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# NUMERICAL AND EXPERIMENTAL STRESS ANALYSIS OF LAYERED COMPOSITE STRUCTURES SUBJECT TO MECHANICAL AND HYGROTHERMAL LOADS

# NUMERIČKA I EKSPERIMENTALNA ANALIZA NAPONSKOG STANJA VIŠESLOJNIH KOMPOZITNIH STRUKTURA POD DEJSTVOM MEHANIČKOG I HIDROTERMIČKOG OPTEREĆENJA

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**Keywords**
- layered composites
- mechanical properties
- hygrothermal effects
- finite elements

*Abstract*

*Research is focused to stress and strength analysis of layered composite structures under mechanical and thermo-mechanical loads. This investigation has been carried out within the framework of laminated plate theory. The stress analysis of laminated plates or shells is carried out by finite element method (FEM). The second aspect of this work considers the change performance of layered fibrous CFC composite structures due to temperature and moisture. The presented finite element analysis, computations and experimental results give a general procedure for stress analysis of layered composite structures under combined mechanical and hygrothermal loads. Mechanical and thermal properties are determined experimentally. The numerical simulation by FEM uses the classical diffusion equation of Fickian type and theory of moisture diffusion to take into account the hygrothermal behaviour of the material. Numerical and experimental results show that the influence of hygrothermal effects on stress and strength of layered composite structures is very important.*

**Ključne reči**
- višeslojni kompozitni materijali
- mehaničke karakteristike
- hidrotermički efekti
- konačni elementi

*Izvod*

*Pažnja u radu je usmerena na analize naponskog stanja i čvrstoće višeslojnih kompozitnih struktura pod dejstvom mehaničkih i termomehaničkih opterećenja. Ovo istraživanje se odvijalo u okviru teorije višeslojnih ploča i ljuski. Za analize naponskih stanja korišćen je metod konačnih elemenata (MKE). Drugi aspekt ovog rada odnosi se na promene mehaničkih i termičkih karakteristika višeslojnih vlaknima ojačanih CFC kompozitnih materijala zbog uticaja temperature i vlage. Primenjeni metod konačnih elemenata, proračunski i eksperimentalni rezultati daju opštu procedure za analizu naponskog stanja višeslojnih kompozitnih struktura pod dejstvom mehaničkog i hidrotermičkog opterećenja. Mehaničke i termičke karakteristike su dobijene eksperimentalno. Numerička simulacija primenom MKE koristi klasičnu jednačinu difuzije Fikovog tipa i teoriju difuzije vlažnosti radi uzimanja u obzir hidrotermičko ponašanje materijala. Numerički i eksperimentalni rezultati pokazuju veoma važan hidrotermički uticaj na naponsko stanje i čvrstoću višeslojnih kompozitnih struktura.*

## INTRODUCTION

Compared to conventional metal structures, fibrous composite materials continue to experience increased application in aerospace structures due to their superior strength and stiffness to weight ratios; however, due to material anisotropy, analysing and designing these materials is more complicated than metallic materials. The aim of this work is to model some moderately thick structures in the form of plates or shells in order to simulate the behaviour of composite structures exposed of environmental effects. As a matter of fact, the use of polymer matrix composites in aircraft structural applications requires a careful evaluation of the effects of environmental exposure on material properties. Influences of common environmental mechanisms of moisture and temperature have been found to significantly degrade the matrix-dominated mechanical properties /1,2/. As a consequence there has been a pressing need to quantify the degree of degradation that can occur during a typical service life of an aircraft (usually 25 years), and to make allowance for it in component design and structural testing. The deterioration that occurs in CFC composites during the service life is, in general, linked with the level of moisture (water) that is absorbed. This is usually confined to the resin matrix, although some absorption by fiber can take place with aramid fibers.

In order to evaluate the influence of moisture and temperature on the structural characteristics of such structures, analytical investigations are performed. A general shell type structure is chosen for this analysis.

The shell is modelled by a displacement approach and higher-order shear deformation theory which allows to take boundary conditions into account on the top and bottom surfaces of the shell, and parabolic distribution of transverse shear stresses.

The hygroscopic nature of the laminated shell is approximated by means of Fourier theory of heat transfer to composite materials and Fick's diffusion model is adopted for moisture distribution.

## MOISTURE AND TEMPERATURE DISTRIBUTIONS

Matrix materials such as epoxy resins, commonly used for present-day composites, absorb moisture from the atmosphere by what is essentially a diffusion process. Under ambient temperature conditions, the rate of diffusion is quite slow: it takes time the order of months for a laminate kept in a humid atmosphere to achieve an equilibrium moisture distribution. The reverse process called desorption also occurs when a laminate containing moisture undergoes long-term exposure in a dry atmosphere. It will give up its moisture also by a diffusion process. The amount of moisture in a laminate is generally expressed in terms of the percentage increase in laminate weight: for graphite/epoxy laminates exposed for long times in humid atmospheres, weight increase of the order of 1% is encountered.

As it will be seen later, since the presence of moisture in a laminate can significantly affect its structural properties, considerable attention has been devoted to establishing theoretical procedures for predicting the moisture content.

Moisture and temperature distributions inside composites can readily be calculated when moisture penetrates into the material by Fickian diffusion. Solutions are presented for moisture and temperature distributions in single and multilayered composites under Fickian diffusion, /3,4/. It is important to be able to quantitatively determine $\Delta T(z)$ and $\Delta m(z)$ given a set of surface conditions. Of course, these quantities could also be a function of $x$ and $y$ as well, but for many applications with thin-walled structures such as plates and shells, the most important direction for heat or moisture flux is through thickness. The following problem is considered:

- temperature and moisture content inside the multilayered composite vary only in the direction normal to the face of the plate,
- the temperature inside the material equilibrates much faster than the moisture concentration, and hence at each instant of time, the temperature distribution inside the material corresponds to the instantaneous ambient temperature,
- material properties depend only on temperature, and are independent of moisture concentration and stress level,
- environmental conditions (temperature, moisture level) vary in arbitrary, but known manner,
- the hygroscopic nature of the plate/shell is approximated by means of the classical diffusion equation /5/,

$$D_z \frac{\partial^2(\Delta m)}{\partial z^2} - \frac{\partial(\Delta m)}{\partial t} = 0, \quad (1)$$

where: $D_z$ is the moisture diffusion constant for the composite in the thickness direction, and $t$ is the time. For the case that the thin-walled structure is instantaneously exposed to a moisture concentration $\Delta m_0$, at the time $t = 0$, on both the top and bottom surfaces, the transient solution of Eq.(1) is the following solution:

$$\Delta m(z,t) = \Delta m_0 + \sum_{n=0}^{\infty} m_n \cos a_n z, \quad (2)$$

where: $m_n = \frac{4}{\pi}\left\{\frac{(-1)^n}{2n+1} e^{-a_n^2 D_z t}\right\}$, and $a_n = \frac{(2n+1)\pi}{h}$.

If only the upper surface of the thin-walled structure is suddenly exposed to moisture concentration $\Delta m_0$ at $t = 0$, the transient solution is:

$$\Delta m(z,t) = \Delta m_0 \left[1 - \sum_{n=0}^{\infty} p_n \cos q_n \left(z + \frac{h}{2}\right)\right], \quad (3)$$

where: $m_n = \frac{4}{\pi}\left\{\frac{(-1)^n}{2n+1} e^{-a_n^2 D_z t}\right\}$, and $a_n = \frac{(2n+1)\pi}{2h}$.

The solutions Eq.(2) and Eq.(3) are also valid for transient thermal distributions if $m$ is replaced by $T$ and $D_z$ becomes the thermal diffusion.

For a steady state case the transient solution die out can be represented by simple linear functions in the $z$-direction, such as

$$\Delta m(z) = \frac{1}{2}\left[\Delta m\left(\frac{h}{2}\right) - \Delta m\left(-\frac{h}{2}\right)\right] + \frac{z}{h}\left[\Delta m\left(\frac{h}{2}\right) - \Delta m\left(-\frac{h}{2}\right)\right]. \quad (4)$$

There is obviously an analogous expression for $\Delta T(z)$.

The temperature distribution inside the multilayered plate/shell is defined as:

$$T_{i-1} - T_i = qR_i \quad (i = 0,1,\ldots,n), \quad (5)$$

where: $R_i = \frac{k_i}{h_i}$; $q = \frac{T_n - T_1}{(R_1 + R_2 + \ldots R_n)}$; $T_0$ through $T_n$ are surface temperatures; $k_i$ is the thermal conductivity of $i$-th layer (in the direction normal to the face of the plate/shell). From these solutions all of the thermal and moisture stress resultants and stress couples can be formulated straightforwardly.

The heat transfer in composite materials is applied by Fourier theory. Fourier's equitation of heat transfer is given in the form

$$\rho_c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial T}{\partial x_j}\right) = (k_{ij} T_{,j})_{,j}, \quad (6)$$

where: $\rho$ is material density; $c$ is specific heat; $T$ is temperature; $x_i$ are material coordinates; and $k_{ij}$ is not a function of temperature, Eq.(6) simplifies to:

$$\rho_c \frac{\partial T}{\partial t} = k_{ij} \frac{\partial^2 T}{\partial x_i \partial x_j}. \quad (7)$$

The rate of heat transfer per unit area per unit time is the heat flux vector

$$-\rho_i = k_{ij}\frac{\partial^2 T}{\partial x_j}. \quad (8)$$

The equation is well known from second and first Fourier laws:

$$[\mathrm{k}] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \quad (9)$$

where:

$$\begin{aligned} k_{\alpha\beta} &= a_{\lambda\alpha} a_{\gamma\beta} k'_{\lambda\gamma} \\ k_{11} &= k'_{11}\cos 2\theta + k'_{22}\sin 2\theta \\ k_{12} &= (k'_{22} - k'_{11})\cos\theta + \sin\theta \\ k_{22} &= k'_{11}\sin 2\theta + k'_{22}\cos 2\theta \\ k_{33} &= k'_{22} = k'_{33} \end{aligned} \quad (10)$$

where: $k'_{11}$ is the conductivity in composite material along the fiber; and $k'_{22}$ is the conductivity in composite material to the fiber. Approximated values of $k'_{11}$ and $k'_{22}$ are given as /6/:

$$k'_{11} = (1 - V_f)k_r + V_f k_f, \quad (11)$$

$$k'_{22} = \left(1 - \sqrt{\frac{V_f}{\pi}}\right)k_r + \frac{k_r}{\beta_k}\left(\pi - \frac{4}{\sqrt{1 - \left(\beta_k^2 \frac{V_f}{\pi}\right)}}\tan^{-1}\frac{\sqrt{1 - \left(\beta_k^2 \frac{V_f}{\pi}\right)}}{1 + \beta_k\sqrt{\frac{V_f}{\pi}}}\right),$$

$$\beta_k = 2\left(\frac{k_r}{k_f} - 1\right). \quad (12)$$

In above equations, $V_f$ is volume fraction of fiber; $k_f$ is the thermal conductivity of fiber; and $k_r$ is the thermal conductivity of resin.

## STRESS ANALYSIS BY FINITE ELEMENTS

The finite element method is used for stress analysis of layered composite structures. Attention is focused on the stress analysis of general composite structures under combined mechanical and higrothermal loading. A detailed FEM description of layered shell structures is given in references /7-9/. Here are given constitutive relations for layered composite structures subjected to combined mechanical and higrothermal loading.

The stress resultants N can be written as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{N}\left\{[\bar{\mathrm{Q}}]_k \begin{bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \varepsilon_{xy_0} \end{bmatrix}\int_{h_{k-1}}^{h_k} dz + [\bar{\mathrm{Q}}]_k \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}\int_{h_{k-1}}^{h_k} zdz - \right.$$
$$\left. - \int_{h_{k-1}}^{h_k}[\bar{\mathrm{Q}}]_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}\Delta T dz - \int_{h_{k-1}}^{h_k}[\bar{\mathrm{Q}}]_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}\Delta m dz \right\} \quad (13)$$

where: $\alpha$ and $\beta$ are the coefficient of thermal and higrothermal expansions, in respect; $\Delta T$ is the increase temperature; and $\Delta m$ is the increase from zero moisture measured in percentage weight increase. The previous equation can be written as

$$[\mathrm{N}] = [\mathrm{A}][\varepsilon_0] + [\mathrm{B}][\mathrm{k}] - [\mathrm{N}]^T - [\mathrm{N}]^m, \quad (14)$$

where:

$$\mathrm{A}_{ij} = \sum_{k-1}^{N}(\bar{\mathrm{Q}}_{ij})_k[h_k - h_{k-1}] \quad (i,j = 1,2,6), \quad (15)$$

$$\mathrm{B}_{ij} = \frac{1}{2}\sum_{k-1}^{N}(\bar{\mathrm{Q}}_{ij})_k[h_k^2 - h_{k-1}^2] \quad (i,j = 1,2,6), \quad (16)$$

$$\mathrm{N}_{ij}^T = \sum_{k-1}^{N}\int_{h_{k-1}}^{h_k}(\bar{\mathrm{Q}}_{ij})[\alpha_{ij}]_k \Delta T dz \quad (i,j = 1,2,6), \quad (17)$$

$$\mathrm{N}_{ij}^m = \sum_{h_{k-1}}^{h_k}(\bar{\mathrm{Q}}_{ij})[\beta_{ij}]_k \Delta m dz \quad (i,j = 1,2,6), \quad (18)$$

where: $h_k$ is the vectoral distance from the plate mid-plane to the upper surface of the $k$-th lamina, i.e. any dimension and any below the midsurface is a negative dimension, and any dimension above the midsurface is positive. Relations that include bending deformations can be written in a similar manner:

$$[\mathrm{M}] = [\mathrm{B}][\varepsilon_0] + [\mathrm{D}][\mathrm{k}] - [\mathrm{M}]^T[\mathrm{M}]^m, \quad (19)$$

where:

$$\mathrm{D}_{ij} = \frac{1}{3}\sum_{k=1}^{N}(\bar{\mathrm{Q}}_{ij})_k[h_k^3 - h_{k-1}^3] \quad (i,j = 1,2,6), \quad (20)$$

$$\mathrm{M}_{ij}^T = \sum_{k=1}^{n}(\bar{\mathrm{Q}}_{ij})_k[\alpha_{ij}]_k \Delta T z dz \quad (i,j = 1,2,6), \quad (21)$$

$$\mathrm{M}_{ij}^m = \sum_{h_{k=1}}^{h_k}\int_{h_{k-1}}^{h_k}(\bar{\mathrm{Q}}_{ij})_k[\beta_{ij}]_k \Delta m z dz \quad (i,j = 1,2,6). \quad (22)$$

In the above equations, the standard notation /8-10/ is used. The layered shell is modelled by a displacement FE approach and a higher-order shear deformation that allows us to take boundary conditions into account on the top and bottom surfaces of the shell and the parabolic distribution of transverse shear deformations.

## EXPERIMENTAL PROCEDURE

It is well known that the exposure of composite structures in a wet environment may cause water absorption. The absorbed water in polymeric composite not only produces changes in chemical and physical nature, but also causes degradation of mechanical properties of materials. The degradation of strength and stiffness is the result of weakening three composite phases: the fiber, the matrix and the interface. The degree of degradation depends on the moisture content in the composite and varies within structural components so remarkably that it should not be ignored in the application or design of composite structures when they are to be exposed in a humid environment for a long period. The natural process of moisture absorption in epoxy matrices is normally very slow, and this makes it very difficult to reach an adequate degree of degradation in a structural test element in a practical real time. It has been found necessary, therefore, to speed up the moisture diffusion process by employing an accelerated conditioning technique that can ensure a representative level of degradation in a much reduced time. The degree of degradation that occurs in composite structures when in service is linked directly to the amount of moisture that is absorbed. All tests are carried out on test machine SCHRENCK TREBL RM 100 and Climate chamber for simulating ambient conditions.

In this study the effects of moisture and temperature on mechanical properties of composite materials are examined experimentally by means of tensile tests of a typical carbon-epoxy system.

*Effect of moisture on tensile strength and module elasticity*

Tested are unidirectional standard CFC specimens with sequence $[(90°)_4]_s$ made from the graphite/epoxy (T300/263) fiber resin system. Figure 1 shows absorption and desorption curves of the considered CFC composite material. The curves are experimentally determined, and absorption tests are carried out with distilled water. The water immersion absorption tests with CFC specimens are conducted at 50 °C. Desorption tests of composite specimens are also conducted at 50 °C. It is observed that tensile strength $F_{22}$ and modulus of elasticity $E_{22}$ of these specimens are reduced after moisture absorption. The influence of moisture and temperature on tensile ultimate strength and module of elasticity are given in Figs. 2 and 3.

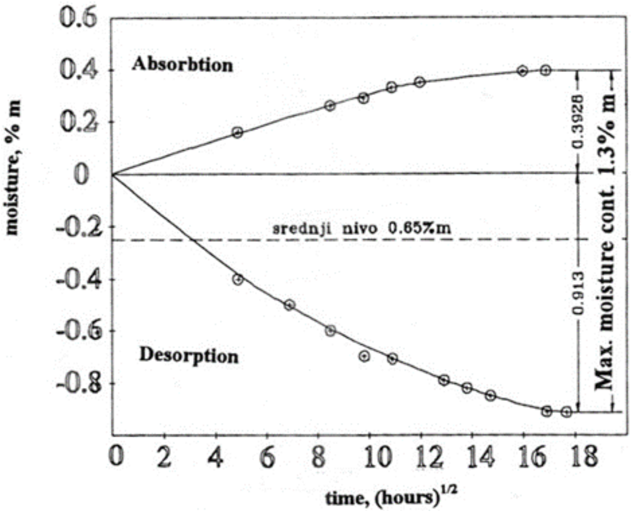


Figure 1. Absorption and desorption curves for CFC composite specimens.

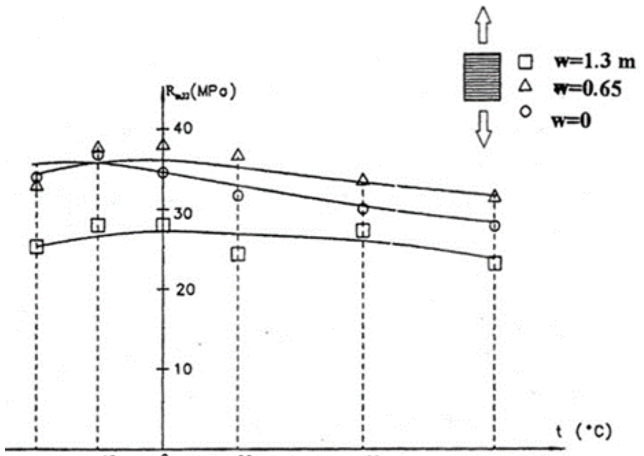


Figure 2. Effects of temperature and moisture on ultimate tensile strength $F_{22}$.

The rates at which water is absorbed by specimens are determined by weighting the specimens after immersion time. It is well known that degradation of the resin matrix and interface and change of specimen weight may arise during immersion in water. The weight change is considered to be produced by an increase of the absorbed water in the specimen and from the dissolved matrix in water.

The tensile tests are carried out after immersion tests by Schrenck universal testing machine. The climate chamber is directly connected to the Schrenck machine and is possible to directly measure strains under temperature effects.

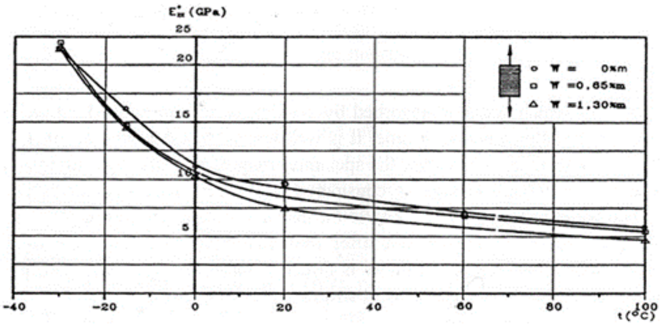


Figure 3. Effects of temperature and moisture on module of elasticity $E_{22}$.

Figures 2 and 3 show the importance of the influence of temperature and moisture on mechanical properties of CFC. Evident is a drastic degree of degradation of mechanical properties that occurs in composite specimens with 90° orientation. This is about 35% for the tensile ultimate strength and 50% for the $E_{22}$.

## NUMERICAL EXAMPLES

To demonstrate the numerical approach in stress analysis of composite structures under combined mechanical and hygrothermal loads, here are included simple numerical tests. Considered is a layered composite panel subjected to mechanical and hygrothermal loading. The geometry and FE model are shown in Fig. 4.

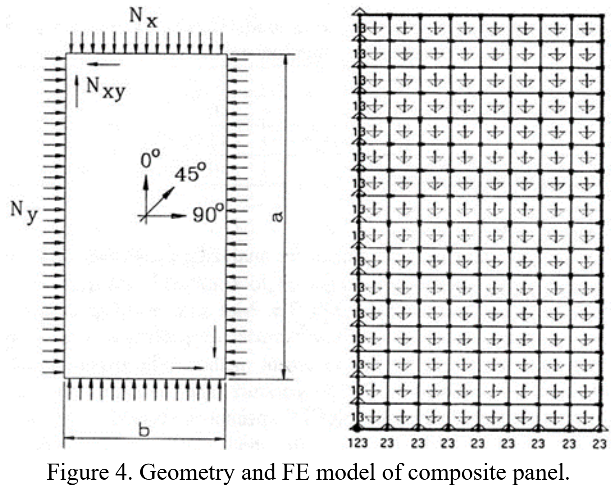


Figure 4. Geometry and FE model of composite panel.

Mechanical properties of the composite material are: $E_{11}$ = 122 GPa, $E_{22}$ = 9.43 GPa, $G_{12}$ = 4.62 GPa, $t_{layer}$ = 0.125 mm, $\nu_{12}$ = 0.31, $\alpha_1$ = 0, $\alpha_2$ = 0.000003. These properties are given for standard ambient conditions. The hygrothermal effects, based on the previous experiments, are included in numerical analysis. Dimensions of the panel are $a$ = 300 mm and $b$ = 150 mm.

The composite panel was under uniaxial load $N_x$ = 45000 N and subject to $\Delta T$ = 800 °C. The complete results of stresses and displacements of this panel are given in Tables 1 and 2.

Table 1. Stresses at layered composite panel under combined mechanical and hygrothermal loads.

| No. | Sequences | Stresses in layers | $\sigma_1$ [MPa] | $\sigma_2$ [MPa] | $\tau_{12}$ [MPa] |
|---|---|---|---|---|---|
| 1 | [(0°/90°)$_4$]s | 0°<br>90° | 305.91<br>21.46 | 8.42<br>23.96 | 0<br>0 |
| 2 | [±45°/0°$_4$/90°$_2$]s | ±45°<br>0°<br>90° | 133.62<br>273.42<br>-6.19 | 13.22<br>5.58<br>20.85 | 11.76 |
| 3 | [(±45°)$_4$]s | ±45° | 163.68 | 16.19 | 75 |

Table 2. Displacements of composite panel under combined mechanical and hygrothermal loads (simply supported boundary conditions).

| No. | Sequences | $f_x$ [mm] | $f_y$ [mm] |
|---|---|---|---|
| 1 | [(0°/90°)$_4$]s | -0.0173 | 0.7458 |
| 2 | [±45°/0°$_4$/90°$_2$]s | -1.0225 | 2.8253 |
| 3 | [(±45°)$_4$]s | -0.0156 | 0.6681 |

## CONCLUSION

Methods of analysis have been developed for analysing the stresses in moderately thick laminated plates and shells subjected to combined elevated temperature, absorbed moisture and mechanical loads. The heat and moisture transfer are carried out by Fourier theory and Fick's diffusion model, respectively. The stresses and displacements are calculated by finite element method. The experimental results have shown that the effect of the temperature and moisture on the mechanical characteristic is evident. Test results from CFC specimens (T300/263 graphite/epoxy system) due to ambient conditions (moisture and temperature) have shown that the degree of degradation of tensile strength $F_{22}$ is about 35% and module of elasticity $E_{22}$ about 50%. The stresses in the composite material due to ambient conditions are incorporated in the calculation. Numerical simulations for hygrothermal effects, stability analysis, postbuckling behaviour are in progress, as well as some experimental simulation hygrothermal environments.

## ACKNOWLEDGEMENTS

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## REFERENCES

1. Vinson, J.R., Walker, W.U., Pipes, R.B., Ulrich, O.R., The Effects of Relative Humidity and Elevated Temperatures on Composite Structures, Air Force Office of Scientific Technical Report 77-0030, February 1977.
2. Tsai, S.W., Hahn, H.T., Introduction to Composite Materials, Technomic Publishing Co., Inc., Westport, CT, 1980.
3. Agarwal, B.D., Broutman L.J., Chandrashekhara, K., Analysis and Performance of Fiber Composites, 3rd Ed., Wiley, New York, 2006.
4. Brischeto, S. (2013), *Hygrothermoelastic Analysis of multi-layered composite and sandwich shells*, J Sandwich Struct. & Mater. 15(2):168-202. doi: 10.1177/1099636212471358
5. Pipes, R.B., Vinson, J.R., Chou, T.W. (1976), *On the hygrothermal response of laminated composite systems*, J Compos. Mater. 10(2): 129-148. doi: 10.1177/002199837601000203
6. Springer, G.S. (ed.), Environmental Effects on Composite Materials, Technomic Press, Westport, CT, 1981.
7. Maksimović, S., *Structural analysis/synthesis of laminated fiber composite structures*, in Proc. of STRUCAD 86, Structural Analysis Systems, World Conf., Paris, France (ed. A-Niku-Lari), Pergamon Press, 1986.
8. Maksimović, S., *A simple and effective finite element for nonlinear analysis of layered composite shells using refined higher-order theory*, in Proc. 6th World Congress on FEM (ed. J. Robinson), Banff, Canada, Robinson and Associates, 1990.
9. Maksimović, S., *Buckling and postbuckling analysis of laminated shell structures by finite elements based on the third order theory*, in Computer Structures 6 (ed. I.H. Marshall), Elsevier Science Publ. LTD, London, 1991.
10. Maksimović, S., et al. (2014), *Determination of load distributions on main helicopter rotor blades and strength analysis of its structural components*, J Aerosp. Eng. 27(6). doi: 10.1061/(ASCE)AS.1943-5525.0000301