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A PRACTICAL METHOD FOR CALCULATING STRUCTURAL RELIABILITY WITH A MIXTURE OF RANDOM AND FUZZY VARIABLES

PRAKTIČNA METODA ZA PRORAČUN POUZDANOSTI KONSTRUKCIJE SA MEŠAVINOM SLUČAJNIH I FAZI PROMENLJIVIH

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Keywords
- maximum specificity
- insufficient reason
- possibility theory
- fuzzy reliability
- structural reliability

Abstract

The objective of this article is to present a practical method for calculating structural reliability in case of a mixture of random and fuzzy input variables. In order to facilitate a unified framework for handling fuzzy and random variables, the novel formulas are proposed for establishing normal random variables equivalent to the symmetric triangular fuzzy number. From these equivalent random ones, the original problem is converted to the basic structural reliability problem, then the methods of the traditional reliability theory should be applied for calculation. Simultaneously, this article presents two notions in terms of central reliability and marginal reliability, and a technique to define them. Numerical results are compared with that of the existing methods, to demonstrate the accuracy and effectiveness of the proposed method.

Ključne reči
- maksimalna specifičnost
- nedovoljan razlog
- teorija mogućnosti
- fazi pouzdanost
- pouzdanost konstrukcija

Izvod

Cilj rada je da se predstavi praktična metoda za proračun pouzdanosti konstrukcije za slučaj mešavine ulaznih slučajnih i fazi promenljivih. Predložene su nove formule koje obezbeđuju integrisani okvir za obradu fazi i slučajnih promenljivih, gde se ostvaruju normalne slučajne promenljive, ekvivalentne simetričnom trougaonom fazi broju. Iz ovih ekvivalentnih slučajnih promenljivih, originalni problem se prevodi u osnovni problem pouzdanosti konstrukcije, a zatim se za proračun mogu primeniti metode tradicionalne teorije pouzdanosti. Istovremeno, u radu su date dve pretpostavke koje se odnose na centralnu pouzdanost i na marginalnu pouzdanost, kao i postupak za njihovo definisanje. Numerički rezultati su upoređeni sa postojećim metodama, radi demonstracije tačnosti i efikasnosti predložene metode.

INTRODUCTION

In engineering structures, most of the input data, such as load characteristics, material properties, boundary conditions, geometric dimensions, load-carrying capacities, contain non-deterministic quantities, which are described as uncertainty variables. In /1/, uncertainties present in a structural system can be categorized as either aleatory or epistemic. If the input data are random parameters, the limit state function is a random parameter, expressed as follows

$$M = R - S\,, \qquad (1)$$

where: M - limit state function; R - resistance function; S - load effect function.

The structural reliability is defined as /2-6/

$$\mathrm{P_s} = \ \mathrm{Prob}(M > 0)\,. \qquad (2)$$

Due to using the probability theory, which is the most complete theory, assessing structures by the reliability index is prescribed in structural design codes /7, 8/. When the input data in structural systems are epistemic uncertainties, depending on how to describe the uncertainties, the structural reliability can be derived by different approaches, as using either the fuzzy set theory /9-11/ or the fuzzy random theory /12, 13/.

For realistic systems, aleatory and epistemic uncertainties exist simultaneously in the reliability assessment. In this context, the difference for the basic feature of uncertainties will create the challenging problems.

Unique and provably correct solutions to the problems cannot be demonstrated, /14/. When epistemic uncertainties are represented as fuzzy numbers, three classes of approaches commonly are pursued evaluating the structural reliability $\mathrm{P_s}$ which can be recognised as fuzzy stress - random strength interference model, or transformation random variables to fuzzy variables for calculating the fuzzy reliability, and transformation of fuzzy variables to random variables for calculating the basic reliability. These approaches are analysed below in detail.

In /16/, the authors have proposed a formula for fuzzy reliability analysis of mechanical structures when the stress S is modelled as a fuzzy number with a given membership function $\mu_S(x)$ and the strength R is modelled as a random variable with given distribution function $f_\delta(x)$. The authors idealized that fuzzy reliability in the fuzzy stress-random strength interference model is a real value, and used the Zadeh's notion, /15/, to calculate the probability of a fuzzy event. The drawback of this approach is that two functions under the integral in the formula (5) of /16/ have different measurements: the area between the curve $f_\delta(x)$ and the abscissa is unit, but for the fuzzy number $\mu_S(x)$ it is not. Different from /16/, considering that the α-cuts of a fuzzy number are the probability density functions, for instance, the linear distribution /17/, or uniform distributions /18/, the authors /17, 18/ computed the conventional probability at various α-cuts, and the reliability probability is the average at the α-cuts. However, as can be seen different distribution assumptions lead to totally different reliability results, so as to innovate, the authors /19/ define the reliability in the case of a mixture of random and interval variables, at the worst case a combination of interval variables, by solving optimisation problems for reliability subjected to marginal of interval variables. This opinion is extended in the presence of random and fuzzy uncertainties. In /20/, the authors consider that reliability is a fuzzy number, and combine methods: high dimensional model representation (HDMR) method to approximate the limit state function; the method interval variable transformation to reduce a high amount of optimisation problems and estimation of failure probability bounds using fast Fourier transform (FFT). Different from /20/, the authors in /21/ propose three reliability indices with the mixture of random and fuzzy variables. These reliability indices are: the interval reliability index defined by upper bound $\overline{P}_r^{(I)}$ and lower bound $\underline{P}_r^{(I)}$, that represent expectations of the maximum and minimum reliability P_r at the given membership level α; the mean reliability index is the average of upper bound $\overline{P}_r^{(I)}$ and lower bound $\underline{P}_r^{(I)}$; the numerical reliability index is an expectation of reliability. In order to define the interval reliability index and the mean one, needed to solve optimisation problems at each membership level α when the fuzzy numbers are transformed to interval numbers (similar to /20/), the integral is then calculated at α-cuts to determine these reliability indices. In order to define the numerical reliability index, the interval variables are assumed as uniform distribution at the given α-cuts, then the traditional reliability analysis methods could be applied. Similar to the interval reliability index and the mean one, the numerical one is defined according to integral formula at the α-cuts, the details could be seen in /21/. In order to reduce computational cost, the probability density evolution method (PDEM) is applied.

From the analysis above, one realizes that use of the traditional reliability analysis theory is a basis for assessing fuzzy reliability, when the input parameters are the mixture of random and fuzzy variables, is a reasonable approach, because traditional probabilistic methods remain dominant in the field of measurements, /22/, and are well established in decision making problems, /23/. However, determining reliability at the α-cuts by either solving optimisation problems /19-21/ or assuming equivalent probability density functions of interval numbers /17-19/, is very time-consuming, because the traditional reliability analysis has to be carried out for every evaluation. In order to overcome this drawback, based on the innovation for the conservatism of the principles of probability- possibility transformations, the article presents a new method to transform a symmetric triangular fuzzy number into three equivalent normal distributions. Then, a fuzzy reliability problem will be converted to traditional reliability problem and given two definitions and assessments of reliability: central reliability, is the 'belief' value of reliability; and marginal reliability, that represents an estimator of interval reliability [$P_{s\min}$, $P_{s\max}$]. These two assessments, as in /21/, can give an intuitive of reliability, and easily compare with prescribed reliability in structural design codes /7, 8/. Numerical results are compared with those of the existing methods, when the limit state functions are explicit, to demonstrate the accuracy and effectiveness of the proposed method.

TRANSFORMATION PRINCIPLES AND THE INNOVATION APPROACH

Transformation principles and the basic idea of the innovation approach

Probability measure (probability density function) and fuzzy number (possibility distribution function) are not yet two equivalent representations of uncertainty. The transformation from fuzzy measures into probability measures and conversely can be useful in any problem where heterogeneous uncertain and imprecise data must be dealt with, /24/. Formally, the transformation from fuzzy measure to probability measure needs to add information to some probabilistic knowledge, besides, the converse transformation from probability measure to fuzzy measure, some information is lost. The transformation principles are proposed by Dubois et al. /22, 24, 25/, and Klir G. /26/. In /22, 24, 25/, Dubois proposed to use the principle of the insufficient reason for the transformation from a fuzzy measure to a probability measure, and the principle of the maximum specificity for the converse transformation from a probability measure to a fuzzy measure. The principle of insufficient reason aims at finding a probability measure which preserves the uncertainty of choice between outcomes. The principle of maximum specificity aims at finding the most informative possibility function, which has the narrowest in the predefined α-cuts.

Apart from Dubois's transforms, in /26/, Klir proposed the principle of uncertainty invariance for the transformation from possibility to probability and the converse transformation from probability to possibility. The advantage of this principle is information preservation of fuzzy measure and the equivalent probability one. However, see /27/ (pp. 258-259), the results of using the one may conflict with the probability/possibility consistency principle. Besides, the Klir's approach, /26/, should be based on three assumptions, but the Dubois's approach /22, 24, 25/ does not make any assumption. Nevertheless, for an initial fuzzy number, a transform forward to probability measure by the principle of insufficient reason, then, from this probability measure, a transform backward to fuzzy measure by the principle of maximum specificity, the last fuzzy number is different from the initial one. Hence, transformations (from possibility to probability and converse) based on these two principles make non-conservative information, and the calculation result of reliability is hard to prescribe.

From the discussions above, the paper proposes an innovation to determine normal random variables that are equivalent fuzzy variable, based on the total least deviations (deviation of probability measurement between normal random variables and random variables are transformed by the principle of insufficient reason, deviation of possibility measurement between the initial fuzzy variable and the fuzzy variables are transformed from normal random variables by the principle of maximum specificity). This approach is applied for symmetric triangular fuzzy number, which is normally used to represent input data in fuzzy reliability problems. Using normal density function is natural because it looks like the most important distribution in the traditional reliability problem. In the next section, the formulas are established to determine these normal density functions.

Establishing formulas in terms of the innovation idea

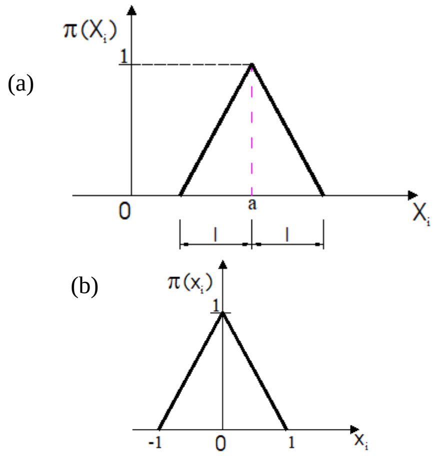

Figure 1. Transformation to a standardized fuzzy variable: a) original fuzzy variable; b) standardized fuzzy variable.

For a symmetric triangular fuzzy number $\tilde{X}_i = (a,l)_{LR}$ (Fig. 1), where: a - belief value (at the membership level $\mu = 1$) of the fuzzy number; l - spread of fuzzy number; the standardized fuzzy variable $\tilde{x}_i = (0,1)_{LR}$ is defined by using transformation, /28/,

$$\tilde{x}_i = \frac{\tilde{X}_i - a}{l} . \tag{3}$$

From the standardized fuzzy number $\tilde{x}_i = (0,1)_{LR}$, the density distribution function is derived by using the principle of the insufficient reason

$$p(x) = \begin{cases} -\frac{1}{2}\ln(-x); & x \in [-1,0) \\ -\frac{1}{2}\ln(x); & x \in (0,1] \end{cases} \tag{4}$$

Consider the normal density distribution function $p_1(x)$, where the expectation $\mu = 0$ (is equal to the belief value of the standardized fuzzy number), and the variance is σ^2,

$$p_1(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)} . \tag{5}$$

For event {A} : $-1 \le x_0 \le x$. Because of the symmetry of the two density distribution probability functions $p(x)$ and $p_1(x)$, we only consider the case $x \le 0$.

Probabilities of event A for density distribution functions $p(x)$ and $p_1(x)$ are

$$P(\text{A}) = \int_{-1}^{x} -\frac{1}{2}\ln(-x)dx = \frac{1}{2}\left[x - x\ln(-x) + 1\right] , \tag{6}$$

$$P_1(\text{A}) = \int_{-1}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)} dx . \tag{7}$$

In order to equal the two probability functions, one needs

$$\left|P(\text{A}) - P_1(\text{A})\right| \to \min ,$$

or

$$\left(P(\text{A}) - P_1(\text{A})\right)^2 \to \min, \quad \forall x \in [-1,0) . \tag{8}$$

From Eq.(6), we get

$$F_1(\sigma) = \int_{-1}^{0} \left(P(\text{A}) - P_1(\text{A})\right)^2 dx \to \min , \tag{9}$$

where: P(A), P_1(A) are determined by Eqs.(4) and (5).

Because the domain of density distribution function $p(x)$ is within [–1,1], while the domain of density distribution function $p_1(x)$ is within $(-\infty,+\infty)$, in order that the probability of density distribution function $p_1(x)$ within $(-\infty,+\infty)$ be insignificant, one needs

$$F_2(\sigma) = P_1\left[\text{A*:} \;\; x_0 \in (-\infty,-1)\right] = \int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)} dx \to \min \tag{10}$$

Combine Eqs.(9) and (10), and we have

$$F(\sigma) = F_1(\sigma) + F_2(\sigma) =$$

$$= \int_{-1}^{0} \left(P(\text{A}) - P_1(\text{A})\right)^2 dx + \int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)} dx \to \min . \tag{11}$$

Equation (11) is used to determine the deviation σ of the normal density distribution function $p_1(x)$ which is equivalent to the density distribution function $p(x)$.

Backward transforms from the normal density distribution $p_1(x)$ to equivalent possibility function (membership function) by principle of maximum specificity, /22, 24, 25/,

$$\pi_1(x) = \pi_1(-x) = \int_{-6\sigma}^{x} p_1(y)dy + \int_{-x}^{6\sigma} p_1(y)dy . \tag{12}$$

Because the normal density distribution function $p_1(x)$ is symmetric, so $f(x) = y = -x$, and 3σ rule is applied (limits $-\infty$ and $+\infty$ are replaced by -6σ and 6σ, respectively).

Membership function of the standardized fuzzy number is

$$\pi(x) = \begin{cases} 1 + x \; ; & x \in [-1,0] \\ 1 - x \; ; & x \in [0,1] \\ 0 \; ; & x \in (-\infty,-1] \;\; \text{and} \;\; x \in [1,\infty) \end{cases} \tag{13}$$

A difference from the probability theory, in the fuzzy theory, fuzzy measures (possibility measure, necessity measure ...) is defined according to membership function (possibility distribution function) by the max/min operator. Hence, with the similar reason presented above, we have

$$G(\sigma)=\int_{-1}^{0}\left(\pi_1(x)-1-x\right)^2 dx+\int_{-\infty}^{-1}\pi_1^2(x)dx\rightarrow\min\,. \quad (14)$$

In order to solve the multiobjective optimisation problem Eq.(11) and Eq.(14), transforms of multiple objectives into a scalar objective function are performed by multiplying each objective function by a weighting factor and summing up all contributors

$$\mathrm{H}(\sigma)=\gamma\,\mathrm{F}(\sigma)+(1-\gamma)\,\mathrm{G}(s)\rightarrow\min\,, \quad (15)$$

where: $\gamma\in[0,1]$.

For the mathematical meaning, Eq.(15) is an extension that modifies the equivalent characteristic according to two principles: the principle of the insufficient reason; and the principle of the maximum specificity. As represented in /30/, when the weighting factor γ is changed systematically, the solutions of Eq.(15) present the Pareto front of two objective functions: F(σ) and G(σ).

For solving Eq.(15), a Genetic Algorithm (GA) /29/ is applied using built-in functions in Matlab. Then, one considers three values of the weighting factor γ:

- when γ = 0.5 we get σ = 0.476 , (16a)
- when γ = 1 we get σ = 0.288 , (16b)
- when γ = 0 we get σ = 0.640 . (16c)

The value σ = 0.476 (with γ = 0.5) means the balance for the weighting factors between objective function F(σ) and objective function G(σ), is applied to calculate central reliability. The values σ = 0.288 (with γ = 1) and σ = 0.640 (with γ = 0) are applied to calculate marginal reliability, which is estimator of maximum and minimum of reliability.

Central and marginal reliability

Consider the limit state function with a mixture of random and fuzzy variables given by

$$g(x)=g(x_F,x_R)=g(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n,x_{n+1},x_{n+2},\ldots,x_{n+m}) \quad (17)$$

where: $x_F=(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$ are independent fuzzy variables; and symmetric triangular fuzzy number $\tilde{x}_i=(a_i,l_i)_{LR}$; x_R = (x_{n+1}, x_{n+2}, …, x_{n+m}) are independent random variables.

In order to determine the reliability according to traditional reliability theory, the transform from fuzzy variable $\tilde{x}_i=(a_i,l_i)_{LR}$ to normal random variable x_i ~ N(μ_i,σ_i) according to Eq.(16) means:

- when γ = 0.5 : $\mu_i = a_i$, σ_i = 0.476 l_i (18a)
- when γ = 1.0 : $\mu_i = a_i$, σ_i = 0.288 l_i (18b)
- when γ = 0 : $\mu_i = a_i$, σ_i = 0.640 l_i (18c)

Based on Eq.(18), the notions and the approach to calculating central and marginal reliability are presented below.

Central reliability

As the notion for expectation in the probability theory, the central reliability P_{sc} is the belief value of reliability. In order to calculate central reliability P_{sc}, the expectation and deviation of the normal density distribution, defined by Eq.(18a) are applied. Then, we can utilize any techniques in the traditional reliability theory, such as First-order reliability method (FORM), Second-order reliability method (SORM), Cornell reliability index, Hasofer-Lind reliability index, the Monte Carlo method, to define structural reliability.

Marginal reliability

Marginal reliability [$P_{s\min}$, $P_{s\max}$] is interval estimator of reliability P_s, such as the notion of interval estimator in the probability theory. The ‘true’ value of reliability P_{strue} is in the range from $P_{s\min}$ to $P_{s\max}$. Marginal reliability provides information for lower and upper reliability, so one can estimate the intuitive for reliability.

In order to determine $P_{s\min}$ and $P_{s\max}$, we need to solve optimisation problems. However, one realizes that most of the input data in structural reliability have a certain physical meaning, so increasing deviation either increases or decreases reliability and vice versa. Hence, the vertex method (VT), /31/, can be applied to determine $P_{s\min}$ and $P_{s\max}$ with input data and is defined by combinations of values in Eqs.(18b) and (18c).

When VT is applied, we need to solve 2^n traditional reliability problems, with normal random variables N(μ_i,σ_i) defined by Eqs.(18b) and (18c). Values $P_{s\min}$ and $P_{s\max}$ are given as follows

$$P_{s\min}=\min(P_{s_1},P_{s_2},\ldots,P_{s_{2^n}})\,, \quad (19a)$$

$$P_{s\max}=\max(P_{s_1},P_{s_2},\ldots,P_{s_{2^n}})\,, \quad (19b)$$

where: P_{s_k} - reliability value at the k-th combination of equivalent random variables, which have an expectation μ_i and deviation σ_i, defined by Eqs.(18b) and (18c).

ILLUSTRATIVE EXAMPLES

The illustrative examples including mathematical and engineering examples are used to demonstrate the accuracy and effectiveness of the proposed method. For comparison, the results of the proposed method are referred to those of the existing methods. Limit state functions in the illustrative examples are considered as explicit functions, in order to achieve a resemblance to structural analysis methods.

Example 1

An engineering example is used in /16/, where stress $\tilde{S}$ is a symmetric triangular fuzzy number, strength R is a normal random variable. The Cornell reliability index /2, 3/ is used in the proposed method. Table 1 displays the reliability of the proposed and the existing method, /16/.

Comments: It is found from analysis results that the results of the central reliability of the proposed method have an only small error in comparison with fuzzy reliability /16/ (most of the errors are less than 1%, the largest error is 4.6521%). When the fuzzy threshold H is taken more than 0.5 (is the reasonable value of H according to /16/), similar small errors are given (errors are in the range from 0.0885% to 0.7316%). However, the proposed method is more reasonable than method /16/, because it has overcome the drawback of different measurements between the membership function of stress $\tilde{S}$ and random variable of strength R.

Table 1. Reliability of the proposed method and the existing method, /16/.

γ	μ_S	σ_S	μ_R	σ_R	Cornell reliability index β of proposed method	Reliability P_S of proposed method	Fuzzy reliability P_s in /16/	Error (%) of reliability P_s
0	100.230	20.58880	179.17000	17.17900	2.94393	0.998380	0.999991	0.161138
0.5	100.230	15.31292	179.17000	17.17900	3.43022	0.999698	0.999991	0.029255
1	100.230	9.26496	179.17000	17.17900	4.04444	0.999974	0.999991	0.001722
0	111.720	23.80800	179.17000	17.17900	2.29744	0.989203	0.999844	1.064257
0.5	111.720	17.70720	179.17000	17.17900	2.73397	0.996871	0.999844	0.297326
1	111.720	10.71360	179.17000	17.17900	3.33153	0.999568	0.999844	0.02759
0	121.350	29.38880	179.17000	17.17900	1.69852	0.955295	0.985185	3.033949
0.5	121.350	21.85792	179.17000	17.17900	2.07979	0.981228	0.985185	0.401676
1	121.350	13.22496	179.17000	17.17900	2.66699	0.996173	0.985185	1.115352
0	129.610	28.00640	179.17000	17.17900	1.50843	0.934278	0.960186	2.698275
0.5	129.610	20.82976	179.17000	17.17900	1.83556	0.966789	0.960186	0.687635
1	129.610	12.60288	179.17000	17.17900	2.32609	0.989993	0.960186	3.104314
0	138.270	26.34240	179.17000	17.17900	1.30052	0.903288	0.899886	0.378076
0.5	138.270	19.59216	179.17000	17.17900	1.56963	0.941750	0.899886	4.6521
1	138.270	11.85408	179.17000	17.17900	1.95957	0.974977	0.899886	8.344498

Example 2

The hypothetical limit state function with three variables is indicated in Example 1 from /20/,

$$g(x) = 8.0 - 0.32(x_1 - 1)^2 x_2^2 - x_2 + x_3^3 - 0.2\sin(x_1 x_3)\,, \qquad (20)$$

where: x_1, x_2 are assumed to be independent standard normal variables; x_3 is assumed to be symmetric triangular fuzzy number $(0,1)_{LR}$.

The Hasofer-Lind reliability index /2, 3/ is used in the proposed method. Table 2 displays the reliability of the proposed and the existing method, /20/.

Table 2. Reliability of the proposed method and Example 1 in /20/.

γ	Hasofer-Lind reliability index β of proposed method	Reliability P_S of proposed method	Reliability index β of /20/	Reliability P_s of /20/	Error (%) of reliability P_s
0	2.300351	0.989286	2.155	0.984419	0.49438
0.5	2.300482	0.989290	2.28	0.988696	0.060016
1	2.3006	0.989293	2.42	0.992240	0.296991

Example 3

Consider Example 2 of /20/, a cantilever beam subjected to a tip load P is shown in Fig. 2. The length L, width b, and height h of the beam are random variables with mean values of 30 in., 0.8359 in. and 2.5093 in., respectively, and the elastic modulus E is a certain variable of 10^7 psi. Standard deviations of length, width, and height of the beam are σ_L = 3.0 in., σ_h = 0.25 in., σ_b = 0.08 in., respectively. Both L and h are considered as log-normally distributed and b is considered as a normal distribution. The load $P = (80, 20)_{LR}$ (units: lb) is a symmetric triangular fuzzy number. The prescribed tip displacement is 0.15 in.

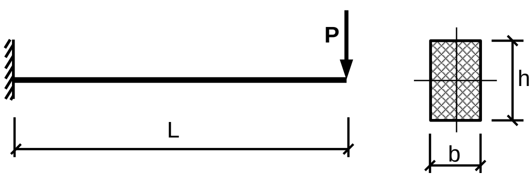

Figure 2. Cantilever beam subjected to tip load P.

The limit state function in terms of tip displacement is given as follows

$$g(x) = 0.15 - \frac{4PL^3}{Ebh^3}\,. \qquad (21)$$

The empirical second-order reliability index of Zhao and Ono /4, 5/ is used in the proposed method. Table 3 displays the reliability of the proposed and the existing method /20/.

<u>Comments:</u> It is found from analysis results of Example 2 and Example 3 that the proposed method has an only small error in comparison with the solutions /20/, the largest error is 5.36 % in Example 2. These errors are emitted from two reasons: i) different ideas for determining structural reliability with a mixture of random and fuzzy variables of the proposed and the method in /20/; ii) accuracy of the traditional reliability methods. The idea of the proposed method, based transformations from fuzzy variable to a family of normal random variables, is similar to the idea of /32, 33/, based on fitting Johnson distributions to an interval variable. In contrast to this idea, the authors /20/ aim at finding the worst combinations of interval variables to determine extreme reliability values $P_{s\min}$ and $P_{s\max}$. Because of this, the interval with reliability defined in /20/ always is wider than marginal reliability in the proposed method. Hence, according to the principle of maximum specificity /22, 24, 25/, the results of the proposed method are more informative than those in /20/. The second reason that makes errors is because the use of Rosenblatt transformation generally increases the nonlinearity of limit state function in standard normal space, /30/. It affects errors between second-order reliability in the proposed method and the Monte Carlo simulation in /20/. Indeed, if the Monte Carlo simulation is used to calculate reliability in the proposed method, with a number of trials $N_s = 10^6$, the structural reliability $P_{s\min}$ = 0.965201, the largest error between the two methods is 5.35 % (decreases 0.015%).

Table 3. Reliability of the proposed method and Example 2 in /20/.

γ	Empirical second-order reliability index β_{SORM} of proposed method	Reliability P_S of proposed method	Reliability index β of /20/	Reliability P_s of /20/	Error (%) of reliability P_s
0	1.8166	0.965361	1.38	0.916207	5.364961
0.5	1.8528	0.968044	1.89	0.970621	0.265452
1	1.8843	0.970238	2.57	0.994915	2.480340

Example 4

The hypothetical limit state function with four variables is indicated in Example 1 of /21/,

$$g(x) = x_1^2 + 5x_1 + 2x_2^2 + 7x_2 + x_3^2 - 8x_3 + x_4^2 - 10x_4 - 200 \quad (22)$$

Where: x_1, x_2 are assumed to be normally distributed with a mean of 10.0 and a standard deviation of 2.0. Variables x_3 and x_4 are assumed to be symmetric triangular fuzzy numbers $(10,5)_{LR}$.

The Hasofer-Lind reliability index is used in the proposed method. Table 4 displays results of central reliability P_{sc} and marginal reliability [P_{smin}, P_{smax}] of the proposed method and that of numerical reliability index $P_r^{(III)}$ and interval reliability index $P_r^{(I)}$, /21/, of the Monte Carlo simulation.

Table 4. Reliability of the proposed method and Example 1 in /21/.

γ	Hasofer-Lind reliability index β of proposed method	Reliability P_S of proposed method	Reliability P_s of Monte Carlo simulation in /21/	Error (%) of reliability P_s
0	2.645614471	0.995923	0.999364	0.34433
0.5	2.520677325	0.994144	0.996130	0.19942
1	2.421615044	0.992274	0.985445	0.69300

Example 5

Consider Example 2 of /21/, a roof truss is shown in Fig. 3. The top boom and compression bars are reinforced by concrete, and the bottom boom and tension bars are of steel. The uniform load q (units: N/m) is applied on the roof truss. It is assumed to be a symmetric triangular fuzzy number $\tilde{q} = (20000,1000)_{LR}$. Section areas A_c, A_s; elastic modules E_c, E_s; lengths of the concrete and steel bars l – are independent normal random variables, given in Table 5.

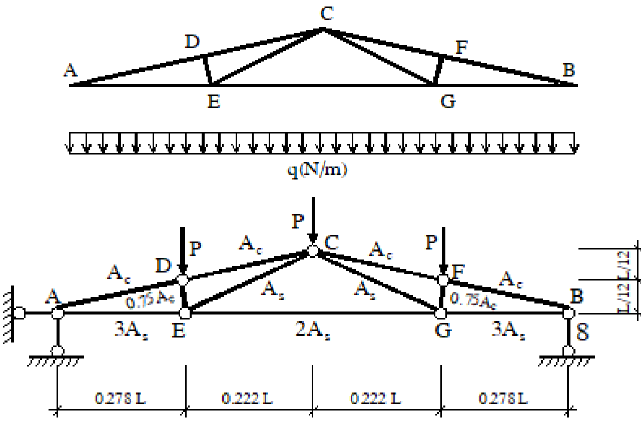

Figure 3. Schematic diagram of roof truss, /21/.

Table 5. Characteristics of random variables of a roof truss.

Random variable	l (m)	A_s (m^2)	A_c (m^2)	E_s (N/m^2)	E_c (N/m^2)
Mean μ	12	9.82×10^{-4}	0.04	1×10^{11}	2×10^{10}
Coefficient of variance v_x	0.01	0.06	0.12	0.06	0.06

The perpendicular deflection Δ_C of the peak of structure node C is not exceeding 3 cm. Applying the methods of structural mechanics, the perpendicular deflection Δ_C is expressed as follows

$$\Delta_C = \frac{ql^2}{2}\left(\frac{3.81}{A_c E_c} + \frac{1.13}{A_s E_s}\right). \quad (23)$$

The limit state function in terms of deflection is given as follows

$$g(x) = 3\times10^{-2} - \frac{ql^2}{2}\left(\frac{3.81}{A_c E_c} + \frac{1.13}{A_s E_s}\right). \quad (24)$$

The empirical second-order reliability index of Zhao and Ono /4, 5/ is used in the proposed method.

Table 6 displays results of central reliability P_{sc} and marginal reliability [P_{smin}, P_{smax}] of the proposed method and that of numerical reliability index $P_r^{(III)}$ and interval reliability index $P_r^{(I)}$, /21/, of the Monte Carlo simulation.

Table 6. Reliability of proposed method and Example 2 in /21/.

γ	Reliability P_S of proposed method	Reliability P_s /21/	Error (%) of reliability P_s
0	0.9999999962	0.996573	0.34388
0.5	0.9999999994	0.998780	0.12215
1	0.9999999999	0.999618	0.03821

<u>*Comments:*</u> It is found from analysis results of Example 4 and Example 5 that the proposed method has an only small error in comparison with the solutions /21/. The largest error is 0.7 % in Example 4. Although the approach and the idea for determining the reliability in the two methods are different, the results indicate that central reliability P_{sc} and marginal reliability [P_{smin}, P_{smax}] of the proposed method are similar meaning to numerical reliability index $P_S^{(III)}$ and interval reliability index $P_S^{(I)}$ of the method /21/. However, the proposed method has more computational efficiency and less computational complexity than the method /21/: the proposed method requires only 3 runs to determine central reliability and marginal reliability, while the PDEM in /21/ requires 192 runs to determine reliability indices, but also ensures the accuracy of calculated quantities. When Monte Carlo simulation is used to calculate reliability in the proposed method with a number of trials $N_s = 10^6$, the

central reliability P_{sc} = 0.998102, and the error between two methods is reduced to 0.0679 %.

Example 6

Consider a two-story frame structure system in Fig. 4, where the two girders are infinitely rigid. Elastic modulus E, ceiling height H, width b, and height h of the column are assumed to be normal random variables with mean values of 2×10^7 kN/m^2, 3 m, 0.20 m, 0.30 m, and standard deviation values of 1.2×10^5 kN/m^2, 0.1 m, 0.02 m, 0.03 m, in respect. Loads P_1 and P_2 (unit: kN) are assumed to be symmetric triangular fuzzy numbers: $\tilde{P}_1=(10,1)_{LR}$, $\tilde{P}_2=(15,2)_{LR}$.

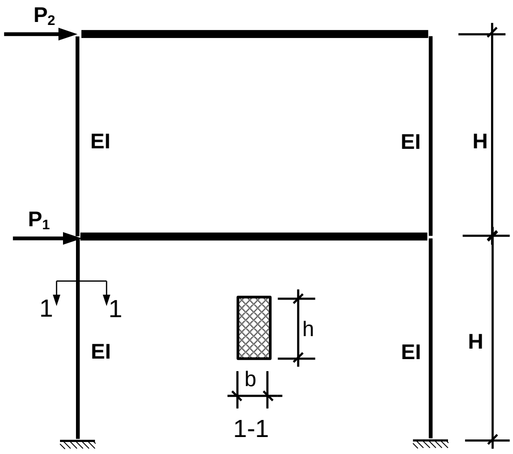

Figure 4. Two-story frame structural system.

By applying the shear forces distribution method, the horizontal displacement at the top of the structural system is expressed as follows

$$\Delta_t=\frac{(2P_2+P_1)H^3}{24EI}, \tag{25}$$

where: $I = bh^3/12$.

Because the prescribed horizontal displacement at the top of frame structure is $2H/500$, the limit state function in terms of horizontal displacement at the top is given as follows

$$g(x)=\frac{H}{250}-\frac{(2P_2+P_1)H^3}{24EI}. \tag{26}$$

The second-order reliability method SORM is used in the proposed method, where the values of P_{smin}, P_{smax} are defined according to Eqs.(19a) and (19c). The derived results are compared with the method /20/ and represented in Table 7.

Table 7. Results of proposed method and that of method in /20/.

Second order reliability P_{SORM}	Proposed method	Reliability according to method in /20/	Error (%) of reliability P_s
P_{sc}	0.991486	0.990084	0.14163
P_{smin}	0.991211	0.978582	1.29060
P_{smax}	0.991699	0.996089	0.44078

Example 7

Determine the reliability in terms of the stability of the frame structural system, as shown in Fig. 5. Consider two cases as follows:

1. Inertia moment I = 215 cm^4, length l = 5 m. Elasticity modulus E (units: kN/cm^2), concentrated load P (units: kN) are assumed as symmetric triangular fuzzy numbers given as: $\tilde{E}=(2.1\times10^4,2.5\times10^3)_{LR}$, $\tilde{P}=(410,60)_{LR}$.
2. Inertia moment I = 215 cm^4. Length l (units: m), elasticity modulus E (units: kN/cm^2) and concentrated load P (units: kN) are assumed as symmetric triangular fuzzy numbers given as: $\tilde{l}=(5,0.5)_{LR}$, $\tilde{E}=(2.1\times10^4,2.5\times10^3)_{LR}$, $\tilde{P}=(410,60)_{LR}$.

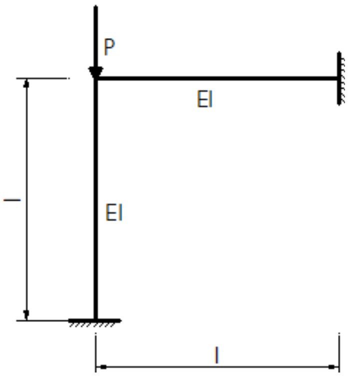

Figure 5. The structural frame system.

According to the displacement method /33/, the structural frame system is kinematically indeterminate to the first degree. The primary structural frame system in terms of the displacement method and the bending moment diagram are shown in Fig. 6. Set unit bending stiffness is $i = EI/l$.

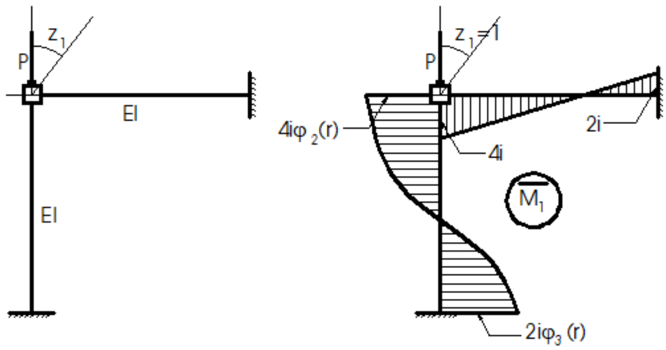

Figure 6. The primary structural frame system and bending moment diagram.

We get the equilibrium equation for the unknown degree of freedom

$$r_{11}=4i[1+\varphi_2(\nu)]. \tag{27}$$

The equation in terms of stability is given as follows

$$r_{11}=0 \;\Rightarrow\; \varphi_2(\nu)=-1. \tag{28}$$

The solution of Eq.(28) is: ν = 5.3223.

The critical load is

$$P_{cr}=\nu^2\frac{EI}{l^2}=28.3269\frac{EI}{l^2}. \tag{29}$$

The limit state function in terms of stability is given as follows

$$g(x) = 28.3269\frac{EI}{l^2} - P\,. \quad (30)$$

Table 8 displays the results of central reliability and marginal reliability of the proposed method and that of fuzzy reliability of the method in /11/. The FORM is applied in terms of case 1 and the SORM is applied in terms of case 2 in the proposed method.

Table 8. Results of proposed method and that of method in /11/.

Case	Proposed method			Method in /11/
	P_{sc}	P_{smin}	P_{smax}	
1	0.993748	0.968223	0.999981	0.996827
2	0.958488	0.902323	0.997850	0.892180

Comments: It is found from analysis results in Example 7 that the results of fuzzy reliability of the method in /11/ has an only small error in comparison with that of reliability P_{sc}, P_{smin}, P_{smax} of the proposed method. When the variables $\tilde{R}$ and $\tilde{S}$ are symmetric triangular fuzzy numbers (case 1), the method /11/ produces results close to the results of central reliability P_{sc} of the proposed method. When the membership functions of $\tilde{R}$ and $\tilde{S}$ are nonlinear (case 2), the method /11/ produces results close to the results of marginal reliability. Because of the different meaning between fuzzy reliability /11/ and reliability in the traditional reliability theory, the errors have occurred, as were analysed in /33/.

CONCLUSIONS

This article presents a practical method for calculating structural reliability with a mixture of fuzzy and random variables. In order to overcome the different measurements of fuzzy and random ones, the novel formulas for determining normal random variables are given based on innovation and combination of the principles of the insufficient reason and maximum specificity. From these equivalent random ones, the basic structural reliability problems are determined, then the two novel notions of structural reliability can be applied. The proposed central reliability needs to be compared with the prescribed reliability in the structural design codes. Simultaneously, the proposed marginal reliability should be used to estimate reasonable bounds of reliability. Numerical results are utilized to demonstrate the accuracy and effectiveness of the proposed method.

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ESIS ACTIVITIES

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

January 16, 2020	1st Virtual Conference on Structural Integrity	virtual	https://www.vcsi1.eu/
January 23, 2020	Novel materials and engineering solutions for high temperature power generation plant. FESI	Bristol, UK	link
January 23-24, 2020	TC8 Meeting – Numerical Methods for Fracture	Munich, Germany	Announcement
February 26-28, 2020	MedFract1 1st Mediterranean Conference on Fracture and Structural Integrity	Athens, Greece	http://www.medfract1.eu/
March 25-27, 2020	5th Iberian Conference on Structural Integrity – IbSCI 2020	Coimbra, Portugal	https://ibcsi.pt/
March 30 - April 3, 2020	VAL4, 4th International Conference on Material and Component Performance under Variable Amplitude Loading	Darmstad, Germany	First Announcement
April 1-3, 2020	TC4 Meeting	Twente, The Netherlands	Announcement
April 30, 2020	Structural Integrity Developments for a Competitive UK Nuclear Industry	Cambridge, UK	Initial Flyer
May 26-28, 2020	4th International Symposium on Fatigue Design and Material Defects	Potsdam, Germany	link
June 27-28, 2020	7th Summer School on "Fracture Mechanics and Structural Integrity"	Funchal, Madeira, Portugal	https://www.ecf23.eu/
June 29-July 3, 2020	23rd European Conference on Fracture - ECF23	Funchal, Madeira, Portugal	https://www.ecf23.eu/
July 12-15, 2020	ICEFA IX, Ninth International Conference on Engineering Failure Analysis	Shanghai, China	Elsevier link
August 31-September 4, 2020	8th International Conference on Very High Cycle Fatigue (VHCF8)	Sapporo, Hokkaido, Japan	https://www.vhcf8.jp/
September 2-4, 2020	20th International Colloquium on Mechanical Fatigue of Metals	Wrocław, Poland	http://icmfmxx.pwr.edu.pl/
September 6-10, 2020	TC4 Meeting - Fracture of Polymers, Composites and Adhesives	Les Diablerets, Switzerland	link
September 15-18, 2020	4th International Conference on Structural Integrity and Durability & Summer School	Dubrovnik, Croatia	http://www.icsid2020.fsb.hr
June 22-24, 2021	LCF9, 9th International Conference on Low Cycle Fatigue	Berlin, Germany	link
September 8-10, 2021	TC15 ESIAM21	Vienna, Austria	