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ELECTROHYDRODYNAMIC THERMAL INSTABILITY IN A POROUS MEDIUM LAYER SATURATED BY A WALTERS' (MODEL B') ELASTICO-VISCOUS NANOFLUID

ELEKTROHIDRODINAMIČKA TERMIČKA NESTABILNOST SLOJA POROZNE SREDINE KOJI JE ZASIĆEN VALTERS (MODEL B) ELASTOVISKOZNIM NANOFLUIDOM

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- nanofluid
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Abstract

In this paper, the effect of vertical AC electric field on the onset of electrohydrodynamic thermal instability in a horizontal layer of an elastico-viscous nanofluid saturated by a porous medium stimulated by the dielectrophoretic force due to the variation of dielectric constant with temperature is investigated. Walters' (model B') elastico-viscous fluid model is used to describe rheological behaviour of nanofluid and for porous medium, the Darcy model is employed. The model used for nanofluid incorporates the effects of thermophoresis and Brownian diffusion. It is assumed that nanoparticle flux is zero on the boundaries. Linear stability analysis based on perturbation theory and normal mode analysis method is applied. The resulting eigenvalue problem is solved for isothermal free-free boundaries analytically and numerically by using the Galerkin method. For the case of stationary convection, it is observed that Walters' (model B') elastico-viscous nanofluid behaves like an ordinary nanofluid. The oscillatory convection does not exist under the realistic boundary conditions.

Ključne reči
- nanofluid
- električno polje naizmenične struje
- Rejlejev broj
- Valters (model B)
- elektrohidrodinamika

Izvod

U radu je istražen uticaj vertikalnog električnog polja naizmenične struje na elektrohidrodinamičku termičku nestabilnost u horizontalnom sloju elastoviskoznog nanofluida zasićenog poroznom sredinom stimulisanom dielektroforeznom silom, usled promene dielektrične konstante sa temperaturom. Primenjen je Valters (model B) elastoviskozni model fluida za opisivanje reološkog ponašanja nanofluida, a Darsi model za poroznu sredinu. Model nanofluida sadrži efekte termoforeze i Braunove difuzije. Pretpostavlja se da je fluks nanočestica jednak nuli na granicama. Urađena je analiza linearne stabilnosti na bazi teorije perturbacije i analiza normalnog moda. Rezultujući problem sopstvenih vrednosti je rešen analitički i numerički za izotermalne slobodne-slobodne granice primenom metode Galerkina. U slučaju stacionarne konvekcije, uočava se da se Valters (model B) elastoviskozni nanofluid ponaša kao običan nanofluid. Oscilatorna konvekcija ne postoji u uslovima realističnih granica.

INTRODUCTION

Electrohydrodynamic thermal instability in a porous medium is a phenomenon related to various fields. It has various applications in different areas such as EHD enhanced thermal transfer, EHD pumps, EHD in microgravity, micromechanic systems, drug delivery, micro-cooling system, nanotechnology, oil reservoir modelling, petroleum industry, building of thermal insulation, biomechanics, engineering etc. Chandrasekhar /1/ has given a comprehensive account of thermal instability of Newtonian fluid under the various assumptions of hydrodynamics and hydromagnetics. The investigation in porous media has been started with the Darcy model. A good account of convection problems in a porous medium is given in /2-4/. Electrodynamics of continuous media and electrohydrodynamic convection in fluids has been studied by /5–7/. Electrohydrodynamics is a branch of fluid mechanics which deals with the motion of fluid under the influence of electrical forces. It can also be considered as that part of electrodynamics which is necessitated with the influence of moving media on electric fields. Electrohydrodynamics involve both the effect of fluid in motion and the influence of the field in motion /8-9/.

Nanofluid was first coined by Choi /10/. Nanofluid is a mixture of nano-sized metallic particles immersed in common fluids such as water, ethanol or engine oils, typically used as base fluids in nanofluids and the nanoparticles may be taken as oxide ceramics such as Al_2O_3 or CuO, nitride ceramics such as AlN or SiN and several metals such as Al or Cu. Nanofluid has various applications in automotive industries, energy saving etc. Further, suspensions of nanoparticles are being developed in medical applications including cancer therapy. The detailed study of thermal convection in a layer of nanofluid in porous medium based upon Buongiorno /11/ model has been discussed by different authors /12-22/.

All the studies referred above deal with Newtonian nanofluids. However, with the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry attracted widespread interest in the study on non-Newtonian nanofluids. One such type of fluids is Walters' (model B') /22/ elastico-viscous fluid having relevance in chemical technology and industry. Walters' (model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products. A good account of thermal instabilities problems in a Walters' (Model B') elastico-viscous fluid in a porous medium is given in /24-27/.

Recently, considerable interest has been evinced in the study of electrohydrodynamic thermal instability in viscous and viscoelastic fluid. Takashima /28/ discussed the effect of uniform rotation on the onset of convective instability in a dielectric fluid under the simultaneous action of AC electric field. The onset of electrohydrodynamic instability in a horizontal layer of viscous and viscoelastic fluid was studied by /29-36/.

The growing number of applications of electrohydrodynamic thermal instability in an elastico-viscous nanofluid fluid in a porous medium which include several engineering and medical fields, such as automotive industries, energy saving and cancer therapy, motivated the current study. Our main aim is to study the effect of vertical AC electric field on the onset of thermal instability problem in a horizontal layer of an elastico-viscous Walters' (Model B') nanofluid in a porous medium.

FORMULATION OF THE PROBLEM AND MATHEMATICAL MODEL

Here we consider an infinite horizontal porous layer of a Walters' (model B') elastico-viscous nanofluid of thickness d, bounded by the planes $z = 0$ and $z = d$ and subject to a uniform vertical AC electric field applied across the layer; the lower surface is grounded and the upper surface is kept at an alternating (60 Hz) potential whose root mean square value is V_1 (see Fig. 1). The layer is heated from below, which is acted upon by a gravity force $\mathbf{g} = (0, 0, -g)$ aligned in the z direction. The temperature, T, and the volumetric fraction of nanoparticles, φ, at the lower (upper) boundary is assumed to take constant values T_0, and $\boldsymbol{\varphi_0}$ (T_1, and φ_1), respectively. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation. However, we assumed these conditions, which have also been previously adopted by several authors /12-19/.

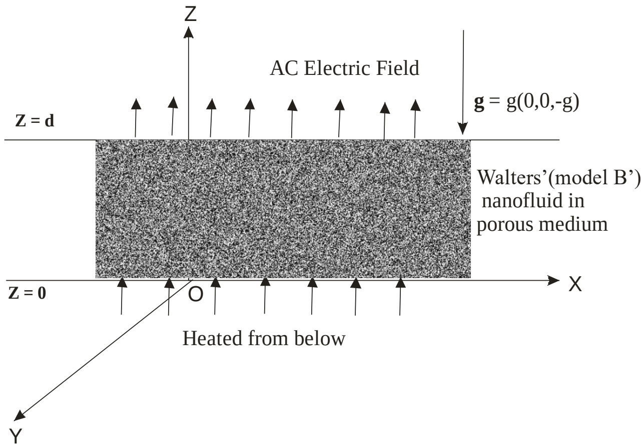

Figure 1. Physical configuration.

Assumptions

The mathematical equations describing the physical model are based upon the following assumptions
- all thermophysical properties, except for the density in the buoyancy term, are constant (Boussinesq hypothesis);
- base fluid and nanoparticles are in thermal equilibrium state;
- nanofluid is incompressible and laminar;
- negligible radiative heat transfer;
- size of nanoparticles is small as compared to pore size of the matrix;
- nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix;
- the temperature and volumetric fraction of the nanoparticles are constant on the boundaries;
- the base fluid of the nanofluid is a Walters' (model B') elastico-viscous fluid;
- there is no nanoparticle flux at the plate and that the particle fraction value adjusts accordingly.

Governing equations

Let T_{ij}, τ_{ij}, e_{ij}, μ, μ', p, δ_{ij}, q_i, x_i and d/dt denote, respectively, the total stress tensor, shear stress tensor, rate-of-strain tensor, viscosity, viscoelasticity, isotropic pressure, Kronecker delta, velocity vector, position vector and convective derivative. Then the Walters' (model B') elastico-viscous fluid is described by the constitutive relations

$$T_{ij} = -p\delta_{ij} + \tau_{ij}\,, \tag{1}$$

$$\tau_{ij} = 2\left(\mu - \mu'\frac{d}{dt}\right)e_{ij}\,, \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right). \tag{3}$$

The above relations were proposed and studied by Walters' /18/.

The equations of mass-balance and momentum-balance for Walters' (model B') elastico-viscous with vertical AC electric field /1, 23-27/ under the Oberbeck-Boussinesq approximation in a porous medium are

$$\nabla.\mathbf{q} = 0 \tag{4}$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\nabla P+\rho\mathbf{g}-\frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\nabla K-$$
$$-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q} \qquad (5)$$

where: $\mathrm{P}=p-\frac{\rho}{2}\frac{\partial K}{\partial \rho}(\mathbf{E}\cdot\mathbf{E})$ is the modified pressure /27/ and ρ, μ, μ', p, ε, k_1, $\mathbf{E}$, K, $\mathbf{q}(u,v,w)$ denote respectively, density, viscosity, viscoelasticity, pressure, medium porosity, medium permeability, root mean square value of the electric field and Darcy velocity vector, respectively.

The density ρ of the nanofluid can be written /11/ as

$$\rho=\varphi\rho_p+(1-\varphi)\rho_f \qquad (6)$$

where: φ is the volume fraction of nanoparticles; ρ_p is the density of nano particles; and ρ_f is the density of base fluid. We approximate the density of the nanofluid by that of the base fluid, that is we consider $\rho = \rho_f$ /13, 15/. Now introducing the Boussinesq approximation for the base fluid, the specific weight, $\rho\mathbf{g}$ in Eq.(5) becomes

$$\rho\mathbf{g}\simeq\left\{\varphi\rho_p+(1-\varphi)\left[\rho(1-\alpha(T-T_0))\right]\right\}\mathbf{g}\,, \qquad (7)$$

where: α is the coefficient of thermal expansion.

If one introduces a buoyancy force, the equation of motion for Walters' (model B') nanofluid by using Boussinesq approximation and Darcy model for porous medium (e.g. /18/) is given by

$$0=-\nabla P+\left\{\varphi\rho_p+(1-\varphi)\left[\rho\left(1-\alpha(T-T_0)\right)\right]\right\}\mathbf{g}-$$
$$-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q}-\frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\nabla K\,, \qquad (8)$$

The mass-balance equation for the nanoparticles (Buongiorno /11/) is

$$\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_0}\nabla^2 T\,. \qquad (9)$$

The thermal energy equation for a nanofluid is

$$(\rho c)_m\left[\frac{\partial T}{\partial t}+\mathbf{q}\cdot\nabla T\right]=k_m\nabla^2 T+$$
$$+\varepsilon(\rho c)_p\left(D_B\nabla\varphi\cdot\nabla T+\frac{D_T}{T_0}\nabla T\cdot\nabla T\right), \qquad (10)$$

where: $(\rho c)_m$ is heat capacity of fluid in porous medium; $(\rho c)_p$ is heat capacity of nanoparticles; and k_m is thermal conductivity.

The Maxwell equations are

$$\nabla\times\mathbf{E}=0\,, \qquad (11)$$
$$\nabla\cdot(K\mathbf{E})=0\,. \qquad (12)$$

Let V be root mean square value of electric potential. The electric potential can be expressed as

$$\mathbf{E}=-\nabla V\,. \qquad (13)$$

The dielectric constant is assumed to be a linear function of temperature and is of the form

$$K=K_0[1-\gamma(T-T_0)]\,, \qquad (14)$$

where: $\gamma > 0$, is the thermal coefficient of expansion of dielectric constant and is assumed to be small.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. The boundary conditions /1, 18/ are

$$w=0,\ T=T_0,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_0}\frac{\partial T}{\partial z}=0 \text{ at } z=0$$
$$\text{and } w=0,\ T=T_1,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \text{ at } z=d. \qquad (15)$$

We introduce non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right),\ (u',v',w')=\left(\frac{u,v,w}{\kappa_m}\right)d\,,\ t'=\frac{t\kappa}{\sigma d^2}\,,$$
$$P'=\frac{Pk_1}{\mu\kappa_m}\,,\ \varphi'=\frac{\varphi-\varphi_0}{\varphi_1-\varphi_0}\,,\ T'=\frac{T-T_1}{T_0-T_1}\,,\ K'=\frac{K}{\gamma E_0\Delta Td}\,,$$

where: $\kappa_m=\frac{k_m}{(\rho c_P)_f}$ is thermal diffusivity of the fluid; and $\sigma=\frac{(\rho c_P)_m}{(\rho c_P)_f}$ is the thermal capacity ratio. Eliminating the modified pressure from the momentum-balance Eq.(8) by operating twice curl and retaining the vertical component, we obtain the equations in non-dimensional form (after dropping the dashes (') for convenience) as

$$\nabla\cdot\mathbf{q}=0\,, \qquad (16)$$

$$0=-\nabla p-\left(1-\mathrm{F}\frac{\partial}{\partial t}\right)\mathbf{q}-\mathrm{Rm}\hat{e}_z+\mathrm{Ra}\,T\hat{e}_z-$$
$$-\mathrm{Rn}\,\varphi\hat{e}_z+\mathrm{R_{ea}}\nabla_H^2\left(T-\frac{\partial K}{\partial z}\right), \qquad (17)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}q.\nabla\varphi=\frac{1}{\mathrm{Le}}\nabla^2\varphi+\frac{\mathrm{N_A}}{\mathrm{Le}}\nabla^2 T\,, \qquad (18)$$

$$\frac{\partial T}{\partial t}+\mathbf{q}\cdot\nabla T=\nabla^2 T+\frac{\mathrm{N_A}}{\mathrm{Le}}\nabla\varphi\cdot\nabla T+\frac{\mathrm{N_A N_B}}{\mathrm{Le}}\nabla T\cdot\nabla T\,, \qquad (19)$$

$$\nabla^2 V=\frac{\partial T}{\partial z}\,. \qquad (20)$$

Here

$$\mathrm{Le}=\frac{\kappa_m}{D_B}\,, \qquad (21)$$

$$\mathrm{F}=\frac{\mu'\kappa_m}{\mu\sigma d^2}\,, \qquad (22)$$

$$\mathrm{Ra}=\frac{\rho g\alpha dk_1(T_0-T_1)}{\mu\kappa_m}\,, \qquad (23)$$

$$\mathrm{Rm}=\frac{\rho_p\varphi_0+\rho(1-\varphi_0)gk_1d}{\mu\kappa_m}\,, \qquad (24)$$

$$\mathrm{Rn}=\frac{(\rho_p-\rho)(\varphi_1-\varphi_0)gk_1d}{\mu\kappa_m}\,, \qquad (25)$$

$$\mathrm{N_A}=\frac{D_T(T_0-T_1)}{D_B T_1(\varphi_1-\varphi_0)}\,, \qquad (26)$$

$$\mathrm{N_B}=\frac{\varepsilon(\rho c)_p(\varphi_1-\varphi_0)}{(\rho c)_f}\,, \qquad (27)$$

$$\mathrm{R_{ea}} = \frac{\gamma^2 K E_0^2 d^2 (\Delta T)^2}{\mu \kappa_m}, \quad (28)$$

denote respectively: thermal Lewis number; kinematic visco-elasticity parameter; density Rayleigh number; nanoparticle Rayleigh number; modified diffusivity ratio; AC electric Rayleigh number; and modified particle-density ratio. ∇^2_H is the two-dimensional Laplace operator in the horizontal plane, that is $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

The dimensionless boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial K}{\partial z} = 0,\ T = 1,\ \frac{\partial \phi}{\partial z} + \mathrm{N_A}\frac{\partial T}{\partial z} = 0 \ \text{ at } z = 0$$

$$\text{and } w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial K}{\partial z} = 0,\ T = 0,\ \frac{\partial \phi}{\partial z} + \mathrm{N_A}\frac{\partial T}{\partial z} = 0 \ \text{ at } z = 1 \quad (29)$$

Basic solutions

We assume that the basic state is quiescent /15, 16, 18, 34, 36, 38/ and is given by

$$u = v = w = 0,\ p = p(z),\ K = K_b(z),\ T = T_b(z),$$
$$\varphi = \varphi_b(z),\ E = E_b(z),\ \psi = \psi_b(z),$$
$$T_b = T_0 - \frac{\Delta T}{d} z,\ \phi_b = \phi_0 + \left(\frac{D_T \Delta T}{D_B T_1 d}\right) z, \quad (30)$$
$$K_b = K_0\left(1 + \frac{\gamma \Delta T}{d} z\right)\hat{k},\ E_b = \frac{E_0}{1 + \frac{\gamma\Delta T}{d} z}\hat{k}.$$

Also, we have

$$V_b(z) = -\frac{E_0 d}{\gamma \Delta T}\log\left(1 + \frac{\gamma \Delta T}{d}\right)\hat{k},$$

where: $E_0 = -\dfrac{V_1 \frac{\gamma \Delta T}{d}}{\log(1 + \gamma \Delta T)}$ is the root mean square value of the electric field at $z = 0$.

The basic state defined in Eqs.(30) is substituted into Eqs.(18) and (19), these equations reduce to

$$\frac{d^2 \varphi_b(z)}{dz^2} + \mathrm{N_A}\frac{d^2 T_b(z)}{dz^2} = 0, \quad (31)$$

$$\frac{d^2 T_b(z)}{dz^2} + \frac{\mathrm{N_B}}{\mathrm{Le}}\frac{d\varphi_b(z)}{dz}\frac{dT_b(z)}{dz} + \frac{\mathrm{N_A N_B}}{\mathrm{Le}}\left(\frac{dT_b(z)}{dz}\right)^2 = 0. \quad (32)$$

Using boundary conditions Eq.(29) in Eqs.(31) and (32), the integration of Eq.(31) gives

$$\frac{d\varphi_b(z)}{dz} + \mathrm{N_A}\frac{dT_b(z)}{dz} = 0. \quad (33)$$

Using Eq.(22) in Eq.(32), we obtain

$$\frac{d^2 T_b(z)}{dz^2} = 0. \quad (34)$$

Applying the boundary conditions Eq.(29), the solution of Eq.(23) is given by

$$T_b(z) = 1 - z. \quad (35)$$

Integrating Eq.(33) by applying boundary conditions Eq.(29), we get

$$\varphi_b(z) = \phi_0 + \mathrm{N_A} z. \quad (36)$$

These results are identical with the results obtained by Sheu /16/ and Nield and Kuznetsov /15, 18/.

Perturbation solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, so that

$$\mathbf{q}(u,v,w) = q''(u,v,w),\ T = T_b + T'',\ K = K_b + K'',$$
$$p = p_b + p'',\ E = E_b + E'' + V = V_b + V''. \quad (37)$$

Introducing Eq.(37) into Eqs.(16)-(20), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities, and dropping the primes ('') for convenience, the following equations are obtained

$$\nabla \cdot \mathbf{q} = 0, \quad (38)$$

$$0 = -\nabla P - \left(1 - F\frac{\partial}{\partial t}\right)\mathbf{q} + \mathrm{Ra}\, T\hat{e}_z - \mathrm{Rn}\,\varphi\hat{e}_z + \mathrm{R_{ea}}\,\nabla_H^2\left(T - \frac{\partial K}{\partial z}\right) \quad (39)$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{\mathrm{Le}}\nabla^2 \varphi + \frac{\mathrm{N_A}}{\mathrm{Le}}\nabla^2 T, \quad (40)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{\mathrm{N_B}}{\mathrm{Le}}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2\mathrm{N_A N_B}}{\mathrm{Le}}\frac{\partial T}{\partial z}, \quad (41)$$

$$\nabla^2 V = \frac{\partial T}{\partial z}. \quad (42)$$

Boundary conditions for Eqs.(38)-(42) are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial V}{\partial z} = 0,\ T = 0,\ \frac{\partial \phi}{\partial z} + \mathrm{N_A}\frac{\partial T}{\partial z} = 0$$
$$\text{at } z = 0 \text{ and at } z = 1. \quad (43)$$

Normal mode analysis

We express the disturbances into normal modes of the form

$$[w, T, \varphi, V] = [\mathrm{W}(z), \Theta(z), \Phi(z), \Psi(z)]\exp(ilx + imy + \omega t), \quad (44)$$

where: l, m are the wave numbers in the x and y direction, respectively; and ω is the growth rate of the disturbances.

Substituting Eq.(44) into Eqs.(38)-(42), we obtain the following eigenvalue problem

$$(D^2 - a^2)(1 - \omega F)\mathrm{W} + a^2\,\mathrm{Ra}\,\Theta - a^2\,\mathrm{Rn}\,\Phi + a^2\,\mathrm{R_{ea}}(\Theta - D\Psi) = 0 \quad (45)$$

$$\frac{1}{\varepsilon}\mathrm{W} + \mathrm{N_{CT}}(D^2 - a^2)\Theta + \frac{1}{\mathrm{Le}}\left(D^2 - a^2 - \frac{\omega}{\sigma}\right)\Gamma = 0, \quad (46)$$

$$\mathrm{W} + \left(D^2 + \frac{\mathrm{N_A}}{\mathrm{Le}}D - \frac{2\mathrm{N_A N_B}}{\mathrm{Le}}D - a^2 - \omega\right)\Theta - \frac{\mathrm{N_B}}{\mathrm{Le}}D\Phi = 0, \quad (47)$$

$$\frac{1}{\varepsilon}\mathrm{W} - \frac{\mathrm{N_A}}{\mathrm{Le}}(D^2 - a^2)\Theta - \left(\frac{1}{\mathrm{Le}}(D^2 - a^2) - \frac{\omega}{\sigma}\right)\Phi = 0, \quad (48)$$

$$(D^2 - a^2)\Psi = D\Theta.$$

where: $D = d/dz$ and $a^2 = l^2 + m^2$ is the dimensionless horizontal wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$\mathrm{W} = 0,\ D^2\mathrm{W} = 0,\ \Theta = 0,\ D\Psi = 0,\ D\Phi + \mathrm{N_A}D\Theta = 0$$
$$\text{at } z = 0 \text{ and } z = 1. \quad (50)$$

METHOD OF SOLUTION

The Galerkin-type weighted residuals method is used to find an approximate solution of the system of Eqs.(45)-(49) with the corresponding boundary conditions Eqs.(50). In this method the test functions are the same as the base (trial) functions. Thus, we can write

$$\mathrm{W}=\sum_{s=1}^{N} A_s \mathrm{W}_s\,,\ \ \Theta=\sum_{s=1}^{N} B_s\Theta_s\,,$$
$$\Phi=\sum_{s=1}^{N} C_s\Phi_s\,,\ \ \Psi=\sum_{s=1}^{N} D_s\Psi_s\,, \qquad (51)$$

where: A_s, B_s, C_s and D_s are unknown coefficients; s = 1, 2, 3, …, N; and base functions W_s, Θ_s, Φ_s and Ψ_s satisfy boundary conditions Eqs.(50). Using expression for W, Θ, Φ and Ψ in Eqs.(45)-(49) and multiplying the first equation by W_s, second by Θ_s, third by Φ_s and fourth by Ψ_s; then integrating between limits 0 to 1, we obtain a set of 3N homogeneous equations with 3N unknowns A_s, B_s, C_s and D_s; s = 1, 2, 3, … N. For the existence of non-trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in terms of Rayleigh number, Ra.

Linear stability analysis and dispersion relation

We have considered the case of free-free boundaries for which the system of Eqs.(45)-(49) together with boundary conditions Eqs.(50) constitute a linear eigenvalue problem with variable coefficient for the growth rate of disturbance of the system. The resulting eigenvalue problem is solved numerically by the Galerkin method of first order (N = 1), which gives the expression for Rayleigh number Ra as

$$\mathrm{Ra}=\frac{(1-\omega \mathrm{F})(\pi^2+a^2)(\pi^2+a^2+\omega)}{a^2}-$$
$$-\frac{\varepsilon \mathrm{N_A}(\pi^2+a^2)+\mathrm{Le}(\pi^2+a^2+\omega)}{(\pi^2+a^2)\sigma+\omega \mathrm{Le}}\cdot \qquad (52)$$
$$\cdot\frac{\sigma}{\varepsilon}\mathrm{Rn}-\frac{a^2}{(\pi^2+a^2)}\mathrm{R_{ea}}$$

Equation (52) is the required dispersion relation accounting for the effect of Lewis number, kinematic viscoelasticity parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio on thermal instability in a layer of Walters' (model B') elastico-viscous nanofluid saturating a porous medium under vertical AC electric field.

For neutral stability, the real part of ω is zero. Hence on putting $\omega = i\omega$ (where ω is real and is a dimensionless frequency) in Eq.(52), we have

$$\mathrm{Ra}=\Delta_1+i\omega\Delta_2\,, \qquad (53)$$

where

$$\Delta_1=\frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\,\mathrm{F})}{a^2}-\frac{a^2}{(\pi^2+a^2)}\mathrm{R_{ea}}-$$
$$-\frac{(\pi^2+a^2)^2\left(\mathrm{N_A}+\dfrac{\mathrm{Le}}{\varepsilon}\right)+\dfrac{\omega^2}{\sigma\varepsilon}}{(\pi^2+a^2)^2+\left(\dfrac{\omega \mathrm{Le}}{\sigma}\right)^2}\mathrm{Rn} \qquad (54)$$

and

$$\Delta_2=\frac{(\pi^2+a^2)\left(1-\mathrm{F}(\pi^2+a^2)\right)}{a^2}+$$
$$+\frac{(\pi^2+a^2)\left(\dfrac{\mathrm{Le}}{\sigma}\left(\mathrm{N_A}+\dfrac{\mathrm{Le}}{\varepsilon}\right)-\dfrac{\mathrm{Le}}{\varepsilon}\right)+\dfrac{\omega^2}{\sigma\varepsilon}}{(\pi^2+a^2)^2+\left(\dfrac{\omega \mathrm{Le}}{\sigma}\right)^2}\mathrm{Rn}. \qquad (55)$$

Since Ra is a physical quantity, so it must be real. Hence, it follows from Eq.(52) that either ω = 0 (exchange of stability, steady state) or Δ_2 = 0 ($\omega \neq 0$ overstability or oscillatory onset).

STATIONARY CONVECTION

Since oscillatory convection has been ruled out, because of the absence of two opposing buoyancy forces, we need to consider only the case of stationary convection. Put ω = 0 in Eq.(52), we obtain

$$\mathrm{Ra}=\frac{(\pi^2+a^2)^2}{a^2}-\frac{a^2}{(\pi^2+a^2)}\mathrm{R_{ea}}-\left(\mathrm{N_A}+\frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}\,. \qquad (56)$$

Equation (56) expresses the Rayleigh number as a function of the dimensionless resultant wave number a and parameters $\mathrm{R_{ea}}$, ε, Rn, Le, $\mathrm{N_A}$. Since the elastico-viscous parameter F vanishes with ω, the Walters' (model B') elastico-viscous nanofluid fluid behaves like an ordinary Newtonian nanofluid. Equation (56) is identical to that obtained by Kuznetsov and Nield /14/, and Rana and Chand /36/. Also, in Eq.(56) the particle increment parameter $\mathrm{N_B}$ does not appear, and the diffusivity ratio parameter $\mathrm{N_A}$ appears only in association with the nanoparticle Rayleigh number Rn. This implies that the nanofluid cross-diffusion terms approach is to be dominated by the regular cross-diffusion term.

In the absence of AC electric field $\mathrm{R_{ae}}$, Eq.(56) reduces to

$$\mathrm{Ra}=\frac{(\pi^2+a^2)^2}{a^2}-\left(\mathrm{N_A}+\frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}\,, \qquad (57)$$

which is identical with the result derived by Kuznetsov and Nield /10/, Rana et al. /35/, and Rana and Chand /36/.

The critical cell size at the onset of instability is obtained by minimizing Ra with respect to a. Thus, the critical cell size must satisfy

$$\left(\frac{\partial \mathrm{Ra}}{\partial a}\right)_{a=a_c}=0\,,$$

Eq.(56) which gives $a_c = \pi \cong 3.1416$.

The minimum of first term of right-hand side of Eq.(56) is attained at $a_c = \pi$ and minimum value found $4\pi^2$, so the corresponding critical Rayleigh number given by

$$(\mathrm{Ra})_c=4\pi^2-\left(\mathrm{N_A}+\frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}\,. \qquad (58)$$

This is the same result derived by Nield and Kuznetsov /15/.

In the absence of nanoparticles (Rn = Le = $\mathrm{N_A}$ = 0) and AC vertical electric field (Re = 0), Eq.(58) reduces to

$$\mathrm{Ra}=4\pi^2\,. \qquad (59)$$

which is a well-known result of critical Rayleigh number.

Thus, presence of the nanoparticles lowers the value of the critical Rayleigh number usually by substantial amount. Also, parameter N_B does not appear in Eq.(56), thus instability is purely a phenomenon due to buoyancy coupled with conservation of nanoparticles. Thus, average contribution of nanoparticles flux in the thermal energy equation is zero with one-term Galerkin approximation.

To study the effect of AC electric Rayleigh number, Lewis number, nanoparticle Rayleigh number, modified diffusivity ratio and medium porosity, we examine the behaviour of $\partial\mathrm{Ra}/\partial\mathrm{R_{ea}}$, $\partial\mathrm{Ra}/\partial\mathrm{Le}$, $\partial\mathrm{Ra}/\partial\mathrm{N_A}$, $\partial\mathrm{Ra}/\partial\mathrm{Rn}$ and $\partial\mathrm{Ra}/\partial\varepsilon$ analytically.

From Eq.(56), we obtain

$$\frac{\partial \mathrm{Ra}}{\partial \mathrm{R_{ea}}}=-\frac{a^2}{(\pi^2+a^2)}, \tag{60}$$

which is negative, thereby implying the AC electric field inhibits the stationary convection of the system, thereby implying the AC electric field has destabilizing effect on the system which is in an agreement with the results derived by Takashima /31/ and Shivakumara /34/, Rana et al. /35/ and Rana and Chand /36/.

From Eq.(56), we obtain

$$\frac{\partial \mathrm{Ra}}{\partial \mathrm{Le}}=-\frac{\mathrm{Rn}}{\varepsilon}, \tag{61}$$

which is negative if Rn is positive. But for bottom-heavy nanoparticle distribution, Rn is negative, thereby implying Lewis number advances the stationary convection, thereby implying Lewis number has stabilizing effect on the system which is in an agreement with the results derived by Sheu /16/.

From Eq.(56), we obtain

$$\frac{\partial \mathrm{Ra}}{\partial \mathrm{N_A}}=-\mathrm{Rn}, \tag{62}$$

which is positive if Rn is negative, thereby implying modified diffusivity ratio stabilizing effect on the system.

From Eq.(56), we obtain

$$\frac{\partial \mathrm{Ra}}{\partial \mathrm{Rn}}=-\left(\mathrm{N_A}+\frac{\mathrm{Le}}{\varepsilon}\right), \tag{63}$$

which is negative, thereby implying nanoparticle Rayleigh number inhibits the electroconvection, thereby implying nanoparticle Rayleigh number has destabilizing effect on the system which is in an agreement with the results derived by Nield and Kuznetsov /15, 18/, Chand et al. /22/ and Chand and Rana /23/.

From Eq.(56), we obtain

$$\frac{\partial \mathrm{Ra}}{\partial \varepsilon}=\frac{\mathrm{LeRn}}{\varepsilon^2}, \tag{64}$$

which is negative if Rn is negative, thereby implying medium porosity has a destabilizing effect on the system.

RESULTS AND DISCUSSIONS

The thermal Rayleigh number at the onset of stationary convection is given by Eq.(56) and does not depend on the viscoelastic parameter. It takes the same value as the one obtained for an ordinary Newtonian fluid. Furthermore, the critical wave number, a_c, at the onset of steady convection coincides with that reported by Tzou /13/, Nield and Kuznetsov /15/. Note that this critical value does not depend on any thermophysical property of the nanofluid. Consequently, the interweaving behaviours of Brownian motion and thermophoresis of nanoparticles does not change the cell size at the onset of steady instability and the critical cell size a_c is identical to the well-known result for Bénard instability with a regular fluid /1/.

According to the definition of the nanoparticle Rayleigh number Rn in Eq.(25), this corresponds to negative value of Rn for bottom-heavy distribution of nanoparticles ($\varphi_1 < \varphi_0$ and $\rho_p > \rho$). In such cases, values of N_A are also negative according to Eq.(26). The value of the critical Rayleigh number for the nanofluid is larger than that for an ordinary fluid, that is, convection sets earlier in an ordinary fluid than in a nanofluid with bottom-heavy distribution of nanoparticles. This implies that thermal conductivity of this kind of nanofluids is higher than that of ordinary fluids. In the following discussion, negative values of Rn and N_A are presented.

The dispersion relation Eq.(56) is analysed numerically. Graphs have been plotted by giving some numerical values to the parameters to depict the stability characteristics. Stability curves for AC electric Rayleigh number R_{ea}, Lewis number Le, nanoparticles Rayleigh number Rn, modified diffusivity ratio N_A, and porosity parameter ε are shown in Figs. 2-6.

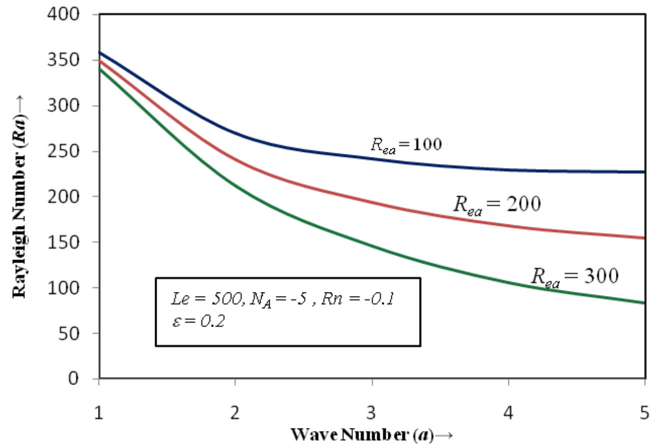

Figure 2. Variations of thermal Rayleigh number Ra with wave number a for values of AC electric Rayleigh number R_{ea} =100, 200 and 300.

Variations of thermal Rayleigh number Ra with the wave number a for three values of AC electric Rayleigh number, namely, R_{ea} = 100, 200 and 300 is plotted in Fig. 2 and it is observed that the thermal Rayleigh number decreases with increase in AC electric Rayleigh number, thereby implying AC electric Rayleigh number destabilizes the system.

In Fig. 3, variations of thermal Rayleigh number Ra with wave number a for three values of nanofluid Lewis number, namely, Le = 500, 1000 and 1500, shows that the thermal Rayleigh number increases with the increase in the Lewis number. Thus, the Lewis number has a stabilizing effect on the system.

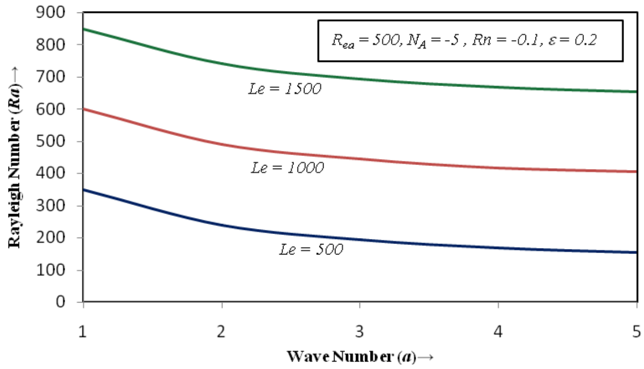

Figure 3. Variations of thermal Rayleigh number Ra with wave number *a* for Lewis number Le = 500, 1000, and 1500.

Variations of thermal Rayleigh number Ra with the wave number *a* for three different values of modified diffusivity ratio, namely, N_A = -5, -45, -85 are plotted in Fig. 4 and it is found that the thermal Rayleigh number increases with the modified diffusivity ratio, thereby implying that the modified diffusivity ratio has a stabilizing effect on the system. For negative values of N_A, an increase of N_A reduces the thermophoresis effect of pushing the heavier nanoparticles upwards. As a result, the stabilizing effects of particle distributions are enhanced. Thus, the effect of increasing N_A is to stabilize the system when R_N is negative.

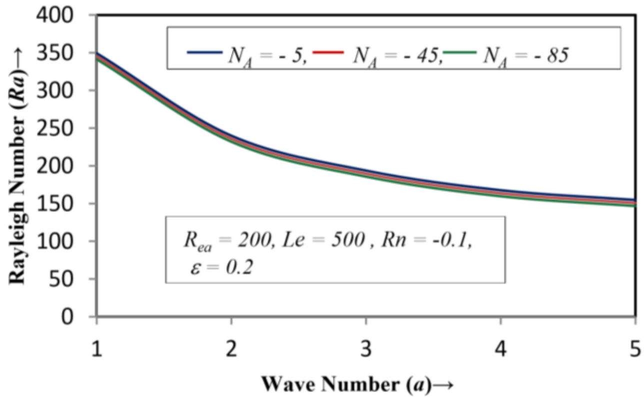

Figure 4. Variations of thermal Rayleigh number Ra with wave number *a* for modified diffusivity ratio N_A = -5, -45 and -85.

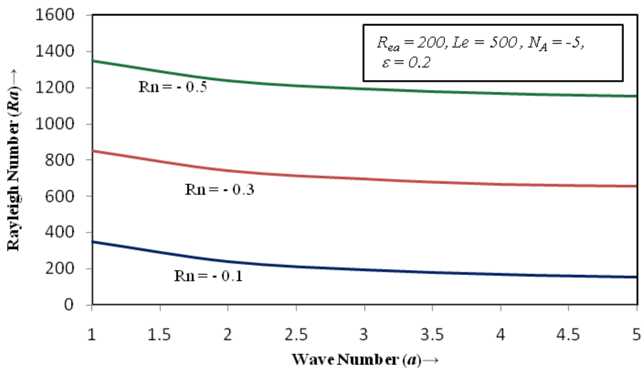

Figure 5. Variations of thermal Rayleigh number Ra with wave number *a* for nanoparticle Rayleigh number Rn = -0.1, -0.3, -0.5.

In Fig. 5, the variations of thermal Rayleigh number Ra with wave number *a* for three different values of nanoparticle Rayleigh number, namely Rn = -0.1, -0.3, -0.5, shows that the thermal Rayleigh number increases with the decrease in nanoparticle Rayleigh number. Thus, nanoparticle Rayleigh number has a destabilizing effect on the system. Variations of thermal Rayleigh number Ra with wave number *a* for three different values of medium porosity, namely ε = 0.2, 0.4 and 0.6, are plotted in Fig. 6 and it is found that thermal Rayleigh number decreases with the increase in medium porosity, thereby implying medium porosity has a destabilizing effect on the onset of stationary convection in a layer of Walters' (model B') elastico-viscous nanofluid saturating a porous medium.

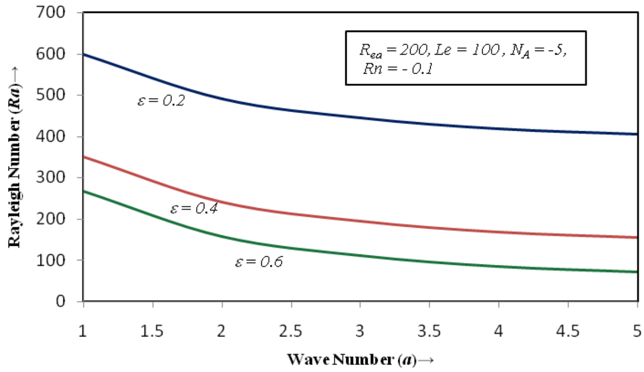

Figure 6. Variations of thermal Rayleigh number Ra with wave number *a* for medium porosity ε = 0.2, 0.4 and 0.6.

CONCLUSION

Electrohydrodynamic thermal instability in a porous medium layer of Walters' (model B') elastico-viscous nanofluid in a porous medium has been investigated by using a linear stability analysis and Galerkin method. The Walters' (model B') elastico-viscous nanofluid incorporates the Brownian motion and thermophoresis. The main conclusions of the present analysis are:

– for the case of stationary convection, the Walters' (model B') nanofluid behaves like an ordinary Newtonian nanofluid,
– kinematic viscoelasticity has no effect on the onset of stationary convection,
– the AC electric field Rayleigh Number has a destabilizing effect on the stationary convection,
– the Lewis number and modified diffusivity ratio have a stabilizing effect,
– nanoparticle Rayleigh number and medium porosity have a destabilizing effect on the system,
– oscillatory convection has been ruled out under the more realistic boundary conditions (e.g. Nield and Kuznetsov /18/).

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