ANALYSIS OF PLANE DEFORMATION IN THERMO-DIFFUSIVE MICROSTRETCH AND MICROPOLAR MEDIA IN CONTEXT OF GL AND LS THEORIES OF THERMOELASTICITY

INTRODUCTION

Eringen /11/ developed the theory of micropolar elastic solid with stretch. He derived the equations of motion, constitutive equations and boundary conditions for a class of micropolar solids which can stretch and contract. This model introduced and explained the motion of certain class of granular and composite materials in which grains and fibres are elastic along the direction of their major axis. This theory is a generalization of the theory of micropolar elasticity and is a special case of the micromorphic theory. Eringen /12/ developed a theory of thermomicrostretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level and a two-phase dipolar solid with a care on the macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous medium where pores are filled with gas or inviscid liquids, asphalt or other inclusions and ‘solid-liquid’ crystals etc., are characterized as microstretch solids. Thus, in these solids, the motion is characterized by seven degrees of freedom namely three for translation, three for rotation and one for microstretch. In the framework of the theory of thermo-microstretch solids, Eringen /8/ established a uniqueness theorem for the mixed initial boundary valued problem. This theory is illustrated with the solution of one-dimensional wave and compared with lattice dynamical results. The asymptotic behaviour of solutions and an existence result are presented by Bofill and Quintanilla, /1/.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g. xenon) and other light isotopes (e.g. carbon).
Tomar and Garg \cite{31} discussed the reflection and refraction of plane waves in microstretch elastic medium. Quintanilla \cite{1} studied the spatial decay for the dynamical problem of thermo-microstretch elastic solids. Singh and Tomar \cite{30} discussed Rayleigh-Lamb waves in a microstretch elastic plate cladded with liquid layers. Cicco \cite{3} discussed the stress concentration effects in microstretch elastic bodies. A spherical inclusion in an infinite isotropic microstretch medium is discussed by Liu and Hu \cite{27}. Kumar and Partap \cite{18} analysed free vibrations for Rayleigh-Lamb waves in a medium is discussed by Liu and Hu \cite{27}. Kumar and Kansal \cite{22}. Gravitational effect on plane waves in generalized thermomicrostretch elastic solid was developed by Kumar and Kansal \cite{22}.

In the present paper general model of the equations of microstretch thermoelastic with mass diffusion for a homogeneous isotropic elastic solid is developed. The normal mode analysis technique is used to obtain the expressions for displacement components, couple stress, temperature, mass concentration and microstress distribution. Microstretch effect is shown on the considered domain graphically. Some special cases have been deduced from the present investigation.

**BASIC EQUATIONS**

The basic equations for homogeneous, isotropic microstretch generalized thermoelastic diffusive solids in the absence of body force, body couple, stretch force and heat source are given by:

\[
(\lambda + \mu)\nabla(\nabla u) + (\mu + K)\nabla^2 u + K\nabla \times \phi + \lambda_0 \nabla \phi^* - \beta_1 \left(1 + r_1 \frac{\partial}{\partial t}\right) \nabla T - \beta_2 \left(1 + r^1 \frac{\partial}{\partial t}\right) \nabla C = \rho \ddot{u}
\]

(1)

\[
(\gamma \nabla^2 - 2 K) \phi + (\alpha + \beta) \nabla(\nabla \phi) + K V \times u = \rho \ddot{\phi}
\]

(2)

\[
(\alpha_0 \nabla^2 - \lambda_0) \phi^* - \lambda_0 \nabla u + v_1 \left(1 + r_1 \frac{\partial}{\partial t}\right) T + v_2 \left(1 + r^1 \frac{\partial}{\partial t}\right) C = \frac{D f_0}{2} \phi^*
\]

(3)

\[
K \cdot \nabla^2 T = \rho C e \left(\frac{\partial}{\partial t} + r_0 \frac{\partial^2}{\partial t^2}\right) T + \beta_1 T_0 \left(\frac{\partial}{\partial t} + \epsilon T_0 \frac{\partial^2}{\partial t^2}\right) \nabla u + v_1 T_0 \left(\frac{\partial}{\partial t} + \epsilon T_0 \frac{\partial^2}{\partial t^2}\right) \phi^* + \alpha T_0 \left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C
\]

(4)

\[
D \beta_2 \nabla^2 (\nabla u) + D a \left(1 + r_1 \frac{\partial}{\partial t}\right) \nabla^2 T + \left(\frac{\partial}{\partial t} + \epsilon T_0 \frac{\partial^2}{\partial t^2}\right) \nabla u + v_1 T_0 \left(\frac{\partial}{\partial t} + \epsilon T_0 \frac{\partial^2}{\partial t^2}\right) \phi^* + \alpha T_0 \left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C = 0
\]

(5)

\[
t_{ij} = (\alpha_0 \phi^* + \lambda \sigma_{ij,x}) \delta_{ij} + \mu (u_{ij,x} + u_{ij,j}) + K (u_{ij,j} - e_{ijk} \phi_k) - \beta_1 \left(1 + r_1 \frac{\partial}{\partial t}\right) \delta_{ij} T - \beta_2 \left(1 + r^1 \frac{\partial}{\partial t}\right) \delta_{ij} C
\]

(6)

\[
m_{ij} = \alpha \delta_{ij,r} \delta_{ij} + \beta \phi_{ij,j} + \gamma \phi_{ij,j} + b_0 \delta_{ij} \phi^*_m
\]

(7)

where: \(\lambda, \mu, \alpha, \beta, K, \gamma, \lambda_0, \lambda_1, \alpha_0, \beta_1, \) are material constants; \(\rho\) is mass density; \(u = (u_1, u_2, u_3)\) is the displacement vector and \(\phi = (\phi_x, \phi_y, \phi_z)\) is the microrotation vector; \(\phi^*\) is the scalar microstretch function; \(T\) is the temperature and \(T_0\) is the reference temperature of the body chosen; \(C\) is the concentration of the diffusion material in the elastic body; \(K^*\) is the coefficient of the thermal conductivity; \(C^*\) is the specific heat at constant strain; \(D\) is the thermoelastic diffusion constant; \(\epsilon\) is the coefficient describing the measure of thermal diffusion and \(b\) is the coefficient describing the measure of mass diffusion effects; \(j\) is the microinertia; \(\beta_1 = (3\lambda + 2\mu + K)\alpha_{ij}\), \(\beta_2 = (3\lambda + 2\mu + K)\alpha_{ij}\), \(v_1 = (3\lambda + 2\mu + K)\alpha_{ij}\), \(v_2 = (3\lambda + 2\mu + K)\alpha_{ij}\), \(\alpha_{ij}\), \(\alpha_{ij}\), \(\alpha_{ij}\) are coefficients of linear thermal expansion; and \(\alpha_{ij}\), \(\alpha_{ij}\), \(\alpha_{ij}\) are coefficients of linear diffusion expansion; \(f_0\) is the microinertia for the microelements; \(t_{ij}\) are components of stress; \(m_{ij}\) are components of couple stress; \(\lambda^*\) is the microstress tensor; \(e_{ij}\) are components of strain; \(\delta_{ij}\) is the dilatation; \(\delta_{ij}\) is the Kroncker delta function; \(t^i, t^j\) are the diffusion relaxation times and \(t_0, t_1\) are thermal relaxation times with \(t_0 > t_1 > 0\). Here \(t^i = t^j = t_0 = t_1 = \gamma = 0\) for Coupled thermoelastic theory (CT) model, \(t_1 = t^1 \neq 1, \gamma = 0\) for Lord-Shulman (LS) model, and \(\epsilon = 0, \gamma = 1\) where \(\epsilon > 0\), for Green-Lindsay (GL) model.
In the above equations symbol \((\cdot)\) followed by a suffix denotes differentiation with respect to spatial coordinates, and a superposed dot \(\dot{\cdot}\) denotes the derivative with respect to time.

FORMULATION OF THE PROBLEM

We consider a rectangular Cartesian coordinate system \(OX_1X_2X_3\) with \(x_3\)-axis pointing vertically outward the medium. We consider a normal or tangential force to be acting at the free surface of microstretch thermoelastic medium with mass diffusion half space.

For two-dimensional problems the displacement and microrotation vectors are of the form:

\[ \mathbf{u} = (u_1, 0, u_3), \quad \mathbf{\phi} = (0, \phi_2, 0) \]  

For further consideration it is convenient to introduce in Eqs.(1)-(5) the dimensionless quantities defined as:

\[ m^*_j = \frac{\rho^*}{\rho^*_c} m_j, \quad C^* = \frac{\beta^*_j}{\rho^*_c}. \]

With the aid of Eqs.(8) and (9), the Eqs.(1)-(5) reduce to:

\[ (a_1 \frac{\partial}{\partial x_1} + a_2 \frac{\partial^2}{\partial x_1^2}) u_1 + a_3 \frac{\partial}{\partial x_3} \phi = 0, \quad \frac{\partial}{\partial x_3} \psi = 0, \]  

\[ \frac{\partial}{\partial t} \left( a_1 \frac{\partial}{\partial t} + a_2 \frac{\partial^2}{\partial t^2} \right) \phi + a_3 \frac{\partial}{\partial x_3} \phi = 0, \]  

\[ \frac{\partial}{\partial t} \left( a_1 \frac{\partial}{\partial t} + a_2 \frac{\partial^2}{\partial t^2} \right) \psi + a_3 \frac{\partial}{\partial x_3} \psi = 0. \]

Here,

\[ a_1 = \frac{A + \mu}{\beta^*_j T_0}, \quad a_2 = \frac{\mu + K}{\beta^*_j T_0}, \quad a_3 = K \rho^*_c, \quad a_4 = \frac{\rho^*_c}{\beta^*_j T_0}, \quad a_5 = \frac{\rho^*_c}{\gamma \omega^*}, \quad a_6 = \frac{\epsilon K^*_j}{\gamma}, \quad a_7 = \frac{\rho^*_c}{\gamma}, \quad a_8 = \frac{\gamma \omega^*}{\gamma^2}, \]

\[ a_9 = \frac{\lambda \rho^*_c}{\omega^* K^*}, \quad a_{10} = \frac{\nu_1 \rho^*_c}{\omega^*}, \quad a_{11} = \frac{\nu_2 \rho^*_c}{\gamma}, \quad a_{12} = \frac{\rho^*_c \nu_1}{2 \alpha_0}, \quad a_{13} = \frac{\rho^*_c \nu_2}{\gamma}, \quad a_{14} = \frac{\beta^*_j \rho^*_c}{\omega^* K^*}. \]

The displacement components \(u_1\) and \(u_3\) are related to potential functions \(\phi\) and \(\psi\) as:

\[ u_1 = \frac{\partial \phi}{\partial x_1}, \quad u_3 = \frac{\partial \phi}{\partial x_3}, \quad u_5 = -\frac{\partial \psi}{\partial x_3}. \]

Using the relation Eq.(16), in the Eqs.(10)-(15), we obtain:

\[ (a_1 + a_2) \frac{\partial^2 \phi}{\partial x_1^2} - a_3 \frac{\partial}{\partial x_3} \phi = 0, \]

\[ (a_1 + a_2) \frac{\partial^2 \psi}{\partial x_1^2} + a_3 \frac{\partial}{\partial x_3} \psi = 0. \]
Making use of Eq.(23), the relations Eqs.(17)-(22) yield:
\[
V^4 \phi + a_1 \left[ 1 + r_1 \frac{\partial}{\partial t} \right] V^2 T + a_8 \left[ \frac{\partial}{\partial t} + \epsilon \frac{\partial^2}{\partial x^2} \right] C - a_9 \left[ 1 + r_1 \frac{\partial}{\partial t} \right] V^2 C = 0 ,
\]
Here \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is the Laplacian operator.

**SOLUTION OF THE PROBLEM:**

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:
\[
\{ \phi, \psi, T, \phi^*, \psi^* \} (x_1, x_2, t) = e^{i (\omega x_1 - \omega t)} \{ \phi, \psi, T, \phi^*, \psi^* \} (x_1)
\]
Here \( \omega \) is the angular velocity and \( k \) is a complex constant. Making use of Eq.(23), the relations Eqs.(17)-(22) yield:
\[
\begin{align*}
& b_8 \left[ \frac{d^2}{dx_1^2} - k^2 \right] + \omega^2 \left[ \phi + b_2 \phi^* + b_3 T + b_4 C = 0 , \right. \\
& b_5 \left[ \frac{d^2}{dx_1^2} - k^2 \right] \phi + \left[ \frac{d^2}{dx_1^2} + b_5 \right] \phi^* + b_7 T + b_8 C = 0 , \left. \right. \\
& b_6 \left[ \frac{d^2}{dx_1^2} - k^2 \right] \phi^* + b_9 T + b_{10} C = 0 , \\
& b_20 \left[ \frac{d^2}{dx_1^2} - k^2 \right] \psi + b_{21} T + b_{22} C = 0 .
\end{align*}
\]
Here,
\[
\begin{align*}
b_1 &= \frac{\rho c_1^2}{\beta \rho T_0} , & b_2 &= \frac{\rho c_1^2}{\beta \rho c_1^2} , & b_3 &= -(1 - i \omega \tau_1) , & b_4 &= -b_1 (1 - i \omega \tau_1) , \\
b_5 &= -\frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 + \frac{\rho^2 \rho c_2^4}{2 \alpha_0} , \\
b_6 &= \frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 + \frac{\rho^2 \rho c_2^4}{2 \alpha_0} , \\
b_7 &= \frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 (1 - i \omega \tau_1) , & b_8 &= \frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 (1 - i \omega \tau_1) , \\
b_9 &= -\frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 (1 - i \omega \tau_1) , & b_{10} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} \omega c_1^2 (1 - i \omega \tau_1) , \\
b_{11} &= \frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , & b_{12} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , \\
b_{13} &= \frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , & b_{14} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , \\
b_{15} &= \frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , & b_{16} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , \\
b_{17} &= \frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , & b_{18} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , \\
b_{19} &= \frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) , & b_{20} &= -\frac{\rho c_4^4}{\beta \rho c_1^2} (1 - i \omega \tau_1) .
\end{align*}
\]

**BOUNDARY CONDITIONS**

We consider normal and tangential force acting on the surface \( x_1 = 0 \) along with vanishing of couple stress, microstress and temperature gradient with insulated and impermeable boundary at \( x_1 = 0 \). Mathematically this can be written as:
\[
\begin{align*}
t_{33} &= -F_1 e^{-(k_x \omega - \omega t)} , & t_{31} &= -F_2 e^{-(k_x \omega - \omega t)} , \\
m_{32} &= 0 , & \lambda_3 &= 0 , & \frac{\partial T}{\partial x_1} &= 0 , & \frac{\partial C}{\partial x_1} &= 0 .
\end{align*}
\]
Here \( F_1 \) and \( F_2 \) are the magnitude of the applied force.

Using these boundary conditions and solving the linear equations formed, we obtain:
\[
\begin{align*}
t_{33} &= \sum_{i=1}^{N} G_{i1} e^{-(m_{x1} \omega - \omega t)} e^{-(k_x \omega - \omega t)} , & i &= 1, 2, \ldots , 6 \\
t_{31} &= \sum_{i=1}^{N} G_{i2} e^{-(m_{x1} \omega - \omega t)} e^{-(k_x \omega - \omega t)} , & i &= 1, 2, \ldots , 6 \\
m_{32} &= \sum_{i=1}^{N} G_{i3} e^{-(m_{x1} \omega - \omega t)} e^{-(k_x \omega - \omega t)} , & i &= 1, 2, \ldots , 6 \\
\lambda_3 &= \sum_{i=1}^{N} G_{i4} e^{-(m_{x1} \omega - \omega t)} e^{-(k_x \omega - \omega t)} , & i &= 1, 2, \ldots , 6
\end{align*}
\]
NUMERICAL RESULTS AND DISCUSSIONS

The computations are carried out for a single value of \( \omega = 1 \) and on the surface of the plane \( z = 1 \). The numerical values for the normal stress \( t_{33} \), tangential couple stress \( m_{32} \), temperature distribution \( T \) and microstress \( \lambda_3^* \) on the surface of plane due to applied concentrated and uniformly distributed normal sources are shown in Figs. 1-8. The comparison of two theories of generalized thermoelasticity, namely, Lord-Shulman (L-S) and Green-Lindsay (G-L) are shown in graphs. Also, the diffusion effect and microstretch effect are shown in graphs.

\[ u_1 = \sum_{i=1}^{6} G_{ij} e^{-m_i x_i} e^{-(k_i x_i - w_i)} \quad i = 1, 2, \ldots, 6 \]  
\[ u_2 = \sum_{i=1}^{6} G_{ij} e^{-m_i x_i} e^{-(k_i x_i - w_i)} \quad i = 1, 2, \ldots, 6 \]  
\[ T = \sum_{i=1}^{6} G_{ij} e^{-m_i x_i} e^{-(k_i x_i - w_i)} \quad i = 1, 2, \ldots, 6 \]  
\[ C = \sum_{i=1}^{6} G_{ij} e^{-m_i x_i} e^{-(k_i x_i - w_i)} \quad i = 1, 2, \ldots, 6 \]

Here \( G_{ij}, i = 1, 2, \ldots, 6, \) and \( j = 1, 2, \ldots, 8 \) are the constants.

Case 1- Normal stress

To obtain the expressions due to normal stress we must set \( F_2 = 0 \) in the boundary conditions Eqs.(36).

Case 1- Tangential stress

To obtain the expressions due to tangential stress we must set \( F_1 = 0 \) in the boundary conditions Eqs.(36).

Particular cases

(i) If we take \( \tau_1 = \tau^1 = 0, \epsilon = 1, \gamma_1 = \gamma_0 \) in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(ii) If we take \( \epsilon = 0, \gamma_1 = \tau^0 \) in Eqs.(37)-(44), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(iii) Taking \( \tau^0 = \tau^1 = \gamma_0 = \gamma_1 = 0 \) in Eqs. (37)-(44) yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

Special cases

(a) Microstretch thermoelastic solid

If we neglect the diffusion effect in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar thermoelastic diffusive solid

If we neglect the microstretch effect in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.
Analysis of plane deformation in thermo-diffusive microstretch ... Analiza ravnih deformacij kod termofuzionih mikrorastegljivih ...
and Green Lindsay (G-L) theory of thermoelasticity decreases mechanical source. This behaviour of normal stress with the increase of the parameter served from Figs. 1 and 2 that the value of normal force tremors and drilling into the crust of the earth. It is ob-
useful to analyze the deformation field around mining Vol.

CONCLUSIONS

The results of the problem may be applied to a wide class of geophysical problems involving temperature change. The deformation at any point of the medium at any point is useful to analyse the deformation field around mining tremors and drilling into the crust of the earth. It is observed from Figs. 1 and 2 that the value of normal force stress for Lord Shulman (L-S) theory of thermoelasticity and Green Lindsay (G-L) theory of thermoelasticity decreases with the increase of the parameter k under the effect of mechanical source. This behaviour of normal stress is same for microstretch thermoelastic solid with mass diffu-
sion, microstretch thermoelastic solid and micropolar thermoelastic solid with mass diffusion.

In Figs. 3 and 4 the variation of tangential stress (t_{x1}) is shown. For microstretch thermoelastic solid with mass diffusion the tangential stress first increase until k reaches value 2 and then decreases. The same behaviour is shown by micropolar thermoelastic solid with mass diffusion, but in case of microstretch thermoelastic solid the behaviour of variation of tangential stress (t_{x1}) is slightly different, in this case for initial values of k the tangential stress (t_{x1}) shows some variable behaviour, but for higher values of k it shows a uniform behaviour which is decreasing. Also, the same behaviour is shown in G-L theory and L-S theory of thermoelasticity. In Figs. 5 and 6 the variation of tangential couple stress (m_{12}) is shown. For microstretch thermoelastic solid with mass diffusion the tangential couple stress first increases until k reaches value 2 and then decreases. The same behaviour is shown by micropolar thermoelastic solid with mass diffusion, and in the case of microstretch thermoelastic solid the behaviour of variation of couple tangential stress is also alike. In Figs. 7 and 8 the variation of microstress (\lambda_3) is shown. For microstretch thermoelastic solid with mass diffusion the tangential stress first increases until k reaches value around 1.5 and then decreases. The same behaviour is shown in case of microstretch thermoelastic solid. Hence, we conclude that normal stress decreases with increase of k value, but the values of tangential stress, couple stress and microstress first increase for lower values of k and then decrease for higher values of k.

REFERENCES