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CHARACTERIZATION OF MATERIAL IN A ROTATING DISC SUBJECTED TO THERMAL GRADIENT BY USING SETH TRANSITION THEORY

KARAKTERIZACIJA MATERIJALA ROTIRAJUĆEG DISKA OPTEREĆENOG TEMPERATURNIM GRADIJENTOM PRIMENOM TEORIJE PRELAZNIH NAPONA SETA

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Ključne reči
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- disk

Abstract

The purpose of this paper is to present the study of material characterization in a rotating disc subjected to thermal gradient by using Seth's transition theory. It has been observed that a disc made of materials as: saturated clay, copper, or cast iron, yields at the outer surface at higher angular speed as compared to the disc of rubber material at steady state temperature, whereas the disc made of clay, copper, cast iron, as well as rubber material, yields at a lesser angular speed as compared to the rotating disc at room temperature. With the introduction of temperature, the radial- as well as the hoop stress, both decrease with the increased value of temperature at the elastic-plastic stage, but with the reverse result obtained for a fully plastic case.

Izvod

Cilj ovog rada je prezentacija studije karakterizacije materijala rotirajućeg diska koji je opterećen temperaturskim gradijentom, primenom teorije prelaznih napona Seta. Uočeno je da se kod diska, izrađenog od materijala: zasićena glina, bakar ili liveno gvožđe, javlja tečenje na spoljnjoj površini pri većoj ugaonoj brzini rotacije u poređenju sa diskom od gume, a pri ravnomernoj raspodeli temperature; dok se kod diska od gline, bakra, livenog gvožđa ili čak i od gume, javlja tečenje pri manjoj ugaonoj brzini rotacije u poređenju sa rotirajućim diskom na sobnoj temperaturi. Uvođenjem porasta temperature, radijalni- kao i obimski napon, opadaju sa povećanjem temperature pri elastoplastičnom ponašanju materijala, dok je obrnut slučaj kod plastičnog ponašanja.

INTRODUCTION

Rotating discs form an essential part in the design of rotating machinery, namely: rotors, turbines, flywheel, compressors, and high-speed gear engines, etc. Use of rotating discs in machines and structural applications has generated considerable interest in many problems in the domain of mechanics of solids. The solution for thin isotropic discs can be found in most of the standard elasticity and plasticity textbooks /1-3, 6, 7, 9/. Parmaksizoğlu et al. /10/ analysed the problem of plastic stress distribution in a rotating disc with a rigid inclusion with a radial temperature gradient under the assumptions of Tresca's yield condition, its associated flow rule, and linear strain hardening. To obtain the stress distribution, they matched the plastic stresses at the same radius $r = z$ of the disc. Seth's transition theory /6/ includes classical macroscopic problem solving in elasticity, plasticity, creep and relaxation and assumes semi-empirical yield conditions. The nonlinear transition region through which yielding occurs is neglected. The transition theory, used in solving problems of generalized strain measure, and the asymptotic solution at critical points of differential equations, defining the deforming field, has been successfully applied to a large number of problems /8, 11-40/. In this paper, we investigate the characterization of material in a rotating disc subjected to a thermal gradient. Results are discussed and depicted graphically.

MATHEMATICAL MODEL AND GOVERNING EQUATION

Consider a thin rotating disc having constant density with the central bore of radius r_i and external radius r_0. The rotating disc is mounted on a rigid shaft as shown in Fig. 1. The disc is rotating with angular velocity ω about an axis perpendicular to its plane and passing through the centre. The thickness of the disc is assumed to be constant and is taken to be sufficiently small so that the disc is effectively in a state of plane stress that is, the axial stress τ_{zz} is zero. Let a uniform temperature Θ_0 be applied at the inner surface of the rotating disc.

Displacement coordinates: for this problem displacement components in cylindrical polar co-ordinates are given as /4/

$$u = r(1 - \eta);\ v = 0;\ w = dz, \qquad (1)$$

where: η is position function, depending on the value of $r=\sqrt{x^2+y^2}$ only; and d be a constant.

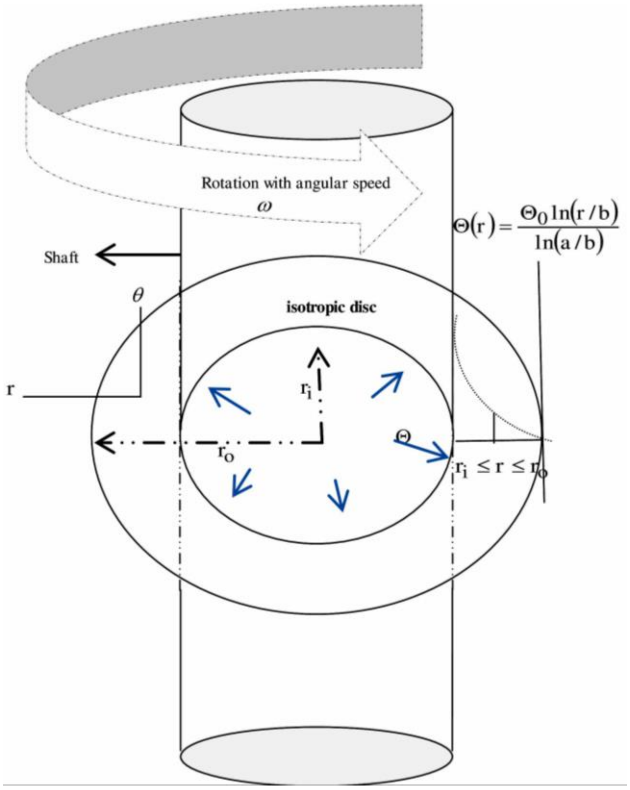

Figure 1. Geometry of disc with thermal gradient.

Generalized strain components: generalized strain components are given /5/ as

$$e_{rr}=\frac{1}{n}\left[1-\{2(r\eta'+\eta)-1\}^{n/2}\right],\ e_{\theta\theta}=\frac{1}{n}\left[1-\{2\eta-1\}^{n/2}\right],$$

$$e_{zz}=\frac{1}{n}\left[1-(1-2d)^{n/2}\right],\ e_{r\theta}=e_{r\theta}=e_{zr}=0\,, \qquad (2)$$

where: $\eta'=d\eta/dr$.

Stress-strain relation: stress-strain relations for thermoplastic in an isotropic media are given by, /7/:

$$\tau_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij}-\xi\Theta\delta_{ij}\,,\ (i,j=1,2,3)\,, \qquad (3)$$

where: e_{ij}, τ_{ij} are strain and stress tensor; $I_1=e_{kk}$ ($k=1,2,3$) is strain invariant; δ_{ij} is Kronecker's delta; Θ be a temperature; $\xi=\alpha(3\lambda+2\mu)$; α being the coefficient of thermal expansion and λ, μ are Lame's constants. Further, Θ has to satisfy the heat equation, which gives /7/:

$$\nabla^2\Theta=0\,. \qquad (4)$$

Equation (3) for this problem becomes

$$\tau_{rr}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{rr}-\frac{2\mu\xi\Theta}{\lambda+2\mu}\,,$$

$$\tau_{\theta\theta}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{\theta\theta}-\frac{2\mu\xi\Theta}{\lambda+2\mu}\,,$$

$$\tau_{zz}=\tau_{zr}=\tau_{r\theta}=\tau_{\theta z}=0\,. \qquad (5)$$

From Eq.(3), strain components in terms of stresses are obtained as

$$e_{rr}=\frac{1}{E}\left[\tau_{rr}-\nu\tau_{\theta\theta}\right]+\alpha\Theta\,,\ e_{\theta\theta}=\frac{1}{E}\left[\tau_{\theta\theta}-\nu\tau_{rr}\right]+\alpha\Theta\,,$$

$$e_{zz}=-\frac{\nu}{E}\left[\tau_{rr}-\tau_{\theta\theta}\right]+\alpha\Theta\,,\ e_{rr}=e_{\theta z}=e_{zr}=0\,, \qquad (6)$$

where: $E=\mu(3\lambda+2\mu)/(\lambda+\mu)$ is the Young's modulus and $\nu=\lambda/2(\lambda+\mu)=1-c/2-c$ be Poisson's ratio in terms of compressibility factor and Lame's constants. From Eq.(2) and Eq.(5), we get the stresses as

$$\tau_{rr}=\frac{2\mu}{n}\left[3-2c-\{2\eta(T+1)-1\}^{n/2}(2-c)-(2\eta-1)^{n/2}(1-c)-\frac{nc\xi\Theta}{2\mu}\right],$$

$$\tau_{\theta\theta}=\frac{2\mu}{n}\left[3-2c-\{2\eta(T+1)-1\}^{n/2}(1-c)-(2\eta-1)^{n/2}(2-c)-\frac{nc\xi\Theta}{2\mu}\right],$$

$$\tau_{zz}=\tau_{zr}=\tau_{r\theta}=\tau_{\theta z}=0\,, \qquad (7)$$

where: $c=2\mu/\lambda+2\mu$ be the compressibility factor in terms of λ, μ and $r\eta'=\eta T$.

Equation of equilibrium: equations of equilibrium are all satisfied except

$$\frac{d}{dr}(r\tau_{rr})-\tau_{\theta\theta}+\rho\omega^2r^2=0\,. \qquad (8)$$

where: τ_{rr} be the radial stress; $\tau_{\theta\theta}$ circumferential stresses; and ρ be the constant density of the rotating disc.

Asymptotic solution at transition points: using Eq.(7) and Eq.(12) in Eq.(8), we get a nonlinear differential equation in η given as

$$(2-c)n\eta^2T\{2\eta(T+1)-1\}^{\frac{n}{2}-1}\frac{dT}{d\eta}=\left[\left\{\frac{n\rho\omega^2r^2}{2\mu}-\{2\eta(T+1)-1\}^{n/2}\times\left[1+\frac{n\eta T(T+1)(2-c)}{2\eta(T+1)-1}\right]+\{2\eta-1\}^{n/2}\left[1-\frac{n\eta T(1-c)}{2\eta-1}\right]\right\}-\frac{nc\xi\bar{\Theta}_0}{2\mu}\right] \qquad (9)$$

where: $\bar{\Theta}_0=\dfrac{\Theta_0}{\log(a/b)}$.

Critical or transition points: transition points of η in Eq.(9) are $T\to 0$ and $T\to\pm\infty$. At transition point $T\to 0$ nothing is of importance.

Boundary condition: the rotating disc considered in the study is subjected to a temperature gradient field and infinitesimal deformation. The inner surface of the disc is assumed to be fixed to a shaft so that isothermal conditions prevail on it. The inner surface of the disc has a uniform temperature gradient. Thus, the boundary conditions of the problem are

$$R=r_i,\ u=0;\quad r=b,\ \tau_{rr}=0\ \text{ at }\ r=r_0, \qquad (10)$$

where: u and T_{rr} denote displacement and stress along the radial direction applied at the external surface. The temperature field satisfying Eq.(4) and

$$\Theta=\Theta_0\ \text{ at }\ r=r_i\,,\ \Theta=0\ \text{ at }\ r=r_0\,, \qquad (11)$$

where: Θ_0 is constant, is given by

$$\Theta=\frac{\Theta_0\log(r/r_0)}{\log(r_i/r_0)}\,. \qquad (12)$$

SOLUTIONS

It has been shown /4, 5, 8, 11-42/ that the asymptotic solution through the principal stress leads from elastic state to plastic state at transition point $T \to \pm\infty$. We define the transition function Z as

$$Z = \frac{n}{2\mu}\left[\tau_{\theta\theta} + c\xi\Theta\right] = $$
$$= \left[(3-2c) - \{2\eta(T+1)-1\}^{n/2}(1-c) - (2\eta-1)^{n/2}(2-c)\right]. \quad (13)$$

By taking the logarithmic differentiation of Eq.(13) with respect to r and using Eq.(9), we get

$$\frac{d(\ln Z)}{dr} = -\frac{\left(\frac{1-c}{2-c}\right)\left[\frac{n\rho\omega^2 r^2}{2\mu\beta^n} - \{2\eta(T+1)\}^{n/2} + (2\eta-1)^{n/2} - \frac{nc\xi\bar{\Theta}_0}{2\mu\eta^n} - n\eta T(1-c)(2\eta-1)^{n/2}\right] + n\eta T(2-c)(2\eta-1)^{n/2}}{r\left[3-2c-\{2\eta(T+1)-1\}^{n/2} \times (1-c) - (2\eta-1)^{n/2}(2-c)\right]}. \quad (14)$$

The asymptotic value from Eq.(14) as $T \to \pm\infty$, and by integrating we get

$$Z = Lr^{-1/(2-c)}, \quad (15)$$

where: L is a constant of integration.

From Eq.(13) and Eq.(15), we have

$$\tau_{\theta\theta} = \left(\frac{2\mu}{n}\right)Lr^{-1/(2-c)} - \frac{c\xi\Theta_0 \log(r/r_0)}{\log(r_i/r_0)}. \quad (16)$$

Using Eq.(16) into Eq.(8) and integrating, we get

$$\tau_{rr} = \left\{\frac{2\mu(2-c)}{n(1-c)}\right\}Lr^{-1/(2-c)} - \frac{c\xi\Theta_0 \log(r/r_0)}{\ln(r_i/r_0)} + \frac{c\xi\Theta_0}{\ln(r_i/r_0)} - \frac{\rho\omega^2 r^2}{3} + \frac{M}{r}, \quad (17)$$

where: M is a constant of integration.

Substituting Eq.(16) and Eq.(17) in Eq.(6), we get

$$\frac{\partial u}{\partial r} = \frac{1}{E}\left[\left(\frac{2\mu}{n}\right)Lr^{-1/(2-c)}\left\{\frac{3-2c}{(1-c)(2-c)}\right\} + \frac{\alpha E\Theta_0(2-c)}{\ln(r_i/r_0)} - \frac{\rho\omega^2 r^2}{3} + \frac{M}{r}\right], \quad (18)$$

$$\frac{u}{r} = \frac{(1-c)}{E(2-c)}\left[\frac{\rho\omega^2 r^2}{3} - \frac{\alpha E\Theta_0(2-c)}{\ln(r_i/r_0)} - \frac{M}{r}\right], \quad (19)$$

where: $c\xi = 2\mu\alpha(3-2c)$; and $E = 2\mu(3-2c)/(2-c)$ is the Young's modulus.

By integrating Eq.(18) with respect to r, we get

$$u = \frac{1}{E}\left[\left(\frac{2\mu}{n}\right)Lr^{\frac{1-c}{2-c}}\left\{\frac{(3-2c)}{(1-c)^2}\right\} + \frac{\alpha E\Theta_0(2-C)r}{\ln(r_i/r_0)} - \frac{\rho\omega^2 r^3}{9} + M\log r\right] + N, \quad (20)$$

where: D is a constant of integration.

From Eq.(19) and Eq.(20), we get

$$\left(\frac{2\mu}{n}\right)Lr^{\frac{1-c}{2-c}}\left\{\frac{3-2c}{(1-c)^2}\right\} = \left[\frac{\rho\omega^2 r^3}{9}\left(\frac{5-4c}{2-c}\right) - \frac{\alpha E\Theta_0 r(3-2c)}{\ln(r_i/r_0)} - NE - M\left\{\frac{(1-c)+(2-c)\ln r}{(2-c)}\right\}\right], \quad (21)$$

and

$$u = \frac{(1-c)}{E(2-c)}\left[\frac{\rho\omega^2 r^3}{3} - \frac{\alpha E\Theta_0(2-c)r}{\ln(r_i/r_0)} - M\right]. \quad (22)$$

Using boundary conditions from Eq.(10) in Eq.(22), we get

$$M = \frac{\rho\omega^2 r_i^3}{3} - \frac{\alpha E\Theta_0 r_i(2-c)}{\ln(r_i/r_0)}. \quad (23)$$

Substituting Eq.(21) into Eq.(17) and using boundary condition from Eq.(10) and Eq.(13), we get

$$N = \frac{1}{E}\left\{\frac{\rho\omega^2 r_o^3}{9}\left(\frac{5-4c}{2-c}\right) - \frac{\alpha E\Theta_0 r_o(3-2c)}{\ln(r_i/r_0)} - \left(\frac{\rho\omega^2 r_i^3}{3} - \frac{\alpha E\Theta_0 r_i(2-c)}{\ln(r_i/r_0)}\right)\left[\left(\frac{1-c}{2-c}\right) + \ln r_o\right] + \frac{(3-2c)}{(1-c)(2-c)}\left[\frac{\rho\omega^2(r_i^3 - r_0^3)}{3} + \frac{\alpha E\Theta_0(2-C)(r_0-r_i)}{\ln(r_i/r_0)}\right]\right\}. \quad (24)$$

By using Eqs.(21), (23) and (24) in Eqs.(16) and (17), in respect, we get

$$\tau_{\theta\theta} = \frac{\rho\omega^2}{3r}\left[\frac{1}{3}\left(\frac{5-4c}{2-c}\right)\frac{(1-c)^2(r^3-r_0^3)}{(3-2c)} - r_i^3\ln\left(\frac{r}{r_0}\right)\frac{(1-c)^2}{(3-2c)} + \left(\frac{1-c}{2-c}\right)(r_o^3 - r_i^3)\right] - \frac{\alpha E\Theta_0}{\ln(r_i/r_0)} \times$$
$$\times\left[\frac{(1-c)^2(r-r_o)}{r} - \frac{r_i}{r}\ln\left(\frac{r}{r_0}\right)\frac{(2-c)(1-c)^2}{(3-2c)} + \frac{(r_0-r_i)(1-c)}{r} + (2-c)\ln\left(\frac{r}{r_o}\right)\right],$$

$$\tau_{rr} = \left\{\frac{\rho\omega^2}{3r}\left[\frac{1}{3}\left(\frac{5-4c}{3-2c}\right)(1-c)(r^3-r_0^3) - r_i^3\ln\left(\frac{r}{r_0}\right)\frac{(1-c)(2-c)}{(3-2c)} + r_0^3 - r^3\right] - \frac{\alpha E\Theta_0(2-c)}{\ln(r_i/r_0)}\left[c\left(\frac{r_0}{r}-1\right) + \ln\left(\frac{r}{r_0}\right) - \frac{r_i}{3r(2-c)}\ln\left(\frac{r}{r_0}\right)\right]\right\},$$

and

$$u = \frac{(1-c)}{(2-c)}\left[\frac{\rho\omega^2}{3}(r^3 - r_i^3) - \frac{\alpha E\Theta_0(2-c)}{\ln(r_i/r_0)}(r-r_i)\right]. \quad (25)$$

Yielding at the initial stage: it has been seen from Eq.(25) that $|\tau_{\theta\theta}|$ is maximum at the outer surface (that is at $r = r_0$), therefore, yielding will take place at the outer surface of the rotating disc and Eq.(25) becomes

$$\left|\tau_{\theta\theta}\right|_{r=r_0} = \left|\frac{\rho\omega^2(r_0^3 - r_i^3)}{3r_0}\left(\frac{1-c}{2-c}\right) + \alpha E\theta_0\left[\frac{(1-c)(r_0-r_i)}{r_0\ln(r_0/r_i)}\right]\right| = Y \text{ (say)}.$$

The angular velocity ω_i necessary for the initial yielding stage is given by

$$\Omega_i^2 = \frac{\rho\omega_i^2 r_0^2}{Y} = \frac{3(2-c)}{(1-r_i^3/r_0^3)(1-c)} - 3(2-c)\left(\frac{\alpha E\Theta_0}{Y}\right)\left[\frac{(1-r_i/r_0)}{(1-r_i^3/r_0^3)\ln(r_0/r_i)}\right], \quad \text{and} \quad \omega_i = \frac{\Omega_i}{r_0}\sqrt{\frac{Y}{\rho}}\,. \tag{26}$$

Fully-plastic stage: the angular velocity ω_f for which the disc becomes fully-plastic ($c \to 0$) at $r = r_i$ is given by Eq.(25) as

$$\Omega_f^2 = \frac{\rho\omega_f^2 r_0^2}{Y^*} = \frac{4\left(\frac{r_i}{r_0}\right)\left(\frac{\alpha E\Theta_0}{Y^*}\right) + 3\left(\frac{r_i}{r_0}\right)}{\frac{2}{9}\left(1-\frac{r_i^3}{r_0^3}\right) - \frac{1}{3}\left(\frac{r_i}{r_0}\right)^3 \ln\left(\frac{r_i}{r_0}\right)}, \tag{27} \quad \text{and} \quad \omega_f = \frac{\Omega_f}{r_o}\sqrt{\frac{Y^*}{\rho}}\,.$$

Non-dimensional components: we introduce the following non-dimensional components as: $R = r/r_0$; $R_0 = r_i/r_0$; $\sigma_r = \tau_{rr}/Y$; $\sigma_\theta = \tau_{\theta\theta}/Y$; $\Theta_1 = \alpha E\Theta_0/Y$; and $\bar{u} = uE/Yr_0$.

Stresses, displacement and angular speed at initial stage: elastic-plastic stresses, angular velocity and displacement from Eqs.(25) and (26) in non-dimensional form become

$$\sigma_\theta = \frac{\Omega_i^2}{3R}\left[\frac{1}{3}\left(\frac{5-4c}{2-c}\right)\frac{(1-c)^2}{(3-2c)}(R^3-1) - R_0^3 \ln R \frac{(1-c)^2}{(3-2c)} + \left(\frac{1-c}{2-c}\right)(1-R_0^3)\right] - \frac{\Theta_1}{\ln R_0}\left[\frac{(1-c)^2(R-1)}{R} + \frac{R_0}{R}\ln R \frac{(2-c)(1-c)^2}{(3-2c)} + \frac{(1-R_0)(1-c)}{R} + (2-c)\ln R\right],$$

$$\sigma_r = \frac{\Omega_i^2}{3R}\left[\frac{1}{3}\left(\frac{5-4c}{3-2c}\right)(1-c)(R^3-1) - R_0^3 \log R \frac{(1-c)(2-c)}{(3-2c)} + 1 - R^3\right] - \frac{\Theta_1(2-c)}{\ln R_0}\left[c\frac{(1-R)}{R} + \ln R - \frac{R_0 \ln R}{3R(2-c)}\right],$$

$$\bar{u} = \left(\frac{1-c}{2-c}\right)\left[\frac{\Omega_i^2}{3}(R^3 - R_0^3) - \frac{\Theta_1(2-c)}{\log R_0}(R - R_0)\right], \tag{28}$$

and

$$\Omega_i^2 = \frac{3(2-c)}{(1-R_0^3)(1-c)} - \frac{3\Theta_1(2-c)}{(1-R_0^3)}\frac{(1-R_0)}{\ln(1/R_0)}\,. \tag{29}$$

Stresses, displacement and angular speed at fully plastic stage: stresses, displacement and angular speed for the fully-plastic state ($c \to 0$), are obtained from Eqs.(28) and (27) as

$$\sigma_r = \frac{\Omega_f^2}{3R}\left[\frac{5}{9}(R^3-1) - \frac{2}{3}R_0^3 \ln R + 1 - R^3\right] - \frac{2\Theta_1^*}{\ln R_0}\left[\log R - \frac{R_0 \ln R}{6R}\right],$$

$$\sigma_\theta = \frac{\Omega_f^2}{3R}\left[\frac{5}{18}(R^3-1) - \frac{R_0^3 \ln R}{3} + \frac{1}{2}(1-R_0^3)\right] - \frac{\Theta_1^*}{\ln R_0}\left[\frac{(R-1)}{R} + \frac{2}{3}\frac{R_0}{R}\ln R + \frac{(1-R_0)}{R} + 2\ln R\right],$$

$$\bar{u}_f = \frac{\Omega_f^2}{6}(R^3 - R_0^3) - \frac{\Theta_1^*}{\ln R_0}(R - R_0)\,, \tag{30}$$

and $$\Omega_f^2 = \frac{3R_0}{\frac{2}{9}(1-R_0^3) - \frac{R_0^3 \ln R_0}{3}} + \frac{4R_0\Theta_1^*}{\frac{2}{9}(1-R_0^3) - \frac{R_0^3 \ln R_0}{3}}, \tag{31}$$ where $\Theta_1^* = \frac{\alpha E\Theta_0}{Y^*}$; $\bar{u}_f = \frac{uE}{Y^* b}$.

NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating stresses, strain rates, based on the above analysis, the following values have been taken: $\nu = 0.5$ ($c = 0$, incompressible material, i.e. rubber); $\nu = 0.42857$ ($c = 0.25$, compressible material, *i.e.* saturated clay); $\nu = 0.33$ ($c = 0.5$, compressible materials, i.e. copper); and $\nu = 0.21$ ($c = 0.75$, compressible materials, i.e. cast iron), /1/, and temperature: $\Theta_1 = 0, 0.3, 0.45, 0.85$, respectively.

Curves are produced in Fig. 2, between angular speeds along with the radius ratio $R_0 = r_i/r_0$ at the initial yielding stage $R_0 = r_i/r_0$. It has been seen that rotating disc made of compressible materials (say saturated clay, copper and cast iron) and of smaller radii ratio, yields at the inner surface with required higher angular speed, as compared to disc made of incompressible material (say rubber) at room temperature. With thermal effects, the disc yields in the external surface at a lesser angular speed as compared to the rotating disc at room temperature. Curves are produced in Fig. 3, between angular speed and various radii ratio $R_0 = r_i/r_0$ for fully plastic. The disc of smaller radii ratio requires higher angular speed to become fully plastic in comparison to rotating disc of higher thickness ratio, and the angular speed increases with increase in temperature.

In Fig. 4, curves are drawn between stresses and radii ratio $R = r/r_0$ at elastic-plastic transition state and the fully plastic state. It is observed that radial stresses are maximum at the inner surface. With the introduction of temperature, the radial, as well as the hoop stresses, decrease with increased value of temperature at the elastic-plastic state, but the reverse result is obtained for a fully plastic case.

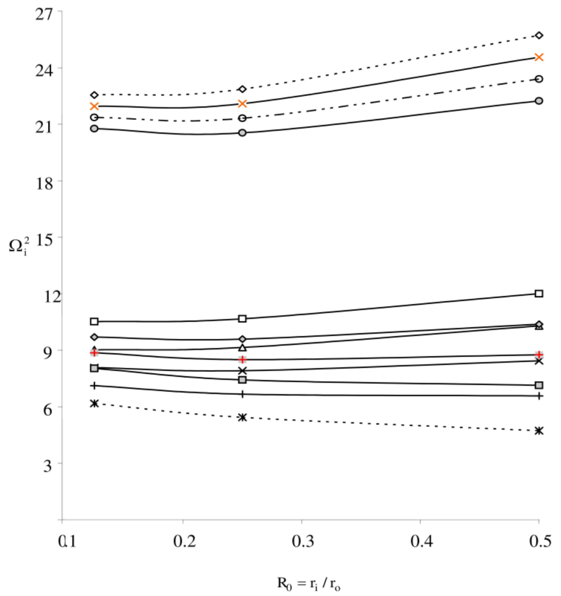

Figure 2. Graph between angular speed along the radii ratio $R_0 = r_i/r_0$ at initial yielding stage.

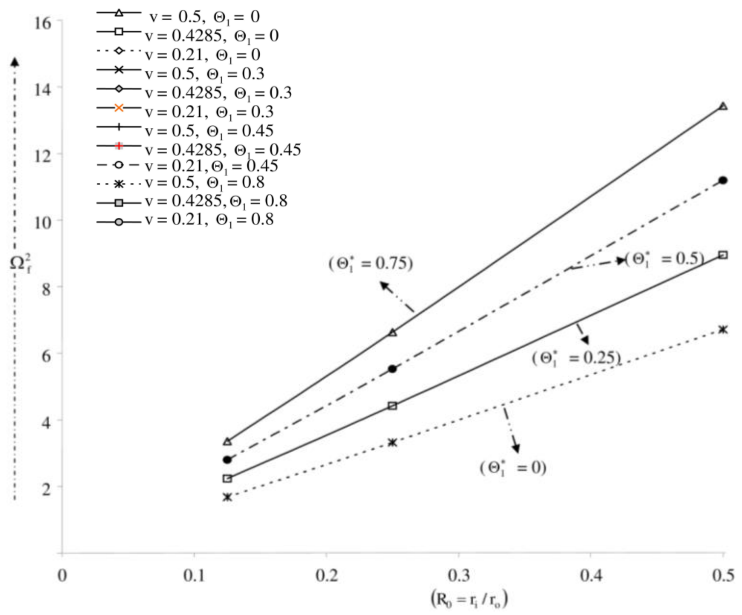

Figure 3. Graph between angular speed along the radii ratio $R_0 = r_i/r_0$ for the fully-plastic stage.

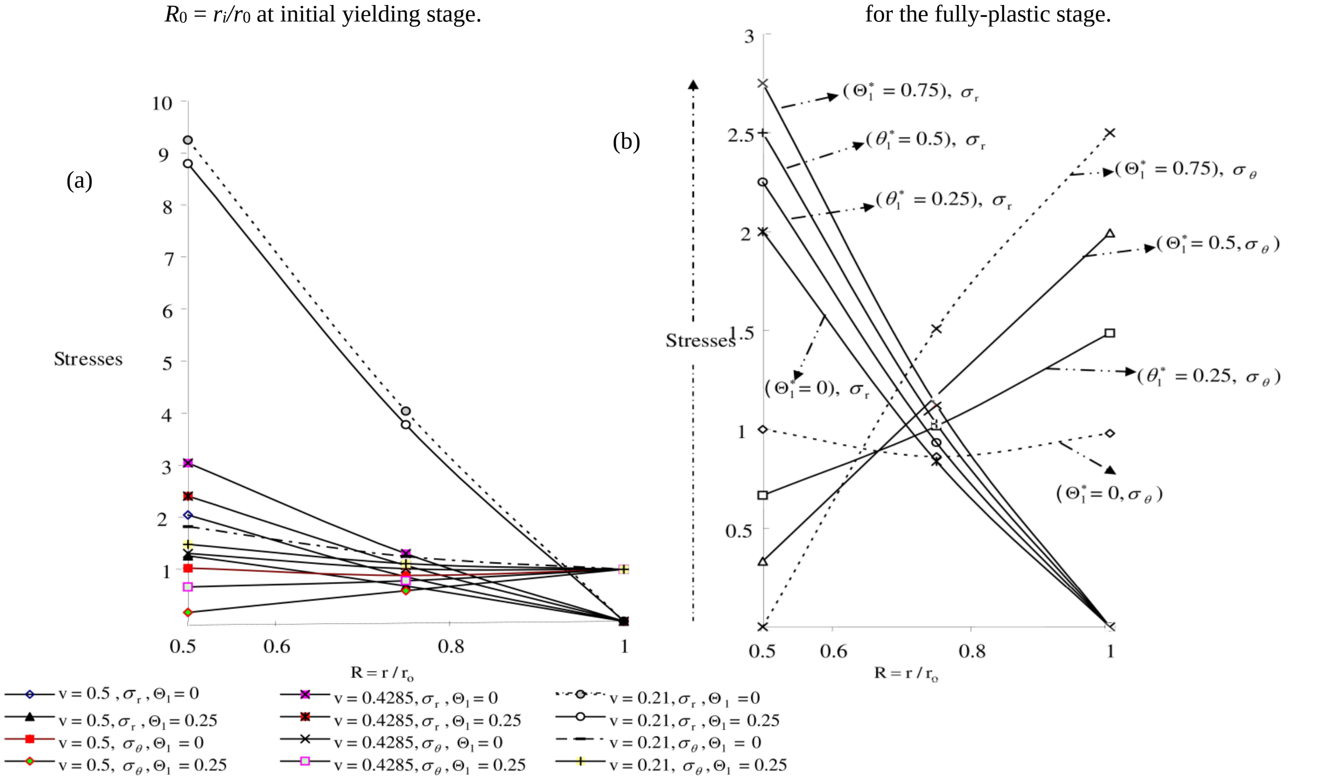

Figure 4. Graph between stresses along the radii ratio $R_0 = r_i/r_0$ for: (a) the initial yielding; (b) fully-plastic stage.

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