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ANALYSIS OF PLANE DEFORMATION IN THERMO-DIFFUSIVE MICROSTRETCH AND MICROPOLAR MEDIA IN CONTEXT OF GL AND LS THEORIES OF THERMOELASTICITY

ANALIZA RAVNIH DEFORMACIJA KOD TERMODIFUZIONIH MIKRORASTEGLJIVIH I MIKROPOLARNIH SREDINA U OKVIRU GL I LS TEORIJA TERMOELASTIČNOSTI

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Abstract

The present investigation deals with generalized model of the equations for a homogeneous isotropic microstretch thermoelastic half space with mass diffusion medium. Theories of generalized thermoelasticity Lord-Shulman (LS) and Green-Lindsay (GL) are applied to investigate the problem. The deformation in the considered medium have been studied due to normal force and tangential force. The normal mode analysis technique is used to calculate the normal stress, shear stress, couple stress and microstress. A numerical computation has been performed on the resulting quantity. The computed numerical results are shown graphically for specific quantities. Some particular cases of interest are deduced from the present investigation.

Izvod

U ovom radu je prikazan generalisani model jednačina za homogeni mikrorastegljivi termoelastični poluprostor sa masenom difuzionom sredinom. Teorije generalisane termoelastičnosti (Lord-Shulman - LS i Green-Lindsay - GL) su primenjene za istraživanje ovog problema. Deformacije u posmatranoj sredini su proučavane s obzirom na normalne i tangencijalne sile. Metoda analize normalnog režima je korišćena za proračun normalnih, smičućih, spregnutih i mikro-napona. Izvedena je numerička analiza na osnovu dobijenih vrednosti. Dobijeni numerički rezultati su prikazani grafički za pojedine veličine. Takođe, na osnovu ovih rezultata su izvedeni slučajevi od posebnog interesa.

INTRODUCTION

Eringen /11/ developed the theory of micropolar elastic solid with stretch. He derived the equations of motion, constitutive equations and boundary conditions for a class of micropolar solids which can stretch and contract. This model introduced and explained the motion of certain class of granular and composite materials in which grains and fibres are elastic along the direction of their major axis. This theory is a generalization of the theory of micropolar elasticity and is a special case of the micromorphic theory. Eringen /12/ developed a theory of thermomicrostretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level and a two-phase dipolar solid with a care on the macroscopic level. For example, composite materials reinforce with chopped elastic fibres, porous medium where pores are filled with gas or inviscid liquids, asphalt or other inclusions and 'solid-liquid' crystals etc., are characterized as microstretch solids. Thus, in these solids, the motion is characterized by seven degrees of freedom namely three for translation, three for rotation and one for microstretch. In the framework of the theory of thermo-microstretch solids, Eringen /8/ established a uniqueness theorem for the mixed initial boundary valued problem. This theory is illustrated with the solution of one-dimensional wave and compared with lattice dynamical results. The asymptotic behaviour of solutions and an existence result are presented by Bofill and Quintanilla, /1/.

The transmission of the load across a differential element of the surface of a microstretch elastic solid is described by a force vector, a couple stress vector and a microstretch vector. The theory of microstretch elastic solid differs from the theory of micropolar elasticity in the sense that there is an additional degree of freedom called stretch and there is an additional kind of stress called microstretch vector. The materials like porous elastic materials filled with gas or inviscid fluid, asphalt, composite fibres etc. lie in the category of microstretch elastic solids.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g. xenon) and other light isotopes (e.g. carbon).

Tomar and Garg /31/ discussed the reflection and refraction of plane waves in microstretch elastic medium. Quintanilla /1/ studied the spatial decay for the dynamical problem of thermo-microstretch elastic solids. Singh and Tomar /30/ discussed Rayleigh-Lamb waves in a microstretch elastic plate cladded with liquid layers. Cicco /3/ discussed the stress concentration effects in microstretch elastic bodies. A spherical inclusion in an infinite isotropic microstretch medium is discussed by Liu and Hu /27/. Kumar and Partap /18/ analysed free vibrations for Rayleigh-Lamb waves in a microstretch thermoelastic plate with two relaxation times. They discussed the dispersion of axisymmetric waves in thermo microstretch elastic plate. Othman /23/ studied the effect of rotation on plane wave propagation in the context of Green-Naghdi (GN) theory type-II by using the normal mode analysis. Ezzat and Awad adopted the normal mode analysis technique to obtain the temperature gradient, displacement, stresses, couple stress, micro rotation etc. Othman et al. /25/ studied the effect of diffusion on 2-dimensional problem of generalized thermo-elastic with Green-Naghdi theory and obtained the expressions for displacement components, stresses, temperature fields, concentration and chemical potential by using normal mode analysis. Othman et al. /26/ used normal mode analysis technique to obtain the expressions for displacement components, force, stresses, temperature, couple stress and microstress distribution in a thermomicrostretch elastic medium with temperature dependent properties for different theories. Othman and Lotfy studied the plane wave propagation in microstretch thermoelastic half space by using normal mode analysis. Kumar et al. /21/ investigated the disturbance due to force in normal and tangential direction and porosity effect by using normal mode analysis in fluid saturated porous medium. Kumar et al. /32/ investigated the effect of viscosity on plane wave propagation in heat conducting transversely isotropic micropolar viscoelastic half space. Fundamental solution in the theory of thermomicrostretch elastic diffusive solids was developed by Kumar and Kansal /22/. Gravitational effect on plane waves in generalized thermomicrostretch elastic solid under Green Naghdi theory was studied by Othman et al. /26/.

In the present paper general model of the equations of microstretch thermoelastic with mass diffusion for a homogeneous isotropic elastic solid is developed. The normal mode analysis technique is used to obtain the expressions for displacement components, couple stress, temperature, mass concentration and microstress distribution. Microstretch effect is shown on the considered domain graphically. Some special cases have been deduced from the present investigation.

BASIC EQUATIONS

The basic equations for homogeneous, isotropic microstretch generalized thermoelastic diffusive solids in the absence of body force, body couple, stretch force and heat source are given by:

$$(\lambda+\mu)\nabla(\nabla\mathbf{u})+(\mu+K)\nabla^2\mathbf{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{\mathbf{u}} \tag{1}$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla\boldsymbol{\phi})+K\nabla\times\mathbf{u}=\rho j\ddot{\boldsymbol{\phi}} \tag{2}$$

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla\mathbf{u}+v_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+v_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\ddot{\phi}^* \tag{3}$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\beta_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla\mathbf{u}+v_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C \tag{4}$$

$$D\beta_2\nabla^2(\nabla\mathbf{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0 \tag{5}$$

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\in_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C \tag{6}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\in_{mji}\phi^*_{,m} \tag{7}$$

where: λ, μ, α, β, K, γ, λ_0, λ_1, α_0, β_0, are material constants; ρ is mass density; $\mathbf{u}=(u_1, u_2, u_3)$ is the displacement vector and $\boldsymbol{\phi}=(\phi_1, \phi_2, \phi_3)$ is the microrotation vector; ϕ^* is the scalar microstretch function; T is temperature and T_0 is the reference temperature of the body chosen; C is the concentration of the diffusion material in the elastic body; K^* is the coefficient of the thermal conductivity; C^* is the specific heat at constant strain; D is the thermoelastic diffusion constant; a is the coefficient describing the measure of thermal diffusion and b is the coefficient describing the measure of mass diffusion effects; j is the microinertia; $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$, $\beta_2=(3\lambda+2\mu+K)\alpha_{c1}$, $v_1=(3\lambda+2\mu+K)\alpha_{t2}$, $v_2=(3\lambda+2\mu+K)\alpha_{c2}$, α_{t1}, α_{t2} are coefficients of linear thermal expansion; and α_{c1}, α_{c2} are coefficients of linear diffusion expansion; j_0 is the microinertia for the microelements; t_{ij} are components of stress; m_{ij} are components of couple stress; λ_i^* is the microstress tensor; e_{ij} are components of strain; e_{kk} is the dilatation; δ_{ij} is Kronecker delta function; τ^0, τ^1 are the diffusion relaxation times and τ_0, τ_1 are thermal relaxation times with $\tau_0\geq\tau_1\geq 0$. Here $\tau^0=\tau^1=\tau_0=\tau_1=\gamma_1=0$ for Coupled Thermoelastic theory (CT) model. $\tau_1=\tau^1=0$, $\varepsilon=1$, $\gamma_1=\tau_0$ for Lord-Shulman (LS) model, and $\varepsilon=0$, $\gamma_1=\tau^0$ where $\tau^0>0$, for Green-Lindsay (GL) model.

In the above equations symbol (,) followed by a suffix denotes differentiation with respect to spatial coordinates, and a superposed dot (˙) denotes the derivative with respect to time.

FORMULATION OF THE PROBLEM

We consider a rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically outward the medium. We consider a normal or tangential force to be acting at the free surface of microstretch thermoelastic medium with mass diffusion half space.

For two-dimensional problems the displacement and microrotation vectors are of the form:

$$\mathbf{u}=(u_1,0,u_3),\quad \boldsymbol{\phi}=(0,\phi_2,0) \tag{8}$$

For further consideration it is convenient to introduce in Eqs.(1)-(5) the dimensionless quantities defined as:

$$\begin{gathered}
u_r'=\frac{\rho\omega^* c_1}{\beta_1 T_0}u_r,\quad x_r'=\frac{\omega^*}{c_1}x_r,\quad t'=\omega^* t,\quad \phi^{*\prime}=\frac{\rho c_1^2}{\beta_1 T_0}\phi^*,\quad T'=\frac{T}{T_0},\quad \tau_1'=\omega^*\tau_1,\quad \tau_0'=\omega^*\tau_0,\quad \gamma_1'=\omega^*\gamma_1,\quad t_{ij}'=\frac{1}{\beta_1 T_0}t_{ij},\\
\omega^*=\frac{\rho c^* c_1^2}{K^*},\quad \phi_i'=\frac{\rho c_1^2}{\beta_1 T_0}\phi_i,\quad \tau^{1\prime}=\omega^*\tau^1,\quad c_1^2=\frac{\lambda+2\mu+k}{\rho},\quad c_2^2=\frac{\mu+k}{\rho},\quad c_3^2=\frac{\gamma}{\rho j},\quad c_4^2=\frac{2\alpha_0}{\rho j_0},\quad \varepsilon=\frac{\gamma^2 T_0}{\rho^2 c^* c_1},\\
m_{ij}^*=\frac{\omega^*}{c\beta_1 T_0}m_{ij},\quad C'=\frac{\beta_2}{\rho c_1^2}C.
\end{gathered} \tag{9}$$

With the aid of Eqs.(8) and (9), the Eqs.(1)-(5) reduce to:

$$a_1\frac{\partial e}{\partial x_1}+a_2\nabla^2 u_1-a_3\frac{\partial\phi_2}{\partial x_3}+a_4\frac{\partial\phi^*}{\partial x_1}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}-\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_1}=\ddot{u}_1, \tag{10}$$

$$a_1\frac{\partial e}{\partial x_3}+a_2\nabla^2 u_3-a_3\frac{\partial\phi_2}{\partial x_1}+a_4\frac{\partial\phi^*}{\partial x_3}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}-\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_3}=\ddot{u}_3, \tag{11}$$

$$\nabla^2\phi_2-2a_6\phi_2+a_6\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=a_7\ddot{\phi}_2, \tag{12}$$

$$\nabla^2\phi^*-a_8\phi^*-a_9 e+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)C=a_{12}\ddot{\phi}^*, \tag{13}$$

$$\nabla^2 T=a_{13}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)e+a_{15}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+a_{16}\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C, \tag{14}$$

$$\nabla^2 e+a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T+a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C=0. \tag{15}$$

Here,

$$\begin{gathered}
a_1=\frac{\lambda+\mu}{\beta_1 T_0},\quad a_2=\frac{\mu+K}{\beta_1 T_0},\quad a_3=\frac{K}{\rho c_1^2},\quad a_4=\frac{\lambda_0}{\rho c_1^2},\quad a_5=\frac{\rho c_1^2}{\beta_1 T_0},\quad a_6=\frac{K c_1^2}{\gamma\omega^{*2}},\quad a_7=\frac{\rho j c_1^2}{\gamma},\quad a_8=\frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}},\\
a_9=\frac{\lambda_0\rho c_1^4}{\beta_1 T_0\alpha_0\omega^{*2}},\quad a_{10}=\frac{\nu_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}},\quad a_{11}=\frac{\nu_2\rho c_1^6}{\beta_1\beta_2 T_0\alpha_0\omega^{*2}},\quad a_{12}=\frac{\rho c_1^2 j_0}{2\alpha_0},\quad a_{13}=\frac{\rho c^* c_1^2}{\omega^* K^*},\quad a_{14}=\frac{\beta_1 c_1^2}{\omega^* K^*},\\
a_{15}=\frac{\nu_1\beta_1 T_0}{\omega^*\rho K^*},\quad a_{16}=\frac{a\rho c_1^4}{\omega^*\beta_2 K^*},\quad a_{17}=\frac{aT_0}{\beta_2},\quad a_{18}\frac{\rho c_1^4}{\omega^* D\beta_2^2},\quad a_{19}=\frac{b\rho c_1^2}{\beta_2^2}.
\end{gathered}$$

The displacement components u_1 and u_3 are related to potential functions ϕ and ψ as:

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3},\quad u_3=\frac{\partial\phi}{\partial x_3}-\frac{\partial\psi}{\partial x_1} \tag{16}$$

Using the relation Eq.(16), in the Eqs.(10)-(15), we obtain:

$$(a_1+a_2)\nabla^2\phi-\ddot{\phi}+a_4\phi^*-\left(1+\tau_1\frac{\partial}{\partial t}\right)T-a_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0, \tag{17}$$

$$\left(\nabla^2-a_8-a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^*-a_9\nabla^2\phi+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(a+\tau^1\frac{\partial}{\partial t}\right)C=0, \tag{18}$$

$$a_{13}\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial}{\partial t^2}\right)\nabla^2\phi+a_{15}\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\phi}^*+a_{16}\left(1+\gamma_1\frac{\partial}{\partial t}\right)\dot{C}-\nabla^2 T=0, \tag{19}$$

$$\nabla^4\phi + a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C - a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0\,, \tag{20}$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0\,, \tag{21}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2\,. \tag{22}$$

Here $\nabla^2 = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

SOLUTION OF THE PROBLEM:

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\begin{aligned}&\{\phi,\psi,T,\phi_2,\phi^*,C\}(x_1,x_3,t) = \\ &= \{\overline{\phi},\overline{\psi},\overline{T},\overline{\phi_2},\overline{\phi^*},\overline{C}\}(x_3)e^{i(kx_1-\omega t)}\end{aligned} \tag{23}$$

Here ω is the angular velocity and k is a complex constant. Making use of Eq.(23), the relations Eqs.(17)-(22) yield:

$$\left[b_1\left(\frac{d^2}{dx_3^2}-k^2\right)+\omega^2\right]\overline{\phi} + b_2\overline{\phi^*} + b_3\overline{T} + b_4\overline{C} = 0\,, \tag{24}$$

$$b_5\left(\frac{d^2}{dx_3^2}-k^2\right)\overline{\phi} + \left(\frac{d^2}{dx_3^2}+b_6\right)\overline{\phi^*} + b_7\overline{T} + b_8\overline{C} = 0\,, \tag{25}$$

$$b_9\left(\frac{d^2}{dx_3^2}-k^2\right)\overline{\phi} + b_{10}\overline{\phi^*} + \left(b_{11}-\frac{d^2}{dx_3^2}\right)\overline{T} + b_{12}\overline{C} = 0\,, \tag{26}$$

$$\left(\frac{d^2}{dz^2}-k^2\right)^2\overline{\phi} + b_{13}\left(\frac{d^2}{dx_3^2}-k^2\right)\overline{T} + \left(b_{14}+b_{15}\left(\frac{d^2}{dx_3^2}-k^2\right)\right)\overline{C} = 0, \tag{27}$$

$$\left[b_{16}\left(\frac{d^2}{dx_3^2}-k^2\right)+\omega^2\right]\overline{\psi} + b_{17}\overline{\phi_2} = 0\,, \tag{28}$$

$$b_{20}\left(\frac{d^2}{dx_3^2}-k^2\right)\overline{\psi} + \left[b_{18}\left(\frac{d^2}{dx_3^2}-k^2\right)+b_{19}\right]\overline{\phi_2} = 0\,. \tag{29}$$

Here,

$$b_1 = \frac{\rho c_1^2}{\beta_1 T_0},\quad b_2 = \frac{\lambda_0}{\rho c_1^2},\quad b_3 = -(1-i\omega\tau_1),\quad b_4 = -b_1(1-i\omega t^1),$$

$$b_5 = -\frac{\lambda_0\rho c_1^4}{\beta_1 T_0\alpha_0\omega^{*2}},\quad b_6 = -k^2 - \frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}} + \frac{\omega^2\rho j_0 c_1^2}{2\alpha_0}\,,$$

$$b_7 = \frac{\nu_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}}(1-i\omega\tau_1),\quad b_8 = \frac{\nu_2\rho^2 c_1^6}{\beta_1\beta_2 T_0\alpha_0\omega^{*2}}\,,$$

$$b_9 = -\frac{\beta_1 c_1^2}{\omega^* K^*}(i\omega+\tau_0\in\omega^2),\quad b_{10} = -\frac{\nu_1\beta_1 T_0}{\omega^*\rho K^*}(i\omega+\tau_0\in\omega^2)\,,$$

$$b_{11} = -\frac{\rho c^* c_1^2}{\omega^* K^*}(i\omega+\tau_0\omega^2)+k^2,\quad b_{12} = -\frac{a\rho c_1^4}{\omega^*\beta_2 K^*}(i\omega+\gamma_1\omega^2),$$

$$b_{13} = \frac{aT_0}{\beta_2}(1-i\omega\tau_1),\quad b_{14} = -\frac{\rho c_1^4}{\omega^* D\beta_2^2}(i\omega+\tau^0\in\omega^2),$$

$$b_{15} = -\frac{b\rho c_1^2}{\beta_2^2}(1-i\omega\tau^1),\quad b_{16} = \frac{\mu+K}{\beta_1 T_0},\quad b_{17} = -\frac{K}{\rho c_1^2},$$

$$b_{18} = \frac{\gamma}{\gamma\rho j c_1^2},\quad b_{19} = -\frac{2K}{\rho j\omega^{*2}}+\omega^2,\quad b_{20} = -\frac{Kc_1^2}{j\omega^{*2}\beta_1 T_0}. \tag{29}$$

On solving Eqs.(24)-(27), we obtain:

$$\left[\mathrm{A}_0\frac{d^8}{dx_3^8}+\mathrm{A}_1\frac{d^6}{dx_3^6}+\mathrm{A}_2\frac{d^4}{dx_3^4}+\mathrm{A}_3\frac{d^2}{dx_3^2}+\mathrm{A}_5\right](\overline{\phi},\overline{\phi^*},\overline{T},\overline{C}) = 0 \tag{30}$$

and on solving Eqs.(28)-(29), we obtain:

$$\left[\frac{d^4}{dx_3^4}+\mathrm{A}_6\frac{d^2}{dx_3^2}+\mathrm{A}_7\right](\overline{\phi_2},\overline{\psi}) = 0\,. \tag{31}$$

The solution of the above system satisfying the radiation conditions that $(\overline{\phi},\overline{\psi},\overline{T},\overline{\phi_2},\overline{\phi^*},\overline{C})\to 0$ as $x_3\to\infty$ are given as following:

$$(\overline{\phi},\overline{\phi^*},\overline{T},\overline{C}) = \sum_{i=1}^{4}(1,\alpha_{1i},\alpha_{2i},\alpha_{3i})M_i e^{-m_i x_3} \tag{32}$$

$$(\overline{\psi},\overline{\phi_2}) = \sum_{i=5}^{6}(1,\beta_{1i})M_i' e^{-m_i x_3} \tag{33}$$

Here M_i and M'_i are functions depending on k and ω; m_i^2 (i = 1, 2, 3, 4) are the roots of the Eq.(30) and m_i^2 (i = 5, 6) are the roots of Eq.(31).

Here,

$$\alpha_{1i} = \frac{D_{1i}}{D_{0i}},\quad \alpha_{2i} = \frac{D_{2i}}{D_{0i}},\quad \alpha_{3i} = \frac{D_{3i}}{D_{0i}},\quad i = 1,2,3,4 \tag{34}$$

$$\beta_{1i} = -\frac{\delta_3(m_i^2-k^2)}{(m_i^2-k^2)+\delta_4},\quad i = 5,6 \tag{35}$$

BOUNDARY CONDITIONS

We consider normal and tangential force acting on the surface $x_3 = 0$ along with vanishing of couple stress, microstress and temperature gradient with insulated and impermeable boundary at $x_3 = 0$. Mathematically this can be written as:

$$\begin{aligned}&t_{33} = -F_1 e^{-(kx_1-\omega t)},\quad t_{31} = -F_2 e^{-(kx_1-\omega t)},\\ &m_{32} = 0,\quad \lambda_3 = 0,\quad \frac{\partial T}{\partial x_3} = 0,\quad \frac{\partial C}{\partial x_3} = 0.\end{aligned} \tag{36}$$

Here F_1 and F_2 are the magnitude of the applied force.

Using these boundary conditions and solving the linear equations formed, we obtain:

$$t_{33} = \sum_{i=1}^{6} G_{1i}e^{-m_i x_3}e^{-(kx_1-\omega t)},\quad i = 1,2,\ldots 6 \tag{37}$$

$$t_{31} = \sum_{i=1}^{6} G_{2i}e^{-m_i x_3}e^{-(kx_1-\omega t)},\quad i = 1,2,\ldots 6 \tag{38}$$

$$m_{32} = \sum_{i=1}^{6} G_{3i}e^{-m_i x_3}e^{-(kx_1-\omega t)},\quad i = 1,2,\ldots 6 \tag{39}$$

$$\lambda_3 = \sum_{i=1}^{6} G_{4i}e^{-m_i x_3}e^{-(kx_1-\omega t)},\quad i = 1,2,\ldots 6 \tag{40}$$

$$u_1 = \sum_{i=1}^{6} G_{5i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, \quad i = 1,2,\ldots 6 \tag{41}$$

$$u_3 = \sum_{i=1}^{6} G_{6i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, \quad i = 1,2,\ldots 6 \tag{42}$$

$$T = \sum_{i=1}^{6} G_{7i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, \quad i = 1,2,\ldots 6 \tag{43}$$

$$C = \sum_{i=1}^{6} G_{8i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, \quad i = 1,2,\ldots 6 \tag{44}$$

Here G_{ij}, $i = 1,2,\ldots,6$, and $j = 1,2,\ldots,8$ are the constants.

Case 1- Normal stress

To obtain the expressions due to normal stress we must set $F_2 = 0$ in the boundary conditions Eqs.(36).

Case 1- Tangential stress

To obtain the expressions due to tangential stress we must set $F_1 = 0$ in the boundary conditions Eqs.(36).

Particular cases

(i) If we take $\tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma_1 = \tau_0$ in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(ii) If we take $\varepsilon = 0$, $\gamma_1 = \tau^0$ in Eqs.(37)-(44), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ in Eqs. (37)-(44) yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

Special cases

(a) Microstretch thermoelastic solid

If we neglect the diffusion effect in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar thermoelastic diffusive solid

If we neglect the microstretch effect in Eqs.(37)-(44), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

NUMERICAL RESULTS AND DISCUSSIONS

The analysis is conducted for a magnesium crystal-like material. The values of physical constants are:

$\lambda = 9.4\times10^{10}$ Nm^{-2}, $\mu = 4.0\times10^{10}$ Nm^{-2}, $K = 1.0\times10^{16}$ Nm^{-2}, $\rho = 1.74\times10^{3}$ kg/m^3, $j = 0.2\times10^{-19}$ m^2, $\gamma = 0.779\times10^{-9}$ N.

Thermal and diffusion parameters are given by

$c^* = 1.04\times10^{3}$ JKg^{-1}K^{-1}, $K^* = 1.7\times10^{6}$ Jm^{-1}s^{-1}K^{-1}, $\alpha_{t1} = 2.33\times10^{-5}$ K^{-1}, $\alpha_{t2} = 2.48\times10^{10}$ K^{-1}, $T_0 = 0.298\times10^{3}$ K, $\tau_0 = 0.02$, $\tau_1 = 0.01$, $\alpha_{c1} = 2.65\times10^{-4}$ m^{3}Kg^{-1}, $\alpha_{c2} = 2.83\times10^{-4}$ m^{3}Kg^{-1}, $a = 2.9\times10^{4}$ m^{2}s^{-2}K^{-1}, $b = 32\times10^{5}$ Kg^{-1}m^{5}s^{-2}, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $D = 0.85\times10^{-8}$ Kgm^{-3}s,

and the microstretch parameters are taken as:

$j_0 = 0.19\times10^{-19}$ m^2, $\alpha_0 = 0.779\times10^{-9}$ N, $b_0 = 0.5\times10^{-9}$ N, $\lambda_0 = 0.5\times10^{10}$ Nm^{-2}, $\lambda_1 = 0.5\times10^{10}$ Nm^{-2}.

The computations are carried out for a single value of $\omega = 1$ and on the surface of the plane $z = 1$. The numerical values for the normal stress t_{33}, tangential couple stress m_{32}, temperature distribution T and microstress λ_3^* on the surface of plane due to applied concentrated and uniformly distributed normal sources are shown in Figs. 1-8. The comparison of two theories of generalized thermoelasticity, namely, Lord-Shulman (L-S) and Green-Lindsay (G-L) are shown in graphs. Also, the diffusion effect and microstretch effect are shown in graphs.

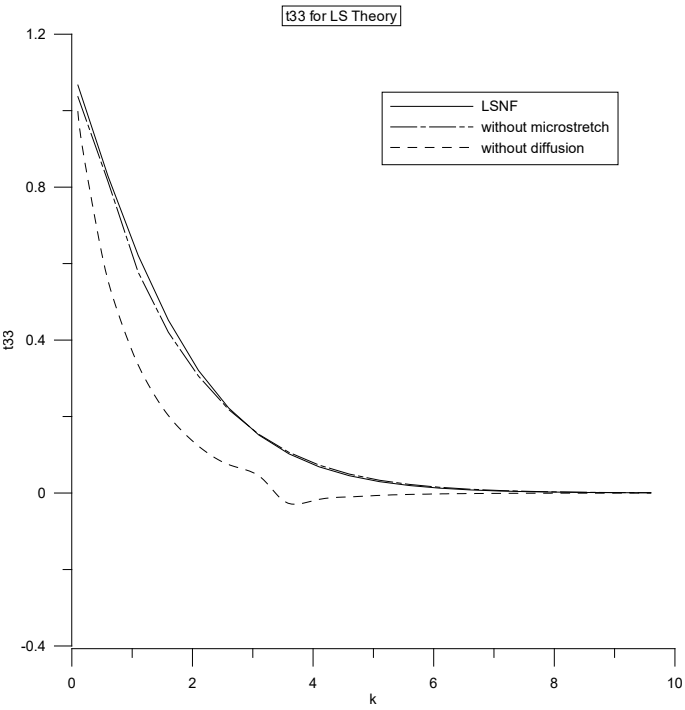

Figure 1. Variation of normal stress (t_{33}) with k in L-S theory.

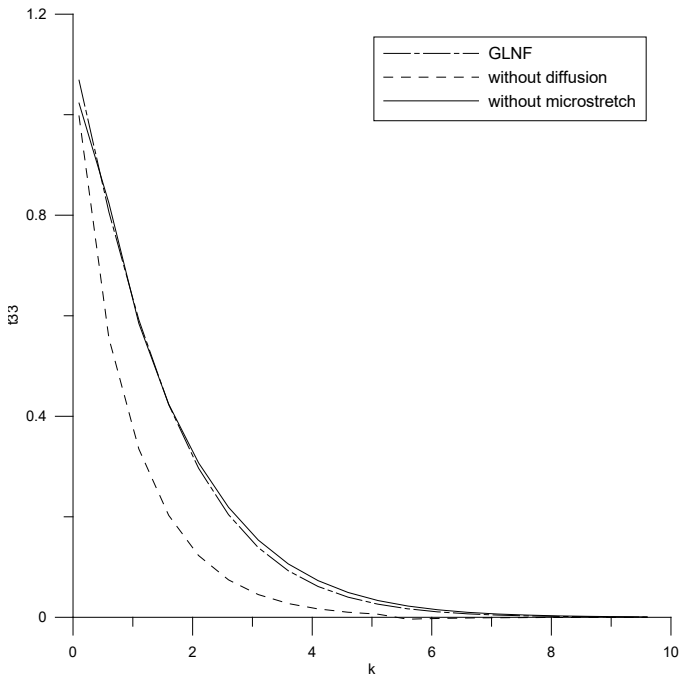

Figure 2. Variation of normal stress (t_{33}) with k in G-L theory.

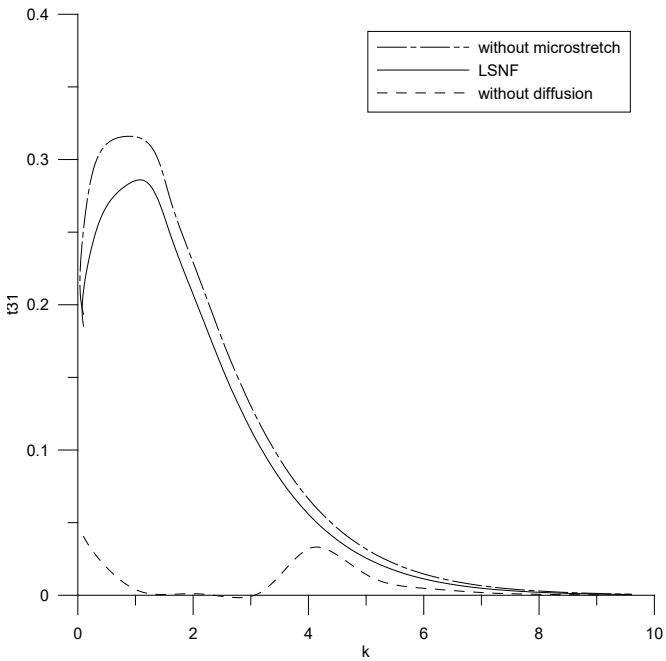

Figure 3. Variation of tangential stress (t_{31}) with k in L-S theory.

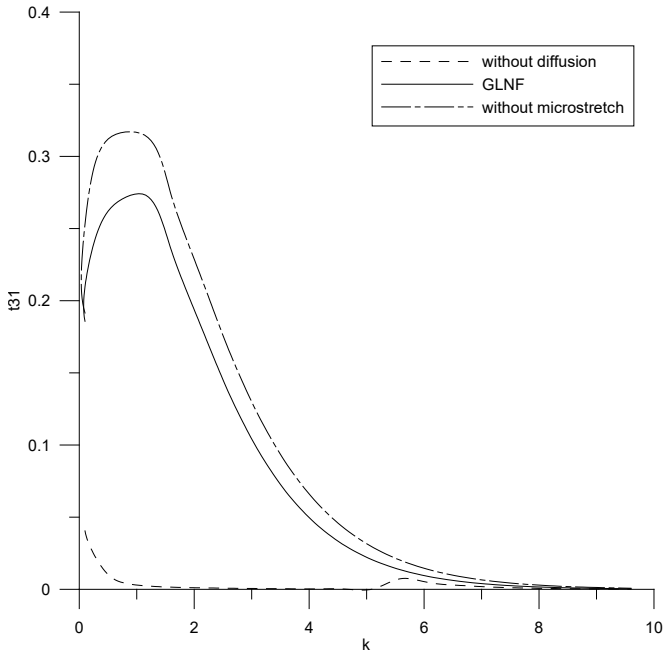

Figure 4. Variation of tangential stress (t_{31}) with k in G-L theory.

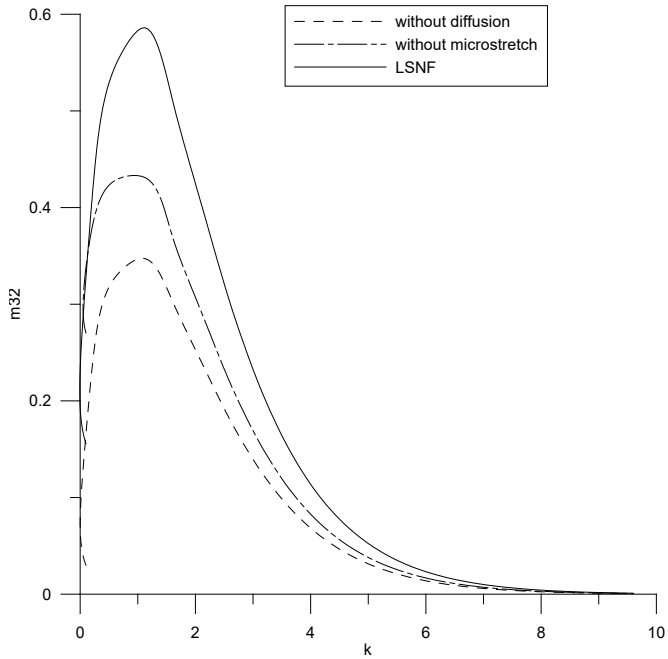

Figure 5. Variation of couple tangential stress (m_{32}) with k in L-S theory.

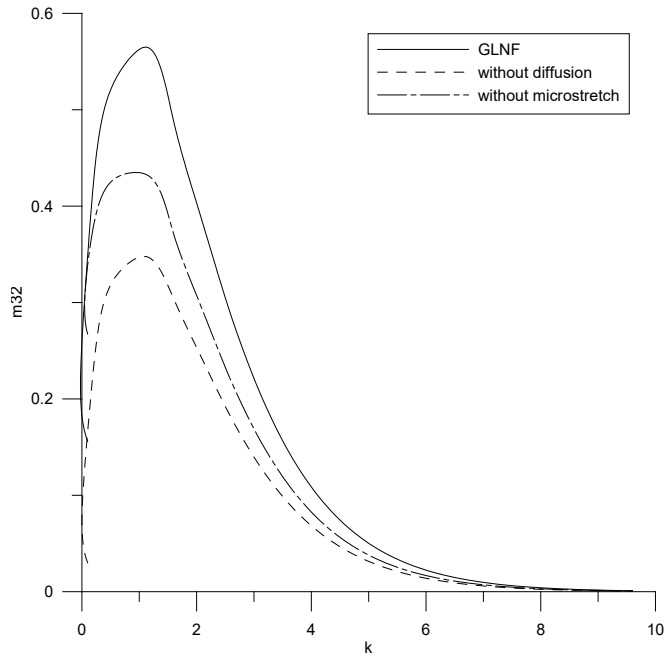

Figure 6. Variation of couple tangential stress (m_{32}) with k in G-L theory.

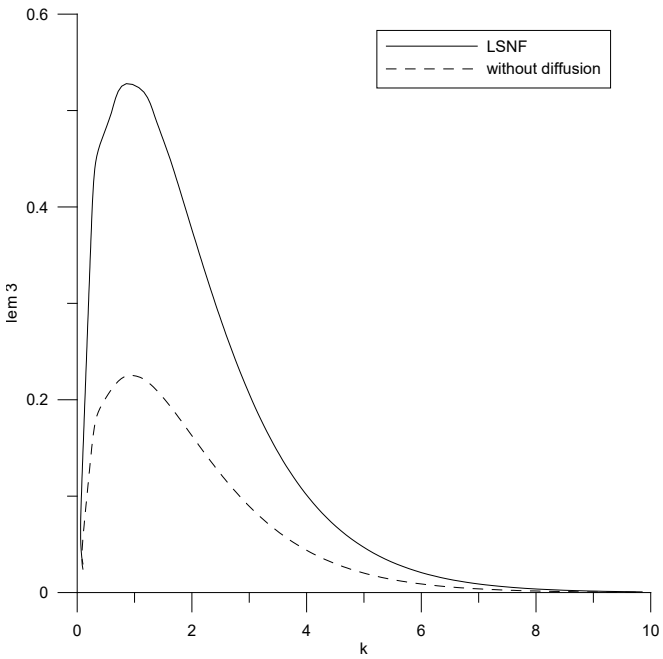

Figure 7. Variation of microstress (λ_3) with k in L-S theory.

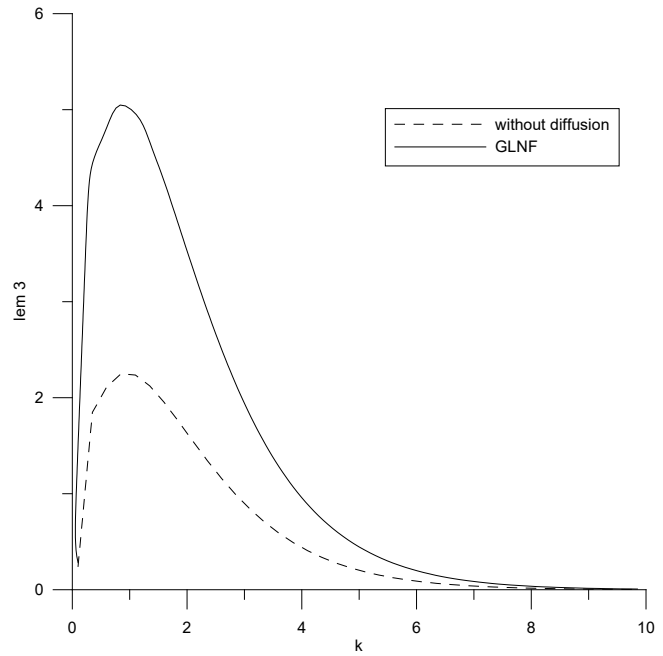

Figure 8. Variation of microstress (λ_3) with k in G-L theory.

CONCLUSIONS

The results of the problem may be applied to a wide class of geophysical problems involving temperature change. The deformation at any point of the medium at any point is useful to analyse the deformation field around mining tremors and drilling into the crust of the earth. It is observed from Figs. 1 and 2 that the value of normal force stress t_{33} for Lord Shulman (L-S) theory of thermoelasticity and Green Lindsay (G-L) theory of thermoelasticity decreases with the increase of the parameter k under the effect of mechanical source. This behaviour of normal stress t_{33} is same for microstretch thermoelastic solid with mass diffusion, microstretch thermoelastic solid and micropolar thermoelastic solid with mass diffusion.

In Figs. 3 and 4 the variation of tangential stress (t_{31}) is shown. For microstretch thermoelastic solid with mass diffusion the tangential stress first increase until k reaches value 2 and then decreases. The same behaviour is shown by micropolar thermoelastic solid with mass diffusion, but in case of microstretch thermoelastic solid the behaviour of variation of tangential stress (t_{31}) is slightly different, in this case for initial values of k the tangential stress (t_{31}) shows some variable behaviour, but for higher values of k it shows a uniform behaviour which is decreasing. Also, the same behaviour is shown in G-L theory and L-S theory of thermoelasticity. In Figs. 5 and 6 the variation of tangential couple stress (m_{32}) is shown. For microstretch thermoelastic solid with mass diffusion the tangential couple stress first increases until k reaches value 2 and then decreases. The same behaviour is shown by micropolar thermoelastic solid with mass diffusion, and in the case of microstretch thermoelastic solid the behaviour of variation of couple tangential stress is also alike. In Figs. 7 and 8 the variation of microstress (λ_3) is shown. For microstretch thermoelastic solid with mass diffusion the tangential stress first increases until k reaches value around 1.5 and then decreases. The same behaviour is shown in case of microstretch thermoelastic solid. Hence, we conclude that normal stress decreases with increase of k value, but the values of tangential stress, couple stress and microstress first increase for lower values of k and then decrease for higher values of k.

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