$$\tau_{\theta\theta} = \frac{2\lambda\mu}{\lambda+2\mu}[\varepsilon_{rr}+\varepsilon_{\theta\theta}] + 2\mu\varepsilon_{\theta\theta}\,,$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0\,. \tag{7}$$

Substituting Eq.(5) in Eq.(7), the stresses are obtained:

$$\tau_{rr} = \frac{2\mu}{n}\left[3-2c-\eta^n\left\{1-c+(2-c)(T+1)^n\right\}\right],$$

$$\tau_{\theta\theta} = \frac{2\mu}{n}\left[3-2c-\eta^n\left\{2-c+(1-c)(T+1)^n\right\}\right]$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0\,. \tag{8}$$

where: c is the compressibility factor of the material in term of Lame's constant given by $c = 2\mu/(\lambda+2\mu)$.

Equation of equilibrium: the equations of equilibrium are all satisfied except:

$$\frac{d}{dr}(r\tau_{rr}) - \tau_{\theta\theta} + \rho(r)\omega^2 r^2 = 0\,, \tag{9}$$

where: τ_{rr} is the radial stress; $\tau_{\theta\theta}$ is circumferential stress; and $\rho(r)$ is the material density of the rotating disk.

Asymptotic solution at transition points: by using Eq.(8) in Eq.(9), we get a nonlinear differential equation for η as:

$$(2-c)n\eta^{n+1}T(T+1)^{n-1}\frac{dT}{d\eta} =$$

$$= \left[\frac{n\rho\omega^2 r^2}{2\mu} + \eta^n\left\{1-(T+1)^n - nT\left[1-c+(2-c)(T+1)^n\right]\right\}\right] \tag{10}$$

where: $r\eta' = \eta T$; and T is a dependent function of η; and η is a dependent function of r. The transition points of η are $T = -1$ and $T \to \pm\infty$.

Boundary conditions: the boundary conditions of the rotating disk are

$$\tau_{rr} = 0, \quad r = r_i; \quad \tau_{rr} = 0, \quad r = r_0\,, \tag{11}$$

where: τ_{rr} denotes stress along the radial direction.

SOLUTION OF THE PROBLEM

For finding the plastic deformation, the transition function is taken through the principal stress (see /3, 4, 11, 13-21/) at the transition point $T \to \pm\infty$, we define the transition function R as, Eq.(11):

$$R = \frac{n}{2\mu}\tau_{\theta\theta} = \left[(3-2c)-\eta^n\left\{2-c+(1-c)(T+1)^n\right\}\right]. \tag{12}$$

Taking the logarithmic differentiation of Eq.(12) with respect to r, we get:

$$\frac{d(\log R)}{dr} =$$

$$= -\left(\frac{n\eta^n T}{r}\right)\frac{2-c+(T+1)^{n-1}(1-c)\left\{(T+1)+\eta\dfrac{dT}{d\eta}\right\}}{3-2c-\eta^n\left\{2-c+(1-c)(T+1)^n\right\}}\,. \tag{13}$$

By substituting the value of $dT/d\eta$ from Eq.(10) into Eq.(13) and by taking asymptotic value $T \to \pm\infty$, we get after integration:

$$R = K_1 r^{\nu-1}\,, \tag{14}$$

where: K_1 is a constant of integration which can be determined by boundary conditions; and $\nu = (1-c)/(2-c)$ is the Poisson ratio.

From Eqs.(12) and (14), it follows:

$$\tau_{\theta\theta} = \left(\frac{2\mu}{n}\right)K_1 r^{\nu-1}\,. \tag{15}$$

By substituting Eq.(15) into Eq.(9) and using Eq.(2), then integrating, we get:

$$\tau_{rr} = \left(\frac{2\mu}{n\nu}\right)K_1 r^{\nu-1} - \frac{\rho_0\omega^2 r^{2-m}r_0^m}{3-m} + \frac{K_2}{r}\,, \tag{16}$$

where: K_2 is a constant of integration which can be determined by boundary condition.

By applying the boundary condition from Eq.(11) in Eq.(16), we get:

$$K_1 = \frac{\rho_0\omega^2 n\nu r_0^m\left(r_0^{3-m}-r_i^{3-m}\right)}{2\mu(3-m)\left(r_0^\nu - r_i^\nu\right)}\,, \quad \text{and}$$

$$K_2 = \frac{\rho_0\omega^2 r_0^m r_i^{3-m}}{3-m} - \frac{\rho_0\omega^2 r_0^m\left(r_0^{3-m}-r_i^{3-m}\right)}{(3-m)(r_0^\nu - r_i^\nu)}r_i^\nu\,.$$

By substituting the values of K_1 and K_2 into Eqs.(15) and (16), we get:

$$\tau_{rr} = \frac{\rho_0\omega^2 r_0^m}{(3-m)r}\left[\frac{\left(r_0^{3-m}-r_i^{3-m}\right)}{r_0^\nu - r_i^\nu}(r^\nu - r_i^\nu) - r^{3-m} + r_i^{3-m}\right], \tag{17}$$

$$\tau_{\theta\theta}=\frac{\rho_0\omega^2 r_0^m v}{3-m}\left[\frac{\left(r_0^{3-m}-r_i^{3-m}\right)}{r_0^v-r_i^v}r^{v-1}\right]. \quad (18)$$

Equations (17) and (18) are elasto-plastic stresses in an isotropic material disk having variable density.

Initial yielding of rotating disk: it is seen from Eq.(18) that $|\tau_{\theta\theta}|$ is maximal at the inner surface ($r = r_i$). Therefore, yielding will take place at the inner surface and Eq.(18) becomes:

$$\left|\tau_{\theta\theta}\right|_{r=r_i}=\left|\frac{\rho_0\omega^2 r_0^m\left(r_0^{3-m}-r_i^{3-m}\right)}{(3-m)(r_0^v-r_i^v)}r_i^{v-1}\right|\cong Y \text{ (yielding)},$$

where: Y is yielding stress for initial yielding. The angular speed ω_i necessary for initial yielding is given by:

$$\Omega_i^2=\frac{\rho_0\omega_i^2 r_0^2}{Y}=\frac{(3-m)(r_0^v-r_i^v)r_0^2}{v\left(r_0^{3-m}-r_i^{3-m}\right)r_0^m r_i^{v-1}}, \quad (19)$$

where: $\omega_i = (1/r_0)\Omega_i(Y/\rho_0)^{1/2}$. We introduce the following non-dimensional components as: $R = r/r_0$, $R_0 = r_i/r_0$, $\Omega^2 = \rho_0\omega^2 r_0^2/Y$, $\sigma_r = \tau_{rr}/Y$, $\sigma_\theta = \tau_{\theta\theta}/Y$. Equations (17), (18) and (19) become:

$$\sigma_r=\frac{\Omega_i^2}{(3-m)R}\left[\frac{(1-R_0^{3-m})}{(1-R_0^v)}(R^v-R_0^v)-R^{3-m}+R_0^{3-m}\right],$$

$$\sigma_\theta=\frac{\Omega_i^2 v}{(3-m)}\frac{(1-R_0^{3-m})}{(1-R_0^v)}R^{v-1},$$

$$\Omega_i^2=\frac{(3-m)(1-R_0^v)}{(1-R_0^{3-m})v}R_0^{1-v}, \quad (20)$$

when $m = 3$, Eq.(20) becomes:

$$\sigma_r=-\frac{\Omega_i^2}{R}\left[\frac{R_0^{3-m}\ln R_0}{1-R_0^v}(R^v-R_0^v)+R^{3-m}\ln R-R_0^{3-m}\ln R_0\right],$$

$$\sigma_\theta=\Omega_i^2 v\frac{R_0^{3-m}R^{v-1}\ln R_0}{R_0^v-1},$$

$$\Omega_i^2=\frac{R_0^v-1}{vR_0^{3-m}\ln R_0}R_0^{1-v}. \quad (21)$$

Fully plastic state of rotating disk: the angular speed $\omega_f > \omega_i$ for which the rotating disk becomes fully plastic ($v \to 1/2 = 0.5$) at the outer surface $r = r_0$, Eq.(18) becomes:

$$\left|\tau_{\theta\theta}\right|_{r=r_0}=\left|\frac{\rho_0\omega^2 r_0^m\left(r_0^{3-m}-r_i^{3-m}\right)}{2(3-m)\sqrt{r_0}\left(\sqrt{r_0}-\sqrt{r_i}\right)}\right|\cong Y^* \text{ (say)},$$

where: Y^* is yielding stress for fully-plastic state. The angular speed ω_f, necessary for initial yielding is given by:

$$\Omega_i^2=\frac{\rho_0\omega_i^2 r_0^2}{Y^*}=\frac{2(3-m)\sqrt{r_0}\left(\sqrt{r_0}-\sqrt{r_i}\right)r_0^2}{r_0^m\left(r_0^{3-m}-r_i^{3-m}\right)}. \quad (22)$$

Stresses and angular speed obtained from Eqs.(20) and (22) for fully plastic state ($v \to 1/2 = 0.5$) become:

$$\sigma_r=\frac{\Omega_f^2}{(3-m)R}\left[\frac{1-R_0^{3-m}}{1-\sqrt{R_0}}\left(\sqrt{R}-\sqrt{R_0}\right)-R^{3-m}+R_0^{3-m}\right],$$

$$\sigma_\theta=\frac{\Omega_f^2\left(1-R_0^{3-m}\right)}{2(3-m)\sqrt{R}\left(1-\sqrt{R_0}\right)},$$

$$\Omega_f^2=\frac{\rho_0\omega_f^2 b^2}{Y^*}=\frac{2(3-m)\left(1-\sqrt{R_0}\right)}{1-R_0^{3-m}}. \quad (23)$$

when $m = 3$, Eq.(23) becomes:

$$\sigma_r=-\frac{\Omega_f^2}{R}\left[\frac{R_0^{3-m}\ln R_0}{1-\sqrt{R_0}}\left(\sqrt{R}-\sqrt{R_0}\right)+R^{3-m}\ln R-R_0^{3-m}\ln R_0\right],$$

$$\sigma_\theta=\frac{\Omega_f^2 R_0^{3-m}\ln R_0}{2\sqrt{R}\left(\sqrt{R_0}-1\right)},$$

$$\Omega_f^2=\frac{2\left(\sqrt{R_0}-1\right)}{R_0^{3-m}\ln R_0}. \quad (24)$$

NUMERICAL RESULTS AND DISCUSSION

For calculating stress and angular speed based on the above analysis, the following values have been taken: $v = 0.5$ (incompressible material, i.e. rubber); $v = 0.42857$ (compressible material, i.e. saturated clay); $v = 0.333$ (compressible material, i.e. copper), /1/; $m = 0$, 1 and 3 in respect. Curves are drawn, between angular speed Ω_i^2 and required initial yielding and radii ratios $R_0 = r_i/r_0$ (see Fig. 1) for a disk of compressible and incompressible material, having Poisson's ratio $v = 0.5$, 0.42857, 0.333; density (i.e. $m = 0$, 1, 3). It has been observed that the rotating disk of incompressible material (i.e. $v = 0.5$) requires higher angular speed to yield at the inner surface as compared to disk of compressible material (i.e. $v = 0.42857$, $v = 0.333$). With the introduction of density parameter, the value of angular speed decreases at the inner surface of the rotating disk.

Curves are drawn between the stress and radii ratios $R = r/r_0$ (see Figs. 2 and 3) for elasto-plastic transition and fully plastic state, respectively. Curves are drawn between the stress and radii ratios $R = r/r_0$ (see Figs. 2 and 3) for elasto-plastic transition and fully plastic state, respectively. It has been seen that the values of hoop stress (Fig. 2) are maximum at the inner surface of the rotating disk made of incompressible material (i.e. rubber $v = 0.5$) as compared to compressible materials (i.e. saturated clay $v = 0.42857$ and copper $v = 0.333$). With the introduction of the density parameter in the rotating disk, it quite decreases the value of hoop and radial stresses at the inner surface for elasto-plastic transitional state and the fully-plastic state.

It has been seen from Fig. 3, that hoop stress is maximal at the inner surface for fully-plastic state. With the introduction of the density parameter, the values of radial, as well as circumferential stresses, are decreased at the inner surface.

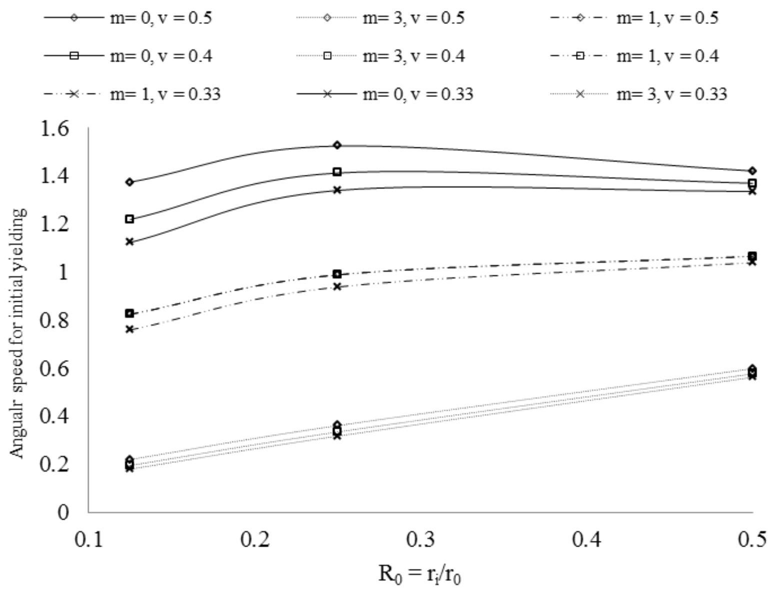

Figure 1. Angular speed required for initial yielding at inner surface of rotating disk along the radii ratio $R_0 = r_i/r_0$.

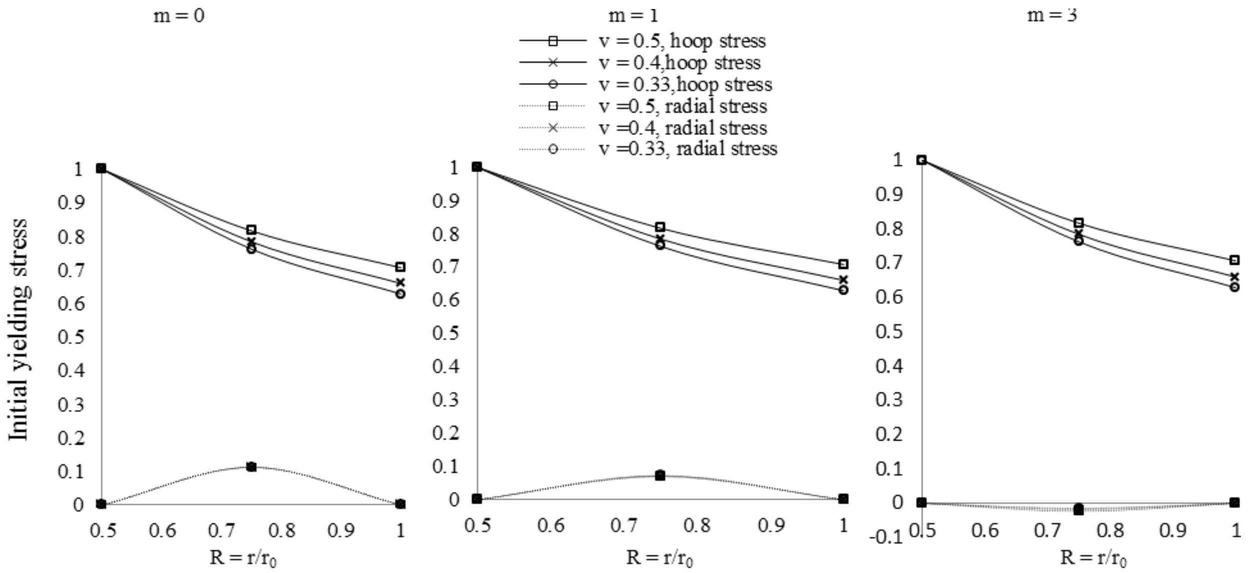

Figure 2. Stress distribution at the elasto-plastic state along radii ratio $R = r/r_0$.

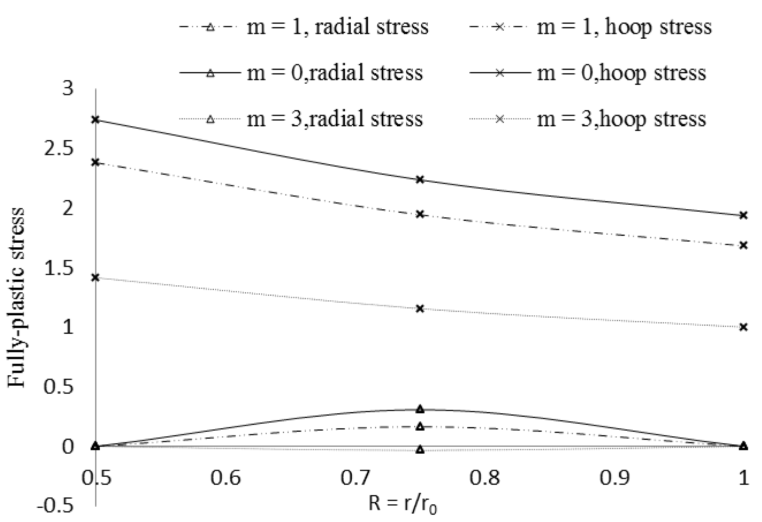

Figure 3. Stress distribution at the fully-plastic state along the radii ratio $R = r/r_0$.

Table 1. Angular speed required for initial yielding and fully plastic state.

	Density parameter m	Poisson ratios for different materials v	Angular speed required for initial yielding Ω_i^2	Angular speed required for fully-plastic state Ω_f^2	Percentage increase in angular speed $(\sqrt{(\Omega_f^2/\Omega_i^2)} - 1)\times 100$
$0.5 \le R \le 1.0$	0	0.5	1.420160785	3.88627	65.4237%
	1		1.1044695	2.626831	54.21947%
	3		0.59758	0.845111	18.92108%
	0	0.42857	1.369319618	3.88627	68.4667%
	1		1.065026369	2.626831	57.04926%
	3		0.57619	0.845111	21.10833%
	0	0.33333	1.335134413	3.88627	70.60981%
	1		1.303837877	2.626831	41.93983%
	3		0.56181	0.845111	22.64847%

It can also be seen from Table 1, that for the disk made of incompressible material (i.e. v = 0.422857 and 0.333), a required higher percentage for values in angular speed to become fully plastic in comparison to the disk made of compressible material (i.e. v = 0.333). With increased values of the density parameter, the ratio of angular velocity for fully plastic state with respect to the initial plastic is increased to large values.

REFERENCES

1. Sokolnikoff, I.S., Mathematical Theory of Elasticity, 2nd Ed., McGraw-Hill Inc., New York, 1953.
2. Swainger, K.H. (1956), *Analysis of deformation*, Chapman & Hall, London; Macmillan, U.S.A., Vol. III, Fluidity, 3.1: 67-68.
3. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi:10.1038/195896a0.
4. Seth, B.R. (1966), *Measure-concept in mechanics*, Int. J Non-linear Mech., 1(1): 35-40. doi: 10.1016/0020-7462(66)90016-3
5. Timoshenko, S.P., Goodier, J.N., Theory of Elasticity. McGraw-Hill, New York, 1970.
6. Johnson, W., Mellor, P.B., Engineering Plasticity, London: Van Nostrand Reinhold Co., 1973.
7. Chakrabarty, J., Theory of Plasticity, McGraw-Hill, New York, 1987. doi: 10.1002/crat.2170240705
8. Reddy, T.Y., Srinath, H. (1974), *Elastic stresses in a rotating anisotropic annular disk of variable thickness and variable density*, Int. J Mech. Sci. 16(2): 85-89. doi.org/10.1016/0020-7403(74)90078-2
9. You, L.H., Tang, Y.Y., Zhang, J.J., Zheng, C.Y. (2000), *Numerical analysis of elastic-plastic rotating disks with arbitrary variable thickness and density*, Int. J Solids and Struc. 37(52): 7809-7820. doi: 10.1016/S0020-7683(99)00308-X
10. Alexandrova, N., Alexandrov, S. (2004), *Elastic-plastic stress distribution in a rotating annular disk*, Mech. Based Design Struct. Machines, An Int. J, 32(1):1-15. doi.org/10.1081/SME-120026587
11. Gupta, S.K., Thakur, P. (2007), *Thermo elastic-plastic transition in a thin rotating disc with inclusion*, Thermal Sci. 11(1): 103-118. doi: 10.2298/TSCI0701103G
12. Arefi, M., Rahimi, G.H. (2013), *Linear thermoelastic analysis of a functionally graded (FG) rotating disk with different boundary conditions using Adomian's decomposition method*, Scient. Res. Essays, 8(21): 858-866. doi: 10.5897/SRE10.341
13. Thakur, P., Kaur, J., Singh, S.B. (2016), *Thermal creep transition stresses and strain rates in a circular disc with shaft having variable density*, Eng. Comput. 33(3): 698-712. doi: 10.1108/EC-05-2015-0110
14. Thakur, P., Gupta, N., Singh, S.B. (2017), *Creep strain rates analysis in cylinder under temperature gradient materials by using Seth's theory*, Eng. Comput. 34(3): 1020-1030. doi: 10.1108/EC-05-2016-0159
15. Thakur, P., Pathania, D., Verma, G., Singh, S.B. (2017), *Elastic-plastic stress analysis in a spherical shell under internal pressure and steady state temperature*, Struc. Integ. and Life, 17(1): 39-43.
16. Thakur, P., Shahi, S., Gupta, N., Singh, S.B. (2017), *Effect of mechanical load and thickness profile on creep in a rotating disc by using Seth's transition theory*, AIP Conf. Proc., Amer. Inst. of Physics, USA, 1859(1): 020024. doi: 10.1063/1.4990177
17. Thakur, P. (2017), *Creep stresses in a circular cylinder subjected to torsion*, Struc. Integ. and Life, 17(3): 183-186.
18. Thakur, P., Mahajan, P., Kumar, S. (2018), *Creep stresses and strain rates for a transversely isotropic disc having the variable thickness under internal pressure*, Struc. Integ. and Life, 18(1): 15-21.
19. Thakur, P., Sethi M. (2018), *Creep damage modelling in a transversely isotropic rotating disc with load and density parameter*, Struc. Integ. and Life, 18(3): 207-214.
20. Thakur, P., Sethi M. (2020), *Elastoplastic deformation in an orthotropic spherical shell subjected to temperature gradient*, Math. Mech. Solids, 25(1): 26-34. doi: 10.1177/1081286519857128
21. Thakur, P., Sethi, M. (2020), *Creep deformation and stress analysis in a transversely material disc subjected to rigid shaft*, Mat. Mech. Solids, 25(1): 17-25. doi: 10.1177/1081286519857109.