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TWO-DIMENSIONAL MECHANICAL STRESSES FOR A PRESSURIZED CYLINDER MADE OF FUNCTIONALLY GRADED MATERIAL

DVODIMENZIONALNI MEHANIČKI NAPONI ZA CILINDAR POD PRITISKOM OD FUNKCIONALNOG KOMPOZITNOG MATERIJALA

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Abstract

In the initial years of development of functionally graded materials, its use is confined to heat resisting materials. But with the passage of time, their use has grown to many other fields of nature and engineering structures. In functionally graded material the material properties vary along the surface of the material. The research papers related to one-dimensional problem is solved for annular disk, cylinders and plates but few have attempted for two-dimensional. In this paper, we have developed the solutions for two-dimensional pressurized cylinder made of functionally graded material with nonlinear variation of Young's modulus in the radial direction and its numerical analysis is done.

Izvod

U prvim godinama razvoja funkcionalnih kompozitnih materijala, njihova primena je bila ograničena na materijale za rad na povišenim temperaturama. Tokom vremena se primena proširila na mnoge druge oblasti i inženjerske konstrukcije. Kod funkcionalnih kompozitnih materijala osobine se menjaju duž površine materijala. Problemi jednodimenzionalnog slučaja se rešavaju za prstenasti disk, cilindre i ploče, ali je redak pokušaj u slučaju dvodimenzionalnih problema. U radu smo razvili rešenja za dvodimenzionalni cilindar pod pritiskom od funkcionalnog kompozitnog materijala sa nelinearnom varijacijom Jungovog modula u radijalnom pravcu, a obuhvaćena je i numerička analiza.

INTRODUCTION

Functionally graded materials (FGMs) are those materials which are formed of two or more materials and the variation is smooth in transitioning from one surface to another. Earlier, the idea of functionally graded material was introduced to make a thermal barrier, /2/. Two-dimensional elasticity problem for long FGM hollow cylinder is solved analytically under steady-state mechanical and thermal loading by Jabbari et al. /3/. The power law distributions for ceramic volume fraction are considered for 2D functionally graded material by Aragh and Hedayati, /1/. The variation of mechanical properties along different directions helps engineers to design a flexible structure to meet its multipurpose requirements /1/. Nejad and Fatehi /4/ gave a systematic prediction to the partially plastic stress reactions of FGM thick-walled cylindrical pressure vessels with fixed ends due to uniform internal and external pressure in radial direction.

Ruhi et al. /5/ showed a semi-analytical solution for thermoelasticity of finite length thick-walled FGM cylinder. He converted the governing equation of partial differential equations to ordinary differential equations assuming Fourier series in the axial variable. Horgan and Chan /6/ solved the problem of linear non-homogeneous isotropic elastostatics in pressurized hollow cylinder or disc. They observed the consequences of material diversity in essential boundary-value problem. Safari et al. /7/ have discussed the progress in the behaviour of thermoelastic stresses in a finite length functionally graded thick hollow cylinder using the technique of Laplace transform and power series method under the load of thermal shock.

Gharibi et. al /8/ have applied Frobenius series method to analyze the elastic nature of cylindrical pressure vessels made of functionally graded material under the consideration that material properties are changing exponentially. A general analytical solution for 1D steady state thermal stresses in FGM thick-walled cylinder is developed by Jabbari et. al /9/. Dehghan et al. /10/ obtained the 3D multi-field equations of functionally graded piezoelectric shells of revolution under thermomechanical loading. Ma and Wang /11/ observed the post-buckling and nonlinear bending of annular/circular plates composed of functionally graded material with thermal and mechanical loadings using higher order shear deformation and classical theories on plates. Green's function approach based on the laminate theory is used to discuss 2D unsteady thermoelastic problems of infinite length FG hollow cylinder /12-13/. Mehditabar and Rahimi /14/ have worked out for the axisymmetric dynamic problem of spinning FG piezoelectric hollow cylinder under vigorous load by using the finite difference method (FDM) and numerical differential quadrature method. Mahbadi /15/ derived equations for stress potency factor of functionally graded solid cylinders carrying radial fracture, situated at edge or inside the cylinder, by an approximation method. Stress potency factors for both plane stress and plane strain conditions of a spinning cylinder subjected to thermomechanical loading is derived by him. Nejad et al. /16/ have done analysis on rotating thick cylindrical shell made of axially functionally graded material subjected to non-uniform internal pressure by utilizing multi-layered method.

Bose and Rattan /17/ examined steady state creep behaviour of isotropic rotating disc of FGM and this functionally graded material follows parabolic variation in the existence of thermal gradient. Sahni and Mehta /18/ have solved a problem by using iterative method of a functionally graded annular disk with thickness varying in the quadratic and cubic form and have evaluated radial stresses, hoop stresses and strains. Moheimani et al. /19/ used nonlinear kinematic hardening model and return mapping algorithm for the problem. They investigated a thick-walled functionally graded cylinder subjected to internal pressure and loading of temperature gradient under plane strain condition. Gupta and Talha /20/ have presented an ample review associated to structural response of FGMs and structures. Sburlati /21/ developed an analytical solution representing the effects of the different profiles narrating the graded properties of the materials on the displacement fields and stresses. In addition to that, the author shows the comparison between graded coating and conventional homogeneous coating. This comparison emphasizes the benefit of the graded material on the interface stress reduction. Xu et al. /22/ analysed the dynamics response of thick hollow 2D FGM cylinder in the time domain. They showed the effectiveness of the spectral finite element method for the evaluation of elastic wave propagation in functionally graded solids with symmetry along axial direction. He further investigated that volume fraction has major consequence on the nature of structural wave propagation. Nkene et al. /23/ have followed numerical and analytical way to develop solutions for strains, stresses and displacements in a rotating hollow cylinder with internal and external loading where walls of cylinder are constructed by using functionally graded material. They also demonstrated the effects of separate profiles addressing the gradual change of mechanical properties in the deformation of the cylinder.

Ebrahimi and Najafizadeh /24/ have solved a problem on the vibration of 2D functionally graded right circular cylindrical shells using Love's first approximation shell theory. Some investigations on the study of static and dynamic analyses of structures formed of 2D FGM have been done by many researchers /25-30/. The solutions of elasticity are obtained for functionally graded rotating solid and annular disks with varying thickness by Zafarmand and Hassani /31/. The analysis of creep nature of pressurized tank made of inhomogeneous material is presented /32/. Thawait /33/ displayed the effects of functional gradation in the parameters of material properties on the stress distribution of shell for two, metal-ceramic and ceramic-metal functionally graded material. Ying and Wang /34/ have derived an exact solution to examine the elasto-dynamic behaviour of finite length hollow cylinder with non-uniform thermal shock where cylinder is fixed at two ends and is independent of traction at the internal and external surfaces of cylinder.

Ghannad and Parhizkar /35/ obtained a solution of thermoelasticity for the steady state response of thick-walled cylinder with pressure and external heat flux in inner surface of cylinder. Durmuş et al. /36/ highlighted the effects of changing Poisson's ratio on the dissemination of stresses and displacements for pressurized thick-walled cylindrical and spherical vessel made of functionally graded material with the change in power law properties. Naki and Beytullah /37/ used complementary function method and plane elasticity theory to demonstrate the axisymmetric displacements and stresses in functionally graded hollow cylinders, spheres and disks subjected to uniform internal loading. Celal and Müfit /38/ investigated a long FGM hollow cylinder subjected to uniform heat generation and internal pressure. Sharma et al. /39/ in 2013 using Seth transition theory solved the elastic-plastic problem under temperature variation. Sahni et al. /40/ has done creep stress analysis for a rotating cylinder made of silicon carbide particle varying exponentially along the radii. With variable thickness, the behaviour of a rotating disc was studied by Sahni et al. /41/. The numerical discussion and its graphical representation was shown for rotating disc and cylinder with variable thickness, Poisson and Young's modulus by Sahni /42-43/.

In this paper, we have derived analytical solutions for two-dimensional pressurized cylinder made of FGMs with nonlinear variation of Young's modulus in the radial direction by using power series method and its numerical analysis is done. The Poisson ratio is taken as constant. We have demonstrated the effects of variations in Young's modulus on the displacements and stress distribution. Partial differential equations are written in terms of radial and axial displacements utilizing governing equations for axisymmetric 2D FGM (isotropic) hollow cylinder and strain displacement relations. Further, displacements functions are expressed in sine series and obtained partial differential equations are solved by power series method. Application of such cylinders can be seen in the formation of CNG cylinder and other gas cylinders in the aerospace industry.

Problem description

A functionally graded axisymmetric hollow cylinder of finite length L and inner radius r_a and outer radius r_b (Fig. 1) is considered for analysis. Cylindrical coordinate system is considered for the cylinder, where axes r, θ and z represent radial, circumferential and axial directions in respect. The cylinder is subjected to internal and external pressures $p_a(z)$ and $p_b(z)$ respectively and simply supported on its two ends. Poisson's ratio is taken as constant for this functionally graded material. In addition to that, we choose modulus of elasticity Y of FGM with smooth and continuous variation over the thickness of the cylinder and follow the nonlinear form as

$$Y(r) = Y_0 r_b^{\alpha \log(r)}, \ \alpha \neq 0 \quad (1)$$

where: Y_0 is constant and α can be positive or negative, representing non homogenous material constants.

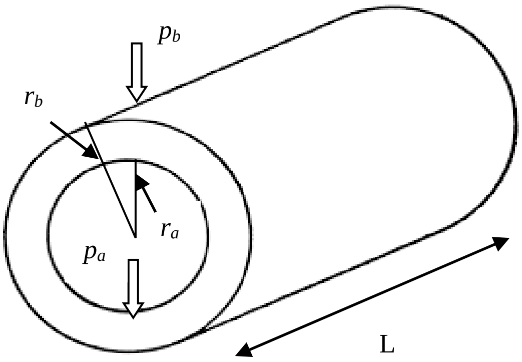

Figure 1. Functionally graded cylinder.

In the case of small deformation, strain displacement equations defined as /5/:

$$\varepsilon_{rr}=\frac{\partial u_1}{\partial r},\ \varepsilon_{\theta\theta}=\frac{u_1}{r},\ \varepsilon_{zz}=\frac{\partial u_2}{\partial z},\ \varepsilon_{rz}=\frac{1}{2}\left(\frac{\partial u_1}{\partial z}+\frac{\partial u_2}{\partial r}\right), \qquad (2)$$

where: u_1 and u_2 are the radial and axial displacement components. On the other side, ε_{rr}, $\varepsilon_{\theta\theta}$, ε_{zz} and ε_{rz} represent the radial-, tangential-, axial- and shear strain respectively.

Constitutive equations for FGM cylinder are /44/

$$\sigma_{rr}=2\eta\varepsilon_{rr}+\rho(\varepsilon_{rr}+\varepsilon_{\theta\theta})\,, \qquad (3a)$$

$$\sigma_{\theta\theta}=2\eta\varepsilon_{\theta\theta}+\rho(\varepsilon_{\theta\theta}+\varepsilon_{zz})\,, \qquad (3b)$$

$$\sigma_{zz}=2\eta\varepsilon_{zz}+\rho(\varepsilon_{zz}+\varepsilon_{rr})\,, \qquad (3c)$$

$$\sigma_{rz}=2\eta\varepsilon_{rz}\,, \qquad (3d)$$

where: σ_{mn} and $\varepsilon_{mn}(m,\ n = r,\ \theta,\ z)$ are the stress and strain tensors; and ρ, η are Lame's coefficients. Lame's constants in terms of Young's modulus $Y(r)$ and Poisson's ratio μ are defined as /45/:

$$\rho=\frac{\mu Y(r)}{(1+\mu)(1-2\mu)}\,, \qquad (4a)$$

$$\eta=\frac{Y(r)}{(1+\mu)}\,. \qquad (4b)$$

Using Newton's second law, the equilibrium equations for an infinitesimal element of axisymmetric FGM cylinder in cylindrical coordinates of radial and circumferential directions, ignoring the body forces and inertia, are /45/

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0\,, \qquad (5a)$$

$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=0\,. \qquad (5b)$$

Boundary conditions for the problem can be expressed as /39/

$$(\sigma_{zz},u)=(0,0)\quad \text{at}\quad z=0,L \qquad (6a)$$

$$(\sigma_{rr},\sigma_{rz})=(p_a(z),0)\quad \text{at}\quad r=r_a \qquad (6b)$$

$$(\sigma_{rr},\sigma_{rz})=(p_b(z),0)\quad \text{at}\quad r=r_b \qquad (6c)$$

where: $p_a(z)$ and $p_b(z)$ are internal and external pressures varying periodically in axial direction. Moreover σ_{rr}, σ_{zz}, and σ_{rz} are radial, axial, and shear stresses.

Using boundary conditions, this problem can be treated as a boundary value problem and the series solution can be applied to get the analytical solution.

From Eqs.(1), (2), (3) and (4), one can obtain

$$\frac{1-2\mu}{2-2\mu}\frac{\partial^2 u_1}{\partial z^2}+\frac{\mu\alpha\log r_b}{1-\mu}\frac{1}{r}\frac{\partial u_2}{\partial z}+\frac{1}{2-2\mu}\frac{\partial^2 u_2}{\partial z\partial r}+\frac{\partial^2 u_1}{\partial r^2}+$$
$$+(\alpha\log r_b+1)\frac{1}{r}\frac{\partial u_1}{\partial r}+\left(\frac{\mu}{1-\mu}\alpha\log r_b-1\right)\frac{u_1}{r^2}=0 \qquad (7a)$$

$$\frac{2-2\mu}{1-2\mu}\frac{\partial^2 u_2}{\partial z^2}+\left(\frac{1}{1-2\mu}+\alpha\log r_b\right)\frac{1}{r}\frac{\partial u_1}{\partial z}+$$
$$+\frac{1}{1-2\mu}\frac{\partial^2 u_1}{\partial z\partial r}+\frac{\partial^2 u_2}{\partial r^2}+(\alpha\log r_b+1)\frac{1}{r}\frac{\partial u_2}{\partial r}=0 \qquad (7b)$$

Furthermore, since pressures are varying periodically, we can represent internal and external pressure by following the sine series to the boundary conditions Eqs.(5b) and (5c), thus one obtains

$$\left(p_a(z),p_b(z)\right)=\left(\sum_0^\infty A_n\sin(\gamma z),\sum_0^\infty B_n\sin(\gamma z)\right),$$

where:

$$\gamma=\frac{n\pi}{L}\,,\ A_n=\frac{2}{L}\int_0^L p_a(z)\sin(\gamma z)dz\,,\ B_n=\frac{2}{L}\int_0^L p_b(z)\sin(\gamma z)dz\,.$$

Without loss of any generality, solutions of Eq.(7) satisfying simply supported boundary conditions Eq.(6) can be expressed as

$$u_1=\sum_{n=0}^{\infty}\phi_n(r)\sin(\gamma z)\,, \qquad (8a)$$

$$u_2=\sum_{n=0}^{\infty}\xi_n(r)\sin(\gamma z)\,. \qquad (8b)$$

Substituting Eqs.(8) into Eqs.(7) results into

$$\left(\frac{1-2\mu}{2-2\mu}\right)\frac{\partial^2\phi_n}{\partial z^2}+\frac{\mu\alpha\log r_b}{1-\mu}\frac{1}{r}\frac{\partial\xi_n}{\partial z}+\frac{1}{2-2\mu}\frac{\partial^2\xi_n}{\partial z\partial r}+$$
$$+\frac{\partial^2\phi_n}{\partial r^2}+\left(\alpha\log r_b+1\right)\frac{1}{r}\frac{\partial\phi_n}{\partial r}+\left(\frac{\mu\alpha\log r_b}{1-\mu}-1\right)\frac{\phi_n}{r^2}=0 \qquad (9a)$$

$$\frac{2-2\mu}{1-2\mu}\frac{\partial^2\xi_n}{\partial z^2}+\left(\frac{1}{1-2\mu}+\alpha\log r_b\right)\frac{1}{r}\frac{\partial^2\xi_n}{\partial z}+$$
$$+\frac{\partial^2\xi_n}{\partial r^2}+\left(\alpha\log r_b+1\right)\frac{1}{r}\frac{\partial\xi_n}{\partial r}+\frac{1}{1-2\mu}\frac{\partial^2\phi_n}{\partial z\partial r}=0 \qquad (9b)$$

Solutions of Eq.(9) can be expressed as

$$\phi_n(r)=\sum_0^\infty C_k r^{s+k}\,,\ \ \xi_n(r)=\sum_0^\infty D_k r^{s+k}\,. \qquad (10)$$

Substituting Eqs.(10) into Eqs.(9) and assuming the coefficients of r^{s+k} equal to zero, then the following recurrence relations are obtained

$$C_k=\frac{\left(\frac{1-2\mu}{2-2\mu}\right)\gamma^2 C_{k-2}+\left(\frac{\gamma(s+k-1)}{2-2\mu}+\frac{\mu\alpha\gamma\log r_b}{1-\mu}\right)D_{k-1}}{(s+k)(s+k+\alpha\log r_b)+\frac{\alpha\log r_b\mu}{1-\mu}-1} \qquad (11a)$$

$$D_k=\frac{\left(\frac{2-2\mu}{1-2\mu}\right)\gamma^2 D_{k-2}-\left(\frac{\gamma(s+k-1)}{1-2\mu}+\frac{\gamma}{1-2\mu}+\gamma\alpha\log r_b\right)C_{k-1}}{(s+k)^2+(s+k)\alpha\log r_b} \qquad (11b)$$

where: $C_{k1} = 0$ and $D_{k2} = 0$ for $k_1, k_2 = -1, -2$.

Equations (11) are valid for arbitrary k. For $k = 0$, Eqs.(11) result into nontrivial solutions as given below

$$s^2 + s\alpha \log r_b + \frac{\alpha\mu \log r_b}{1-\mu} - 1 = 0\,, \tag{12a}$$

$$s^2 + s\alpha \log r_b = 0\,. \tag{12b}$$

Solving Eqs.(12), we get

$$s_1, s_2 = \frac{1}{2}\left(-\alpha \log r_b \pm \sqrt{(\alpha \log r_b)^2 - 4\left(\frac{\mu\alpha \log r_b}{1-\mu} - 1\right)}\right), \tag{13a}$$

$$s_3 = 0,\quad s_4 = -\alpha \log r_b\,. \tag{13b}$$

If material constant α is not an integer, solutions of Eqs.(9) can be expressed as

$$\phi_n(r) = \sum_{i=1}^{4}\sum_{k=0}^{\infty} \psi_i C_k(s_i) r^{s_i+k}\,,$$

$$\xi_n(r) = \sum_{i=1}^{4}\sum_{k=0}^{\infty} \psi_i D_k(s_i) r^{s_i+k}\,,$$

where: $C_k(s_i)$, $D_k(s_i)$ are eigenvectors corresponding to eigenvalues s_i ; and k is an integer.

Also ψ_1, ψ_2, ψ_3 and ψ_4 are unknown constants and can be determined from boundary conditions Eqs.(6). For $k = 0$ the eigenvectors corresponding to four eigenvalues can be written as

$$C_0(s_i) = D_0(s_j) = 1 \quad \text{for } i = 1,2 \text{ and } j = 3,4$$

$$C_0(s_i) = D_0(s_j) = 0 \quad \text{for } i = 1,2 \text{ and } j = 3,4.$$

Recurrence relations from Eqs.(11) can be used to derive eigenvectors for $k > 0$.

NUMERICAL DISCUSSION

A cylinder made up of functionally graded material is considered. For analysis, the numerical values are taken as $r_a = 0.5$, $r_b = 0.9$ and $L = 10$. Values of material constants are considered as $Y_0 = 100$, $\mu = 0.3$, $\alpha = 0.6$. We assume $p_a = 100\sin(\pi z/L)$, $p_b = 0$.

For simplification, we can consider the following non-dimensional parameters in numerical calculations

$$\tilde{r} = \frac{r}{L},\ \tilde{z} = \frac{z}{L},\ \tilde{\sigma}_{rr} = \frac{\sigma_{rr}}{Y_0},\ \tilde{\sigma}_{\theta\theta} = \frac{\sigma_{\theta\theta}}{Y_0},\ \tilde{\sigma}_{zz} = \frac{\sigma_{zz}}{Y_0},\ \tilde{\sigma}_{rz} = \frac{\sigma_{rz}}{Y_0}\,.$$

By assuming above numerical values, normal stresses - radial, hoop and axial are depicted in Figs. 2-4, and shear stress is drawn in Fig. 5.

In Fig. 2, we observe that as radius changes from 0.05 to 0.09, i.e. moving from inner to outer radii and without change in axial length, the radial stress decreases in magnitude but is compressive in nature. The same behaviour is seen throughout the axial length from inner to outer radii as the external surface is free from pressure and the internal surface is subjected to periodic pressure. In Fig. 3, hoop stress against radial and axial values is plotted. The hoop stress acts as a resistance against component failure. The radial stress pulls the components outward, caused by internal pressure but the circumferential stress resists, so that the components remain intact. It is observed from Fig. 3 that the circumferential stress is higher than that of the radial stress. It can also be seen in Table 1, which depicts the all the stresses - radial, hoop, axial and shear. Axial and shear stresses are shown in Figs. 4 and 5. Axial and shear stresses are much smaller than circumferential and radial stresses. From Fig. 5, the shear stress decreases from inner to outer radii and the same behaviour is seen throughout the axial length.

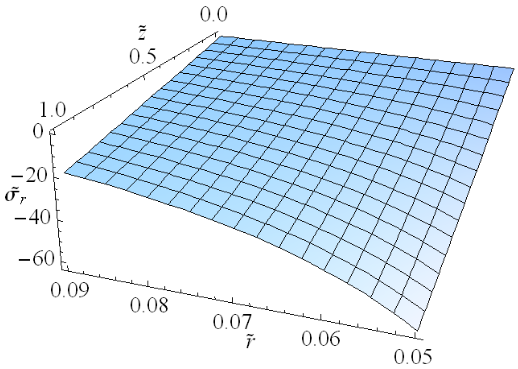

Figure 2. Dimensionless radial stress at $\alpha = 0.6$.

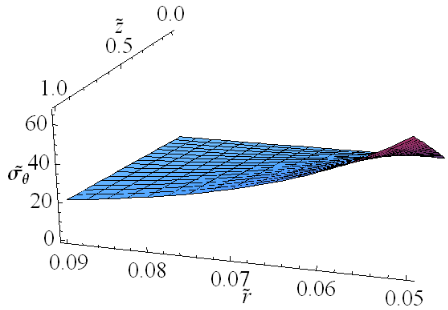

Figure 3. Dimensionless hoop stress $\alpha = 0.6$.

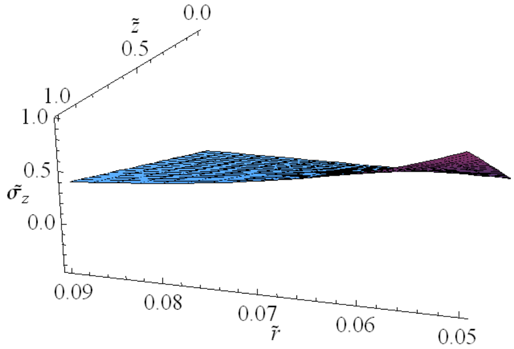

Figure 4. Dimensionless axial stress $\alpha = 0.6$.

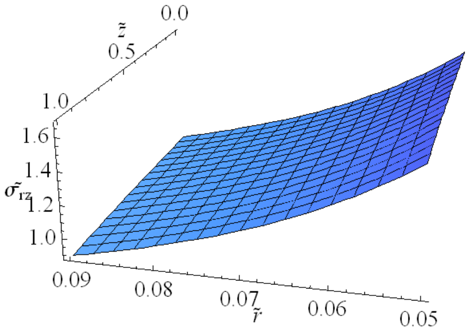

Figure 5. Dimensionless shear stress $\alpha = 0.6$.

There are some small variations in the stresses that are not visible properly by above graphs. To see these variations, we constructed Table 1 for the increasing values of (r, z). It is seen that values of the circumferential stress are always greater than the radial stress. In Table 1, we can see the variation in dimensionless stresses for some random increasing points on radial and axial axes.

Table1. Dimensionless stresses for α = 0.6 at some random increasing values of (r, z).

r z	.05 .1	.0512 .12	.0513 .15	.0514 .6	.0812 .65	.0879 .9	.09 1
$\tilde{\sigma}_{rr}$	-6.5329	-7.4348	-9.2079	-35.9121	-14.9962	-17.3871	-18.296
$\tilde{\sigma}_{\theta\theta}$	6.6261	7.6070	9.517	38.2762	16.5049	19.4609	20.592
$\tilde{\sigma}_{zz}$	-0.3032	-0.2788	-0.2375	0.3844	0.1399	0.3173	0.3873
$\tilde{\sigma}_{rz}$	1.6715	1.6313	1.6284	1.6131	1.0052	0.9136	0.8850

From Table 1 we observe that initially the radial stress is rising from -35.9121 and falling down to -14.9962 and again up to -17.3871. This is because of the periodic applied pressure at the internal surface. This shows radial stress is not monotonically increasing or decreasing. Hoop stresses are increasing from -6.5329 to 38.2762, but decrease to 16.5049 at (0.0812, 0.65). Axial stresses increase to 0.3844 and then fall down to 0.1399 and climb to 0.3173. We can investigate that hoop stresses have more amount of variation compared to other stresses. Shear stresses are continuously decreasing along the increasing radial and axial points.

One can see variation in radial stress as α vary for some positive and negative values.

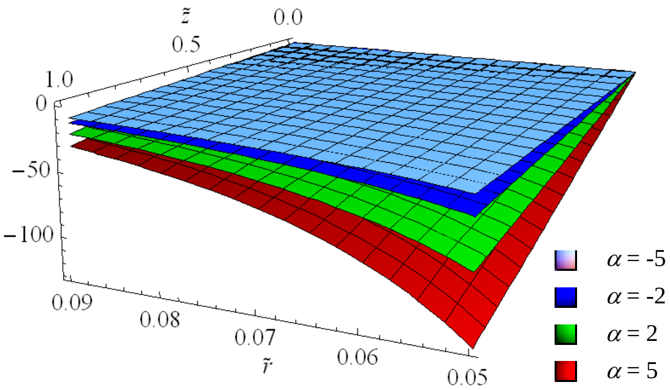

Figure 7. Dimensionless radial stresses at α = -5; -2; 2; 5.

In Fig. 7, peak values of radial stress are increasing in compressive nature as α increases from -5 to 5 because the Young's modulus value is higher for α = 5. The rigidity increases; hence the radial stress increases the resistance stress. If we want to reduce the radial stress then we choose a lower value of α. But peak values of hoop stresses are rising in tensile nature as α increases (Fig. 8). From Fig. 9, it can be observed that the axial stress has more variations at α =5 comparatively for other values of α. In Fig. 10, values of shear stresses at α = -5, -2 are smaller than the values at α = 2, 5.

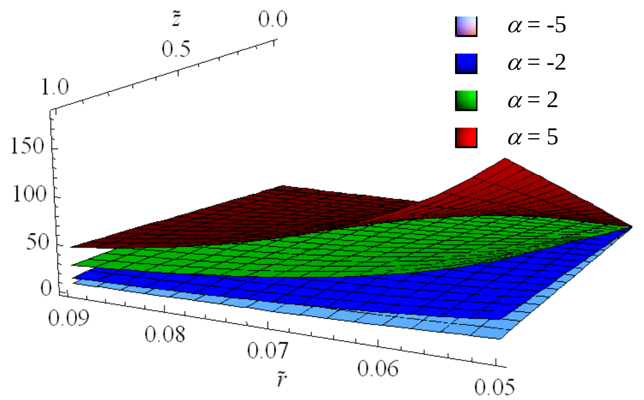

Figure 8. Dimensionless hoop stresses at α = -5, -2, 2, 5.

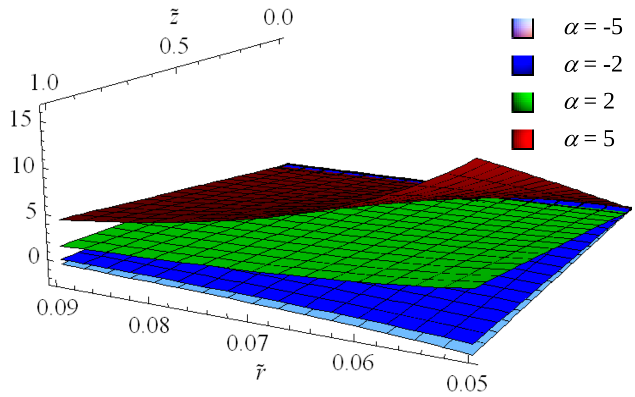

Figure 9. Dimensionless axial stresses at α = -5, -2, 2, 5.

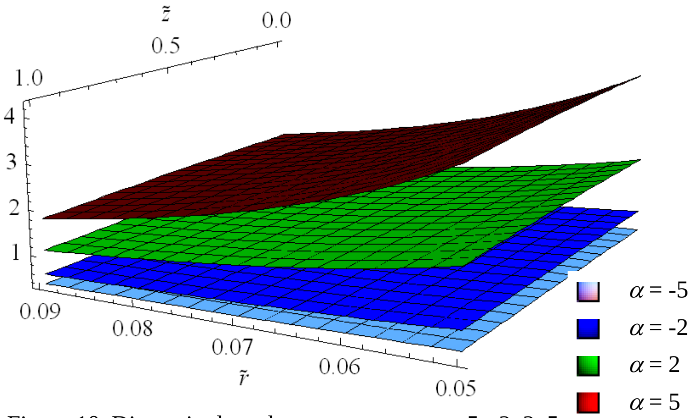

Figure 10. Dimensionless shear stresses at α = -5, -2, 2, 5.

CONCLUSION

In this paper, the analytical solutions for the mechanical stress and displacement in a finite length functionally graded hollow cylinder due to 2D axis-symmetric steady-state load are obtained. The power series method is used to derive the solutions. The study of the behaviour of cylinder under variation of Young's modulus is done. The analysis shows that radial stresses increases for increasing values of α. Hoop stress acts as a restoring force and is always larger than the radial stress. For small value of α, the rigidity is less and hence the hoop stress is less which saves a lot of material usage. The optimum design of a cylinder can be made depending on the strength required.

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