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CRACK DETECTION IN MOVING LOAD DYNAMICS PROBLEM BASED ON STATISTICAL PROCESS CONTROL APPROACH

OTKRIVANJE PRSLINE NA PROBLEMU DINAMIKE POKRETNOG OPTEREĆENJA PRISTUPOM STATISTIČKE KONTROLE PROCESA

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Keywords
- crack
- control chart
- AR (auto regressive) method
- SPC (statistical process control)
- crack location

Abstract

The present work is based on the development of a fault detection method for moving load dynamics problem in the domain of statistical process control (SPC) analysis. The proposed work is aimed to find out the probable existence of cracks and its locations on structure subjected to moving load. In the present analysis, a multi-cracked cantilever beam subjected to a moving load has been considered. The feasible existences and positions of cracks on the structure are spotted from the calculated dynamic responses of the cracked beam. The dynamic responses of the structure are determined using Runge-Kutta approach followed by finite element analysis (FEA) verifications as forward problem analysis. The control chart mechanism has been initially included to find out the potential existences of cracks in the pattern of SPC analogy. The novel auto regressive (AR) technique in the domain of statistical pattern recognition approach is implemented to locate the feasible positions of cracks in the structure as reverse problem. The SPC technique, the reverse problem, analysis has been conducted in a supervised approach. The results obtained from the numerical and FEA methods congregate well those with SPC approach.

Ključne reči
- prslina
- kontrolna karta
- AR (autoregresioni) model
- SPC (statistička kontrola procesa)
- lokacija prsline

Izvod

Rad se zasniva na razvoju metode za otkrivanje grešaka na problemu dinamike pokretnog opterećenja u domenu analize statističkom kontrolom procesa (SPC). Predstavljeni rad ima za cilj iznalaženje verovatnoće postojanja prslina i njihovih lokacija u konstrukciji sa pokretnim opterećenjem. Prema predloženoj analizi, na uklještenu gredu se razmatra pokretno opterećenje. Moguće postojanje i položaj prslina u konstrukciji se nalaze iz proračuna dinamičkog odziva grede sa prslinom. Dinamički odziv konstrukcije se određuje primenom metode Runge-Kuta, uz verifikaciju analizom konačnim elementima (FEA) kao analiza direktnog problema. Prvo je primenjena metoda kontrolnih karata za iznalaženje potencijalnih prslina u analogiji SPC. Novi autoregresioni model (AR), u domenu pristupa statističkog prepoznavanja obrazaca, je implementiran za otkrivanje mogućih lokacija prslina u konstrukciji kao reverzno modeliranje problema. Analiza metodom SPC, reverznim modeliranjem problema, je izvedena supervizornim algoritmom. Dobijeni rezultati numeričkom i FEA metodom se u većoj meri poklapaju sa rezultatima dobijenim metodom SPC.

INTRODUCTION

The presence of a crack in the structure diminishes the local stiffness and turns into the root cause for material discontinuities. In the present scenario, the early detection of cracks in structures is an active research work for condition monitoring of structures. The potential feature of early crack detection techniques are the knowledge based methods that can be used for decision-making analysis for structural health monitoring related issues.

Dyniewicz /1/ has used the velocity formulation of space-time FEM approach for the transit mass problems. He has considered the Euler and Timoshenko beams, and string systems and determined the elemental and moving mass matrices for these structures. The proposed method is functional for the observation of response of structures in the era of transportation, manufacturing and robotics engineering. Esen /2/ has applied the FEM approach to explore the consequences of axial and transverse vibration of plate structures subjected to transit load with erratic speed. The proposed method was implemented to a cantilever plate to observe the response of a planner entry plate wood cutting machine under a traversing load with high speed. Aied and Gonzalez /3/ investigated the mechanism of inconsistency of rate of strain of a simply supported structure under a transit load and discussed the effects of the transit mass on the elasticity of the beam. Zhong et al. /4/ determined the vibratory response of excitation of a multi-span prestressed bridge under the action of vehicle movement by FEM. They have observed that the influence of prestress has considerable impact on the maximum transverse acceleration of the structure. Jena and Parhi /5-7/ have conducted numerical studies followed by FEA and experimental analyses for different types of beams under transit mass to find out the response of a structure and investigated the implication of different parameters on the responses of the beam.

The time series modelling along with Mahalanobis distance-based approaches were applied for detection of damages on structures by Gul and Katbas /8/. Mosavi et al. /9/ implemented the vector autoregressive followed Mahalanobis distances based analogies to identify the location of cracks on the structure as reverse problem. Wang et al. /3/ proposed two steps damage detection technique (statistical moment based method and least square optimization approach) to make out cracks on structures. Using FEM, Nguyen /10/ developed a crack detection methodology on the concepts of mode shape investigation of structure. The properties of the coupling mechanism due to bending vibration (longitudinal and transverse) with the presence of cracks on the mode shapes of the structure are analysed. Kopsaftopoulos and Fassois /11/ investigated an exact method to spot the probable existences and crack positions, and also quantify the crack severities on the structure in perspective of statistical time series analysis. Yu and Zhu /12/ altogether applied the time series and higher statistical moments based approaches for structural damage detection. Reiff et al /13/ have investigated a method based on statistical analysis for damage detection of bridge structure under moving load by taking into account the girder allocation factors. The novelty of their work was to be attentive and observe the condition of structures. In later, Jena and Parhi /14-15/ have also developed crack detection methods for moving load dynamics problem using knowledge-base Recurrent Neural Networks.

So far, the literatures are considered, the concepts of control chart analysis for existence of cracks on structure is scanty. However, the present work introduces the features of control chart analysis along with AR approach to know the existence and positions of cracks, respectively. The AR process has been analysed in the domain of regression analysis to localize the cracks. The present mechanism of the problem is performed with commercial IBM SPSS 2017 software. The entire crack finding procedure has been executed in a supervised manner as a reverse problem.

PROBLEM FORMULATION

In the present problem formulation, the theoretical analysis along with FEA are considered as forward problem, whereas the SPC technique has been considered as the reverse problem. The present problem considers a cantilever structure with double cracks under a transit mass (Fig. 1). The cracks are located at a distance of ‘L_1’ and ‘L_2’ from the fixed end with depth of cracks ‘d_1’ and ‘d_2’, respectively. The beam is considered to follow the hypothesis of Euler-Bernoulli’s beam mechanism by neglecting the effect of damping.

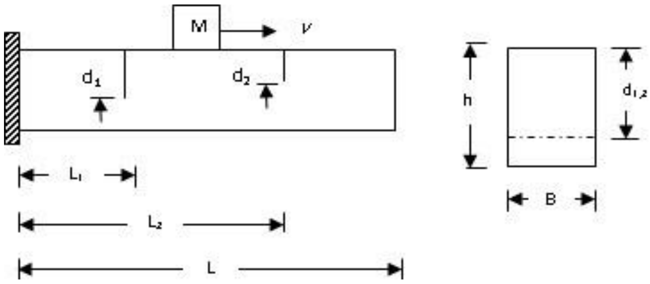

Figure 1. Scheme of cracked cantilever beam under transit mass.

The general equation of motion of moving load dynamics problem is given by

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=f(t)\delta(x-\beta)\,, \quad (1)$$

where: *f(t)*- force due to moving load; δ- Dirac delta function; ρA is beam mass per unit length; *EI* - beam flexural rigidity. The generalized solution for Eq.(1) can be written as in the form of series, i.e.

$$y(x,t)=\sum_{n=1}^{\infty}F_n(x)q_n(t)\,, \quad (2)$$

where: $F_n(x)$ is the amplitude or shape function of the beam; $q_n(t)$ is the function of time; $y(x,t)$ is deflection or response of the structure. The solution of Eq.(1) has already been solved by Jena and Parhi /7/ in their earlier works using Runge-Kutta approach. The Runge-Kutta method is also authenticated by Jena et al. /6/ by using FEA approach. The beam deflections or responses of the structure under traversing load are determined by FEA procedures using ANSYS WORKBENCH 2015®. The equation of motion in FEA domain for moving load dynamic problem is conferred as:

$$M[\ddot{y}]+C[\dot{y}]+K[y]=f(t)\,, \quad (3)$$

where: f(t) - applied force; $K[y]$ - stiffness force; $C[\dot{y}]$ -damping force; $M[\ddot{y}]$ - inertial force; and y, $\dot{y}$ and $\ddot{y}$ are the deflection of the beam structure, speed and acceleration of the traversed mass, respectively. The present investigation ignores the inclusion of damping effects for FEA. Detailed procedures of the numerical analysis and FEA procedures are explained by the authors /5, 7/ in their previous works for the determination of the responses of different types of beams under traversed load. The traversing mass and cantilever beam dynamic interaction and the enhanced view of crack are shown in Figs. 2 and 3, respectively. For the FEA, numerical examples are formulated for the cracked cantilever beam under traversing mass.

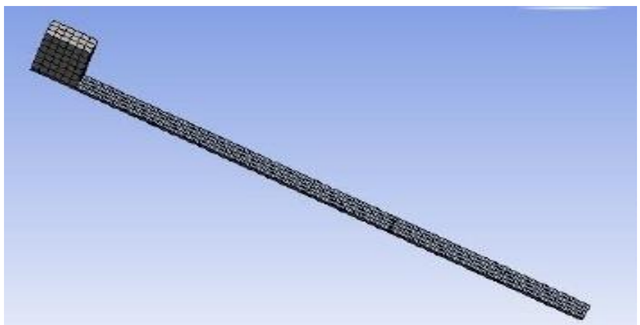
Figure 2. Traversed mass-structure interaction of cracked cantilever beam.

Figure 3. Enhanced view of crack for $\alpha = 0.45$.

NUMERICAL FORMULATION

Beam (mild steel) of size = 150 × 4 × 0.4 cm. Traversing mass M = 1 kg and 1.6 kg. Traversing speed = 5.8 m/s, and 7.1 m/s. Relative crack depth $\alpha_{1,2}$ = 0.45, 0.45 and 0.3, 0.25. Relative crack depth (first and second, respectively) = $\alpha_{1,2} = d_{1,2}/h$. $L_{1,2}$ = crack positions (first and second respectively) = 60, 90 cm, and 80, 115 cm.

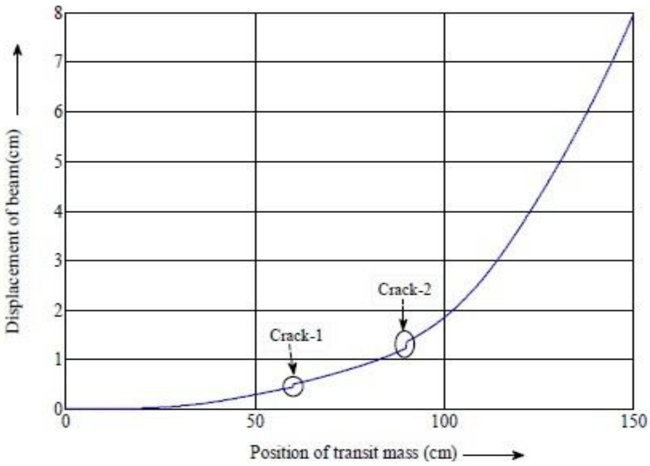

Figure 4. Beam deflections vs. position of transit mass with $\alpha_{1,2}$ = 0.45, 0.45, v = 5.8 m/s, M = 1 kg.

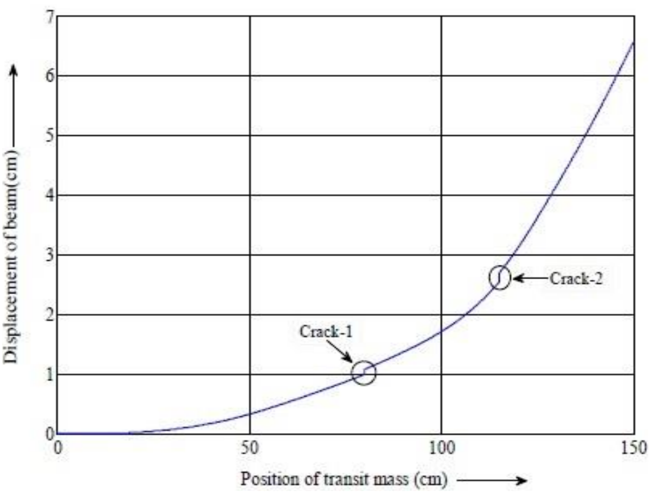

Figure 5. Beam deflections vs. position of transit mass with $\alpha_{1,2}$ = 0.3, 0.25, v = 7.5 m/s, M = 1.6 kg.

From numerical and FEA solutions, the existences and crack positions are recognized by the observed dynamic deflections of the structures. From Figs. 4 and 5, it was remarked that there happened a rapid rise in the deflection of the beam. The abrupt rise in amplitudes or deflections of beam exhibits possible existences of cracks on the structures and from the existence of cracks, the positions of cracks are estimated.

STATISTICAL PROCESS CONTROL (SPC) APPROACH FOR CRACK DETECTION ON STRUCTURE UNDER MOVING LOAD

The analysis of the AR process is presented using the SPSS 2017 software. The concept of control chart ($\bar{x}$) analysis is included in this analysis to predict the existences of cracks on the structure. The control chart executes resembling a significant tool to improvise the different process parameters. The control chart is a graphical depiction of data, which consists of three lines: centerline (CL), upper control line (LCL), and lower control line (LCL). The CL of the chart reveals the average predictable value of the consequent state and, if any chart value falls outside the control limit, then that state is assumed to be out of control.

In the present problem, 500 numbers of samples (pattern) are produced, out of which 350 numbers are from undamaged state while 150 are from damaged state. The time-deflection responses histories of structure under moving load are determined for each of the pattern with 200 numbers of observation intervals. The average of the means and standard deviation of all the patterns are determined. By considering the undamaged state, the CL, UCL and LCL are estimated. For the preparation of control chart for $\bar{x}$ bar chart, the selection of group size is a key matter. If the group size exceeds unity, in that case the mean of the data in that particular group is preferred as the chart variable and be sure that point should be in the subsequent chart. From the statistical analogy by Montgomery /16/, the following relations prepare the control limits:

$$\text{LCL} = \mu_s - \eta_{a/2}\frac{\sigma_s}{\sqrt{n}}, \quad (4)$$

$$\text{UCL} = \mu_s + \eta_{a/2}\frac{\sigma_s}{\sqrt{n}}. \quad (5)$$

where: n - size of the v; s - sample statistics; μ- mean data value; and σ- value of standard deviation. The $\eta_{a/2}$ value is tabulated from the sample sizes, /16/. The control chart analysis for the cracked structure subjected to transit mass is presented in Fig. 6. If any value lies above UCL or below LCL, then it anticipates the prospective subsistence of cracks on the structure. Once the detection of cracks on the structure are over, then the next objectivity is to localize the cracks. The present investigation is based on the concepts of AR model. In this analysis of AR process, the current value of the series is generally expressed as a predetermined, linear cumulative of precedent values of the series with a shock value z_t. The shocks are assumed to be normal with constant variances and zero means. The crack localization on structure under transit mass is investigated using the proposed AR process in the domain of time series analysis.

The present analogy extracts data from the FEA and numerical methods. The data from the time-series histories are compressed from multiple measurement points to single point.

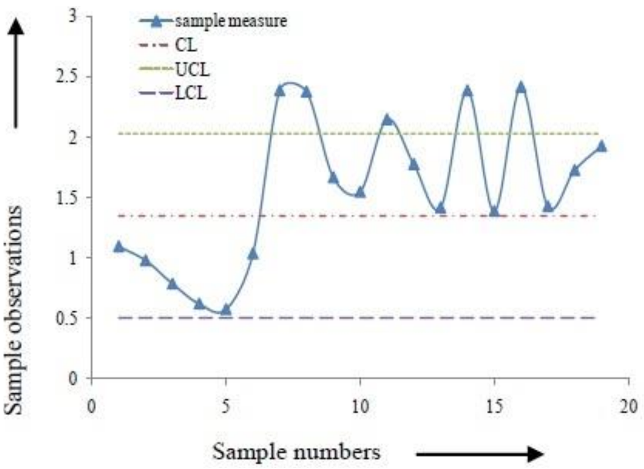

Figure 6. Control chart for cracked structure under transit mass.

Let $z_i(t_j)$ represent the response histories of the structure at r measurement points and samples at n time intervals. The responses (beam deflections vs. time histories) of the structure at a specified time (t_j) are articulated by following relations,

$$z_i(t_j) = \left[z_1(t_j) \quad z_2(t_j) \quad z_3(t_j) \quad \dots \quad z_r(t_j) \right]^T , \qquad (6)$$

where: i varies from 1 to r. 'r' are various locations of the beam where the responses are calculated. The responses data of the structure are not only interrelated to each other but also to each other's preceding data. Then Eq.(6) can be written:

$$z_t = \left[z_{1t} \quad z_{2t} \quad z_{3t} \quad \dots \quad z_{rt} \right]^T . \qquad (7)$$

Afterward, the covariance matrix (ψ) of dimension $r \times r$ amid all the measured positions over the time period can be expressed in the following manner:

$$\psi = \sum_{j=1}^{r} z(t_j) z(t_j)^T . \qquad (8)$$

In the AR (c) model, the present data are represented with the liner combination of the pattern 'c' data.

The AR process with order 'c', has been articulated as:

$$z_t = \sum_{j=1}^{c} \phi_j z_{t-c} + a_t , \qquad (9)$$

where: z_t is the history of responses data at time t; a_t is the arbitrary error (shock value) with constant variance and zero mean. The Eq.(9) may be rewritten as in this form, i.e.

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} + \dots \phi_c z_{t-c} + a_t , \qquad (10)$$

where: $\phi = (\phi_1, \phi_2, \dots \phi_j, \dots \phi_c)$; ϕ_j are coefficients of AR (c) process that represent the sensitive features of damage. By the feature extraction process, the damage sensitive factors are determined from the observed dynamic deflections or responses of the beam. The damage sensitive factors distinguish between cracked and uncracked structure. The values of ϕ_j are estimated by incorporating the AR (c) model to the response data set of the beam with respect to moving time by implementing the Yule-Walker method, /17/. The coefficients of the AR (c) form or sensitive factors of damage are established with the linear least squares regression study in the domain of SPSS 17 software package. The order of the AR (c) model is evaluated by authenticating Gaussianity and arbitrariness of the estimated errors with trial and error analyses. The 'c' matrices can be formulated for each set of the displacement-time responses data that constitutes of ϕ_j ($r \times r$ matrix). Damaged sensitive factors are extracted from the beam deflections-time history response data by comparing the response data between damaged and undamaged structures. The damaged sensitive factors (ϕ_j) are approximated by appropriating the AR (c) process to the deflections-time history data of each of the data pattern,

$$\phi_j = \begin{pmatrix} \phi_{11j} & \phi_{12j} & \phi_{13j} & \cdots & \phi_{1rj} \\ \phi_{21j} & \phi_{22j} & \phi_{23j} & \cdots & \phi_{2rj} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{r1j} & \phi_{r2j} & \phi_{r3j} & \cdots & \phi_{rrj} \end{pmatrix} . \qquad (11)$$

The undamaged data sets are now separated in two factions explicitly reference set data ($Data^R$) and healthy set data ($Data^H$), while those of damaged set data are named as $Data^D$. The $Data^R$ are developed to extort the factors of damage sensitive for all the reference set data for assessments of damaged states. The damage sensitive factors for $Data^H$ and $Data^D$ are also found out individually. The coefficients of AR (c) model or damage sensitive factors of healthy ($Data^H$) and damaged ($Data^D$) are compared with those of reference data set ($Data^R$). Observing the disparity of coefficients of AR (c) or damage sensitive factors that arises in the estimated coefficients of AR (c) model is caused by the possible existences of cracks on structures.

The concept of Fisher's Criterion is introduced to determine the magnitude of variations of damage sensitive factors or coefficients of AR(c) process relating to the healthy set data ($Data^H$) of the structure. The factors of damage sensitive or coefficients of AR (c) process give the deviation of the response history data of the structure. The amounts of deviation acquired in the coefficients of the AR(c) process at special positions of structures provide the information about the probable locations of damages on the structures. The concept of Fisher Criterion is included to determine the actual deviation of coefficients of AR process or factors of damage sensitive for both the damaged and healthy states of the structure. The concept of Fisher's criterion has been explained as follows:

$$\text{Fisher}_{\text{criterion}} = \frac{(\mu_C - \mu_H)^2}{V_C + V_H} , \qquad (12)$$

where: μ_C and μ_H are mean values of factors of damaged sensitive of damaged and healthy data states, correspondingly; V_H and V_C are the variance values of healthy states and damaged sensitive factors of healthy and damaged states, respectively. It has been observed that at the possible location of cracks, the value of the Fisher's criterion is more. The values of the Fisher's principle present the information concerning the sudden amplification of responses of structure and able to detect locations of cracks.

RESULTS AND DISCUSSIONS

In this investigation, an inverse approach in the concepts of statistical process control has been explored for crack detection on structure under transit load in supervised manner. In the initial part of the problem, the responses of the beam structure are calculated both by numerical and FEA based approaches and as of the observed responses of the structure, the existences and positions of cracks are found out.

In the later part, AR process in province of SPC based approach has been applied to predict the possible locations of cracks as a reverse problem in supervised manner. A numerical problem has been formulated to ensure the convergence of the present method. The results obtained from the SPC based approach are represented in Tables 1 and 2. It has been obtained that the average percentage deviation between the FEA and SPC based AR process is nearly about 1.55 %, which seems to be convergent.

Table 1. Results analysis for crack locations (cm) between FEA and AR process for M = 1.6 kg, v = 7.5 m/s, $\alpha_{1,2}$ = 0.3, 0.25.

Numerical		FEA		AR Process	
L_1	L_2	L_1	L_2	L_1	L_2
60	90	59.82	89.75	58.83	88.79
80	115	79.78	114.89	78.89	113.76
Total average deviation percentage between FEA and AR process = 1.5 %					

Table 2. Results analysis for crack locations (cm) between FEA and AR process for M = 1 kg, v = 5.8 m/s. $\alpha_{1,2}$ = 0.45, 0.45.

Numerical		FEA		AR Process	
L_1	L_2	L_1	L_2	L_1	L_2
60	90	59.91	89.76	58.782	88.241
80	115	79.81	114.84	78.818	113.664
Total average deviation percentage between FEA and AR process = 1.6%					

CONCLUSIONS

In the current investigation, an innovative crack detection procedure using the concepts AR process in the province of SPC based methodology has been developed for transit mass dynamics problem. In the preliminary part, the existences and positions of cracks are calculated from the observed responses of the structure. The control chart mechanism has been concerned to predict the possible existences of cracks on the beam. The AR process has been implemented as a reverse problem to predict the likely positions of cracks on the structure. The results achieved from the AR process are evaluated with those of FEA and found to be convergent. It has been proved that the proposed AR process yields well with FEA and can be very constructive for structural health and condition monitoring of structures under moving load.

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