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EXACT SOLUTION OF ROTATING DISC WITH SHAFT PROBLEM IN THE ELASTOPLASTIC STATE OF STRESS HAVING VARIABLE DENSITY AND THICKNESS

TAČNO REŠENJE PROBLEMA ROTIRAJUĆEG DISKA SA OSOVINOM PROMENLJIVE GUSTINE I DEBLJINE PRI ELASTOPLASTIČNIM NAPONIMA

Originalni naučni rad / Original scientific paper
UDK /UDC: 539.319

Rad primljen / Paper received: 18.05.2018

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Keywords
- disc
- inclusion
- stresses
- displacement
- density
- thickness

Ključne reči
- disk
- uključak
- naponi
- pomeranje
- gustina
- debljina

Abstract

The purpose of this paper is to present study of the elastoplastic state of stress in a rotating disc with a shaft having a thickness and density profile by using Seth's transition theory. It has been observed that the rotating disc made of the compressible material with an inclusion requires higher angular speed to yield at the internal surface as compared to the disc made of incompressible material, and a much higher angular speed is required to yield with the increase in radii ratio. The thickness and density parameters decrease the value of angular speed at the internal surface of the rotating disc of compressible as well as incompressible materials. The model proposed in this paper is used in mechanical and electronic devices. They have extensive practical engineering applications such as in steam and gas turbines, turbo generators, flywheel of internal combustion engines, turbojet engines, reciprocating engines, centrifugal compressors and brake discs.

Izvod

Cilj rada je predstavljanje elastoplastičnog naponskog stanja u rotirajućem disku sa osovinom, datog profila debljine i gustine, primenom teorije prelaznih napona Seta. Uočeno je da rotirajući disk proizveden od stišljivog materijala sa uključkom zahteva veću ugaonu brzinu za pojavu tečenja na unutrašnjim površinama, u poređenju sa diskom od nestišljivog materijala. Potrebna je i veća ugaona brzina za pojavu tečenja pri porastu odnosa poluprečnika. Parametri debljine i gustine smanjuju veličinu ugaone brzine na unutrašnjoj površini rotirajućeg diska i kod stišljivih i nestišljivih materijala. Model predstavljen u radu se koristi kod mehaničkih i električnih uređaja. Ovi uređaji imaju izrazit značaj u praktičnim inženjerskim primenama, na pr. kod parnih i gasnih turbina, turbogeneratora, zamajca u SUS motorima, mlaznim i klipnim motorima, centrifugalnim kompresorima i disk kočnicama.

INTRODUCTION

Theoretical investigation of deformations in discs induced by centrifugal forces is an important topic due to its various applications in structural engineering components such as in steam and gas turbine rotors, turbo generators, and internal combustion engines, castings of ship propellers, turbojet engines, reciprocating and centrifugal compressors. The stress analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko et al. /5/ in the elastic range and by Chakrabarty /7/ and Heyman /2/ for the plastic range. Their solutions for the problem of fully plastic state do not involve the plane stress condition, that is to say, one can obtain the same stresses and angular speed necessary for the fully plastic stress of the disc without using the plane stress condition (i.e. $T_{zz} = 0$). Elastic-plastic deformations in variable thickness rotating discs have been investigated extensively by Gamer /6/, Güven et al. /9/, You et al. /10/, Eraslan et al. /12/. On the other hand, thermally induced elastic-plastic deformations of stationary discs have been studied analytically by Güven et al. /9, 11/, employing different boundary conditions. Güven /8/ analysed elastic-plastic stresses in a rotating hyperbolic disc with a rigid inclusion under the assumption of Tresca's yield condition, its associated flow rule, and linear strain hardening. Perfect elasticity and ideal plasticity are two extreme properties of the material, and the use of an ad-hoc rule, like the yield condition, amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, a transition takes place. Since this transition is nonlinear in character and difficult to investigate, workers have taken certain ad hoc assumptions like yield condition, incompressibility condition, and a strain law, which may or may not be valid for the problem. Gupta et al. /14, 15/ investigated the problem of elastic-plastic and creep transition in a thin rotating disc with the inclusion by using Seth's transition theory. Seth's transition theory /3/ does not require these ad-hoc assumptions and thus solves a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points, or turning points of the differential equations, defining the deformed field, and has been applied to a large number of problems successively (Gupta et al. /14-16/, Seth /2, 3/, Thakur et al. /13, 17-42). This paper investigates the influence of density and thickness profile on the elastic-plastic distribution in a rotating thin disc with a rigid shaft using Seth's transition theory. The thickness h and density of the disc ρ are assumed to vary along the radius in the form:

$$h = h_0 (r/b)^{-k} \quad \text{and} \quad \rho = \rho_0 (r/b)^{-m} \tag{1}$$

where h_0 and ρ_0 are thickness and density at $r = b$ respectively, m and k are density and thickness parameters, respectively.

GOVERNING EQUATIONS

Consider a thin rotating disc of variable density and thickness with the central bore of radius a and external radius b. The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed ω about an axis perpendicular to its plane and passing through the centre. The disc is thin so that it is effective in a state of plane stress $T_{zz} = 0$ and the variation of the thickness is radial and symmetric with respect to the mid-plane.

Displacement coordinates

The components of displacement in cylindrical polar coordinates are given using:

$$u = r(1-\beta)\,;\; v = 0\,;\; w = dz \tag{2}$$

where: β is position function, depending on $r = \sqrt{x^2+y^2}$ only; and d is a constant. The generalized components of strain are given by /4/ as:

$$\begin{aligned} e_{rr} &= \frac{1}{n}\left[1-(r\beta'+\beta)^n\right], \\ e_{\theta\theta} &= \frac{1}{n}\left[1-\beta^n\right], \\ e_{zz} &= \frac{1}{n}\left[1-(1-d)^n\right], \\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0. \end{aligned} \tag{3}$$

where: n is the measure and $\beta' = d\beta/dr$.

Stress-strain relation

The stress-strain relations for isotropic material are given by /1/:

$$\tau_{ij} = \lambda\delta_{ij} I_1 + 2\mu e_{ij} \quad (i,j = 1,2,3) \tag{4}$$

where: λ and μ are lame's constants; and e_{kk} is the first strain invariant, δ_{ij} is the Kroncecker's delta. Equation (4) for this problem becomes:

$$\begin{aligned} \tau_{rr} &= \frac{2\lambda\mu}{\lambda+2\mu}[e_{rr}+e_{\theta\theta}] + 2\mu e_{rr}, \\ \tau_{\theta\theta} &= \frac{2\lambda\mu}{\lambda+2\mu}[e_{rr}+e_{\theta\theta}] + 2e_{\theta\theta}, \\ \tau_{r\theta} &= \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0. \end{aligned} \tag{5}$$

Strain components in terms of stresses are obtained as:

$$\begin{aligned} e_{rr} &= \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-(r\beta'+\beta)^2\right] = \frac{1}{E}\left[\tau_{rr} - \left(\frac{1-C}{2-C}\right)\tau_{\theta\theta}\right], \\ e_{\theta\theta} &= \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}[1-\beta^2] = \frac{1}{E}\left[\tau_{\theta\theta} - \left(\frac{1-C}{2-C}\right)\tau_{rr}\right], \\ e_{zz} &= \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}[1-(1-d)^2] = -\frac{(1-C)}{E(2-C)}[\tau_{rr} - \tau_{\theta\theta}], \\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0 \end{aligned} \tag{6}$$

where: $E = \dfrac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$; and $C = \dfrac{2\mu}{\lambda+2\mu}$.

Substituting Eq.(3) into Eq.(5), we get

$$\begin{aligned} \tau_{rr} &= \frac{2\mu}{n}\left[3-2C-\beta^n\left\{1-C+(2-C)(P+1)^n\right\}\right], \\ \tau_{\theta\theta} &= \frac{2\mu}{n}\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right], \\ \tau_{r\theta} &= \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0. \end{aligned} \tag{7}$$

Equation of equilibrium

The equations of equilibrium are all satisfied except:

$$\frac{d}{dr}(rh\tau_{rr}) - h\tau_{\theta\theta} + \rho\omega^2 r^2 h = 0 \tag{8}$$

where: ρ is the density; and h be the thickness of the material in the rotating disc.

Critical points or turning points

By substituting Eq.(7) into Eq.(8), we get a nonlinear differential equation with respect to β:

$$\begin{aligned} &(2-C)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \left(\frac{rh'}{h}\right)\times \\ &\times\left[3-2C-\beta^n\left\{(1-C)+(2-C)(P+1)^n\right\}\right] + \\ &+\beta^n\left[1-(P+1)^n - np\left\{1-C+(2-C)(P+1)^n\right\}\right] \end{aligned} \tag{9}$$

where: $r\beta' = \beta P$ (P is function of β, and β is function of r only).

Transition points

Transition points of β in Eq.(9) are $P \to -1$ and $P \to \pm\infty$.

Boundary condition

The boundary conditions of the problem are given by:

$$r = a,\; u = 0,\; r = b,\; \tau_{rr} = 0. \tag{10}$$

SOLUTION THROUGH THE PRINCIPAL STRESSES

For finding the plastic stress, the transition function is taken through the principal stress (see /3, 4, 13-42/) at the transition point $P \to \pm\infty$, we define the transition function ζ as, Eq.(11):

$$\zeta = \frac{n}{2\mu}\tau_{\theta\theta} = \left[(3-2C) - \beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right], \quad (11)$$

where: ζ be the transition function of r only.

Taking the logarithmic differentiation of Eq.(11) with respect to r and using Eq.(9), we get Eq.(12):

$$\frac{d(\log\zeta)}{dr} = -\frac{\frac{1-C}{2-C}\left[\beta^n\left\{1-(P+1)^n - n(1-C)P\right\} + \frac{n\rho\omega^2 r^2}{2\mu\beta^n} + \left(\frac{rh'}{h}\right)\left\{(3-2C)--\beta^n\left[(1-C)+(2-C)(P+1)^n\right]\right\}\right] + (2-C)nP\beta^n}{r\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\} - \frac{nC\xi\theta}{2\mu}\right]}. \quad (12)$$

Taking the asymptotic value of Eq.(12) as $P \to \pm\infty$ and integrating, we get

$$R = (A_1/h)r^{-1/(2-C)}, \quad (13)$$

where: A_1 is a constant of integration which can be determined from the boundary condition. From Eqs. (11) and (13) and using Eq.(1), we have

$$\tau_{\theta\theta} = \frac{A_1 b^{-k}}{h_0 r^{-k}} r^{-1/(2-C)}. \quad (14)$$

Substituting Eq.(14) into Eq.(18) and using Eq.(1) then integrating, we get

$$\tau_{rr} = \frac{A_1 b^{-k}}{h_0 r^{-k}}\left(\frac{2-C}{1-C}\right) r^{-1/(2-C)} - \frac{\rho_0\omega^2 r^{2-m}}{b^{-m}(3-m-k)} + \frac{A_2 b^{-k}}{r h_0 r^{-k}}, \quad (15)$$

where: A_2 is a constant of integration which can be determined from the boundary condition. Substituting Eq.(14) and Eq.(15) into the second equation of Eq.(6), we get

$$\beta = \sqrt{1 - \frac{2(1-C)}{E(2-C)}\left[\frac{\rho_0\omega^2 r^{2-m}}{b^{-m}(3-m-k)} - \frac{A_2 b^{-k}}{r h_0 r^{-k}}\right]}.$$

Substituting the value β from Eq(2), we have

$$u = r - r\sqrt{1 - \frac{2(1-C)}{E(2-C)}\left[\frac{\rho_0\omega^2 r^{2-m}}{b^{-m}(3-m-k)} - \frac{A_2 b^{-k}}{r h_0 r^{-k}}\right]}, \quad (16)$$

where: $E = \frac{2\mu(3-2C)}{(2-C)}$ is the Young's modulus. Using boundary condition Eq.(10) in Eq.(15) and Eq.(16), we get

$$A_1 = \frac{\rho_0\omega^2(1-C)h_0(b^{3-m-k} - a^{3-m-k})}{(2-C)(3-m-k)b^{1-C/2-C}b^{-m-k}},$$

$$A_2 = \frac{\rho_0\omega^2 a^{3-m-k}h_0}{b^{-m-k}(3-m-k)}.$$

Substituting the values of constants A_1 and A_2 from Eqs.(14), (15) and (16), respectively, we get the transitional stresses and displacement as:

$$\tau_{\theta\theta} = \frac{\rho_0\omega^2 b^2(1-C)}{(3-m-k)(2-C)}\left[1-\left(\frac{a}{b}\right)^{3-m-k}\right]\left(\frac{r}{b}\right)^{-\frac{1}{2-C}+k},$$

$$\tau_{rr} = \frac{\rho_0\omega^2 b^2}{(3-m-k)}\left[\left\{1-\left(\frac{a}{b}\right)^{3-m-k}\right\}\left(\frac{r}{b}\right)^{-\frac{1}{2-C}+k} - \left(\frac{r}{b}\right)^{2-m} + \frac{a^{3-m-k}}{b^{2-m}r^{1-k}}\right]$$

$$u = r - r\sqrt{1 - \frac{2}{E}\left(\frac{1-C}{2-C}\right)\frac{\rho_0\omega^2(r^{3-m-k} - a^{3-m-k})}{b^{-m}(3-m-k)r^{1-k}}}, \quad (17)$$

and

$$\tau_{rr} - \tau_{\theta\theta} = \frac{\rho_0\omega^2 b^2}{(3-m-k)}\left[\left\{1-\left(\frac{a}{b}\right)^{3-m-k}\right\}\left(\frac{1}{2-C}\right)\times \left(\frac{r}{b}\right)^{-\frac{1}{2-C}+k} - \left(\frac{r}{b}\right)^{2-m} + \frac{a^{3-m-k}}{b^{2-m}r^{1-k}}\right] \quad (18)$$

Initial yielding

From Eq.(18), it is seen that $|\tau_{rr} - \tau_{\theta\theta}|$ is maximum at the inner surface (at $r = a$), therefore yielding of the disc takes place at the internal surface and Eq.(18) may be written as

$$\left|\tau_{rr} - \tau_{\theta\theta}\right|_{r=a} = \left|\frac{\rho_0\omega^2 b^2}{(3-m-k)}\left[\frac{1}{2-C}\left(\frac{a}{b}\right)^{k-\frac{1}{2-C}}\left\{1-\left(\frac{a}{b}\right)^{3-m-k}\right\} - \left(\frac{a}{b}\right)^{2-m} + 1\right]\right| = Y(\text{say})$$

where: Y is the initial yielding stresses. The angular speed necessary for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho_0\omega_i^2 b^2}{Y} = \left|\frac{3-m-k}{\left[\frac{1}{2-C}\left(\frac{a}{b}\right)^{k-\frac{1}{2-C}}\left\{1-\left(\frac{a}{b}\right)^{3-m-k}\right\} - \left(\frac{a}{b}\right)^{2-m} + 1\right]}\right| \quad (19)$$

where: $\omega_i = (\Omega_i/b)\sqrt{Y/\rho_0}$.

Fully-plastic state

The disc becomes fully plastic ($C \to 0$) at the external surface and Eq.(18) becomes:

$$\left|\tau_{rr} - \tau_{\theta\theta}\right|_{r=b} = \left|\frac{\rho_0\omega^2 b^2}{3-m-k}\left[\frac{1}{2}\left\{1-\left(\frac{a}{b}\right)^{3-m-k}\right\} + \left(\frac{a}{b}\right)^{3-m-k} - 1\right]\right| = Y^*(\text{say})$$

where: Y^* is yielding stress. The angular speed required for fully plastic state is given by:

$$\Omega_f^2=\frac{\rho\omega_f^2 b^2}{Y^*}=\left|\frac{2(3-m-k)}{(a/b)^{3-k-m}-1}\right|, \tag{20}$$

where: $\omega_f=(\Omega_f/b)\sqrt{Y^*/\rho_0}$. We introduce the following non-dimensional components: $R = r/b$, $R_0 = a/b$, $\sigma_r = \tau_{rr}/Y$, $\sigma_\theta = \tau_{\theta\theta}/Y$, $\bar{u} = u/b$, $\Omega_i^2 = \rho_0\omega_i^2 b^2/Y$, and $\Omega_f^2 = \rho_0\omega_f^2 b^2/Y^*$. Elastic-plastic transitional stresses, angular speed and displacement from Eqs.(27) and (19) in non-dimensional form become:

$$\sigma_\theta=\frac{\Omega_i^2(1-C)}{(3-k-m)(2-C)}\left[1-R_0^{3-m-k}\right]R^{k-\frac{1}{2-C}},$$

$$\sigma_r=\frac{\Omega_i^2}{(3-k-m)}\left[\left\{1-R_0^{3-m-k}\right\}R^{k-\frac{1}{2-C}}-R^{2-m}+\frac{R_0^{3-m-k}}{R^{1-k}}\right], \tag{21}$$

$$\bar{u}=R-R\sqrt{1-2\frac{Y}{E}\left(\frac{1-C}{2-C}\right)\frac{\Omega_i^2\left(R^{3-m-k}-R_0^{3-m-k}\right)}{(3-k-m)R^{1-k}}}\ ,\ \forall\ 3-k-m\neq 0$$

and

$$\Omega_i^2=\left|\frac{(3-k-m)}{\left[1/(2-C)\right]R_0^{k-\frac{1}{2-C}}\left(1-R_0^{3-m-k}\right)-R_0^{2-m}+1}\right|,\ \forall\ 3-k-m\neq 0. \tag{22}$$

Stresses, displacement and angular speed for fully plastic state ($C \to 0$), are obtained from Eqs.(21) and (20) as:

$$\sigma_\theta=\frac{\Omega_f^2}{2(3-k-m)}\left[1-R_0^{3-m-k}\right]R^{k-\frac{1}{2}},$$

$$\sigma_r=\frac{\Omega_f^2}{(3-k-m)}\left[\left\{1-R_0^{3-m-k}\right\}R^{k-\frac{1}{2}}-R^{2-m}+\frac{R_0^{3-m-k}}{R^{1-k}}\right], \tag{23}$$

$$\bar{u}=R-R\sqrt{1-\left(\frac{Y}{E}\right)\frac{\Omega_f^2\left(R^{3-m-k}-R_0^{3-m-k}\right)}{(3-k-m)R^{1-k}}}\quad \forall\ 3-k-m\neq 0$$

and

$$\Omega_f^2=\frac{\rho\omega_f^2 b^2}{Y^*}=\left|\frac{2(3-m-k)}{(a/b)^{3-m-k}-1}\right|\ \ \forall\ 3-k-m\neq 0. \tag{24}$$

NUMERICAL RESULTS AND DISCUSSION

For calculating stresses, displacement and angular speed based on the above analysis, the following values have been taken: $C = 0$ (incompressible material, i.e. rubber); $C = 0.25$ (compressible material, i.e. saturated clay); and $C = 0.5$ (compressible material, i.e. copper); $k = -1$, 0 (flat disc), 1.3 (between values 1 and 2); and $m = -1$, 0, and 1, in respect. In Fig. 1, curves are drawn between angular speed required for initial yielding and various radii ratios $R_0 = a/b$ for $C = 0.00$; 0.25; 0.5; $k = -1$, 0, 1.3; and $m = -1$, 0, 1, in respect. It has been observed that the rotating disc made of compressible material with an inclusion has require higher angular speed to yield at the internal surface as compare to the disc of incompressible material, and a much higher angular speed is required to yield with increase in radii ratio. The thickness and density parameters decrease the value of angular speed at the internal surface of the rotating disc of compressible as well as incompressible material. It can also be seen from Table 1, that for higher percentage of increase in angular speed is required to become a fully plastic state for $k = 1.3$ as compared to the disc of thickness $k = -1$ and $m = -1$, 0, 1. Curves are produced between stresses along the radii ratio $R = r/b$ at initial yielding (see Fig. 2) for the rotating disc made of compressible, as well as incompressible material.

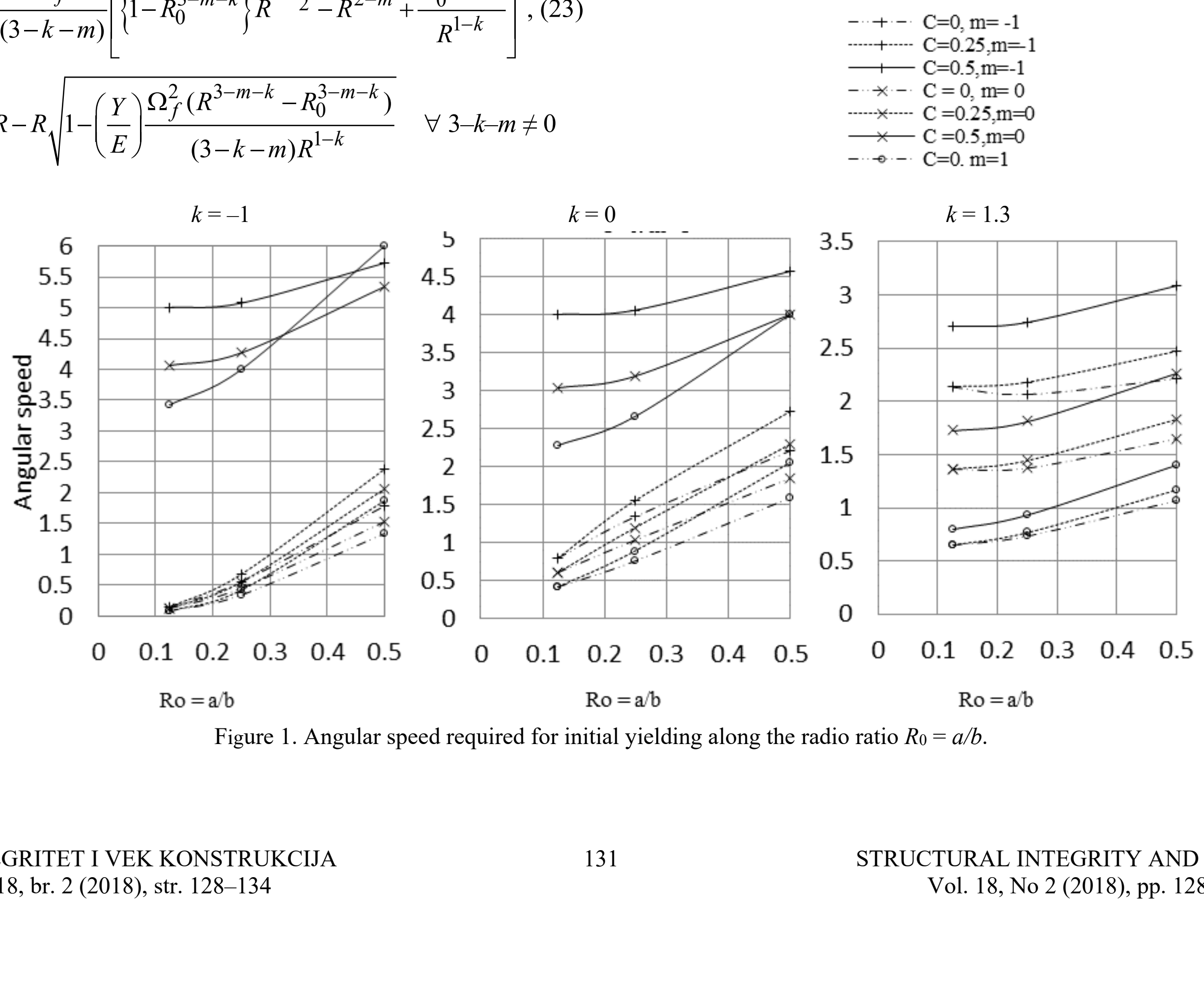

Figure 1. Angular speed required for initial yielding along the radio ratio $R_0 = a/b$.

Table 1. Angular speed required for initial yielding and fully plastic state.

Density m	Angular speed	k = –1			k = 0 (flat disc)			k = 1.3		
		C= 0	C = 0.25	C = 0.5	C = 0	C = 0.25	C = 0.5	C = 0	C = 0.25	C = 0.5
–1	Ω_i^2	1.77777	2.38601	5.7142	2.2068	2.729	4.57142	2.215	2.473726	3.085714
	Ω_f^2	10.3225	10.3225	10.322	8.5333	8.533333	8.533333	6.382172	6.382172	6.382172
0	Ω_i^2	1.52381	2.071277	5.333333	1.846154	2.305537	4	1.648683	1.833678	2.266667
	Ω_f^2	8.533333	8.533333	8.533333	6.857143	6.857143	6.857143	4.911777	4.911777	4.911777
1	Ω_i^2	1.333333	1.872156	6	1.6	2.056618	4	1.066868	1.169877	1.4
	Ω_f^2	6.857143	6.857143	6.857143	5.333333	5.333333	5.333333	3.641776	3.641776	3.641776
–1	P %	140.9658	107.9974	34.4043	96.6384	76.8216	36.62601	69.7218	60.62327	43.81573
0	P %	136.6432	102.9739	26.49111	92.72482	72.45889	30.93073	72.60396	63.66573	47.20599
1	P %	126.7787	91.3818	6.904497	82.57419	61.03585	15.47005	84.75717	76.4357	61.28449

where $P=\left(\sqrt{\Omega_i^2/\Omega_f^2}-1\right)\times 100$ is the percentage increase in angular speed from initial yielding to fully plastic state, having thickness k = –1, 0, 1.3 and density m = –1, 0, 1.

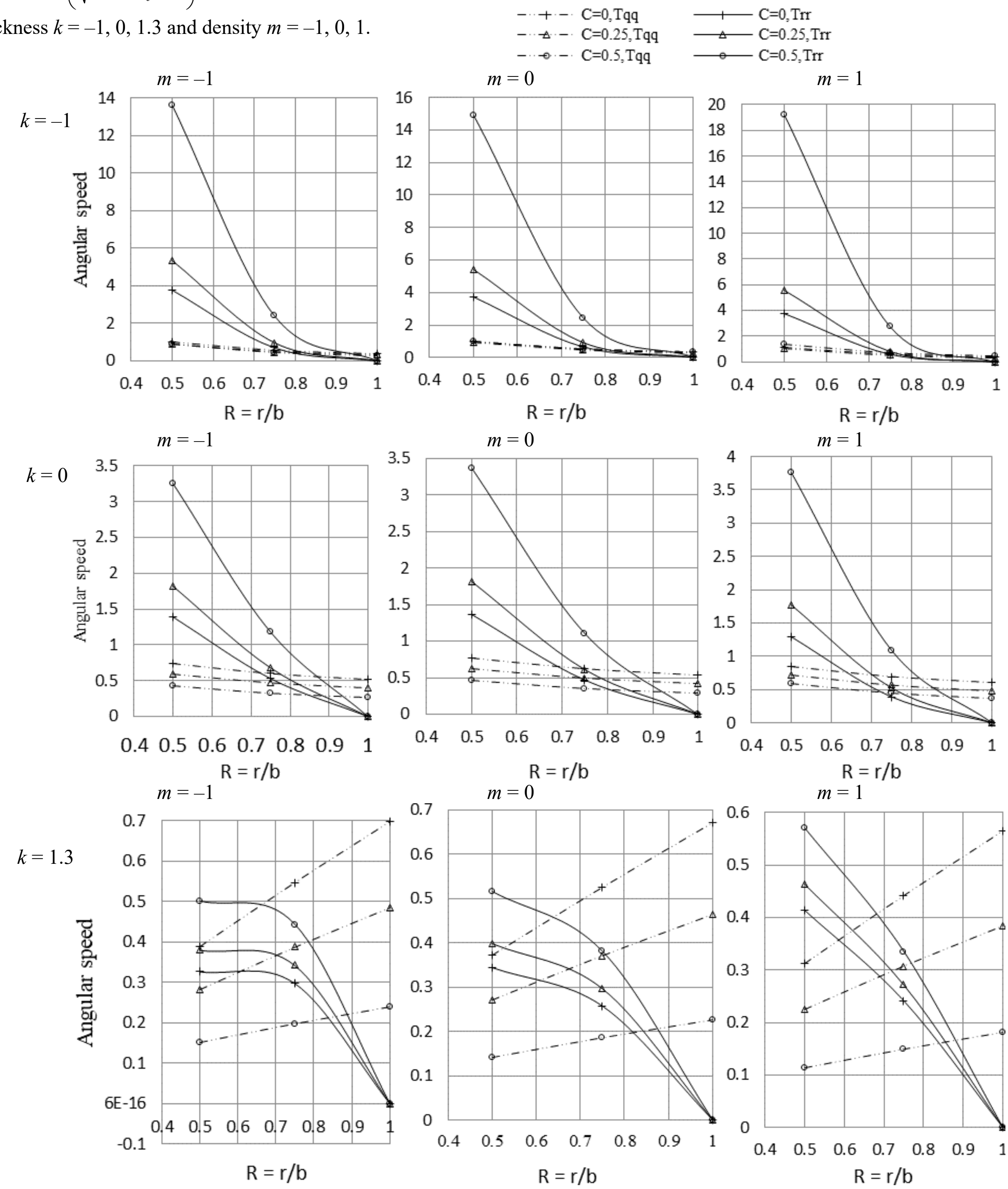

Figure 2. Stresses for initial yielding state.

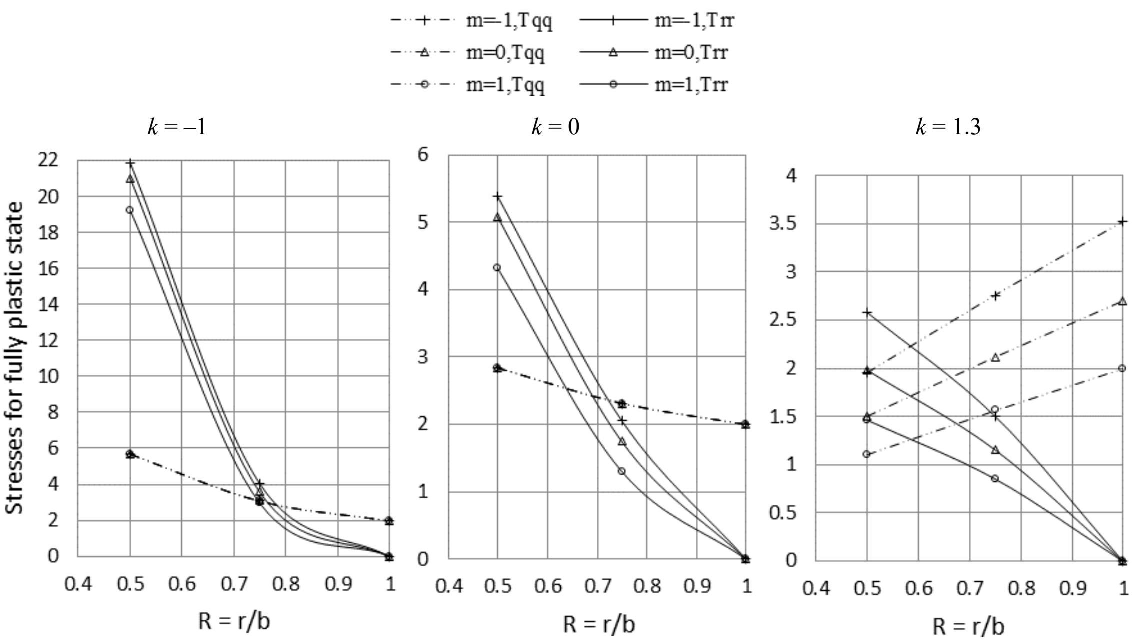

Figure 3. Stresses for the fully-plastic state.

It is also observed from Fig. 2 that the radial stress has a maximum value at the internal surface of the rotating disc made of the compressible materials (i.e. saturated clay or copper) as compared to the disc made of the incompressible material (i.e. rubber) for $k = -1$, 0 and $m = -1$, 0, 1, in respect. When the value of thickness parameter k varies between 1 and 2, the circumferential stress is maximum at the external surface of the disc made of incompressible material as compared to the disc of compressible material, for $m = -1$ and 0, and radial stresses are maximum at the internal surface for compressible material for $m = 1$, in respect. Curves are produced between stresses along the radii ratio $R = r/b$ at fully-plastic state (see Fig. 3) for $k = -1$, 0, 1.3, and $m = -1$, 0, 1. It has been seen from Fig. 3 that radial stresses are maximum at the internal surface at $m = -1$ for $k = -1$, 0, but when the value of thickness parameter k lies between 1 and 2, the results are reversed. The thickness parameter decreases and the density parameter increases the values of stresses at the inner and outer surfaces of the rotating disc of incompressible, as well as compressible material.

ACKNOWLEDGEMENT

The authors gratefully acknowledge UGC, New Delhi, for providing financial support to carry out this research work under UGC-Major Research Project Scheme (MRP-MAJOR-MATH-2013-41603).

REFERENCES

1. Sokolnikoff, I.S., Mathematical Theory of Elasticity, 2nd Ed., McGraw-Hill Inc., New York, 1953.
2. Heyman, J. (1958), *Plastic design of rotating discs*, Proc. Inst. Mech. Engrs, 172(1): 531-547. doi.org/10.1243/PIME_PROC_1958_172_045_02
3. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi:10.1038/195896a0.
4. Seth, B.R. (1966), *Measure concept in mechanics*, Int. J Non-linear Mech., 1(1): 35-40.
5. Timoshenko, S.P., Goodier, J.N., Theory of Elasticity, 3rd Ed., New York, McGraw-Hill Book Co., London, 1951.
6. Gamer, U. (1984), *Elastic-plastic deformation of the rotating solid disk*, Ingenieur-Archiv, 54(5): 345-354.
7. Chakravorty, J., Theory of Plasticity, McGraw-Hill Book Inc., New York, 1987.
8. Güven, U. (1998), *Elastic-plastic stress distribution in a hyperbolic disk with rigid inclusion*, Int. J Mech. Sci., 40(1): 97-109. doi.org/10.1016/S0020-7403(97)00036-2
9. Güven, U., Altay, O. (1998), *Linear hardening solid disk with rigid casing subjected to a uniform heat source*, Mech. Res. Comm., 25(6): 679-684. doi.org/10.1016/S0093-6413(98)00087-1
10. You, L.H., Tang, Y.Y., Zhang, J.J., Zheng, C.Y. (2000), *Numerical analysis of elastic-plastic rotating disks with arbitrary variable thickness and density*, Int. J Solids and Struc. 37(52): 7809-7820. doi: 10.1016/S0020-7683(99)00308-X
11. Güven, U., Altay, O. (2000), *Elastic-plastic solid disk with non-uniform heat source subjected to external pressure*, Int. J Mech. Sci. 42(5): 831-842. doi: 10.1016/S0020-7403(99)0003 2-6
12. Eraslan, A.N., Argeso, H. (2002), *Limit angular velocities of variable thickness rotating disks*, Int. J Solids and Struc. 39 (12): 3109-3130. doi: 10.1016/S0020-7683(02)00249-4
13. Thakur Pankaj (2006), Some problems in elastic-plastic and creep transition, Ph.D. thesis, Department of Mathematics, H.P.U. Shimla, 46-47.
14. Gupta, S.K., Thakur, P. (2007), *Creep transition in a thin rotating disc with rigid inclusion*, Defence Sci. J, 57(2):185-195.
15. Gupta, S.K., Thakur, P. (2007), *Thermo elastic-plastic transition in a thin rotating disc with inclusion*, Thermal Sci., 11(1): 103-118. doi:10.2298/TSCI0701103G
16. Gupta, S.K., Thakur, P. (2008), *Creep transition in an isotropic disc having variable thickness subjected to internal pressure*, Proc. Nat. Acad. of Science, India, Section-A, Vol.78, Part-1, pp.57-66.
17. Thakur, P. (2009), *Elastic-plastic transition stresses in a transversely isotropic thick-walled cylinder subjected to internal pressure and steady-state temperature*, Thermal Sci., 13(4): 107-118. doi: 10.2298/TSCI0904107P

18. Thakur P. (2011), *Elastic-plastic transition stresses in rotating cylinder by finite deformation under steady-state temperature*, Therm. Sci., 15(2): 537-543.
19. Thakur, P. (2012), *Deformation in a thin rotating disc having variable thickness and edge load with inclusion at the elastic-plastic transitional stress*, Struc. Integ. & Life, 12(1):65-70.
20. Thakur, P. (2012), *Thermo creep transition stresses in a thick walled cylinder subjected to internal pressure by finite deformation*, Struc. Integ. & Life, 12(3):165-173.
21. Thakur, P., Singh, S.B., Thakur, J.K. (2013), *Elastic-plastic transitional stresses in a thin rotating disc with shaft having variable thickness under steady state temperature*, Struc. Int. & Life , 13(2):109-116.
22. Thakur, P. (2013), *Finitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft*, J Techn. of Plast. 38(2):143-156.
23. Thakur, P. (2014), *Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature*, Therm. Sci., 18 (Suppl. 1): S93-S106. doi: 10.2298/TSCI1103 18079P
24. Thakur, P. (2014), *Steady thermal stress and strain rates in a circular cylinder with non-homogeneous compressibility subjected to thermal load*, Thermal Sci., 18 (Supp. 1): S81-S92. doi: 10. 2298/TSCI110315080P
25. Thakur, P. (2015), *Analysis of thermal creep stresses in transversely thick-walled cylinder subjected to pressure*, Struc. Integ. and Life, 15(1): 19-26.
26. Thakur, P., Singh, S.B., Lozanović Šajić, J. (2015), *Thermo elastic-plastic deformation in a solid disk with heat generation subjected to pressure*, Struc. Integ. and Life, 15(3):135-142.
27. Thakur, P., Kumar, S., Singh, J., Singh, S.B. (2016), *Effect of density variation parameter in a solid disk*, Struc. Integ. and Life, 16(3): 143-148.
28. Thakur, P., Kumar, S. (2016), *Stress evaluation in a transversely isotropic circular disk with an inclusion*, Struc. Integ. and Life, 16(3): 155-160.
29. Gupta, N., Thakur, P., Singh, S.B. (2016), *Mathematical method to determine thermal strain rates and displacement in a thick-walled spherical shell*, Struc. Integ. and Life, 16(2): 99-104.
30. Kaur, J., Thakur, P., Singh, S.B. (2016), *Steady thermal stresses in a thin rotating disc of finitesimal deformation with mechanical load*, J Solid Mech., 8(1): 204-211.
31. Thakur, P., Kaur, J., Singh, S.B. (2016), *Thermal creep transition stresses and strain rates in a circular disc with shaft having variable density*, Eng. Comput., 33(3): 698-712. doi.org/10.1108/EC-05-2015-0110
32. Thakur, P., Gupta, N., Singh, S.B. (2017), *Creep strain rates analysis in cylinder under temperature gradient materials by using Seth's theory*, Eng. Comput., 34(3): 1020-1030. doi.org/ 10.1108/EC-05-2016-0159
33. Thakur, P., Pathania, D., Verma, G., Singh, S.B. (2017), *Elastic-plastic stress analysis in a spherical shell under internal pressure and steady state temperature*, Struc. Integ. and Life, 17(1): 39-43.
34. Thakur, P., Shahi, S., Gupta, N., Singh, S.B. (2017), *Effect of mechanical load and thickness profile on creep in a rotating disc by using Seth's transition theory*, AIP Conf. Proc., Amer. Inst. of Physics, USA, 1859(1): 020024. doi.org/10.1063/1.49 90177
35. Thakur, P., Pathania, D., Verma, G., Singh, S.B. (2017), *Thermal creep analysis in non-homogeneous spherical shell*, Struc. Integ. and Life, 17(2): 89-95.
36. Thakur, P., Pathania, D., Verma, G., Singh, S.B. (2017), *Creep stresses in a rotating disc having variable density and mechanical load under steady-state temperature*, Struc. Integ. and Life, 17(2): 97-104.
37. Thakur, P., Pathania, D., Verma, G., Singh, Lozanović Šajić, J. (2017), *Non-homogeneity effect in the spherical shell by using Seth's theory*, Struc. Integ. and Life, 17(3): 177-182.
38. Thakur, P. (2017), *Creep stresses in a circular cylinder subjected to torsion*, Struc. Integ. and Life, 17(3): 183-186.
39. Sharma, S., Thakur, P., Sharma, R., et al. (2017), *Thermo-elastic-plastic transition in torsion of composite thick-walled circular cylinder subjected to pressure*, Struc. Integ. and Life, 17(3): 193-201.
40. Thakur, P., Singh, S.B., Pathania, D., Verma, G. (2017), *Thermal creep stress and strain analysis in a non-homogeneous spherical shell*, J Theor. and Appl. Mech., Warsaw, 55(4): 1155-1165. doi: 10.15632/jtam-pl.55.4.1155
41. Thakur, P., Mahajan, P., Kumar, S. (2018), *Creep stresses and strain rates for a transversely isotropic disc having the variable thickness under internal pressure*, Struc. Integ. and Life, 18(1): 15-21.
42. Thakur, P. (2018), *Strain rates analysis in a rotating disc having variable thickness parameter and subjected to mechanical load*, accepted for publication: J Theor. and Appl. Mech., Warsaw, 2018.