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CREEP DAMAGE MODELLING IN A TRANSVERSELY ISOTROPIC ROTATING DISC WITH LOAD AND DENSITY PARAMETER

MODELIRANJE PUZANJA KOD TRANSVERZALNOG IZOTROPNOG ROTIRAJUĆEG DISKA SA PARAMETRIMA OPTEREĆENJA I GUSTINE

Originalni naučni rad / Original scientific paper
UDK /UDC: 66.018.9:539.376
Rad primljen / Paper received: 26.11.2018

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Keywords
- creep stress modelling
- strain rates
- angular speed
- load
- disc

Ključne reči
- modeliranje napona puzanja
- brzina deformacije
- ugaona brzina
- opterećenje
- disk

Abstract

The purpose of this paper is to present the study of creep damage modelling in a transversely isotropic material disc having variable density and mechanical load by using Seth's transition theory. Neither the yield criterion, nor the associated flow rule is assumed here. The results obtained here are applicable to transversely isotropic and isotropic materials. If the additional condition of incompressibility is imposed, then the expression of stresses corresponds to those arising from Tresca yield condition. It has been seen that the disc made of transversely isotropic material and having different mechanical load and whose density decreases radially is on the safer side of the design compared to the disc made of isotropic material.

Izvod

U radu je predstavljena studija modeliranja oštećenja puzanjem kod diska od transverzalnog izotropnog materijala, sa promenljivom gustinom i mehaničkim opterećenjem, primenom Setove teorije prelaznih napona. Ovde se ne pretpostavljaju niti kriterijum tečenja, niti odgovarajući zakon tečenja. Dobijeni rezultati se mogu primeniti na transverzalno izotropne i na izotropne materijale. Ako se primeni dopunski uslov nestišljivosti, tada se izraz za napone odnosi na napone koji potiču iz uslova Treska tečenja. Uočeno je da je disk od transverzalno izotropnog materijala sa različitim mehaničkim opterećenjem, sa gustinom koja radijalno opada, na strani sigurnosti u projektovanju, u poređenju sa diskom od izotropnog materijala.

INTRODUCTION

A rotating disc is an important structural component in machinery such as high-speed gears, rotors, flywheel, turbine, compressors and turbojet engines, etc. In the power generation and aerospace industries, components such as a disc, pipes, pressure vessels, and gas and turbine blades experience high temperatures such that creep deformation will occur. Arnold /5/ has studied transversely isotropic thermoelastic theory. Wu et al. /6/ discussed the problem stress in transversely isotropic half-space with typical loads acting on its surface. Ghadi et al. /14/ discussed the mathematical analysis for an axi-symmetric disc-shaped crack in transversely isotropic half-space. Hajimohammadi et al. /17/ analysed the problem of vertical vibration of a rigid circular disc at the interface of a transversely isotropic bimaterial. Sharma et al. /7/ have analysed the problem spherically symmetric generalized thermoelastic waves transversely isotropic medium. Chen et al. /9/ has studied three-dimensional analytical solution for a rotating disc of functionally graded materials with transverse isotropy. Ghadi et al. /13/ discussed forced vertical vibration of a rigid circular disc on a transversely isotropic half-space. Akinola et al. /16/ has studied the large deformation of transversely isotropic elastic thin circular disk in rotation. Seth's transition theory does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. Seth's transition theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems /2, 3, 10-12, 15, 18-31/. The density of the disc is assumed to vary along the radius in the form:

$$\rho(r)=\rho_0(r/b)^{-m}\,, \tag{1}$$

where: ρ_0 is the constant density at $r = b$; and m is the density variation parameter.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

Consider a thin disc having variable density with a central bore of inner radius a and outer radius b, respectively. The disc is rotating with an angular speed ω of gradually increasing magnitude about an axis perpendicular to its plane and passing through the centre. The thickness of the disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress τ_{zz} is zero.

Displacement coordinate: the displacement components in cylindrical coordinate are given by /2/:

$$u=r(1-\beta),\ v=0,\ w=0\,, \tag{2}$$

where: u, v, w are displacement components; β is position function, depending on $r = \sqrt{x^2+y^2}$ only. The generalized components of strain are given by /3/:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right], \quad e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right],$$
$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right], \quad e_{r\theta} = e_{\theta z} = e_{zr} = 0, \tag{3}$$

where: n is the measure; r, θ, z are polar coordinates; and $\beta' = d\beta/dr$.

Stress-strain relation: the stress-strain relations for the transversely isotropic material are:

$$\tau_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz},$$
$$\tau_{\theta\theta} = c_{21}e_{rr} + c_{22}e_{\theta\theta} + c_{23}e_{zz},$$
$$\tau_{zz} = c_{31}e_{rr} + c_{32}e_{\theta\theta} + c_{33}e_{zz}, \tag{4}$$
$$\tau_{r\theta} = 2c_{66}e_{r\theta} = 0, \quad \tau_{\theta z} = 2c_{44}e_{\theta z} = 0, \quad \tau_{zr} = 2c_{55}e_{zr} = 0,$$

where: $c_{12} = (c_{11} - 2c_{66}) = c_{21}$, $c_{13} = c_{31} = c_{32} = c_{23}$, $c_{55} = c_{44}$ and $c_{11} = c_{22}$, c_{33} are elastic constants respectively. Substituting the strain components from Eq.(3) into Eq.(4), the stresses are obtained as:

$$\tau_{rr} = \frac{1}{n}\left\{\left(\frac{c_{11}c_{33}-c_{13}^2}{c_{33}}\right)\left[2-\beta^n\left\{1+(P+1)^n\right\}\right] - 2c_{66}\left[1-\beta^n\right]\right\},$$

$$\tau_{\theta\theta} = \frac{1}{n}\left\{\left(\frac{c_{11}c_{33}-c_{13}^2}{c_{33}}\right)\right\}\left[2-\beta^n\left\{1+(P+1)^n\right\}\right] -$$
$$-2c_{66}\left[1-\beta^n(1+P)^n\right],$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0, \tag{5}$$

where: $r\beta' = \beta P$; P is a function of β and β is the function of r only.

Equation of equilibrium: due to the fact that a rotating thin disc is an axisymmetric problem, its equilibrium equation is given by /1/:

$$r\frac{d}{dr}(\tau_{rr}) + \tau_{rr} - \tau_{\theta\theta} + \rho(r)\omega^2 r^2 = 0, \tag{6}$$

where: τ_{rr}, $\tau_{\theta\theta}$, ω and $\rho(r)$ are, respectively, radial stress, hoop stress, angular velocity and radially varying material mass density, which is used instead of mass density ρ of the isotropic, homogeneous materials; and r is the radial distance ($a \le r \le b$).

Critical points: using Eq.(5) into Eq.(6), we get a nonlinear differential equation in β as:

$$\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta} = -\beta^n P\left[1+(P+1)^n\right] + \frac{2c_{66}c_{33}\beta^n}{n(c_{11}c_{33}-c_{13}^2)A} \times$$
$$\times\left[1+(P+1)^n\right] + \frac{2c_{66}c_{33}\beta^n P}{(c_{11}c_{33}-c_{13}^2)} + \frac{\rho\omega^2 r^2 c_{33}}{(c_{11}c_{33}-c_{13}^2)}. \tag{7}$$

The transition points of β in Eq.(7) are $P \to -1$ and $P \to \pm\infty$.

Boundary conditions: the boundary conditions are:

$$\tau_{rr} = 0 \text{ at } r = a \text{ and } \tau_{rr} = L \text{ at } r = b, \tag{8}$$

where: L is the mechanical load applied to the external surface of the disc.

SOLUTION OF THE PROBLEM

For finding the creep strain rates, the transition function is taken through principal stress difference (see /2, 3, 10-12, 15, 18-31/) at the transition point $P \to -1$. We define the transition function η as:

$$\eta = \tau_{rr} - \tau_{\theta\theta} = \frac{2c_{66}\beta^n}{n}\left[1-(P+1)^n\right]. \tag{9}$$

Taking the logarithmic differentiation of Eq.(9) with respect to r and substituting Eq.(7) and taking asymptotic value $P \to -1$, we get:

$$\frac{d}{dr}(\ln\eta) = -\frac{1}{r}\left[\frac{2c_{66}c_{33}(n-1)}{c_{11}c_{33}-c_{13}^2} - 2n\right] - \frac{n\rho\omega^2 r^{n+1}c_{33}}{(c_{11}c_{33}-c_{13}^2)D^n}. \tag{10}$$

The asymptotic value of β as $P \to -1$ is D/r, D being a constant. Integrating Eq.(10) with respect to r, we get

$$\eta = \tau_{rr} - \tau_{\theta\theta} = Ar^{A_1}\exp F, \tag{11}$$

where: A_1 is a constant of integration, which can be determined by using boundary conditions:

$$A_1 = -2n + \frac{2c_{66}c_{33}(n-1)}{c_{11}c_{33}-c_{13}^2} \quad \text{and}$$

$$F = -\frac{n\omega^2 c_{33}}{D^n(c_{11}c_{33}-c_{13}^2)}\int \rho r^{n+1}dr.$$

Substituting Eq.(11) in Eq.(6) and integration with respect to r, we get

$$\tau_{rr} = A_2 - A\int r^{A_1-1}\exp F dr - \omega^2\int \rho r dr, \tag{12}$$

where: A_2 is a constant of integration which can be determined by using boundary conditions.

From Eq.(12) and Eq.(11), we have:

$$\tau_{\theta\theta} = A_2 - A\int r^{A_1-1}\exp F dr - \omega^2\int \rho r dr - Ar^{A_1}\exp F. \tag{13}$$

Using boundary condition Eq.(8) in Eq.(13), we get

$$A = -\left[\omega^2\int_a^b \rho r dr + L\right] \Big/ \int_a^b r^{A_1-1}\exp F dr \quad \text{and}$$

$$A_2 = -\frac{\left(\omega^2\int_a^b \rho r dr + L\right)\left[\int r^{A_1-1}\exp F dr\right]_{r=b}}{\int_a^b r^{A_1-1}\exp F dr} + \omega^2\left[\int \rho r dr\right]_{r=b}.$$

Substituting the value of constant A and A_2 in Eq.(13) and using Eq.(1), we get:

$$\tau_{rr} = \frac{\left[L + \frac{\rho_0\omega^2 b^m(b^{2-m}-a^{2-m})}{2-m}\right]\int_a^b r^{A_1-1}\exp F_1 dr}{\int_a^b r^{A_1-1}\exp F_1 dr} +$$
$$+\frac{\rho_0\omega^2 b^m(b^{2-m}-a^{2-m})}{2-m}$$

and

$$\tau_{\theta\theta}=\tau_{rr}+\frac{\left[L+\dfrac{\rho_0\omega^2 b^m (b^{2-m}-a^{2-m})}{2-m}\right] r^{A_1-1}\exp F_1}{\int_a^b r^{A_1-1}\exp F_1 dr}\quad \forall m\neq 2 \quad (14)$$

where: $A_1=-2n+\dfrac{2c_{66}c_{33}(n-1)}{c_{11}c_{33}-c_{13}^2}$ and

$$F_1=-\frac{n\rho_0\omega^2 b^m r^{n-m+2}c_{33}}{D^n(c_{11}c_{33}-c_{13}^2)(n-m+2)}.$$

Equation (14) gives creep stresses in the disc having variation of density parameter and applied mechanical load.

Non-dimensional components: we introduce the following non-dimensional components: $R = r/b$, $R_0 = a/b$, $\sigma_r = \tau_{rr}/L$, $\sigma_\theta = \tau_{\theta\theta}/L$, $L_1 = L/c_{66}$ and $\Omega^2 = \rho\omega^2 b^2/L$. The creep stresses Eq.(14) in non-dimensional form become:

$$\sigma_r=\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]\int_{R_0}^{R} R^{A_1-1}\exp F_2 dr}{\int_{R_0}^{1} R^{A_1-2}\exp F_2 dr}+\frac{\Omega^2(R^{2-m}-R_0^{2-m})}{2-m},$$

$$\sigma_\theta=\sigma_r+\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]R^{A_1}\exp F_2}{\int_{R_0}^{1} R^{A_1-2}\exp F_2 dR}\quad \forall m\neq 2, \qquad (15)$$

where: $F_2=-\dfrac{nL_1\Omega^2 b^n R^{2+n-m}c_{11}c_{33}c_{66}}{c_{11}(c_{33}c_{11}-c_{13}^2)D^n(2+n-m)}$.

Special case: for density parameter $m = 2$, Eq.(15) becomes:

$$\sigma_r=\frac{\left[1-\Omega^2\ln R_0\right]\int_{R_0}^{R} R^{A_1-1}\exp F_3 dR}{\int_{R_0}^{1} R^{A_1-2}\exp F_3 dR}-\Omega^2\ln\left(\frac{R}{R_0}\right),$$

$$\sigma_\theta=\sigma_r+\frac{(1-\Omega^2\ln R_0)R^{A_1}\exp F_3}{\int_{R_0}^{1} R^{A_1-2}\exp F_3 dr}, \qquad (16)$$

where: $F_3=-\dfrac{L_1\Omega^2 b^n R^n c_{11}c_{33}c_{66}}{c_{11}(c_{33}c_{11}-c_{13}^2)D^n}$.

Creep stresses for isotropic case: for isotropic materials, the material constants reduce to $c_{11} = c_{22} = c_{33}$, $c_{12} = c_{21} = c_{13} = c_{31} = c_{23} = c_{32} = (c_{11} - 2c_{66})$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. In term of constants λ and μ, these can be written as: $c_{12} = \lambda$, $c_{11} = \lambda + 2\mu$, $c_{66} = ½(c_{11} - c_{12}) \equiv \mu$, $c = 2\mu/\lambda + 2\mu$ and $1 - c_2 = 1 - c/2 - c$. Equation (15) for isotropic case becomes:

$$\sigma_r=\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]\int_{R_0}^{R} R^{A_3-1}\exp F_4 dR}{\int_{R_0}^{1} R^{A_3-2}\exp F_4 dR}+\frac{\Omega^2(R^{2-m}-R_0^{2-m})}{2-m}$$

$$\sigma_\theta=\sigma_r+\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]R^{A_3}\exp F_4}{\int_{R_0}^{1} R^{A_3-2}\exp F_4 dR},\ \forall m\neq 2, \qquad (17)$$

where: $F_4=-\dfrac{nL_1\Omega^2 b^n R^{2+n-m}}{2(2-c)D^n(2+n-m)}$ and $A_2=-2n+\dfrac{n-1}{2-c}$.

Special case (*m* = 2): Eq.(17) becomes

$$\sigma_r=\frac{\left[1-\Omega^2\ln R_0\right]\int_{R_0}^{R} R^{A_3-1}\exp F_5 dR}{\int_{R_0}^{1} R^{A_3-2}\exp F_5 dR}-\Omega^2\ln\left(\frac{R}{R_0}\right),$$

$$\sigma_\theta=\sigma_r+\frac{(1-\Omega^2\ln R_0)R^{A_3}\exp F_5}{\int_{R_0}^{1} R^{A_3-2}\exp F_5 dR}, \qquad (18)$$

where: $F_5=-\dfrac{L_1\Omega^2 b^n R^n}{2(2-c)D^n}$.

Incompressibility condition: for the incompressible case ($c \to 0$), Eq.(17) becomes

$$\sigma_r=\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]\int_{R_0}^{R} R^{A_3-1}\exp F_6 dr}{\int_{R_0}^{1} R^{A_3-2}\exp F_6 dr}+\frac{\Omega^2(R^{2-m}-R_0^{2-m})}{2-m},$$

$$\sigma_\theta=\sigma_r+\frac{\left[\dfrac{\Omega^2(1-R_0^{2-m})}{2-m}+1\right]R^{A_3}\exp F_6}{\int_{R_0}^{1} R^{A_3-2}\exp F_6 dR}\quad \forall m\neq 2, \qquad (19)$$

where: $F_6=-\dfrac{nL_1\Omega^2 b^n R^{2+n-m}}{4D^n(2+n-m)}$ and $A_3=-\dfrac{1}{2}(3n+1)$.

Special case (*m* = 2): for $m = 2$, Eq.(19) becomes

$$\sigma_r=\frac{\left[1-\Omega^2\ln R_0\right]\int_{R_0}^{R} R^{A_3-1}\exp F_7 dR}{\int_{R_0}^{1} R^{A_3-2}\exp F_7 dR}-\Omega^2\ln\left(\frac{R}{R_0}\right),$$

$$\sigma_\theta=\sigma_r+\frac{(1-\Omega^2\ln R_0)R^{A_3}\exp F_7}{\int_{R_0}^{1} R^{A_3-2}\exp F_7 dR}, \qquad (20)$$

where: $F_7=-\dfrac{L_1\Omega^2 b^n R^n}{4D^n}$.

Investigation of creep parameters: strain rates are given by /29/:

$$\dot{\varepsilon}_{rr} = \frac{c_{33}c_{11}\zeta}{4(c_{66}c_{33} - c_{33}c_{11} + c_{13}^2)}\left[\left(\frac{c_{33}c_{11} - c_{13}^2 - 2c_{33}c_{66}}{c_{33}c_{11}}\right)\sigma_r - \left(\frac{c_{33}c_{11} - c_{13}^2}{c_{33}c_{11}}\right)\sigma_\theta\right],$$

$$\dot{\varepsilon}_{\theta\theta} = \frac{c_{33}c_{11}\zeta}{4(c_{66}c_{33} - c_{33}c_{11} + c_{13}^2)}\left[\left(\frac{c_{33}c_{11} - c_{13}^2 - 2c_{33}c_{66}}{c_{33}c_{11}}\right)\sigma_r - \left(\frac{c_{33}c_{11} - c_{13}^2}{c_{33}c_{11}}\right)\sigma_\theta\right], \quad (21)$$

$$\dot{\varepsilon}_{zz} = \frac{c_{33}c_{11}\zeta}{4(c_{66}c_{33} - c_{33}c_{11} + c_{13}^2)}\left[\left(\frac{c_{33}c_{11} - c_{13}^2 - 2c_{33}c_{66}}{c_{33}c_{11}}\right)(\sigma_r + \sigma_\theta)\right].$$

Creep parameters for isotropic case: Eq.(21) becomes

$$\dot{\varepsilon}_{rr} = \frac{\zeta(2-c)}{2(3-2c)}\left[\sigma_r - \left(\frac{1-c}{2-c}\right)\sigma_\theta\right],$$

$$\dot{\varepsilon}_{\theta\theta} = \frac{\zeta(2-c)}{2(3-2c)}\left[\sigma_\theta - \left(\frac{1-c}{2-c}\right)\sigma_r\right], \quad (22)$$

$$\dot{\varepsilon}_{zz} = \frac{\zeta(2-c)}{2(3-2c)}\left[(\sigma_r + \sigma_\theta)\left(\frac{1-c}{2-c}\right)\right].$$

These are the constitutive equations used /4/ for finding the creep stresses, provided we put $n = 1/N$ and N be the measure.

RESULTS AND DISCUSSION

To illustrate the analysis we have taken numerical values of elastic constants C_{ij} from the existing literature. As a numerical example, elastic constants C_{ij} (units of 10^{10} N/m^2) have been given for transversely isotropic material such as (magnesium $C_2 = 0.64$; $C_{11} = 5.97$, $C_{12} = 2.62$, $C_{13} = 2.17$, $C_{33} = 6.17$, $C_{44} = 1.64$) and isotropic material (brass $C_2 = 0.50$; $C_{11} = 3.00$, $C_{12} = 1.00$, $C_{13} = 1.00$, $C_{33} = 3.00$, $C_{44} = 0.9997$); Haojiang et al. /8/, $\Omega^2 = 0.3, 3$; $n = 1/3, 1/5, 1/7$ (i.e. $N = 3, 5, 7$); $D = 1$, $a = 1$ and $b = 2$, in respect. In classical theory measure N is equal to $1/n$.

Curves are produced in Figs. 1-4, between creep stress distribution in a thin rotating disc having density parameter ($m = 0, 1$), mechanical load $L_1 = 0.2, 20$ and angular speed $\Omega^2 = 0.3, 3$ for the measure $n = 1/3$, 1/5 and 1/7, in respect.

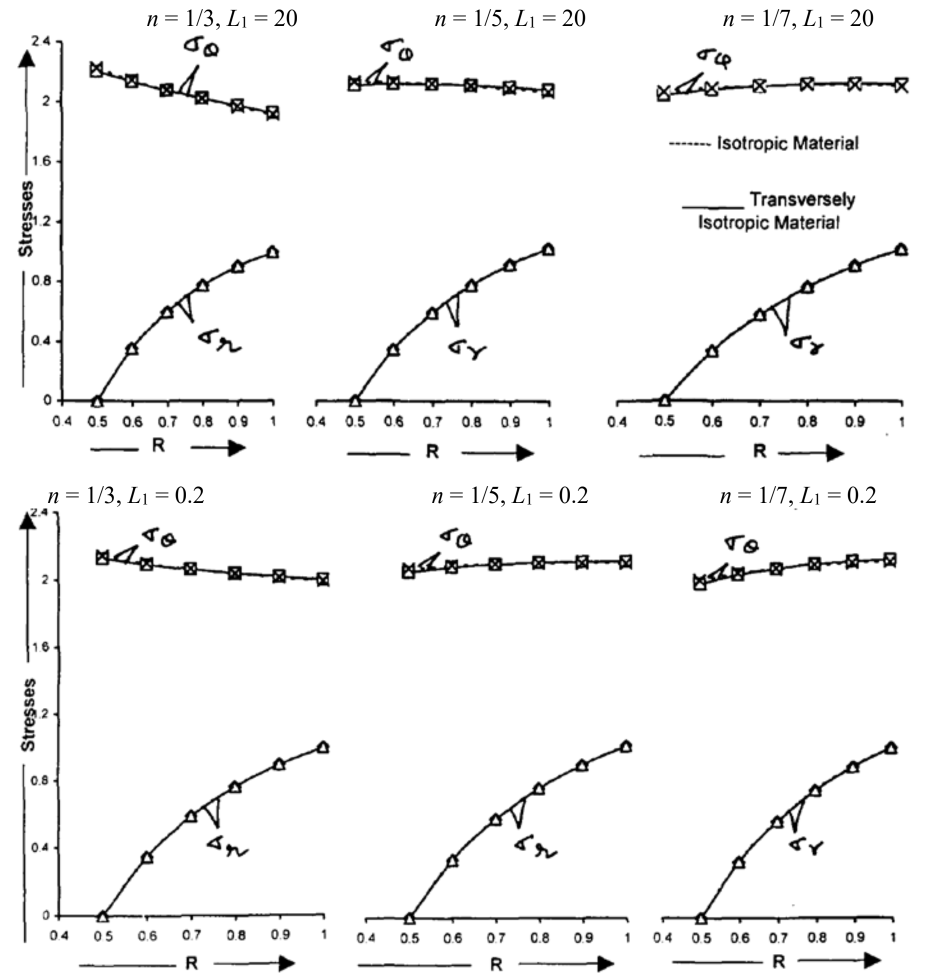

Figure 1. Creep stress distribution in disc with load ($L_1 = 0.2, 20$), density ($m = 0$) and angular speed ($\Omega^2 = 0.3$) along radius ratios $R = r/b$.

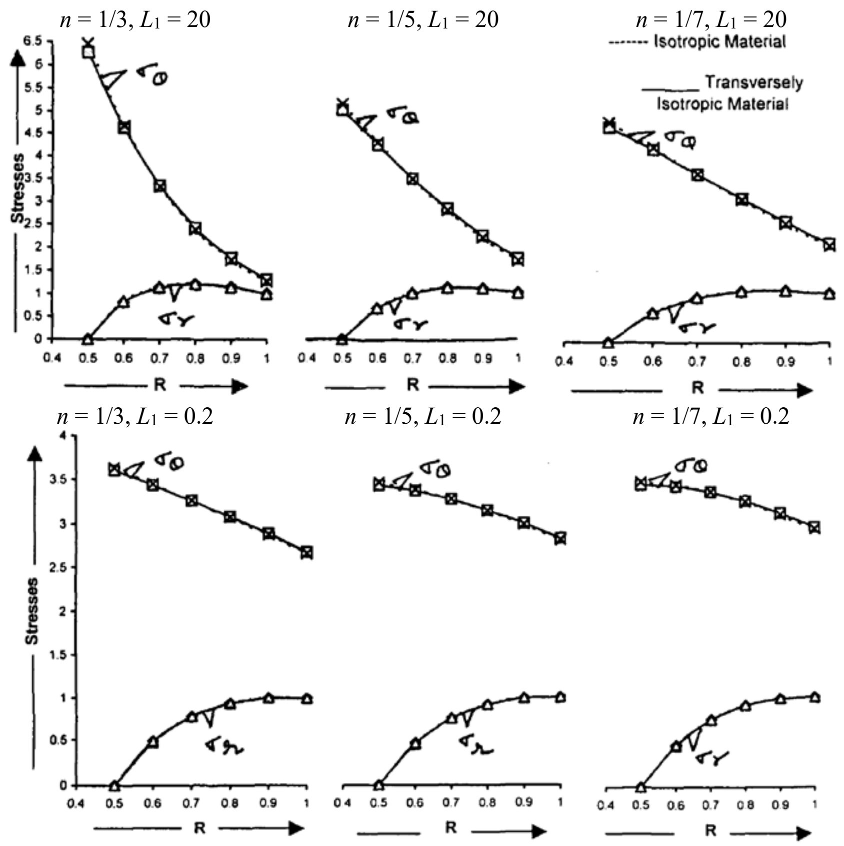

Figure 2. Creep stress distribution in disc with load (L_1 = 0.2, 20), density (m = 0) and angular speed (Ω^2 = 3) along radius ratios $R = r/b$.

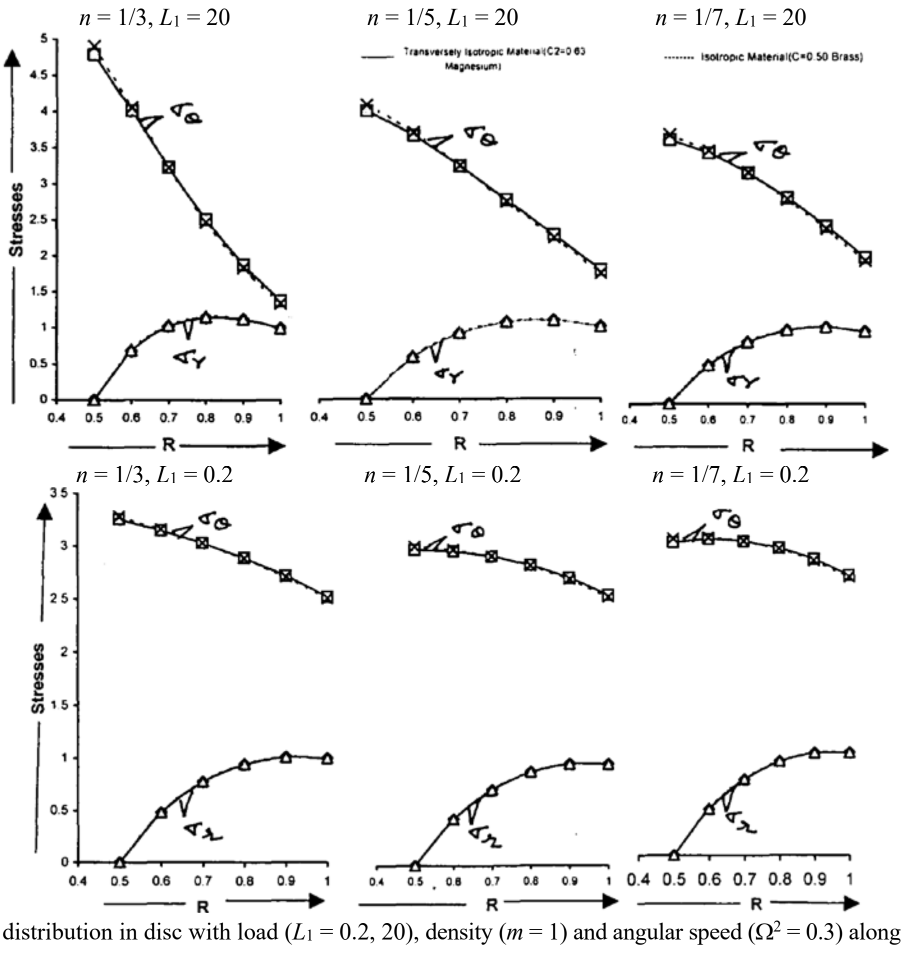

Figure 3. Creep stress distribution in disc with load (L_1 = 0.2, 20), density (m = 1) and angular speed (Ω^2 = 0.3) along radius ratios $R = r/b$.

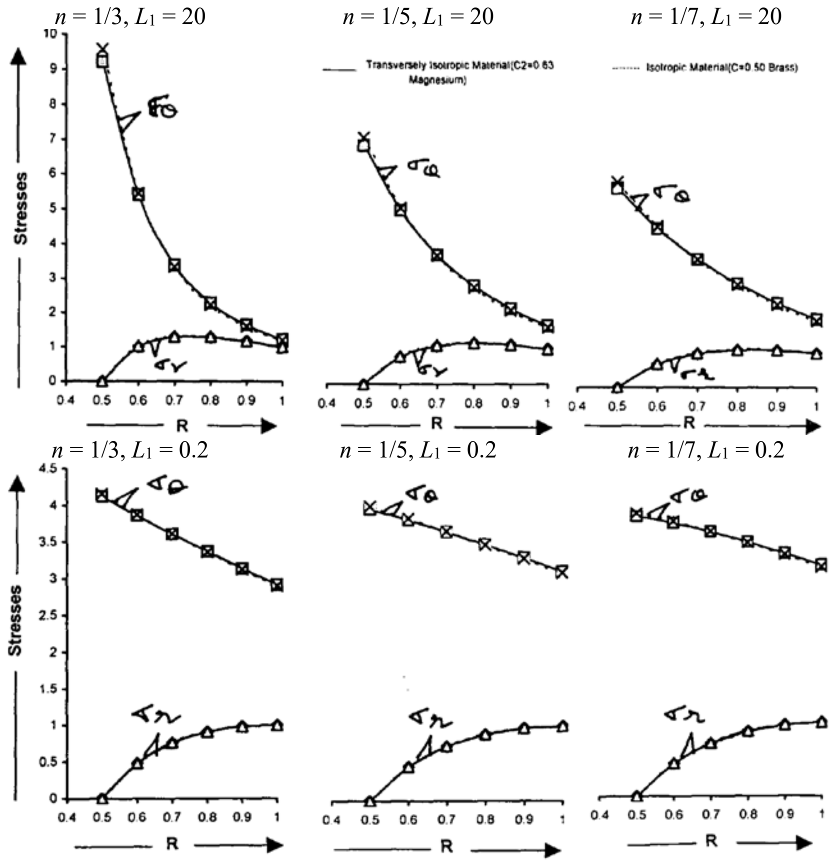

Figure 4. Creep stress distribution in disc with load (L_1 = 0.2, 20), density (m = 1) and angular speed (Ω^2 = 3) along radius ratios $R = r/b$.

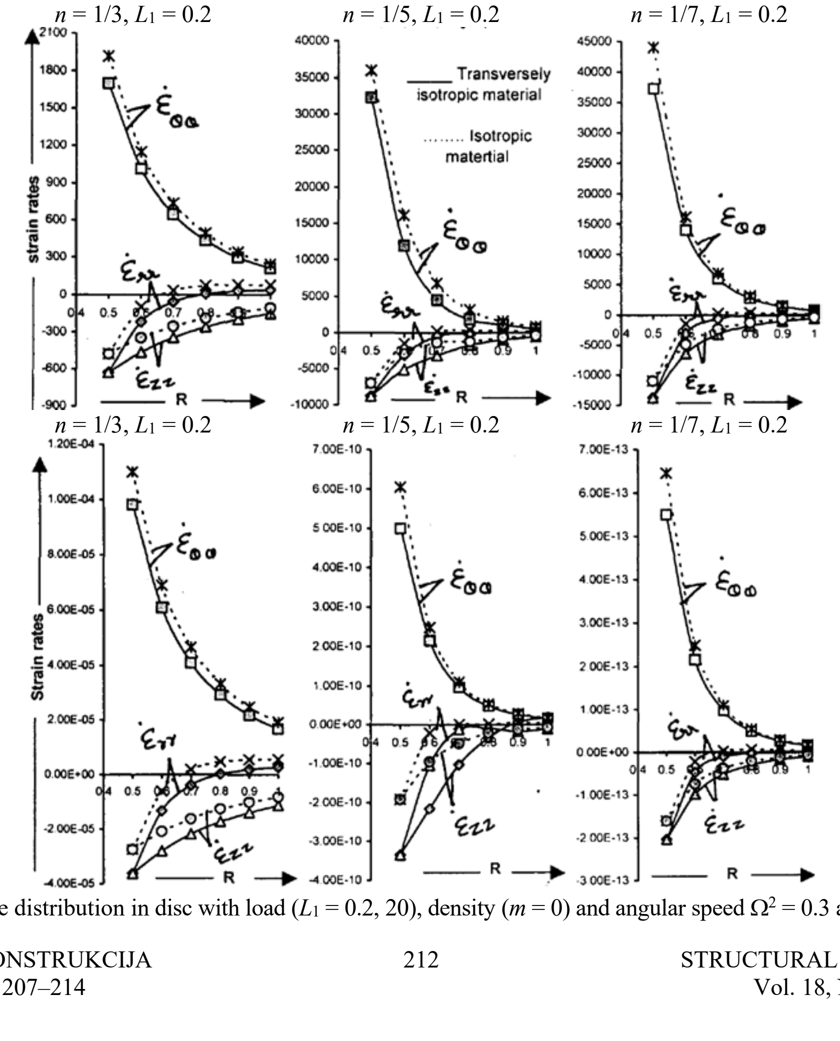

Figure 5. Creep strain rate distribution in disc with load (L_1 = 0.2, 20), density (m = 0) and angular speed Ω^2 = 0.3 along radius ratios $R = r/b$.

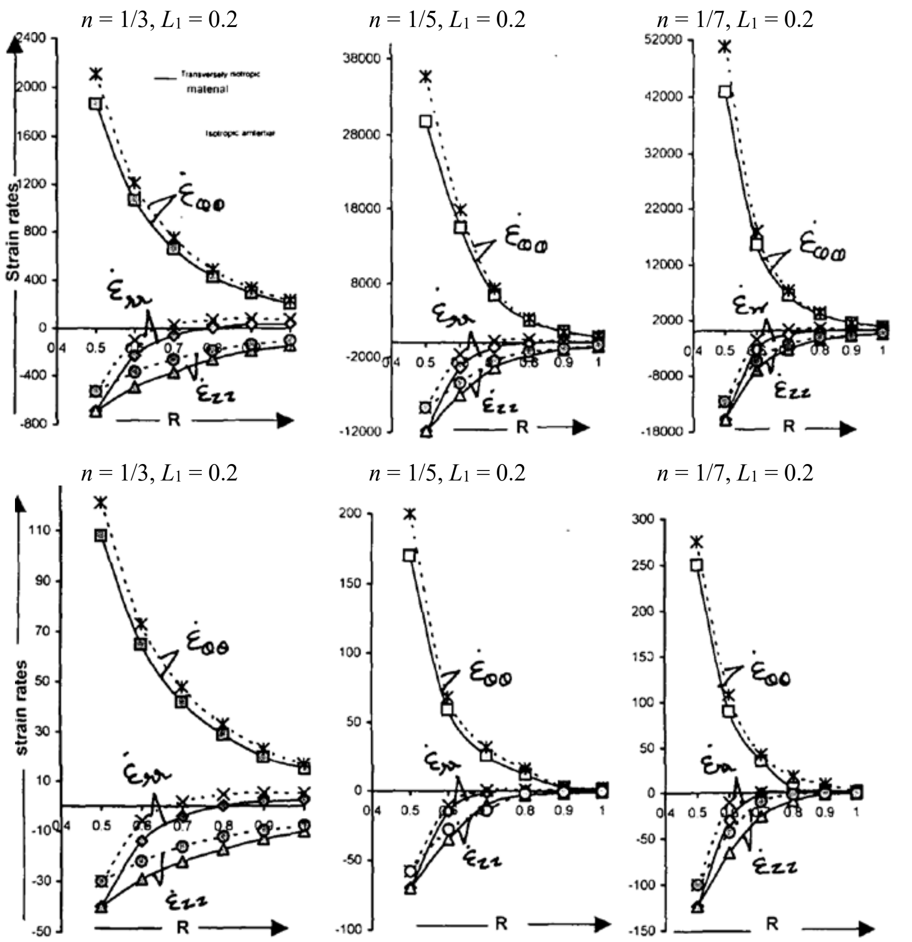

Figure 6. Creep strain rate distribution in disc with load (L_1 = 0.2, 20), density (m = 1) and angular speed Ω^2 = 0.3 along radius ratios $R = r/b$.

It has also been observed from Figs. 2-5, that the hoop stress has a maximum value at the inner surface of the disc made of the isotropic material for measure N = 3, as compared to the disc made of transversely isotropic material. With increase in mechanical load (say L_1 = 0.2 to 20) and angular speed (say Ω^2 = 0.3 to 3), the value of hoop stress will also be increased at the inner surface for measure N = 3, and for the measure N = 5 and 7, the value of hoop stress decreases at the inner surface of the disc. With increase in density parameter (m = 0 to 1), the value of hoop stresses goes on increasing with increase in mechanical load and angular speed. It means that the isotropic rotating disc whose density decreases gradually, increases the possibility for fracture at the bore as compare to the transversely isotropic disc, whereas a density increase radially recedes the possibility for fracture. Curves are produced for strain rates distribution along the radii ratio $R = r/b$ (see Figs. 5-6) for rotating disc of transversely isotropic material as well as isotropic material, having different mechanical load, density and angular speed. It has been seen that isotropic disc has the maximal value of strain rates at the inner surface as compared to the transversely isotropic disc for measure N = (3, 5, and 7), respectively.

REFERENCES

1. Timoshenko, S.P., Goodier, J.N., Theory of Elasticity, 3rd Ed., New York, McGraw-Hill Book Co., London, 1951.
2. Seth, B.R. (1962), Transition theory of elastic-plastic deformation, creep and relaxation, Nature, 195: 896-897. doi:10.1038/195896a0.
3. Seth, B.R. (1966), *Measure concept in mechanics*, Int. J Non-linear Mech., 1(1): 35-40.
4. Odquist, F.K.G., Mathematical Theory of Creep and Creep Rupture, Clarendon Press, Oxford, 1974.
5. Arnold, S.M. (1989), A thermoelastic transversely isotropic thick walled cylinder/disk application: An analytical solution and study, NASA Tech. Memorandum 102320, NASA Lewis Research Center, Cleveland, OH, USA, pp.1-43.
6. Wu, S.-M., Liang, J., Hu, Y. (2000), *Stress in transversely isotropic half-space with typical loads acting on its surface*, Appl. Math. Mech., 21(8): 901-908.
7. Sharma J.N., Sharma R.L. (1996), *On spherically symmetric generalized thermoelastic waves in a transversely isotropic medium*, Indian Journal of Pure and Applied Math, 27(11): 1151-1165.
8. Ding, H., Chen, W., Zhang, L. (2006), Elasticity of Transversely Isotropic Materials, Series: Solid Mechanics and Its Applications, Vol.126, Springer, Vol. 126. doi: 10.1007/1-4020-4034-2
9. Chen, J., Ding, H., Chen, W. (2007), *Three-dimensional analytical solution for a rotating disc of functionally graded materials with transverse isotropy*, Arch. Appl. Mech., 77(4):241-251. doi 10.1007/s00419-006-0098-5

10. Gupta, S.K., Thakur, P. (2007), *Creep transition in a thin rotating disc with rigid inclusion*, Defence Sci. J, 57(2):185-195.
11. Gupta, S.K., Thakur, P. (2008), *Creep transition in an isotropic disc having variable thickness subjected to internal pressure*, Proc. Nat. Acad. of Science, India, Section-A, Vol.78, Part-1, pp.57-66.
12. Thakur, P. (2009), *Elastic-plastic transition stresses in a transversely isotropic thick-walled cylinder subjected to internal pressure and steady-state temperature*, Thermal Sci., 13(4): 107-118. doi: 10.2298/TSCI0904107P
13. Eskandari-Ghadi, M., Fallahi, M., Ardeshir-Behrestaghi, A. (2010), *Forced vertical vibration of rigid circular disc on a transversely isotropic half-space*, J Eng. Mech., 136(7): 913-922. doi: 10.1061/(ASCE)EM.1943-7889.0000114
14. Eskandari-Ghadi, M., Fallahi, M., Ardeshir-Behrestaghi, A. (2013), *Mathematical analysis for an axissymmetric disc-shaped crack in transversely isotropic half-space*, Int. J Mech. Sci., 68: 171-179. doi: 10.1016/j.ijmecsci.2013.01.011
15. Thakur, P. (2011), *Elastic-plastic transition stresses in rotating cylinder by finite deformation under steady-state temperature*, Thermal Sci., 15(2): 537-543.
16. Akinola, A.P., Fadodun, O.O., Olokuntoye, B.A. (2012), *Large deformation of transversely isotropic elastic thin circular disk in rotation*, Int. J Basic & Appl. Sci. IJBAS-IJENS, 12(5): 22-26.
17. Hajimohammadi, A.R., et al. (2013), *Vertical vibration of a rigid circular disc at the interface of a transversely isotropic bi-material*, Int. J Solids Struc., 50(18): 2772-2780. doi: 10.10 16/j.ijsolstr.2013.04.024
18. Thakur, P. (2014), *Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature*, Therm. Sci., 18 (Supp. 1): S93-S106.
19. Thakur, P. (2014), *Steady thermal stress and strain rates in a circular cylinder with non-homogeneous compressibility subjected to thermal load*, Thermal Sci., 18 (Supp. 1): S81- S92. doi: 10. 2298/TSCI110315080P
20. Thakur, P. (2015), *Analysis of thermal creep stresses in transversely thick-walled cylinder subjected to pressure*, Struc. Integ. and Life, 15(1): 19-26.
21. Thakur, P., Singh, S.B., Lozanović Šajić, J. (2015), *Thermo elastic-plastic deformation in a solid disk with heat generation subjected to pressure*, Struc. Integ. and Life, 15(3):135-142.
22. Thakur, P., Kumar, S., Singh, J., Singh, S.B. (2016), *Effect of density variation parameter in a solid disk*, Struc. Integ. and Life, 16(3): 143-148.
23. Thakur, P., Kumar, S. (2016), *Stress evaluation in a transversely isotropic circular disk with an inclusion*, Struc. Integ. and Life, 16(3): 155-160.
24. Gupta, N., Thakur, P., Singh, S.B. (2016), *Mathematical method to determine thermal strain rates and displacement in a thick-walled spherical shell*, Struc. Integ. and Life, 16(2): 99-104.
25. Thakur, P., Kaur, J., Singh, S.B. (2016), *Thermal creep transition stresses and strain rates in a circular disc with shaft having variable density*, Eng. Comput., 33(3): 698-712. doi. org/10.1108/EC-05-2015-0110
26. Thakur, P., Gupta, N., Singh, S.B. (2017), *Creep strain rates analysis in cylinder under temperature gradient materials by using Seth's theory*, Eng. Comput., 34(3): 1020-1030. doi.org/ 10.1108/EC-05-2016-0159
27. Thakur, P., et al. (2017), *Elastic-plastic stress analysis in a spherical shell under internal pressure and steady state temperature*, Struc. Integ. and Life, 17(1): 39-43.
28. Thakur, P., Singh, S.B., Pathania, D., Verma, G. (2017), *Thermal creep stress and strain analysis in a non-homogeneous spherical shell*, accepted for publication, J Theor. and Appl. Mech., Warsaw, 55(4): 1155-1165. doi: 10.15632/jtam-pl.55.4. 1155
29. Thakur, P., Mahajan P., Kumar S. (2018), *Creep stresses and strain rates for a transversely isotropic disc having the variable thickness under internal pressure*, Struc. Integ. and Life, 18(1): 15-21.
30. Thakur, P., et al. (2018), *Exact solution of rotating disc with shaft problem in the elastoplastic state of stress having variable density and thickness*, Struc. Integ. and Life, 18(2): 128-134.
31. Thakur, P., et al. (2018), *Modelling of creep behaviour of a rotating disc in the presence of load and variable thickness by using Seth transition theory*, Struc. Integ. and Life, 18(2): 135-142.