

## A COMPARATIVE STUDY OF SLIDER BEARING PERFORMANCE FOR DISSIMILAR SHAPES OF LUBRICANT FILM WITH A TRANSVERSE ROUGH SURFACE

### KOMPARATIVNA STUDIJA FUNKCIJE KLIZNOG LEŽAJA KOD RAZNIH OBLIKA PODMAZUJUĆEG FILMA SA POPREČNOM GRUBOM POVRŠINOM

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#### Keywords

- Reynolds' type equation
- plane slider bearing
- transverse roughness
- load-tolerating capability

#### Abstract

*In the article at hand, a comparative study of the performance of a slider bearing for dissimilar shapes of lubricant film has been carried out by considering that the roughness of the bearing surface is transverse which is used to examine the behaviour of the load-tolerating capability in various shaped slider bearings, viz. a finite plane, exponential, secant and hyperbolic shaped slider bearing. The examined results for the load-tolerating capability are presented graphically, achieved by solving the underlying averaged Reynolds' type equation numerically by assuming suitable boundary conditions. Indeed, it is made known that the underlying bearing's load-tolerating capability can be enhanced significantly by choosing appropriate lubricant film shape.*

#### INTRODUCTION

In almost all types of bearing systems very important role is played by the roughness of the bearing surface and its roughness pattern. It's observed by some of the researchers (viz. Michell /1/, Pinkus/2/, Burton /3/, Davies /4/, Tzeng-Seibel /5/) that the bearing's performance and its life period depend on the surface roughness, friction, wear, and lubricants.

After that, many researchers viz. Andharia et al. /6-8/, Naduvanamani et al. /9/, Panchal and Patel et al. /10-13/, Sharma et al. /18-21/, Sahni et al. /22-23/ have studied and established that bearing's performance is dependent on its roughness and they have also given some essential results in terms of the bearing's load-tolerating capability and various parameters like mean, standard deviation, and skewness of the underlying rough surface with and without using a magnetic-lubricant. In these investigations, the role of Christensen and Tender /14-17/ could not be underestimated, who have produced stochastically 'Averaged Reynolds' type equation (A.R.T.E.) by applying random averaging process on 'Reynolds' type equation' which gives the mean pressure more willingly than a local-pressure.

#### Ključne reči

- jednačina Reynoldsovog tipa
- ravni klizni ležajevi
- poprečna hrapavost
- sposobnost podnošenja opterećenja

#### Izvod

*U radu je data komparativna studija funkcionalnosti kliznog ležaja kod različitih oblika podmazujućeg filma, razmatranjem poprečne hrapavosti površine ležaja, koja se upotrebljava za analizu ponašanja sposobnosti podnošenja opterećenja kod kliznih ležajeva različitih oblika, na primer, konačna ravan, ili klizni ležaj oblika eksponencijalnog, hiperboličkog, ili oblika odsečka. Rezultati za sposobnost podnošenja opterećenja su predstavljeni grafički, i dobijeni su numeričkim rešavanjem odgovarajuće usrednjene jednačine Reynoldsovog tipa, pod pretpostavkom odgovarajućih graničnih uslova. Zaista, odgovarajuća sposobnost podnošenja opterećenja datog ležaja se može znatno poboljšati izborom odgovarajućeg oblika podmazujućeg filma.*

In this article, a stochastic A.R.T.E. for dissimilar lubricant film shapes and the transverse rough bearing surface are solved numerically, and graphical results are produced accordingly. Indeed, it is revealed at this moment that the load-tolerating capability (L.T.C.) of a particular bearing can be enhanced significantly by applying proper lubricant film shape.

#### MATHEMATICAL ANALYSIS

The A.R.T.E. governing the average pressure in a rough slider bearing, as per Patir /24/ is

$$\frac{\partial}{\partial x} \left[ \psi_x \frac{h^3}{12\eta} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \psi_y \frac{h^3}{12\eta} \frac{\partial \bar{p}}{\partial y} \right] = \frac{V}{2} \frac{\partial \bar{h}}{\partial x} + \frac{V\sigma}{2} \frac{\partial \psi_s}{\partial x}. \quad (1)$$

Here the bearing's surface roughness is thought as transverse ( $\lambda < 1$ ), and so the deviation in roughness heights in the direction of motion of the slider is small compared to the perpendicular direction, and therefore, the shearing flow factor  $\psi_s$  may cause a significant effect to generate pressure. Hence the influence of  $\psi_s$  may be regarded as extensive in comparison to the case of the longitudinally rough surface. The measure of transverseness or longitudinalness is quantified by  $\lambda$ , the parameter of roughness pattern (P.R.P.). In

the case of  $\lambda > 1$ , the roughness is known as longitudinal, while  $\lambda < 1$  shows that the roughness is transverse.

Assuming that the slider moves in the  $x$  direction only, Eq.(1) implies,

$$\frac{d}{dx} \left[ \psi_x \frac{h^3}{12\eta} \frac{d\bar{p}}{dx} \right] = \frac{V}{2} \frac{d\bar{h}}{dx} + \frac{V\sigma}{2} \frac{d\psi_s}{dx} \quad (2)$$

The geometry of a plane shape slider (P.S.B.) bearing is given in Fig. 1a, and the fairly accurate geometry of the exponential, hyperbolic, and secant shape slider bearing is provided in Fig. 1b. The lubricant-film thickness ( $h_x$ ) in these bearings at point  $x$  is given by following equations.

For plane shape slider bearing,

$$h_x = h_0 + (h_1 - h_0) \left( 1 - \frac{x}{l} \right).$$

For exponential shape slider bearing,

$$h_x = h_1 \exp \left( -\frac{x}{l} \ln \frac{h_1}{h_0} \right).$$

For hyperbolic shape slider bearing,

$$h_x = h_1 \left[ 1 + \frac{x}{l} \left( \frac{h_1}{h_0} - 1 \right) \right]^{-1}$$

For secant shape slider bearing,

$$h_x = h_0 \sec \left[ \left( 1 - \frac{x}{l} \right) \sec^{-1} \frac{h_1}{h_0} \right]$$

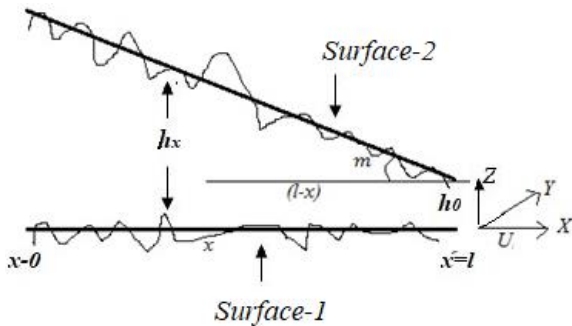


Figure 1a. Bearing geometry-plane shape.

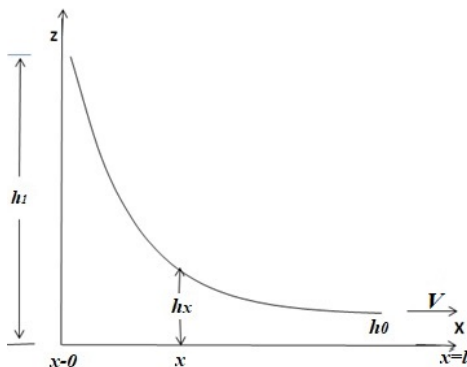


Figure 1b. Bearing geometry-various shapes.

The mean, variance, and skewness of surface roughness have been explained by Deheri et al. /25-26/ and are written in the form of expectancy operator as mean  $\alpha = E(\rho)$ , variance  $\sigma^2 = E[(\rho - \alpha)^2]$ , and skewness  $\varepsilon = E[(\rho - \alpha)^3]$ , respectively, where,  $f(\rho)$ ,  $-c < \rho < c$  is the p.d.f., and where  $E(\Omega)$  represents the expected value of  $\Omega$  and is defined as

$$E(\Omega) = \int_{-c}^c \Omega f(\rho) d\rho.$$

For the case of transverse rough surface ( $\lambda < 1$ ), Eq.(2) is reduced to

$$\frac{d}{dx} \left[ \psi_x \frac{g_h}{12\eta} \frac{d\bar{p}}{dx} \right] = \frac{V}{2} \frac{d\bar{h}}{dx} + \frac{V\sigma}{2} \frac{d\psi_s}{dx}, \quad (3)$$

where:  $g_h = h^3 + 3h^2(\alpha) + 3h(\sigma^2 + \alpha^2) + (\varepsilon + 3\sigma^2\alpha + \alpha^3)$ .

Equation (3) then in dimensionless form is:

$$\frac{d}{dX} \left[ \psi_x G_h^* \frac{dP^*}{dX} \right] = 6 \frac{d}{dX} \left[ h^* + \bar{\sigma} \psi_s \right], \quad (4)$$

where:  $G_h^* = (h^*)^3 + 3\bar{\alpha}(h^*)^2 + 3(\bar{\alpha}^2 + \bar{\sigma}^2)(h^*) + (3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon})$ .

The experimental relation for  $\psi_x$  - Pressure Flow Factor (PFF) and  $\psi_s$  - Shearing Flow Factor (SFF) for transverse ( $\lambda < 1$ ) roughness are established by Patir /24/ as follows,

$$\psi_x = 1 - Ce^{-rH} = 1 - Ce^{-rh^*H_0},$$

$$\psi_s = A_1(h^*H_0)^{\alpha_1} e^{-\alpha_2(h^*H_0) + \alpha_3(h^*H_0)^3},$$

where:  $H = \frac{h}{\sigma}$ ,  $H_0 = \frac{h_0}{\sigma}$ ,  $h^* = \frac{h}{h_0}$ ; and the constants  $r$ ,  $C$ ,

$A_1$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are expressed as a function of  $\lambda$  (Table 1 and Table 2, /24/) in case of the transverse-rough surface.

Table 1. The relation between  $C$ ,  $\lambda$ ,  $r$ , and  $H$ .

| $\lambda$ | $C$  | $r$  | $H$   |
|-----------|------|------|-------|
| 1/3       | 1.16 | 0.42 | > 0.5 |
| 1/6       | 1.38 | 0.42 | > 0.5 |
| 1/9       | 1.48 | 0.42 | > 0.5 |

Table 2. The relation between  $A_1$ ,  $\alpha_i$  ( $i = 1, 2, 3$ ),  $\lambda$ , and  $H$ .

| $\lambda$ | $A_1$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $H$   |
|-----------|-------|------------|------------|------------|-------|
| 1/3       | 1.858 | 1.01       | 0.76       | 0.03       | > 0.5 |
| 1/6       | 1.962 | 1.08       | 0.77       | 0.03       | > 0.5 |
| 1/9       | 2.046 | 1.12       | 0.78       | 0.03       | > 0.5 |

Assuming the B.C.:  $P^* = 0$  at  $X = 0, 1$ ; and  $dP^*/dX = 0$ , where the mean gap between two surfaces is maximum, Eq.(4) can be turned out to be

$$P^*(X) = \int_0^X \frac{1}{\psi_x G_h^*} \left[ 6(h^* + \bar{\sigma} \psi_s) - K^* \right] dX, \quad (5)$$

$$\text{where: } K^* = \frac{\int_0^1 \frac{1}{\psi_x G_h^*} \left[ 6(h^* + \bar{\sigma} \psi_s) \right] dX}{\int_0^1 \frac{1}{\psi_x G_h^*} dX}.$$

The non-dimensional L.T.C. per unit width for transverse rough surface ( $\lambda < 1$ ) is

$$W^* = \frac{wh_0^2}{\eta V l^2} = \int_0^1 P^* dX,$$

$$\Rightarrow W^* = \int_0^1 \int_0^X \frac{1}{\psi_x G_h^*} \left[ 6(h^* + \bar{\sigma} \psi_s) - K^* \right] dX dX. \quad (6)$$

## RESULTS AND DISCUSSION

Figures 2-5 (for transverse rough surface,  $\lambda < 1$ ) show the variation in the dimensionless L.T.C. in accordance with the mean  $\bar{\alpha}$  and S.D.  $\bar{\sigma}$  retaining the skewness  $\bar{\varepsilon} = -0.025$ , predetermined in the underlying bearings.

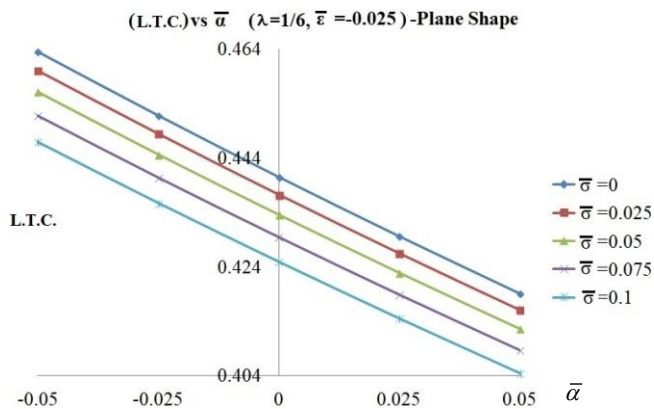


Figure 2. L.T.C. versus  $\bar{\alpha}$  and  $\bar{\sigma}$  - plane slider bearing.

From the achieved comparatives, it is exposed that the bearings (viz. exponential, hyperbolic, and secant shape bearings) produce superior L.T.C. in comparison to P.S.B. Also, it is clear from the figures that the L.T.C. diminishes according to the rising values of  $\bar{\sigma}$  in the case of transverse roughness, which is different from the case of rough longitudinal surfaces, /12/.

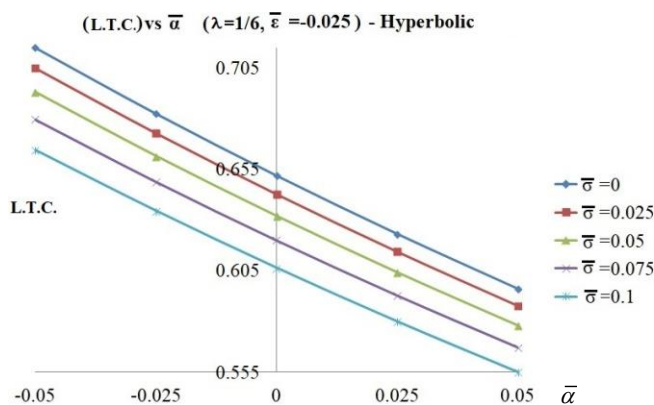


Figure 3. L.T.C. versus  $\bar{\alpha}$  and  $\bar{\sigma}$  - hyperbolic slider bearing.

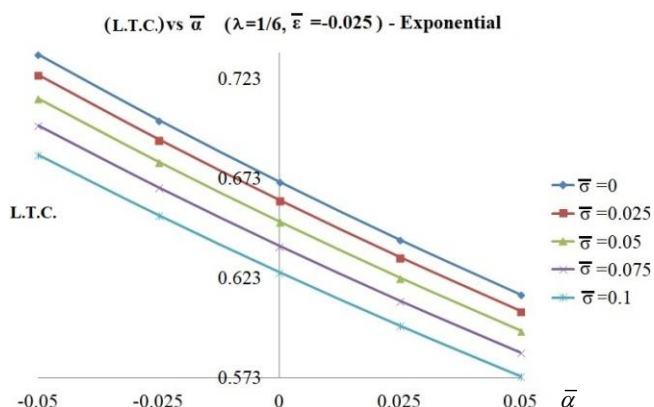


Figure 4. L.T.C. versus  $\bar{\alpha}$  and  $\bar{\sigma}$  - exponential slider bearing.

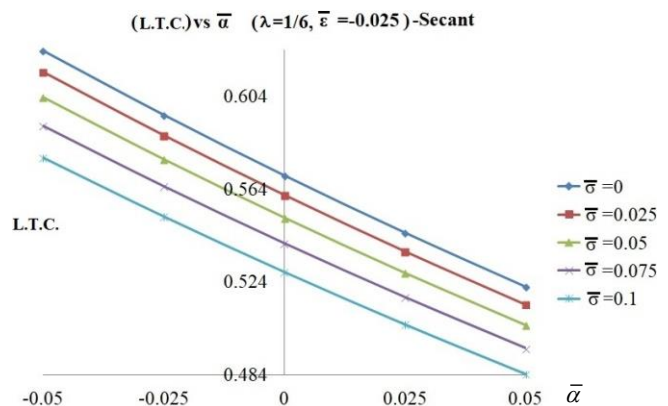


Figure 5. L.T.C. versus  $\bar{\alpha}$  and  $\bar{\sigma}$  - secant slider bearing.

The observations of Panchal and Patel et al. /12/ show that there is an improvement in the L.T.C. by escalating the value of  $\bar{\varepsilon}$  (-ve) and  $\bar{\sigma}$  by assuming the dimensionless  $\bar{\alpha} = -0.05$  (fixed) in the longitudinal rough- bearings of different shapes. Whereas Figs. 6-9 disclose that the trend of L.T.C. for the transverse-rough surface is wholly contradictory to the case of the bearings with longitudinal-rough surfaces w.r.t.  $\bar{\sigma}$ . Also, it is observed that the L.T.C. can be lifted by enlarging the value of  $\bar{\varepsilon}$  (-ve). Hence we can say from this observation that the roughness parameters play a vital job in getting a better lifetime and act of the underlying bearing. Also, the figures show that the lubricant-film of exponential, hyperbolic, and secant shapes gives better performance than plane shape subject to get better L.T.C.

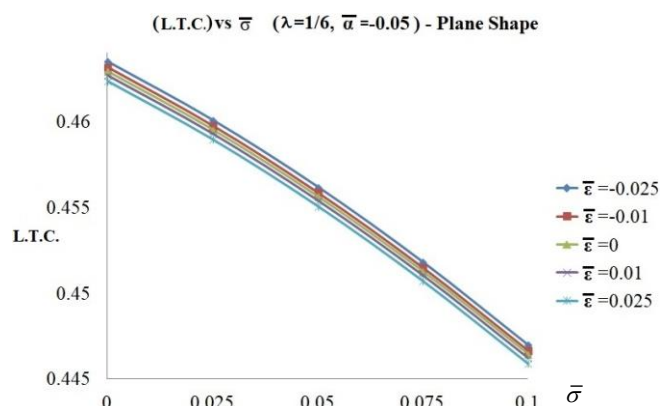


Figure 6. L.T.C. versus  $\bar{\sigma}$  and  $\bar{\varepsilon}$  - plane slider bearing.

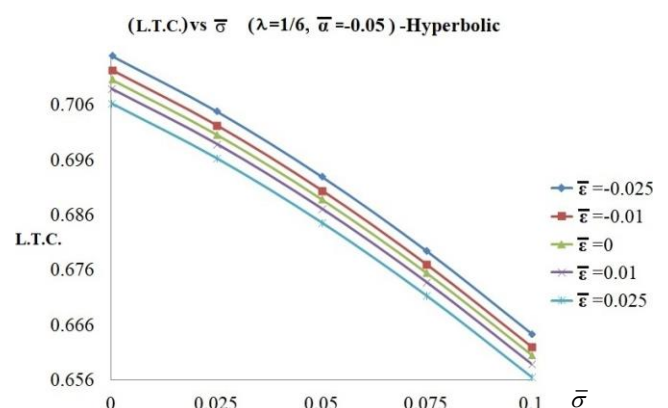
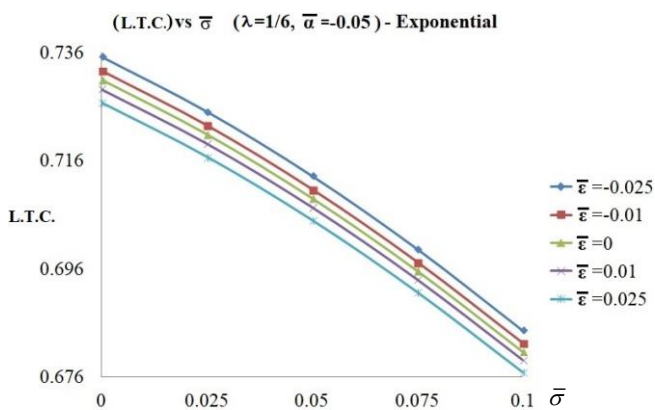
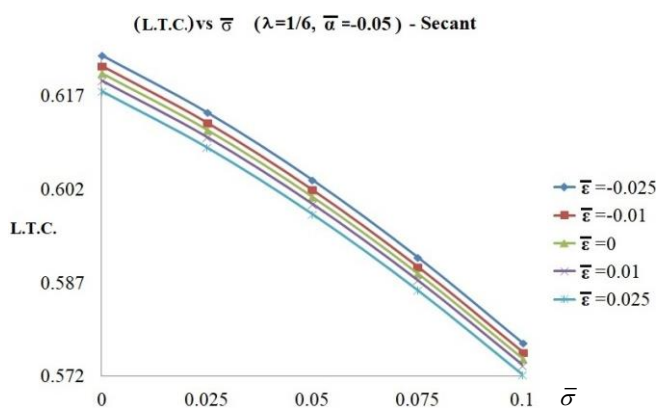
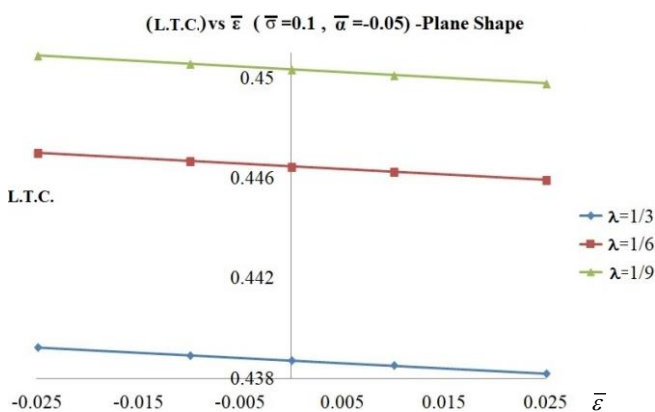
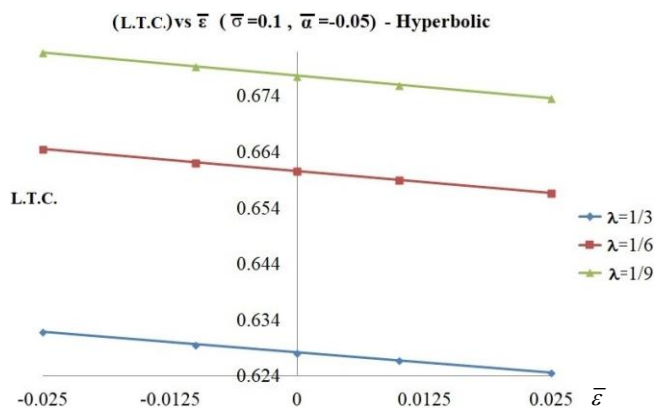
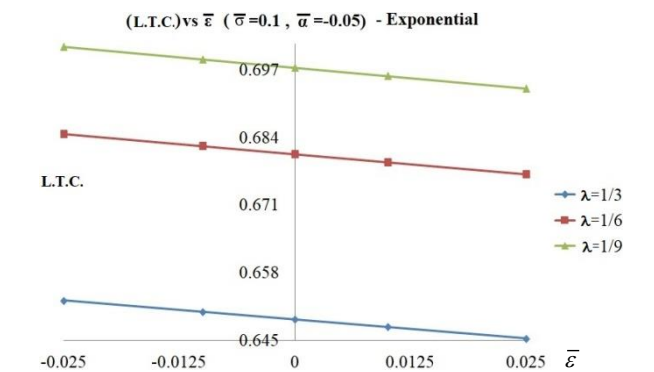
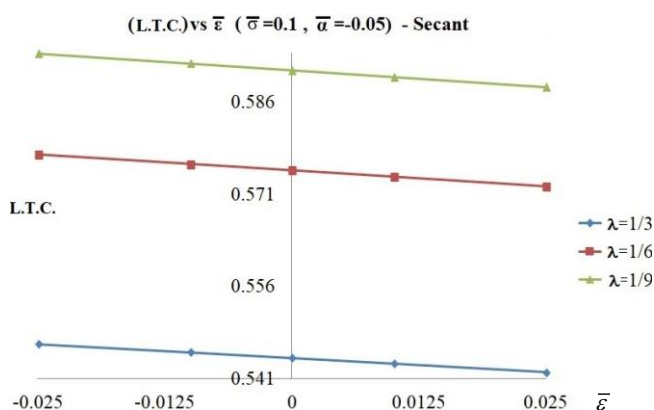


Figure 7. L.T.C. versus  $\bar{\sigma}$  and  $\bar{\varepsilon}$  - hyperbolic slider bearing.

Figure 8. L.T.C. versus  $\bar{\sigma}$  and  $\bar{\epsilon}$  - exponential slider bearing.Figure 9. L.T.C. versus  $\bar{\sigma}$  and  $\bar{\epsilon}$  - secant slider bearing.

Figures 10-13 represent the performance of L.T.C. according to the skewness  $\bar{\epsilon}$  and P.R.P.  $\lambda < 1$ ) by fixing S.D.  $\bar{\sigma} = 0.1$  and mean  $\bar{\alpha} = -0.05$  for the transverse rough slider bearings. They show that the L.T.C. enormously depends on P.R.P. ( $\lambda$ ), and it can be enriched by minimizing the value of P.R.P. ( $\lambda$ ).

Figure 10. L.T.C. versus  $\lambda$  and  $\bar{\epsilon}$  - plane slider bearing.Figure 11. L.T.C. versus  $\lambda$  and  $\bar{\epsilon}$  - hyperbolic slider bearing.Figure 12. L.T.C. versus  $\lambda$  and  $\bar{\epsilon}$  - exponential slider bearing.Figure 13. L.T.C. versus  $\lambda$  and  $\bar{\epsilon}$  - secant slider bearing.

## CONCLUSION

The present analysis is made to compare various shapes, viz. plane, hyperbolic, secant, and exponential shape- of slider bearings and has disclosed that the mean (-ve) and skewness (-ve) may produce an excellent outcome of the underlying bearing irrespective of the P.R.P ( $\lambda$ ) and vice versa. The exponential and secant shape slider-bearings provide enhanced results (in the sense of getting better L.T.C.) than the P.S.B. and the hyperbolic slider bearing. Also, it is noticed that the performance and the lifetime of the given bearing can be enhanced by the appropriate selection of the lubricant-film shape with a particular range of necessary parameters like the mean, standard deviation, skewness, and P.R.P.

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