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CREEP TORSION IN THICK-WALLED CIRCULAR CYLINDER UNDER INTERNAL AND EXTERNAL PRESSURE

PUZANJE PRI UVIJANJU CILINDRIČNIH DEBELOZIDIH POSUDA PRI UNUTRAŠNJEM I SPOLJNOM PRITISKU

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- pritisak

Abstract

In this paper, a closed-form solution has been developed for the creep behaviour of functionally graded isotropic cylinder subjected to twist under internal and external pressure. Seth's transition theory using the concept of generalized strain measure has been applied to study the creep behaviour of the cylinder. The compressibility of the cylinder is considered to be varying in the radial direction. Creep stresses for different values of compressibility parameters and strain measures are plotted against radii ratio with fixed angle of twist under internal and external pressure. Effects of compressibility, pressure and torsion on creep stresses have been discussed. From the analysis, it is found that cylinder made up of functionally graded material is a better choice for designing as compared to a homogeneous cylinder. This is because the shear stresses are on the higher side in a functionally graded cylinder as compared to a homogeneous cylinder.

Izvod

U radu je prikazano dobijanje rešenja zatvorenog oblika za puzanje izotropnog cilindra od funkcionalnog kompozitnog materijala, opterećenog na uvijanje pod dejstvom unutrašnjeg i spoljnog pritiska. Setova teorija prelaznih napona sa konceptom generalisanih mera deformacija je primenjena u proučavanju puzanja cilindra. Razmatrano je promenljivo stišljivo stanje materijala cilindra u radijalnom pravcu. Naponi puzanja kod različitih veličina parametara stišljivosti i mera deformacija su predstavljeni u funkciji odnosa poluprečnika, sa fiksnim uglom uvijanja pri dejstvu unutrašnjeg i spoljnog pritiska. Diskutovani su uticaji stišljivosti, pritiska i torzije na napone puzanja. Prema analizama, utvrđeno je da je cilindar od funkcionalnog kompozitnog materijala bolja varijanta u projektovanju u poređenju sa homogenim cilindrom. Ovo se objašnjava većim naponima smicanja prisutnim kod cilindra od funkcionalnog kompozita u poređenju sa homogenim cilindrom.

INTRODUCTION

Pressurized cylindrical vessels are broadly used as components in the industries such as chemical, military, oil, water and nuclear power plants. These industrial components are commonly subjected to complex loading conditions such as torsion, pressure, temperature etc. Saint-Venant investigated torsion in straight bars and successfully formulated the constitutive equations of torsion in a straight bar. Some analytical solutions to homogeneous cylinder are available in literature: Timoshenko /1/. Functionally graded materials (FGMs) are advanced composite materials which play a vital role in the design of composite structures. Functionally graded materials (FGMs) are non-homogeneous materials whose properties and composition change continuously in certain components operating under complex loading /2-4/. Gupta et al. /5/ investigated steady state creep stresses in circular cylinder subjected to torsion using the concepts of transition theory /6/ and principle of generalized strain measure /7, 8/. Further, Gupta and Rana /9/ studied transitional stresses in a transversely isotropic cylinder subjected to torsion and found that shear stresses are higher for a cylinder made up of transversely isotropic material as compared to the cylinder made up of isotropic material. The torsion problems with homogeneous elastic materials have been studied by several authors /5, 9, 10, 11/ in numerous aspects. Torsion analysis is a major topic of research and its applications are found in the various fields of engineering, Sadd /10/. Nazarov and Puchkov /11/ constructed a class of functions in cylindrical coordinates for shear moduli and obtained general solutions to the problems related to displacement and stresses in a hollow cylinder subjected to torsion. Rooney and Ferrari /12/ studied torsion in bars with inhomogeneous shear moduli in the direction of coordinates. Horgan and Chan /13/ studied the effect of material inhomogeneity in linearly elastic bars and found that the maximum shear stress does not exist on the boundary of the rod. Ting /14, 15/ investigated a pressurized cylindrical elastic tube made up of anisotropic material subjected to torsion. Jiang and Henshall /16/ developed a model for the analysis of prismatic bars subjected to torsion by dividing the bar into small slices. Taliercio /17/ presented an analytical solution to a hollow circular cylinder of isotropic micropolar material subjected to twist. Thermal creep analysis of functionally graded material in pressurized cylinder has been done by Singh and Gupta /18/. They concluded that the circumferential stress decreased at the external surface while increased at the internal surface. Bayata et al. /19/ presented a solution to a hollow circular cylinder made up of functionally graded material subjected to torsion. They determined shear stresses for material whose Poisson's ratio and Young's modulus are varying in the radial direction. Ghannad and Garooni /20/ presented the elastic solution for thick-walled pressurized cylinder of functionally graded material using infinitesimal and first-order shear deformation theory. Zenkour /21/ obtained a closed form solution for heterogeneous circular cylinder under magnetic and thermal load with the assumption that rigidity of the cylinder is varying in the axial direction. Nejad et al. /22/ investigated time-dependent thermal creep stresses in a rotating functionally graded thick-walled pressure vessel under uniform heat flux using Norton's law. From the analysis, they found significant effects of non-homogeneity constants on radial displacement and creep stresses. Further, Kashkoli et al. /23/ investigated time-dependent thermal elastic creep stresses in a functionally graded spherical pressure vessel and concluded that non-homogeneity constants have significant effect on the creep stresses. Kobelev /24/ discussed secondary phase torsion and creep bending of rods. Sharma et al. /25/ investigated elastic and plastic stresses in a pressurized thick-walled cylinder made up of functionally graded material subjected to twist using transition theory. Recently, Kashkoli et al. /26/ performed time-dependent thermal creep analysis of thick-walled cylinder using shear deformation theory. They mainly concluded that the temperature gradient has a significant effect on the creep life of the cylinder. Thermal creep analysis of functionally graded material in pressurized cylinder has been done by Singh and Gupta /27/. They concluded that circumferential stress decreased at the external surface while increased at the internal surface. To determine critical crack initiation properties, Radaković et al. /28/ used magnetic emission and potential drop technique. Analytical method has been used by Sahni and Sahni /29/ to determine radial and hoop stresses in a rotating disk with variable thickness. Thakur /30/ determined thermal stresses in a thick-walled isotropic shell using the concept of transition theory and concluded that spherical shell made up of homogeneous material required high pressure for initial yielding as compared to the shell made up of non-homogeneous material. Sharma /31/ determined thermal elastic and plastic stresses in functionally graded thick-walled cylinder and discussed that yielding in the material starts at any radius in between internal and external surface and full plasticity occurs at the internal, or at the external surface.

Seth /6/ identified the transition state in which the governing differential equation shows some sort of criticality. In this paper, our main aim is to eliminate the requirement of assumptions of semi-empirical laws, yield conditions, creep-strain laws, jump conditions, etc. to analyse the impact of torsion on creep stresses in pressurized functionally graded cylinder using generalized strain measure /7/. A parametric study of the effects of torsion, pressure, compressibility and strain measure over stresses has been investigated. The radial, circumferential and shear stresses are computed and presented with the help of tables and graphs.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a pressurized functionally graded thick-walled circular cylinder with internal and external pressures p_1 and p_2 respectively subjected to torsion with η as the angle of twist per unit length. The pressurized cylinder is considered with an inner radius a and outer radius b. Geometric parameters of the cylinder are shown in Fig. 1.

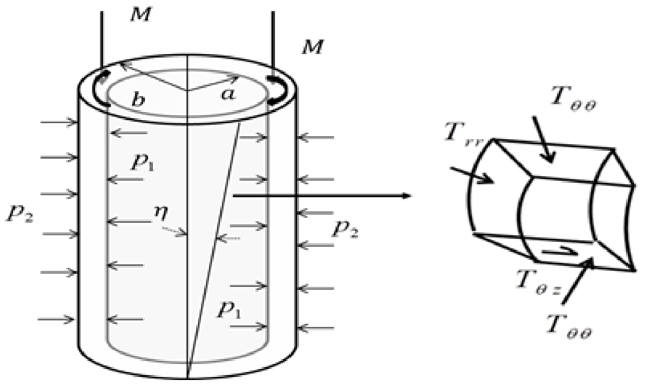


Figure 1. A pressurized thick-walled functionally graded cylinder subjected to torsion.

The components of displacement in cylindrical polar coordinates /5-9, 28-35/ are given by

$$u = r(1-\beta),\ \ v = \eta\, r\, z,\ \ w = zd \tag{1}$$

where: β is a function of $r = \sqrt{x^2+y^2}$; r is radius of the cylinder; d is a constant; and η is the angle of twist per unit length in radians.

The compressibility of functionally graded cylinder is defined as

$$C = C_0 (r)^k\ , \tag{2}$$

where: $a \le r \le b$; C_0 and k (≥ 0) are constants.

The generalized components of strain /26, 7/ are given as

$$e_{rr} = \frac{1}{n}[1-(r\beta'+\beta)^n]\ ,\ \ e_{\theta\theta} = \frac{1}{n}[1-\beta^n]\ ,\ \ e_{zz} = \frac{D^n}{n}\left[1-\left(\frac{\eta r \beta}{D}\right)^n\right],$$

$$e_{\theta z} = \frac{1}{n^m}\left[\eta^{n/2} r^{n/2} \beta^n\right]^m\ ,\ \ e_{r\theta} = e_{zr} = 0\ , \tag{3}$$

where: $D^n = [1-(1-d)^n]$; $n = 1/N$ is strain measure of deformation; and m is a parameter for creep state and $\beta' = d\beta/dr$. Secondary creep or minimum rate of creep holds when $m = 1$.

The constitutive equations for isotropic materials in linear theory of elasticity is given by

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij}\ ,\ \ (i,j = 1, 2, 3) \tag{4}$$

where: T_{ij}, e_{ij} are stress and strain tensors respectively; $I_1 = e_{rr} + e_{\theta\theta} + e_{zz}$ is strain invariant; λ and μ are Lame's constants; δ_{ij} is Kronecker's delta.

Equation (4) using Eq.(3) can be rewritten as

$$T_{rr}=\left(\frac{\lambda+2\mu}{n}\right)\left(1-(r\beta'+\beta)^n\right)+\frac{\lambda}{n}(1-\beta^n)+\frac{\lambda D^n}{n}\left[1-\left(\frac{\eta r\beta}{D}\right)^n\right],$$
$$T_{\theta\theta}=\left(\frac{\lambda}{n}\right)\left(1-(r\beta'+\beta)^n\right)+\left(\frac{\lambda+2\mu}{n}\right)(1-\beta^n)+\frac{\lambda D^n}{n}\left[1-\left(\frac{\eta r\beta}{D}\right)^n\right],$$
$$T_{zz}=\left(\frac{\lambda}{n}\right)\left(1-(r\beta'+\beta)^n\right)+\left(\frac{\lambda}{n}\right)(1-\beta^n)+\left(\frac{\lambda+2\mu}{n}\right)\left[D^n-(\eta r\beta)^n\right],$$
$$T_{\theta z}=\frac{2\mu}{n}\left[\eta^{n/2}r^{n/2}\beta^n\right],\ T_{zr}=T_{r\theta}=0 \tag{5}$$

Equation of equilibrium in the absence of body force is expressed as

$$\frac{d}{dr}(T_{rr})+\frac{(T_{rr}-T_{\theta\theta})}{r}=0\,. \tag{6}$$

OBJECTIVE

According to transition theory, there exists an intermediate state in between elastic and creep state which is known as transition state. Thus, the differential system defining the elastic state should reach a critical value in the transition state. Therefore, the nonlinear differential equation in transition state is obtained by substituting Eq.(5) in Eq.(6) as

$$n\beta P(P+1)^{n-1}\frac{dP}{d\beta}=r\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\left[\left\{(3-2C)-(1-C)(1-d)^n\right\}\frac{1}{\beta^n}-\left\{(1-C)(1+\eta^n r^n)+(P+1)^n\right\}\right]C\left[1-(P+1)^n\right]+$$
$$+rC'\left[(1+\eta^n r^n)-\left\{2-(1-d)^n\right\}\frac{1}{\beta^n}\right]-nP\left[(1-C)+(P+1)^n\right]-n\eta^n r^n(1-C)(P+1)-nr^{n+1}\eta^{n-1}\eta'(1-C). \tag{7}$$

where: $C=\dfrac{2\mu}{(\lambda+2\mu)}$; $\eta=\dfrac{2\eta_1(3-2C)}{(2-C)}$; $P=\dfrac{r\beta'}{\beta}$; η_1 is a constant and C' and μ' are the first order derivatives of C and μ, respectively.

The critical points of Eq.(7) are $P\to -1$ and $P\to\pm\infty$.

The boundary conditions are

$$T_{rr}=-p_1 \text{ at } r=a \text{ and } T_{rr}=-p_2 \text{ at } r=b. \tag{8}$$

The resultant axial force in the circular cylinder is defined by

$$\int_a^b rT_{zz}dr=0\,. \tag{9}$$

Method of Approach

According to transition theory, elastic state switches to creep state through transition region. For finding the creep stresses at the transition point $P\to -1$, the transition function R /5-9, 28-35/ is defined as the difference of principal stresses:

$$R=T_{rr}-T_{\theta\theta}=\frac{2\mu}{n}\beta^n\left[1-(P+1)^n\right]. \tag{10}$$

The distinction between elastic and creep states disappears at the transition point. Further, taking the logarithmic differentiation of Eq.(10) with respect to r, we have

$$\frac{d}{dr}(\log R)=\frac{nP}{r}+\frac{\mu'}{\mu}-\frac{n\beta P(P+1)^{n-1}\dfrac{dp}{d\beta}}{r\left[1-(P+1)^n\right]}\,. \tag{11}$$

Substituting the value of $dP/d\beta$ from Eq.(7) into Eq.(11) and taking the asymptotic value as $P\to -1$, we get

$$\frac{d}{dr}(\log R)=-\frac{2n}{r}+2\frac{\mu'}{\mu}-\frac{C'}{C}+X\,, \tag{12}$$

where:

$$X=\frac{(n-1)C}{r}-C\frac{\mu'}{\mu}+\frac{C'}{\beta^n}\left[2-(1-d)^n\right]-\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\times$$
$$\times\left[(3-2C)-(1-C)(1-d)^n\right]\frac{1}{\beta^n}+(1-C)nr^n\eta^{n-1}\eta'.$$

Integration of Eq.(12) yields

$$R=A\frac{\mu^2}{Cr^{2n}}\exp f\,, \tag{13}$$

where: $f=\int Xdr$; and A is a constant of integration.

Using Eqs.(10) and (13), we have

$$R=T_{rr}-T_{\theta\theta}=A\frac{\mu^2}{Cr^{2n}}\exp f=ArF\,, \tag{14}$$

where: $F=\dfrac{\mu^2}{Cr^{2n+1}}\exp f$.

The asymptotic value of the function from Eqs.(10) and (14) is obtained as

$$\beta^n=A\frac{n\mu}{2Cr^{2n}}\exp f\,. \tag{15}$$

Substituting Eq.(14) in Eq.(6) and integrating, we get

$$T_{rr}=B-A\int Fdr\,, \tag{16}$$

where: B is a constant of integration and asymptotic value of β is D/r when $P\to -1$; D is a constant.

The constants of integration, A and B are calculated by substituting boundary conditions from Eq.(8) in Eq.(16) as

$$A=\frac{p_2-p_1}{\int_a^b F dr}\,;\quad B=-p_2+A\left[\int G dr\right]_{r=b}. \tag{17}$$

Substituting the value of B in Eq.(16), we get

$$T_{rr}=-p_2+A\int_r^b F dr\,. \tag{18}$$

The circumferential, axial and shearing stresses obtained from Eqs.(14), (18) and (5) are

$$T_{\theta\theta}=-p_2+A\left[\int_r^b F\,dr-rF\right],$$

$$T_{zz}=\frac{(1-C)}{(2-C)}(T_{rr}+T_{\theta\theta})+2\mu\frac{(3-2C)}{(2-C)}e_{zz}, \tag{19}$$

$$T_{\theta z}=A\left(\frac{\mu^2\eta^{n/2}r^{-3n/2}}{C}\exp f\right),$$

where: $e_{zz}=\dfrac{-\int_a^b \frac{r(1-C)}{(2-C)}[T_{rr}+T_{\theta\theta}]dr}{\lambda\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}dr}$;

$$D^n=\frac{-\int_a^b \frac{r(1-C)}{(2-C)}[T_{rr}+T_{\theta\theta}]dr}{\frac{\lambda}{n}\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}dr}+A\left(\frac{2\eta_1(3-2C)}{(2-C)}\right)^n\frac{n\mu}{2Cr^n}\exp f\,.$$

The twisting couple M is given by

$$M=2\pi\int_a^b r^2T_{\theta z}dr=2\pi\int_a^b A\left(\frac{\mu^2\eta^{n/2}r^{(-3n+4)/2}}{C}\exp f\right)dr\,. \tag{20}$$

Now we introduce the following non-dimensional components to reduce the radial, circumferential, axial and shear stresses in non-dimensional form as

$R=\dfrac{r}{b}$; $R_0=\dfrac{a}{b}$; $P_2-P_1=\dfrac{p_2-p_1}{E}$; $\sigma_{rr}=\dfrac{T_{rr}}{E}$; $\sigma_{\theta\theta}=\dfrac{T_{\theta\theta}}{E}$; $\sigma_{zz}=\dfrac{T_{zz}}{E}$; $\sigma_{\theta z}=\dfrac{T_{\theta z}}{E}$, where $E=2\mu\dfrac{(3-2C)}{(2-C)}$ is Young's modulus of the material.

The radial, circumferential, axial, shear stresses and twisting couple defined in Eqs.(18)-(20) in non-dimensional form, with the use of compressibility parameter are expressed as

$$\sigma_{rr}=-P_2+A_2\int_R^1 bF_2dR,\quad \sigma_{\theta\theta}=-P_2+bA_2\left[\int_R^1 F_2dR-RF_2\right],$$

$$\sigma_{zz}=\frac{1-C_0b^kR^k}{2-C_0b^kR^k}\left[-2P_2+bA_2\left(\int_R^1 2F_2dR-RF_2\right)\right]+e_{zz},$$

$$\sigma_{\theta z}=bRA_2F_2(2\eta_1 bR)^{\frac{n}{2}}\left[\frac{3-2C_0b^kR^k}{2-C_0b^kR^k}\right]^{\frac{n}{2}}, \tag{21}$$

$$M=2\pi\int_{R_0}^1\left(A_2b^{-k-2n+3}R^{-k-2n+2}\frac{E^2(2-C_0b^kR^k)^2}{4C_0(3-2C_0b^kR^k)^2}\times\right.$$
$$\left.\times\left[\frac{2\eta_1(3-2C_0b^kR^k)bR}{(2-C_0b^kR^k)}\right]^{n/2}\exp f_2\right)dR,$$

where: $A_2=\dfrac{P_2-P_1}{\int_a^b bF_2dR}$;

$$F_2=\frac{E^2(2-C_0b^kR^k)^2}{4C_0(3-2C_0b^kR^k)^2}b^{-k-2n-1}R^{-k-2n-1}\exp f_2;\ f_2=\frac{(n-1)C_0b^kR^k}{k}+\frac{2kC_0b^{n+k}R^{n+k}}{D^n(n+k)}-\frac{C_0k}{D^n}\int\frac{b^{n+k}R^{n+k-1}(3-2C_0b^kR^k)dR}{1-C_0b^kR^k}+$$

$$+\log(1-C_0b^kR^k)-2\eta_1C_0nk\int\frac{b^{n+k}R^{n+k-1}(1-C_0b^kR^k)}{(2-C_0b^kR^k)^2}\left\{\frac{2\eta_1(3-2C_0b^kR^k)}{2-C_0b^kR^k}\right\}^{n-1}dR\ \text{and}\ e_{zz}=\frac{-\int_{R_0}^1\frac{b^2R(1-C_0b^kR^k)}{2-C_0b^kR^k}(\sigma_{rr}+\sigma_{\theta\theta})dr}{\int_{R_0}^1 b^2REdR}.$$

Equations (21) represent radial, circumferential, axial, shear stresses and twisting couple in non-dimensional form for secondary state of creep ($m = 1$) and $n = 1/N$.

RESULTS AND NUMERICAL DISCUSSION

The material properties of the functionally graded cylinder are defined as: Poisson's ratio $\nu = 0.3$; compressibility coefficient $C_0 = 0.5$. The internal and external radii of the cylinder are $a = 1$ m and $b = 1$ m, respectively. To observe the effects of various parameters, i.e. torsion, strain measure N and lateral pressures P_1 and P_2, graphs are plotted between radii ratios and stresses.

When pressure at the internal surface is on the higher side as compared to the pressure at the external surface, then influence of strain measure with angle of twist $\eta_1 = 100$ on creep stresses has been examined which can be seen from Table 1 and Fig. 2.

It is found that circumferential stresses are tensile in nature except for the functionally graded cylinder ($k = 0.5$) at the internal surface with nonlinear measure. These creep stresses are maximum at the internal surface of the cylinder with linear measure ($N = 1$), and maximum at the external surface of the cylinder with nonlinear measure ($N = 7$). The circumferential creep stresses decrease at the internal surface but increase towards the outer surface with the change in measure from linear to nonlinear. The negative values of the stresses indicate their compressive nature. Also, it has been noticed that these stresses increase at the internal surface while decrease at the external surface with increase in non-homogeneity parameter. A significant variation is observed in stresses with the change in measure from linear to nonlinear. With increase in pressure, significant increase in stresses is noticed for functionally graded cylinder with linear and nonlinear measure (see Fig. 3).

Table 1. Circumferential creep stresses in thick-walled functionally graded cylinder for $\eta_1 = 100$.

$\sigma_{\theta\theta}$	$N = 1$			$N = 3$			$N = 7$		
	$P_1 = 1.5$, $P_2 = 0.5$								
k \ R	0.5	0.75	1	0.5	0.75	1	0.5	0.75	1
0	1.167	0.426	0.167	0.500	0.500	0.500	0.329	0.513	0.615
0.5	1.539	0.379	0.013	-0.490	0.568	1.203	-1.376	0.539	2.216
1	1.932	0.330	-0.130	0.316	0.512	0.639	-0.070	0.526	0.966
1.5	2.199	0.317	-0.283	0.879	0.459	0.295	0.463	0.479	0.594
	$P_1 = 0.5$, $P_2 = 1.5$								
0	-3.167	-2.426	-2.167	-2.500	-2.500	-2.500	-2.329	-2.513	-2.615
0.5	-3.539	-2.379	-2.013	-1.510	-2.568	-3.203	-0.624	-2.539	-4.216
1	-3.932	-2.330	-1.870	-2.316	-2.512	-2.639	-1.930	-2.526	-2.966
1.5	-4.199	-2.317	-1.717	-2.879	-2.459	-2.295	-2.463	-2.479	-2.594

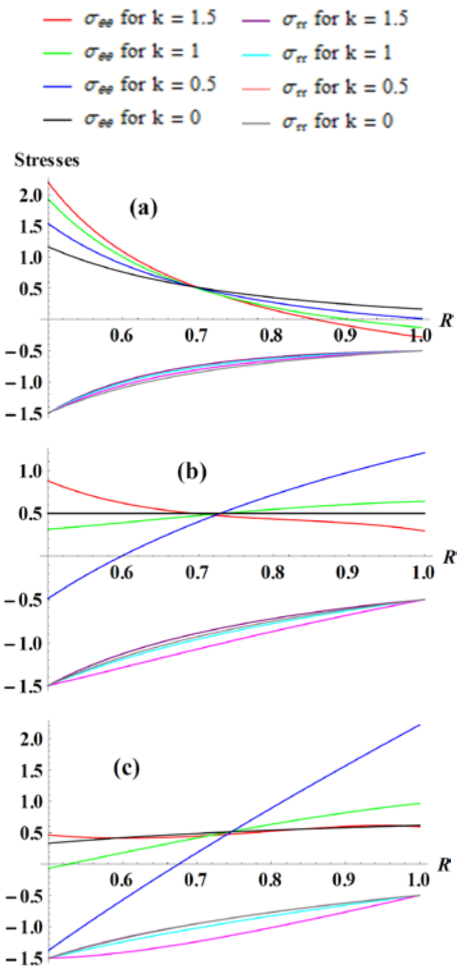
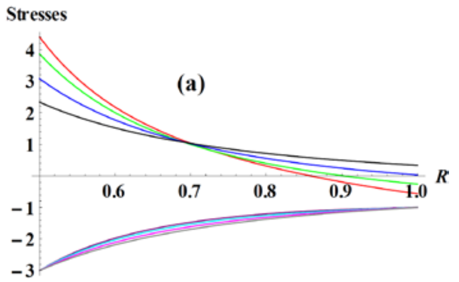


Figure 2. Creep stresses in thick-walled functionally graded cylinder for $P_1 = 1.5$ and $P_2 = 0.5$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.



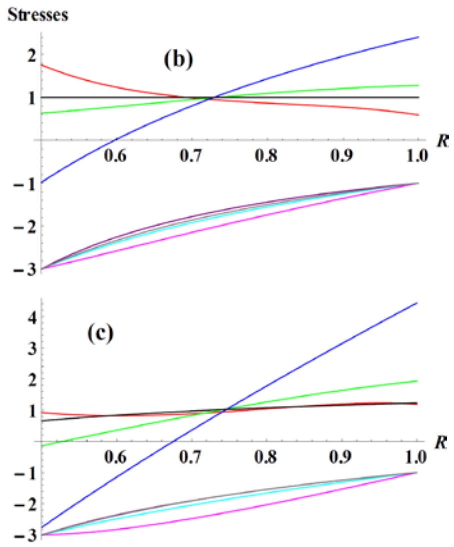


Figure 3. Creep stresses in thick-walled functionally graded cylinder for $P_1 = 3$ and $P_2 = 1$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

The effects of strain measure and angle of twist on creep stresses when the pressure at the internal surface is on the lower side as compared to the pressure at the external surface are shown in Fig. 4 and Table 1. It is examined that circumferential stresses are compressive throughout the radii. These stresses are maximum at the internal surface of the cylinder with linear measure while maximum at the external surface with nonlinear measure. Also, it has been noticed that these stresses increase at the internal surface while decrease at the external surface with the increase in non-homogeneity parameter, whereas they increase with an increase in pressure (see Fig. 5).

The impacts of pressure, torsion and non-homogeneity on shear stresses when internal pressure is greater than that of external pressure for different values of strain measure are shown in Figs. 6-9 and in Table 2. The cylinder is subjected to different forces on each side and shear stress acts as a supporting force. Shear stresses are additive to pressure and if shear stresses are on the higher side, then it will reduce the risk of fracture because those materials are beneficial for the manufacturing purpose which allow the cylinder to sustain larger forces without a risk of failure under creep.

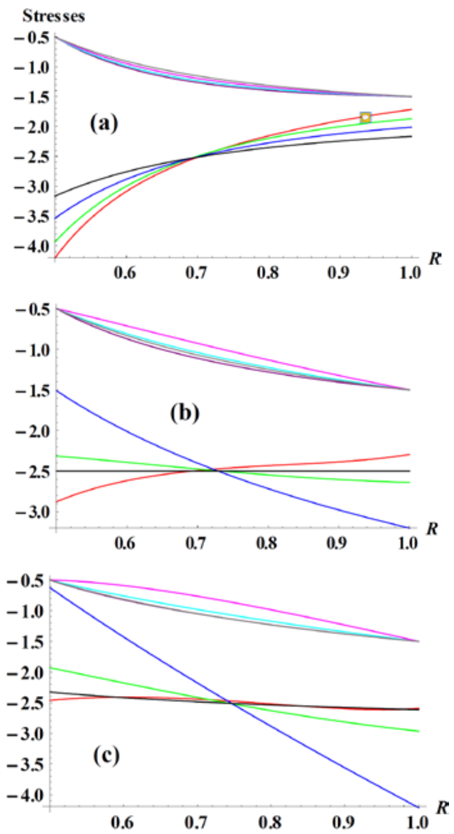


Figure 4. Creep stresses in thick-walled functionally graded cylinder for $P_1 = 0.5$ and $P_2 = 1.5$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

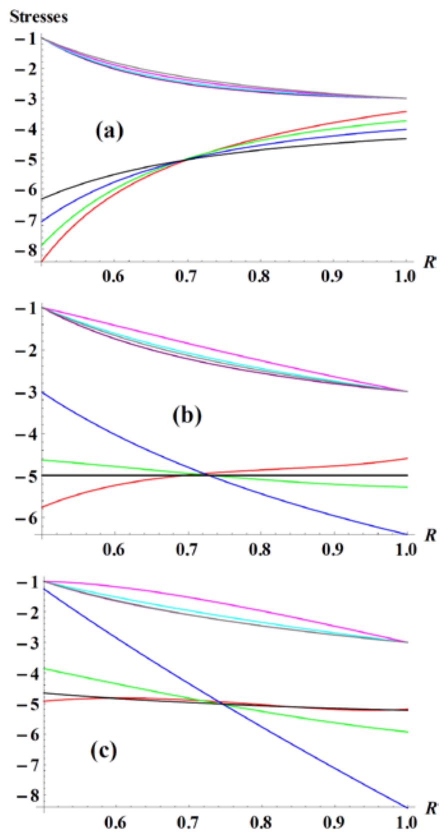
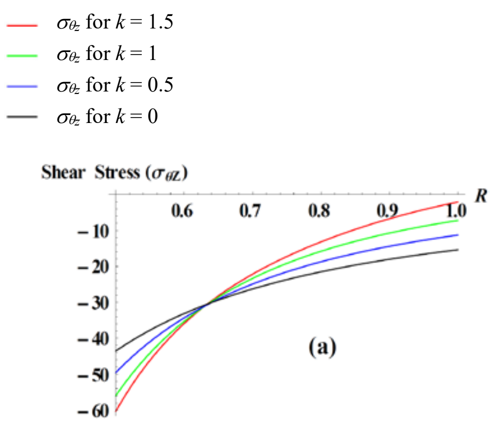


Figure 5. Creep stresses in thick-walled functionally graded cylinder for $P_1 = 1$ and $P_2 = 3$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

Table 2. Creep shear stresses in thick-walled functionally graded cylinder for $\eta_1 = 100$.

$\sigma_{\theta z}$	$N = 1$			$N = 3$			$N = 7$		
	$P_1 = 1.5$, $P_2 = 0.5$								
R / k	0.5	0.75	1	0.5	0.75	1	0.5	0.75	1
0	-43.546	-23.704	-15.396	-5.074	-3.619	-2.848	-2.726	-2.101	-1.746
0.5	-49.564	-21.531	-11.308	-2.561	-4.141	-4.776	-0.185	-2.553	-4.226
1	-55.981	-19.172	-7.238	-4.606	-3.661	-3.075	-2.131	-2.203	-2.244
1.5	-60.328	-17.277	-2.097	-6.034	-3.279	-1.708	-2.925	-2.014	-1.520
	$P_1 = 0.5$, $P_2 = 1.5$								
0	43.546	23.704	15.396	5.074	3.619	2.848	2.726	2.101	1.746
0.5	49.564	21.531	11.308	2.561	4.141	4.776	0.185	2.553	4.226
1	55.981	19.172	7.238	4.606	3.661	3.075	2.131	2.203	2.244
1.5	60.328	17.277	2.097	6.034	3.279	1.708	2.925	2.014	1.520

It has been observed that when pressure at the internal surface is on the higher side as compared to the pressure at the external surface then shear stresses generated are in the clockwise direction throughout the radii of the cylinder (see Fig. 6 and Table 2). These shear stresses are maximum at the internal surface for linear measure and show significant increase when non-homogeneity parameter is increased. It has also been examined that with the change in measure from linear to nonlinear, shear stresses decrease while they increase with the increase in pressure (see Fig. 7). It has also been noticed that shear stresses are on the higher side for functionally graded cylinder as compared to homogeneous cylinder.



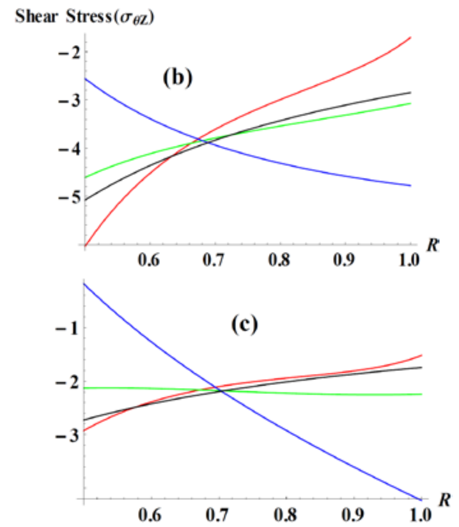


Figure 6. Shear stresses in thick-walled functionally graded cylinder for $P_1 = 1.5$ and $P_2 = 0.5$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

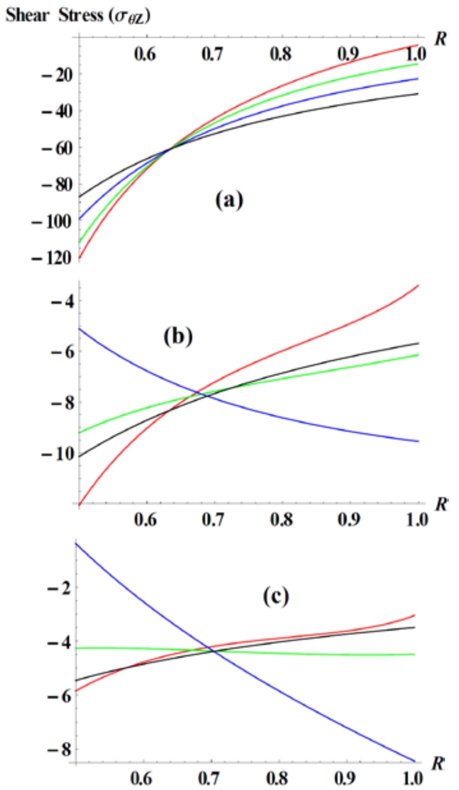


Figure 7. Shear stresses in thick-walled functionally graded cylinder for $P_1 = 3$ and $P_2 = 1$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

The impact of pressure, torsion and non-homogeneity parameter on creep shear stresses when internal pressure in the cylinder is on the lower side as compared to the pressure in the cylinder at external surface for different strain measures are shown in Fig. 8 and Table 2.

It is observed that shear stresses rotate the material of cylinder in anti-clockwise direction and are maximum at the internal surface of cylinder made up of homogeneous and functionally graded material with linear measure. These shear stresses show significant decrease when measure is changed from linear to nonlinear and they are on the higher side for the cylinder made up of functionally graded material as compared to cylinder of homogeneous material. With the increase in pressure, shear stress increases significantly, as can be seen in Fig. 9.

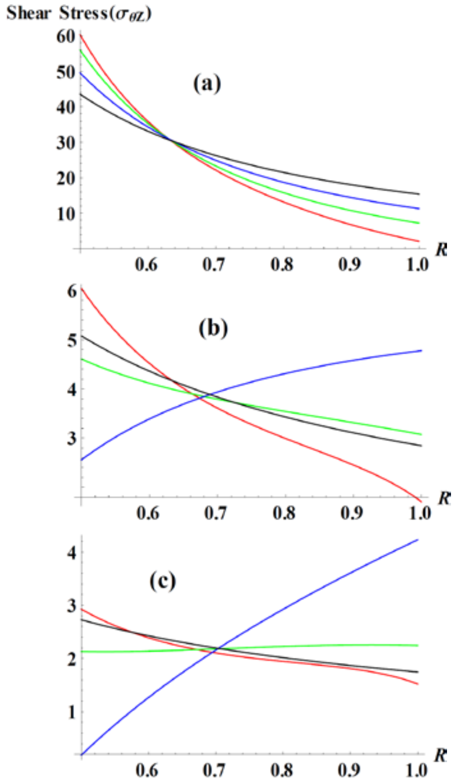


Figure 8. Shear stresses in thick-walled functionally graded cylinder for $P_1 = 0.5$ and $P_2 = 1.5$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

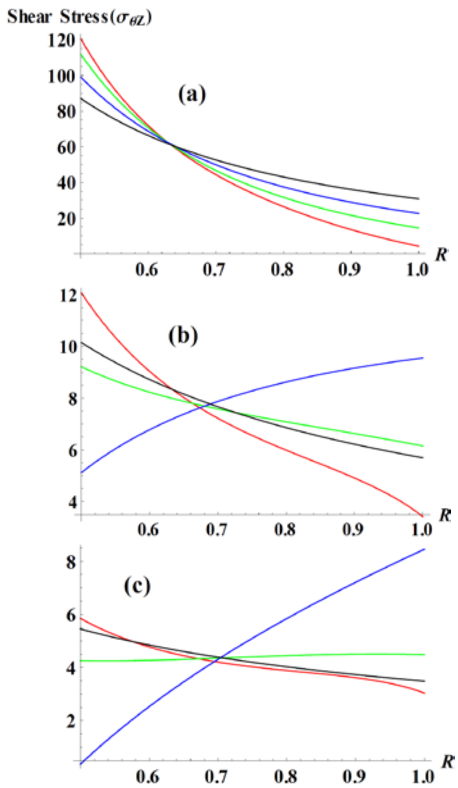


Figure 9. Shear stresses in thick-walled functionally graded cylinder for $P_1 = 1$ and $P_2 = 3$: (a) $N = 1$, (b) $N = 3$, (c) $N = 7$.

It has been observed that shear stresses and normal stresses are high on the same plane for the cylinder. Also, these stresses are on the higher side for the cylinder made up of functionally graded material as compared to cylinder of homogeneous material. It may seem surprising that failure does not occur on the planes that sustain maximum shear stress. The development of a shear fracture on the fracture plane exists when normal stress is minimum and shear stress is maximum. The material shows its maximum strength when material is oriented in a way such that shear fractures develop across the weaker plane. Here, normal stress on the plane is not minimal at the same orientation for which the shear stress is a maximum and the fracture does not occur until the Mohr's circle reaches the outer failure criterion line.

CONCLUSIONS

In this paper, the effects of pressure and strain measure on creep stresses in a thick-walled cylinder made up of functionally graded material subjected to twist are investigated. The closed form solution is obtained for nonlinear differential equation by using transition theory. It has been noticed that stress changes with the change in pressure and strain measure. From the analysis, it has been concluded that pressurized cylinder made up of functionally graded material under torsion is a better choice for manufacturing cylinders as compared to cylinders of homogeneous material. This is because shear stresses are supporting force to pressure and shear stresses are on the higher side for the cylinder made up of functionally graded material as compared to cylinder of homogeneous material. Therefore, functionally graded materials are beneficial for manufacturing purposes as they allow the cylinder to sustain larger forces without a risk of failure under creep.

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