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ELASTODYNAMIC BEHAVIOUR DUE TO MECHANICAL FORCES IN A MICROSTRETCH THERMOELASTIC HALF-SPACE WITH TWO-TEMPERATURES

ELASTODINAMIČKO PONAŠANJE USLED MEHANIČKIH SILA KOD MIKROZATEZNOG TERMOELASTIČNOG DVOTEMPERATURSKOG POLUPROSTORA

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Keywords

- two-temperatures
- Green-Naghdi theory
- microstretch thermoelastic
- internal heat source
- boundary value problem

Abstract

The effect of two temperatures on the elastic properties of a generalized microstretch thermoelastic solid half-space has been investigated. The Green-Naghdi (GN) theory of thermoelasticity is adopted in the present research. The exact solutions of the problem are obtained in terms of the normal modes. Using the normal mode analysis technique, the mathematical expressions of displacement components, normal stress, couple tangential stress, tangential stress, micro stress and the temperature distribution are derived.

Ključne reči

- dvotemperaturni
- Green-Naghdi teorija
- mikrozatezna termoeleastičnost
- unutrašnji izvor toplote
- problem graničnih uslova

Izvod

U ovom radu je prikazan uticaj dve temperature na elastične osobine generalisanog mikrozateznog termoelastčnog čvrstog poluprostora. Za potrebe istraživanja je isvojena Green-Naghdi (GN) teorija termoelastičnosti. Tačna rešenja problema su dobijena za normalni mod. Primenom tehnike analize normalnog moda izvedeni su matematički izrazi za komponente pomeranja, normalni napon, tangencijalni napon, spregnuti tangencijalni napon, mikronapon i raspodelu temperature.

INTRODUCTION

Eringen /1/ developed the theory of thermo-microstretch elastic solids. Microstretch continuum is a model for Bravais lattice with basis on the atomic level and two-phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibers, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and solid-liquid crystals etc. are examples of microstretch solids. Bofill and Quintanilla /2/ verified the existence theorem and some uniqueness theorems for the microstretch thermoelastic materials in the context of linear theory of these materials. Singh and Kumar /3/ investigated the results concerning reflection and transmission in microstretch thermoelastic materials. Mechanical interactions due to the mechanical forces in microstretch mass diffusive half-space were studied by Kumar and Kumar /4/. The elastodynamic interactions due to inclined mechanical forces was investigated by Kumar and Kumar /5/. Thermomechanical interactions of ultra-short laser pulse in generalized microstretch thermoelastic solid were investigated by Kumar et al. /6/. Marin and Vlase /7/ studied the effect of internal state variables in microstretch thermoelasticity. Kumar /8/ investigated the solution of a problem in magneto-microstretch thermoelasticity subjected to inclined loads. Kumar et al. /9/ discussed the pulsed laser heating effect in a dual phase lag mass diffusion thermoelastic medium.

Chen and Gurtin /10/ developed a theory for elastic solids in which the equation of heat conduction involved two distinct temperatures named as thermodynamic temperature and conductive temperature. The difference between these two temperatures is directly proportional to the amount of heat supplied. If no heat is supplied, then these two temperatures are identical. This two-temperature model is used to find the electron and photon temperature distribution in laser processing of metals. Youssef /11/ proved the uniqueness and existence relations in the theory of two-temperature generalized thermoelasticity. Youssef and Bassiouny /12/ discussed a boundary value problem in piezoelectric thermoelastic half-space using state space approach method. Ezzat and Bary /13/ derived solutions of one-dimensional problem in magneto-thermoelasticity with two temperatures. Kumar and Mukhopadhyay /14/ discussed the effects of cylindrical cavity in the generalized theory of two temperatures. A theory of thermoelasticity with two distinct temperatures and without energy dissipation was presented by Youssef and Elsibai /15/. Al-Lehaibi and Eman /16/ derived the generalized solutions of thermal shock problem of nano beam resonator in generalized thermoelasticity with two temperatures. Deswal et al. /17/ presented thermal and mechanical interactions in a micropolar thermoelastic dual phase lag medium with two temperatures. They also studied the effect of gravity on thermal stresses in the considered medium. Sur and Kanoria /18/ presented a 3-dimensional deformation problem in thermoelasticity with two temperatures.

This research deals with the interactions in an isotropic microstretch thermoelastic medium with two temperatures due to the effect of mechanical forces. The normal mode analysis technique is used to obtain the expressions for the thermal stresses and the temperature change.

BASIC EQUATIONS

The basic equations for homogeneous microstretch thermoelastic medium with two temperatures in the absence of body force and body couple are:

stress equation of motion:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\nabla T=\rho\ddot{\boldsymbol{u}} \quad (1)$$

couple stress equation of motion:

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}} \quad (2)$$

equation of balance of stress moments:

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1 T=\frac{\rho j_0}{2}\ddot{\phi}^* \quad (3)$$

equation of heat conduction:

$$K\nabla^2\chi+K^*\nabla^2\dot{\chi}=\rho c^*\frac{\partial^2 T}{\partial t^2}+\beta_1 T_0\frac{\partial^2}{\partial t^2}(\nabla.\boldsymbol{u})+\nu_1 T_0\dot{\phi}^* \quad (4)$$

the constitutive relations are:

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijk}\phi_k)-\beta_1\delta_{ij}T \quad (5)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\varepsilon_{mji}\phi^*_{,m}\,, \quad (6)$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\varepsilon_{ijm}\phi_{j,m}\,, \quad (7)$$

$$T=(1-\kappa\nabla^2)\chi\,. \quad (8)$$

Here λ, μ, α, β, γ, K, λ_0, λ_1, α_0, b_0, are material constants; ρ is mass density; $\boldsymbol{u}=(u_1,u_2,u_3)$ is the displacement vector and $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ is the microrotation vector; ϕ^* is the scalar microstretch function; T is temperature and T_0 is the reference temperature of the body chosen; Q is the input heat source; j is the microinertia; $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$; $\nu_1=(3\lambda+2\mu+K)\alpha_{t2}$; α_{t1}, α_{t2} are coefficients of linear thermal expansion; j_0 is the microinertia for the microelements; t_{ij} are components of stress; m_{ij} are components of couple stress; λ_i^* is the microstress tensor; ε_{ij} are components of strain; ε_{kk} is the dilatation; δ_{ij} is Kronecker's delta function.

FORMULATION OF THE PROBLEM

We consider a microstretch thermoelastic medium with two temperatures with rectangular Cartesian coordinate system $0x_1x_2x_3$ with x_3-axis pointing vertically downward the medium. The geometry of this problem is illustrated in Fig. 1.

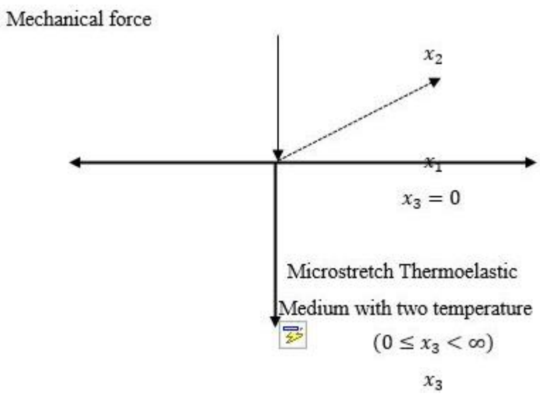

Figure 1. Geometry of the problem.

Here we write the following form of displacement vector and micro-rotational vector for the two-dimensional problem:

$$\boldsymbol{u}=(u_1,0,u_3),\ \ \boldsymbol{\phi}=(0,\phi_2,0)\,. \quad (9)$$

For further consideration it is convenient to introduce in Eqs.(1)-(4) the dimensionless quantities defined by:

$$u_i'=\frac{\rho\omega^* c_1}{\beta_1 T_0}u_i,\ x_i'=\frac{\omega^*}{c_1}x_i,\ t'=\omega^* t,\ T'=\frac{T}{T_0},\ \tau_1'=\omega^*\tau_1,$$

$$\tau_0'=\omega^*\tau_0,\ \gamma_1'=\omega^*\gamma_1,\ t_{ij}'=\frac{1}{\beta_1 T_0}t_{ij},\ \omega^*=\frac{\rho c^* c_1^2}{K^*},\ \phi_i'=\frac{\rho c_1^2}{\beta_1 T_0}\phi_i,$$

$$\tau^{1\prime}=\omega^*\tau^1,\ c_1^2=\frac{\lambda+2\mu+k}{\rho},\ c_2^2=\frac{\mu+k}{\rho},\ c_3^2=\frac{\gamma}{\rho j},\ c_4^2=\frac{2\alpha_0}{\rho j_0},$$

$$\varepsilon=\frac{\gamma^2 T_0}{\rho^2 c^* c_1},\ m_{ij}^*=\frac{\omega^*}{c\beta_1 T_0}m_{ij},\ \phi^{*\prime}=\frac{\rho c_1^2}{\beta_1 T_0}\phi^*. \quad (10)$$

According to Helmholtz representation of a vector into a scalar and vector potentials, the displacement components u_1 and u_3 are related to non-dimensional potential functions ϕ and ψ as:

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3},\ u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}\,. \quad (11)$$

Switching the values of u_1 and u_3 from Eq.(11) in Eqs.(1)-(4) and with utility of Eqs.(9) and (10), after suppressing the primes, we obtain:

$$\nabla^2\phi-\ddot{\phi}+a_1\phi^*-T=0\,, \quad (12)$$

$$(\nabla^2-a_6)\phi^*-a_7\nabla^2\phi+a_8 T=0\,, \quad (13)$$

$$(a_9\nabla^2\chi+\nabla^2\dot{\chi})-[\ddot{T}+a_{10}\nabla^2\ddot{\phi}+a_{11}\ddot{\phi}^*]=0\,, \quad (14)$$

$$a_2\nabla^2\psi-\ddot{\psi}+a_3\phi_2=0\,, \quad (15)$$

$$\nabla^2\phi_2-2a_4\phi_2-a_4\nabla^2\psi=a_5\ddot{\phi}_2\,, \quad (16)$$

where: a_i are mentioned in the Appendix 1; and

$\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$ is the Laplace operator.

SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as expressed in the subsequent equations:

$$\left\{\phi,\psi,\chi,\phi_2,\phi^*\right\}(x_1,x_3,t)=\left\{\bar{\phi},\bar{\psi},\bar{\chi},\bar{\phi_2},\bar{\phi^*}\right\}(x_3)e^{i(kx_1-\omega t)}\,. \quad (17)$$

Here, ω is the angular frequency and k is wave number.

Making use of Eq.(17), Eqs.(12)-(16) reduce to the following relations:

$$(D^2-k^2-\omega^2)\bar{\phi}+a_1\overline{\phi^*}-\bar{T}=0\,, \tag{18}$$

$$(a_2D^2-a_2k^2-\omega^2)\bar{\psi}+a_3\overline{\phi_2}=0\,, \tag{19}$$

$$(D^2-k^2-ga_4-a_5\omega^2)\overline{\phi_2}-a_4(D^2-k^2)\bar{\psi}=0\,, \tag{20}$$

$$(D^2-k^2-a_6)\overline{\phi^*}-a_7(D^2-k^2)\bar{\phi}+a_8\bar{T}=0\,, \tag{21}$$

$$(a_9+\omega)(D^2-k^2)\bar{\chi}-(1-\kappa D^2-\kappa k^2)\bar{\chi}-a_{10}(D^2-k^2)\bar{\phi}+$$

$$+a_{11}\omega\overline{\phi^*}=0\,. \tag{22}$$

The above set of Eqs.(18)-(22) after some simplifications yield:

$$\left[A\boldsymbol{D}^6+B\boldsymbol{D}^4+C\boldsymbol{D}^2+E\right]\bar{\phi}=0\,, \tag{23}$$

$$\left[\boldsymbol{D}^4+F\boldsymbol{D}^2+G\right]\bar{\psi}=0\,, \tag{24}$$

where: $\boldsymbol{D}=\dfrac{d}{dx_3}$; and $A=\dfrac{A_1+A_2\chi}{k_5+a'_{10}\chi}$, $A_1 = k_2 - k_4k_5 - k_5\omega_{11} + a_1a_7k_5 + k_6a'_{10}$, $A_2 = a_8a_{11}\omega + a_1a_8a'_{10} - a_7a_{11}\omega + a'_{10}k^2 + a'_{10}k_4$, $B=\dfrac{B_1+B_2\chi}{k_5+a'_{10}\chi}$, $B_1 = a_8a_{11}a_6\omega - k_2k_4 - \omega_{11}k_2 + \omega_{11}k_4k_5 + a_7k_2 + a_1a_8k_6a'_{10} - a_1a_7k_5k^2 - k_6(\omega a_7a_{11} + a'_{10}k^2 + a'_{10}k_4)$, $B_2 = \omega a_7a_{11}k^2 + a'_{10}k^2k_4 - \omega_{11}a_8a_{11}\omega - a_1a_8a'_{10}k^2$, $C = [k_6(\omega a_7a_{11}k^2 + a'_{10}k^2k_4) - \omega_{11}(\omega a_6a_8a_{11} - k_2k_4) - a_1(a_7k_2k^2 + a_8a'_{10}k_6k^2)]/(k_5 + a'_{10}\chi)$, $E = (a_3a_4 - a_2a_3 - k_7)/a_2$, $F = (k_3k_7 - a_3a_4k^2)/a_2$, $\tau_{11} = (1 + \tau_1 s)$, $\xi_1 = \xi^2 + s^2$.

Also, a_i, $i = 1, \ldots 12$ are defined in appendix A.

The solution of the above system of Eqs.(23)-(24) satisfying the radiation conditions that $(\bar{\phi},\bar{\psi},\bar{\chi},\overline{\phi_2},\overline{\phi^*})\to 0$ as $x_3 \to \infty$ are given as following:

$$\bar{\phi}=\sum_{i=1}^{3}c_ie^{-m_ix_3}\,, \tag{25}$$

$$\overline{\phi^*}=\sum_{i=1}^{3}\alpha_{1i}c_ie^{-m_ix_3}\,, \tag{26}$$

$$\bar{\chi}=\sum_{i=1}^{3}\alpha_{2i}c_ie^{-m_ix_3}\,, \tag{27}$$

$$\left(\bar{\psi},\overline{\phi_2}\right)=\sum_{i=4}^{5}(1,\delta_i)c_ie^{-m_ix_3}\,. \tag{28}$$

Here, m_i^2 (i = 1, 2, 3) are the roots of Eq.(23) and m_i^2 (i = 4, 5) are the roots of characteristic Eq.(24); $\alpha_{1i} = -\Delta_{2i}/\Delta_{1i}$, $\alpha_{2i} = \Delta_{3i}/\Delta_{1i}$, i = 1, 2, 3 and $\delta_i = a_3/(a_2m_i^2 - k_7)$, i = 4, 5.

Here, Δ_{1i}, Δ_{2i}, Δ_{3i} are defined in Appendix B.

Substituting the values of $(\bar{\phi},\bar{\psi},\bar{\chi},\overline{\phi_2},\overline{\phi^*})$ from Eqs.(25) -(28) in Eqs.(5)-(7), and using Eqs.(9)-(11) and Eq.(17) and then solving the resulting equations, we obtain:

$$\bar{t}_{33}=\sum_{i=1}^{5}G_{1i}e^{-m_ix_3}\,, \tag{29}$$

$$\bar{t}_{31}=\sum_{i=1}^{5}G_{2i}e^{-m_ix_3}\,, \tag{30}$$

$$\bar{m}_{32}=\sum_{i=1}^{5}G_{3i}e^{-m_ix_3}\,, \tag{31}$$

$$\lambda_3^*=\sum_{i=1}^{5}G_{4i}e^{-m_ix_3}\,, \tag{32}$$

$$\bar{T}=\sum_{i=1}^{5}G_{5i}e^{-m_ix_3}\,, \tag{33}$$

where: $G_{mi} = g_{mi}C_i$, i, m = 1, 2, … 5; G_{rs} and M_r (r, s = 1, 2, … 5) are described in Appendix C.

BOUNDARY CONDITIONS

We consider that normal force and thermal and mass concentration sources are acting at the surface $x_3 = 0$ along with vanishing of couple stress in addition to thermal and mass concentration boundaries considered at $x_3 = 0$. Mathematically this can be written as

$$t_{33}=-F_1e^{i(kx_1-\omega t)}\,,\ t_{31}=0\,,\ m_{32}=0\,,\ \lambda_3^*=0\,,$$

$$\frac{\partial T}{\partial x_3}=F_2e^{i(kx_1-\omega t)}\,, \tag{34}$$

where: F_1 and F_2 are the magnitude of the applied forces.

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following system of equations,

$$\sum_{i=1}^{6}(G_{1i},G_{2i},G_{3i},G_{4i},m_iG_{5i})c_i=(-F_1,0,0,0,-F_2)\,. \tag{35}$$

The system of Eqs.(35) is solved by using the matrix method as follows,

$$\begin{bmatrix}c_1\\c_2\\c_3\\c_4\\c_5\end{bmatrix}=\begin{bmatrix}g_{11}&g_{12}&g_{13}&g_{14}&g_{15}\\g_{21}&g_{22}&g_{23}&g_{24}&g_{25}\\g_{31}&g_{32}&g_{33}&g_{34}&g_{35}\\g_{41}&g_{42}&g_{43}&g_{44}&g_{45}\\m_1g_{51}&m_2g_{52}&m_3g_{53}&m_4g_{54}&m_5g_{55}\end{bmatrix}^{-1}\begin{bmatrix}-F_1\\0\\0\\0\\-F_2\end{bmatrix}. \tag{36}$$

SPECIAL CASES

(a) Microstretch thermoelastic solid

If we neglect the two-temperature effect in Eq.(35), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar thermoelastic solid with two temperatures

If we neglect the microstretch effect in Eq.(35), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic medium with two-temperatures solid.

(c) Micropolar thermoelastic solid

If we neglect the two-temperature effect and the stretch effect in the final results, we yield the expressions of thermal stresses for the micropolar thermoelastic medium.

NUMERICAL RESULTS AND DISCUSSIONS

The analysis is conducted for a magnesium crystal-like material. The values of the constants are (following Dhaliwal and Singh /19/): $\lambda = 9.4\times10^{10}$ Nm^{-2}, $K = 1.0\times10^{10}$ Nm^{-2}, $\rho = 1.74\times10^3$ kgm^{-3}, $j = 0.2\times10^{-19}$ m^2, $\gamma = 0.779\times10^{-9}$ N, $c^* = 1.04\times10^3$ Jkg^{-1}K^{-1}, $K^* = 1.7\times10^6$ Jm^{-1}s^{-1}K^{-1}, $\alpha_{t1} = 2.33\times10^{-5}$ K^{-1}, $\alpha_{t2} = 2.48\times10^{-5}$ K^{-1}, $T_0 = 0.298\times10^3$ K, $\tau_0 = 0.02$, $\tau_1 = 0.01$, $\alpha_{c1} =$

2.65×10^{-4} m^{3}kg^{-1}, $\alpha_{c2} = 2.83\times10^{-4}$ m^{3}kg^{-1}, $a = 2.9\times10^4$ m^{2}s^{-2}K^{-1}, $b = 32\times10^5$ kg^{-1}m^{5}s^{-2}, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $D = 0.85\times10^{-8}$ kgm^{-3}s, $j_0 = 0.19\times10^{-19}$ m^2, $\alpha_0 = 0.779\times10^{-9}$ N, $b_0 = 0.5\times10^{-9}$ N, $\lambda_0 = 0.5\times10^{10}$ Nm^{-2}, $\lambda_1 = 0.5\times10^{10}$ Nm^{-2}.

A dimensionless form of the field variables for the cases of microstretch thermoelastic medium with two temperatures (MT 2-tmp) subjected to normal force is presented in Figs. 2-8. Values of all physical quantities for both cases are shown in the range $0 \leq x_3 \leq 40$.

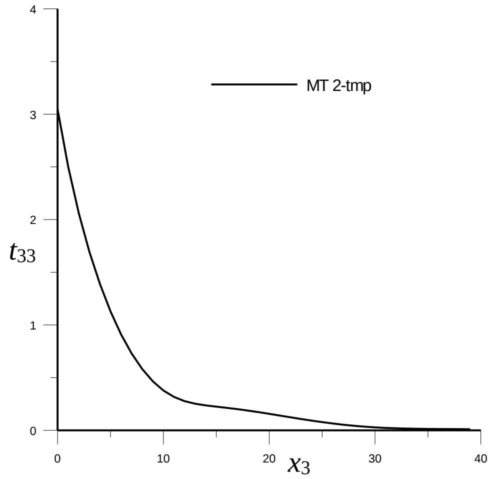

Figure 2. Variation of normal stress.

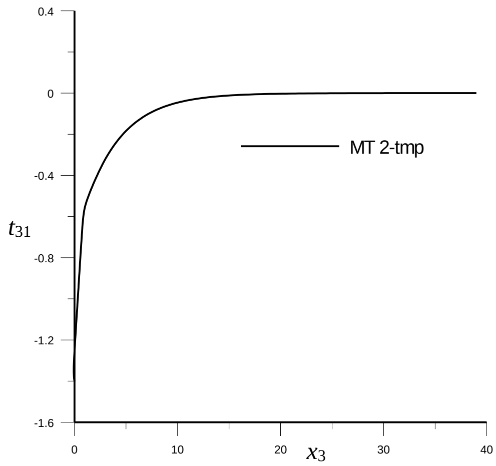

Figure 3. Variation of tangential stress.

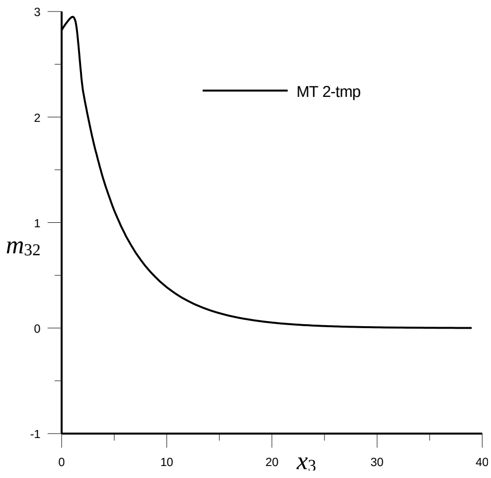

Figure 4. Variation of couple tangential stress.

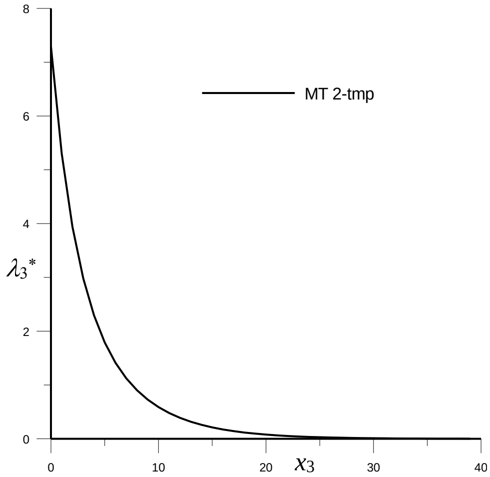

Figure 5. Variation of microstress.

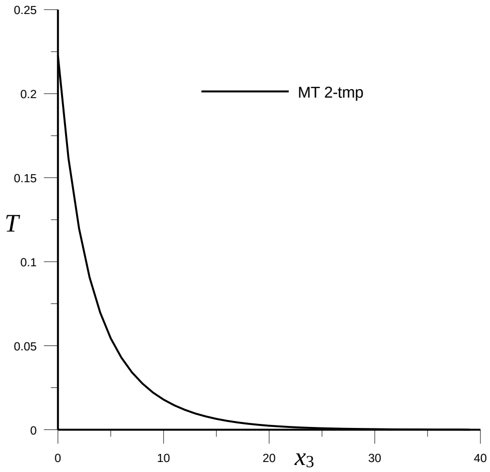

Figure 6. Variation of temperature.

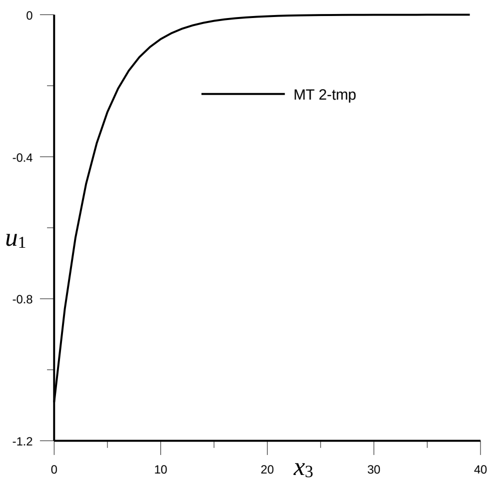

Figure 7. Variation of u_1.

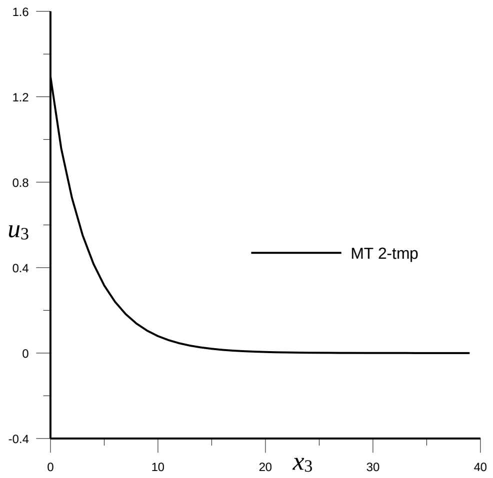

Figure 8. Variation of u_3.

CONCLUSIONS

The problem consists of investigating displacement components, dielectric displacement vector, scalar microstretch, temperature distribution and stress components in a microstretch thermoelastic medium with two temperatures subjected to mechanical forces. Normal mode analysis is employed to express the results. Theoretically obtained field variables are also depicted graphically. Analysis of results permits some concluding remarks:

1. It is clear from the figures that all the field variables have nonzero values only in the bounded region of space indicating that all the results are in agreement with the various theories of thermoelasticity.
2. The trend of variation of physical quantities show similarity with the results of Kumar and Kumar, /9/, after neglecting the two-temperature effect.

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APPENDIX A

$$a_1=\frac{\lambda}{\rho c_1^2},\ a_2=\frac{\mu+K}{\beta_1 T_0},\ a_3=\frac{K}{\rho c_1^2},\ a_4=\frac{Kc_1^2}{\gamma\omega^{*2}},\ a_5=\frac{\rho j c_1^2}{\gamma},\ a_6=\frac{\lambda_1 c_1^2}{\alpha\omega^{*2}}+\frac{\rho j_0 c_1^2}{2\alpha},\ a_7=\frac{\lambda_0 c_1^2}{\alpha\omega^{*2}},\ a_8=\frac{\gamma_1 c_1^2}{\alpha\beta_1\omega^{*2}},\ a_9=\frac{K}{\omega^* K^*},\ a_{10}\frac{\beta_1^2 T_0 c_1}{\rho K^*\omega^{*2}},\ a_{11}\frac{\gamma_1\beta_1 T_0}{\rho K^*\omega^{*2}}$$

$$k_1=a_9+\omega,\ k_2=\chi k^2-1-k_1k^2,\ k_3=k^2+2a_4+a_5\omega^2,\ k_4=k^2+a_6,\ k_5=K+k_1,\ k_6=k^2-1,\ \omega_{11}=k^2+\omega^2,\ a_{10}'=\omega^2 a_{10},\ k_7=\omega^2+a_2k^2,$$

$$k_8=k-k_4k_5+\omega a_8a_{11}\chi,\ k_9=\omega a_8a_{11}k_6-k_2k_4,\ k_{10}=a_1(a_7k_5+a_8a_{10}'\chi),\ k_{11}=a_1(a_7k_2+a_7k_5c-a_8\chi k^2a_{10}'+a_8k_6a_{10}'),\ k_{12}=k^2a_1(a_7k_2+a_8k_6a_{10}'),$$

$$k_{13}=\omega a_{11}a_7+k_4a_{10}'+k^2,\ k_{14}=k^2(a_7\omega a_{11}+a_{10}'k_4)$$

APPENDIX B

$$\Delta_{1i}=\begin{vmatrix} m_i^2-k_4 & -a_8(\chi\omega_i^2+k_6) \\ \omega a_{11} & k_5m_i^2+k_2 \end{vmatrix},\quad \Delta_{2i}=\begin{vmatrix} -a_7(m_i^2-k^2) & -a_8(\chi\omega_i^2+k_6) \\ -a_{10}'(m_i^2-k^2) & k_5m_i^2+k_2 \end{vmatrix},\quad \Delta_{3i}=\begin{vmatrix} -a_7(m_i^2-k^2) & m_i^2-k_4 \\ -a_{10}'(m_i^2-k^2) & \omega a_{11} \end{vmatrix}$$

APPENDIX C

$G_{mi} = g_{mi}C_i,\; C_i = \dfrac{\Delta_i}{\Delta_0},\; i = 1,2,\ldots 5$

also, $g_{1i} = b_1\alpha_{1i} + b_2(m_i^2 - k^2) + b_3 m_i^2 - \beta_1\left(1 - \kappa(m_i^2 - k^2)\right)\alpha_{2i},\; g_{2i} = \iota b_3 k m_i,\; g_{3i} = -\iota b_9 k\alpha_{1i},\; g_{4i} = -\alpha_0 b_{10} m_i \alpha_{1i},\; g_{5i} = \alpha_{2i},\; i = 1,2,3$

and $g_{1l} = -\iota b_3 k m_l,\; g_{2l} = b_6 m_l^2 + b_5 k^2 - b_7\alpha_{3l},\; g_{3l} = -b_8\alpha_{3l} m_l,\; g_{4l} = \iota k b_0 b_{10}\alpha_{3l},\; g_{5l} = 0,\; l = 4,5$

$b_1 = \dfrac{\lambda_0}{\rho c_1^2},\; b_2 = \dfrac{\lambda}{\rho c_1^2},\; b_3 = \dfrac{2\mu + K}{\rho c_1^2},\; b_5 = \dfrac{\mu + K}{\rho c_1^2},\; b_6 = \dfrac{\mu}{\rho c_1^2},\; b_7 = \dfrac{K}{\rho c_1^2},\; b_8 = \dfrac{\omega^{*2}\gamma}{\rho c_1^4},\; b_9 = \dfrac{\omega^{*2} b_0}{\rho c_1^4},\; b_{10} = \dfrac{\omega^{*2}}{\rho c_1^4}.$

ESIS ACTIVITIES

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

January 16, 2020	1st Virtual Conference on Structural Integrity	virtual	https://www.vcsi1.eu/
January 23, 2020	Novel materials and engineering solutions for high temperature power generation plant. FESI	Bristol, UK	link
February 26-28, 2020	MedFract1 1st Mediterranean Conference on Fracture and Structural Integrity	Athens, Greece	http://www.medfract1.eu/
March 25-27, 2020	5th Iberian Conference on Structural Integrity – IbSCI 2020	Coimbra, Portugal	https://ibcsi.pt/
March 30 - April 3, 2020	VAL4, 4th International Conference on Material and Component Performance under Variable Amplitude Loading	Darmstad, Germany	First Announcement
April 1-3, 2020	TC4 Meeting	Twente, The Netherlands	
April 30, 2020	Structural Integrity Developments for a Competitive UK Nuclear Industry	Cambridge, UK	Initial Flyer
May 26-28, 2020	4th International Symposium on Fatigue Design and Material Defects	Potsdam, Germany	link
June 27-28, 2020	7th Summer School on "Fracture Mechanics and Structural Integrity"	Funchal, Madeira, Portugal	https://www.ecf23.eu/
June 29-July 3, 2020	23rd European Conference on Fracture - ECF23	Funchal, Madeira, Portugal	https://www.ecf23.eu/
August 31-September 4, 2020	8th International Conference on Very High Cycle Fatigue (VHCF8)	Sapporo, Hokkaido, Japan	https://www.vhcf8.jp/
September 2-4, 2020	20th International Colloquium on Mechanical Fatigue of Metals	Wrocław, Poland	http://icmfmxx.pwr.edu.pl/
September 6-10, 2020	TC4 Meeting - Fracture of Polymers, Composites and Adhesives	Les Diablerets, Switzerland	link
June 22-24, 2021	LCF9, 9th International Conference on Low Cycle Fatigue	Berlin, Germany	link
September 8-10, 2021	TC15 ESIAM21	Vienna, Austria	