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CREEP TRANSITION IN BENDING OF FUNCTIONALLY GRADED TRANSVERSELY ISOTROPIC RECTANGULAR PLATES

PRELAZNI NAPONI PUZANJA PRI SAVIJANJU FUNKCIONALNE KOMPOZITNE TRANSVERZALNO IZOTROPNE ČETVOROUGAONE PLOČE

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Abstract

Creep stresses have been investigated in transversely isotropic thick-walled cylinder fabricated of functionally graded material subjected to internal pressure. The concept of transition theory is implemented to simplify the set of mechanical equations governing the physical phenomenon. Results have been discussed analytically as well as numerically. On the basis of analysis of the problem, it is concluded that highly functionally graded transversely isotropic cylinder with internal pressure and nonlinear strain measure is a better alternative for designing in comparison to circular cylinder of less functionally graded material.

Ključne reči
- ploča
- funkcionalni kompozit
- pritisak
- puzanje
- transverzalni izotropni materijal

Izvod

Ispitani su naponi puzanja u transverzalno izotropnom debelozidnom cilindru od funkcionalnog kompozitnog materijala, koji je izložen unutrašnjem pritisku. Koncept teorije prelaznih napona je uveden kako bi se uprostio skup jednačina mehanike koje opisuju fizički fenomen. Rezultati su razmatrani analitički, kao i numerički. Na osnovu analize problema, zaključuje se da je cilindar od visoko funkcionalnog kompozitnog transverzalno izotropnog materijala, izložen unutrašnjem pritisku, sa nelinearnom deformacijom, bolja alternativa u poređenju sa kružnim cilindrom od manje funkcionalnog kompozitnog materijala.

INTRODUCTION

Hollow circular cylinders have numerous applications in nuclear industries, aerospace, nuclear reactors, helicopter rotors and submarines etc. In present days, cylinders under pressure have attracted the interest of researchers due to their wide application in nuclear industry, especially in thermal neutron reactors. In general, the cylinders under high pressure require a strict analysis for an optimum design for reliable and secure operational performance and thus efforts are continually made to increase the reliability of such types of cylinders. Creep of thick-walled circular cylinders under internal pressure have been discussed by Altenbach and Skrzypek, /1/. A computational model for the analysis of elastic-plastic and residual stresses in functionally graded rotating solid shafts has been discussed by Argeso and Eraslan, /2/, using Von-Mises' yield criterion, total deformation theory and Swift's nonlinear hardening law. Steady state creep stresses have been investigated by Hoseini et al., /3/, in rotating thick-walled cylindrical shells under internal and external pressure using Norton's law with plane strain conditions to demonstrate the effect of angular speed on stress distribution. Singh and Gupta, /4/, investigated creep stresses in a thick-walled composite circular cylinder under internal pressure by assuming stress exponent and concluded that stresses did not have significant variations with size and contents of reinforcement. The steady state creep stresses have been investigated by Singh and Gupta, /5/, for functionally graded thick-walled composite circular cylinder of silicon carbide particles in a matrix of pure aluminium and concluded that circumferential stresses increase significantly at the inner radius, while they decrease at the outer radius. A closed form analytical solution for steady state creep stresses of non-homogeneous rotating thick cylindrical pressure vessels has been obtained using Norton's law by Zahra et al., /6/, and from results it has been concluded that the property of functionally graded material has a significant effect on the stresses in pressure vessels. In order to analyse the creep stresses in circular cylinder, all the above mentioned authors applied the presumptions of infinitesimal theory of strain, incompressibility and Norton's law, etc. Sharma and Yadav, /7/, determined the numerical solution of the boundary value problem of functionally graded cylinder under internal and external pressure. Sobhaniaragh et al., /8/, discussed thermal stresses in cylindrical shells of ceramic matrix composite and noticed the impact of aggregation factor on circumferential stresses. In transition theory, /9/, there is no requirement of assuming the above mentioned theory and thus it is more convenient to solve a problem using generalized strain measure. Transition theory has been applied by many authors, for example, thermal creep stresses are calculated in a transversely isotropic rotating cylinder under internal pressure by Sharma et al., /10/, and it has been noticed that the transversely isotropic rotating circular cylinder under internal pressure is safe in design. Creep stresses in thick-walled non-homogeneous circular cylinder under external pressure are evaluated by Sharma et al., /11/, which leads to the conclusion that cylinder with nonlinear measure is a better option for the designers. Safety analysis of thermal creep in non-homogeneous thick-walled circular cylinder under internal and external pressure has been done by Aggarwal et al., /12/. Sharma and Panchal, /13/, evaluated creep stresses in pressurized thick-walled rotating spherical shell of functionally graded material and deduced that both rotation and non-homogeneity affect the creep stresses. Sharma et al., /14/, analysed thermal creep stresses for functionally graded thick-walled cylinder subjected to torsion and internal and external pressure and it is found that in the creep - torsion cylinder of less functionally graded material under pressure is a better choice in a designer's point of view as compared to the homogeneous cylinder.

OBJECTIVE

In order to give an insight on design and evaluation of stresses in rectangular plates under pressure, the failure mechanism of plates should be considered carefully. The objective of the present problem is to calculate creep stresses in rectangular plates of heterogeneous transversely isotropic material under the influence of pressure at internal surface.

MATHEMATICAL FORMULATIONS

We consider an initially wide plate of functionally graded material which is formed into a circular cylinder with two of its edges as generators. The bending moment, M, is applied perpendicular to the plane of the paper. Let (x',y',z') coordinates be any point on the plate. The two faces of the plates are bent into a cylinder of inner radius a and outer radius b, and the other two faces into axially terminating planes given by $\theta = \pm z$. The z axis is taken parallel to the axis of the cylinder. Plane sections remain parallel after deformation.

Under these considerations, the components of the displacement are given by

$$u = x - f(r),\ v = y - A\theta,\ w = \alpha z, \tag{1}$$

where: $f(r)$ is any function of $r = \sqrt{x^2+y^2}$; α and A are constants.

The components of generalized strains are

$$\begin{aligned} &e_{rr} = \frac{1}{n}[1-(f')^n],\quad e_{\theta\theta} = \frac{1}{n}[1-(A/r)^n], \\ &e_{zz} = \frac{1}{n}[1-(1-\alpha)^n],\quad e_{r\theta} = e_{\theta z} = e_{zr} = 0, \end{aligned} \tag{2}$$

where: n is the strain measure; and $f' = df/dr$.

Constitutive relations for radial, circumferential and axial stresses with respect to strain for anisotropic materials with five material constants (transversely isotropic material) are

$$\begin{aligned} &T_{rr} = C_{11}e_{rr} + (C_{11}-2C_{66})e_{\theta\theta} + C_{13}e_{zz}, \\ &T_{\theta\theta} = (C_{11}-2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz}, \\ &T_{zz} = C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz}, \\ &T_{zr} = T_{\theta z} = T_{r\theta} = 0. \end{aligned} \tag{3}$$

Using Eqs.(2) in Eqs.(3), we get

$$\begin{aligned} &T_{rr} = (C_{11}/n)\left[1-(f')^n\right] + \left[(C_{11}-2C_{66})/n\right]\left[1-(A/r)^n\right] + C_{13}e_{zz}, \\ &T_{\theta\theta} = \left[(C_{11}-2C_{66})/n\right]\left[1-(f')^n\right] + (C_{11}/n)\left[1-(A/r)^n\right] + C_{13}e_{zz}, \\ &T_{zz} = (C_{13}/n)\left[1-(f')^n\right] + (C_{13}/n)\left[1-(A/r)^n\right] + C_{33}e_{zz}, \\ &T_{r\theta} = T_{\theta z} = T_{rz} = 0. \end{aligned} \tag{4}$$

Taking non-homogeneity in the material (functionally graded material) as

$$\begin{aligned} &C_{11} = C_{011}\left(\frac{r}{b}\right)^{-k},\quad C_{13} = C_{013}\left(\frac{r}{b}\right)^{-k}, \\ &C_{66} = C_{066}\left(\frac{r}{b}\right)^{-k},\quad C_{33} = C_{033}\left(\frac{r}{b}\right)^{-k}. \end{aligned} \tag{5}$$

Using Eq.(5) in Eq.(4), we obtain

$$\begin{aligned} &T_{rr} = (C_{011}/n)\left(\frac{r}{b}\right)^{-k}\left[1-(f')^n\right] + \left[(C_{011}-2C_{066})/n\right]\left(\frac{r}{b}\right)^{-k} \times \\ &\quad \times\left[1-(A/r)^n\right] + C_{013}\left(\frac{r}{b}\right)^{-k}\cdot d, \\ &T_{\theta\theta} = \left[(C_{011}-2C_{066})/n\right]\left(\frac{r}{b}\right)^{-k}\left[1-(f')^n\right] + (C_{011}/n)\left(\frac{r}{b}\right)^{-k} \times \\ &\quad \times\left[1-(A/r)^n\right] + C_{013}\left(\frac{r}{b}\right)^{-k}\cdot d, \\ &T_{zz} = (C_{013}/n)\left(\frac{r}{b}\right)^{-k}\left[1-(f')^n\right] + (C_{013}/n)\left(\frac{r}{b}\right)^{-k} \times \\ &\quad \times\left[1-(A/r)^n\right] + C_{033}\left(\frac{r}{b}\right)^{-k}\cdot d, \\ &T_{r\theta} = T_{\theta z} = T_{rz} = 0 \ \text{ and } \ d = 1-(1-\alpha)^n. \end{aligned} \tag{6}$$

The equilibrium equation is

$$\frac{d}{dr}(T_{rr}) + \left(\frac{T_{rr}-T_{\theta\theta}}{r}\right) = 0. \tag{7}$$

At transition, the differential system defining the elastic state should attain some criticality. The nonlinear differential equation which represents this problem is obtained by substituting Eqs.(6) in Eq.(7),

$$\begin{aligned} &(C_{011}/nr)f^{n+1}P^n\frac{dP}{df} = (-kC_{011}/nr)(2r^n - f^nP^n - A^n) - \\ &-(C_{011}/nr)f^nP^n(P-1) + (2C_{066}-C_{011})A^n r + (2kC_{011}/nr)\times \\ &\times(r^n-A^n) - (2C_{066}/r)(f^nP^n-A^n) - (kC_{011}/n)r^{n-1}d, \end{aligned} \tag{8}$$

where: $rf' = fP$.

The critical point or transitional point of Eq.(8) is $P \to 0$.

Boundary conditions are taken as

$$T_{rr} = 0 \ \text{ at } \ r = a \ \text{ and } \ T_{rr} = 0 \ \text{ at } \ r = b. \tag{9}$$

At plane ends we have

$$2\pi\int_a^b rT_{zz}\,dr = 0, \tag{10}$$

whereas on the straight edges $\theta = \pm z$

$$\int_a^b T_{\theta\theta}\,dr = 0 \ \text{ and } \ M = -\int_a^b rT_{\theta\theta}\,dr. \tag{11}$$

MATHEMATICAL APPROACH

The principal stresses σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} are considered in this problem, so transition from one state to another can be taken through the principal stresses σ_{rr}, $\sigma_{\theta\theta}$ or stress difference $\sigma_{rr} - \sigma_{\theta\theta}$. It has been exhibited in different problems of various authors /7-10/ that transition through $\sigma_{rr} - \sigma_{\theta\theta}$ tends to creep at critical point $P \to 0$.

The transition function R is taken as

$$R = T_{rr} - T_{\theta\theta} = (2C_{066}/n)\left(\frac{r}{b}\right)^{-k}\left[(A/r)^n - (fP/r)^n\right]. \quad (12)$$

On simplifying Eqs.(8) and (12), the following result is obtained

$$\frac{d}{dr}(\log R) = \frac{\begin{bmatrix}\left[(2C_{066}n^2/C_{011}) - k\right][A^n - f^nP^n] + nA^nr^2 - \\ 2nf^nP^n(P-1) - nk(2r^n - A^n - f^nP^n) + \\ +\left[(2C_{066}/C_{011}) - C_{011}\right]n^2r^2A^n + (2C_{066}/C_{011})\times \\ \times kn(r^n - A^n) - nkC_{013}r^n d\end{bmatrix}}{A^n - f^nP^n}. \quad (13)$$

Applying the condition $P \to 0$ in the above Eq.(13) and integrating, we get

$$R = A_1\exp(G), \quad (14)$$

where: A_1 is integration constant; and

$$G = \left[(2C_{066}n^2/C_{011}) - k\right]r + nr^3/3 - nk\left\{2\left[r^{n+1}/(n+1)A^n\right] - 1\right\} + \\ +\left[(2C_{066}/C_{011}) - C_{011}\right]n^2r^3/3 + (2C_{066}/C_{011})kn\left[r^n/(n+1)A^n\right] - \\ -nkd(C_{013}/C_{011})\left[r^{n+1}/(n+1)\right]A^n.$$

Using Eqs.(7), (12) and (13), the stresses in the region of tension ($T_{rr} > T_{\theta\theta}$) are given as

$$T_{rr} = T_{rr}^* + \int_r^A \frac{A_1\exp(G)}{r}dr, \quad T_{\theta\theta} = T_{rr} - A_1\exp(G), \quad (15)$$

where: T_{rr}^* is the radial stress T_{rr} at $r = A$ (neutral axis). Stresses in the region of compression ($T_{\theta\theta} > T_{rr}$) are

$$T_{rr} = T_{rr}^* + \int_A^r \frac{A_1\exp(G)}{r}dr, \quad T_{\theta\theta} = T_{rr} - A_1\exp(G). \quad (16)$$

Applying boundary conditions Eq.(9) in Eqs.(15) and (16) we get

$$T_{rr}^* = -\int_b^A \frac{A_1\exp(G)}{r}dr \text{ and } T_{rr}^* = -\int_A^a \frac{A_1\exp(G)}{r}dr. \quad (17)$$

T_{rr} is continuous at $r = A$, therefore from Eq.(17) we have

$$\int_b^A \frac{A_1\exp(G)}{r}dr = \int_A^a \frac{A_1\exp(G)}{r}dr.$$

From the above expression, the radius of neutral surface A is obtained as

$$A = \left[\frac{a^{n+1} + b^{n+1}}{2}\right]^{1/(n+1)}. \quad (18)$$

Using boundary conditions in Eq.(11) we have

$$M = \int_a^A rT_{\theta\theta}dr + \int_A^b rT_{\theta\theta}dr. \quad (19)$$

The components in non-dimensional form are: $R = (r/b)$, $R_0 = (a/b)$, $\sigma_{rr} = [T_{rr}/C_{066}]$, $\sigma_{\theta\theta} = [T_{\theta\theta}/C_{066}]$, $\sigma_{zz} = [T_{zz}/C_{066}]$.

The stresses in the region of tension ($T_{rr} > T_{\theta\theta}$) in non-dimensional form are

$$\sigma_{rr} = \sigma_{rr}^* + \int_{bR}^{A/b} \frac{(A_1/C_{066})\exp(G^*)}{R}dR, \quad (20)$$
$$\sigma_{\theta\theta} = \sigma_{rr} - A_1\exp(G^*),$$

where $\sigma_{rr}^* = -\int_1^{A/b} \frac{(A_1/C_{066})\exp(G^*)}{R}dR$, and σ_{rr}^* is the radial stress, σ_{rr}, at $r = A$ (neutral axis). In the region of compression ($T_{\theta\theta} > T_{rr}$):

$$\sigma_{rr} = \sigma_{rr}^* + \int_{A/b}^{R_0} \frac{(A_1/C_{066})\exp(G^*)}{R}dR, \quad (21)$$
$$\sigma_{\theta\theta} = \sigma_{rr} - A_1\exp(G^*),$$

where: $\sigma_{rr}^* = -\int_{A/b}^{R_0} \frac{(A_1/C_{066})\exp(G^*)}{R}dR$, and

$G^* = [(2C_{066}n^2/C_{011}) - k]bR + n(bR)^3/3 - nk[(2(bR)^{n+1}/(n+1)A^n) - 1] + [(2C_{066}/C_{011}) - C_{011}]n^2(bR)^3/3 + (2C_{066}/C_{011})kn\cdot[(bR)^{n+1}/(n+1)A^n] - 1) - nk(C_{013}/C_{011})d(bR)^{n+1}/(n+1)A^n$, and

$$M_1 = \int_{R_0}^{A/b} b^2RT_{\theta\theta}dR + \int_{A/b}^{1} b^2RT_{\theta\theta}dR. \quad (22)$$

Particular case: homogeneous transversely isotropic material

Substituting $k = 0$ in Eq.(5), we have

$$C_{11} = C_{011},\ C_{13} = C_{013},\ C_{66} = C_{066},\ C_{33} = C_{033}. \quad (23)$$

Using Eq.(23) in Eq.(16), the radial, circumferential and axial creep stresses for transversely isotropic cylinder become

$$\sigma_{rr} = \sigma_{rr}^* + \int_R^A \frac{(A_1/C_{66})\exp(G^*)}{R}dR, \quad \sigma_{\theta\theta} = \sigma_{rr} - A_1\exp(G^*),$$

in region of tension, and

$$\sigma_{rr} = \sigma_{rr}^* + \int_A^R \frac{(A_1/C_{66})\exp(G^*)}{R}dR, \quad \sigma_{\theta\theta} = \sigma_{rr} - A_1\exp(G^*),$$

in region of compression, (24)

where: $G^* = (2C_{66}n^2/C_{011})bR + n(bR)^3/3 + [(2C_{66}/C_{11}) - C_{11}]\cdot n^2(bR)^3/3$.

Without thermal effects these equations are the same as obtained by Sharma et al., /10/.

NUMERICAL DISCUSSION AND CONCLUSION

In order to calculate the stresses in rectangular plates on the basis of the above-mentioned calculations, the values of measure and pressure are taken as follows: n = 1, 1/3 and 1/5 (i.e. N = 1, 3 and 5); P = 5, 10; D = 1. Elastic constants C_{ij} for steel, magnesium and beryl are given in Table 1.

Graphs are plotted in Figs. 1-8 by showing stresses on one axis and radii ratio ($R = r/b$) on the other for cylinders made up of beryl, magnesium and steel with internal pressure.

Table 1. Elastic constants C_{ij} used (in units of 10^{10} N/m^2).

Material	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}
Steel (isotropic)	2.908	1.27	1.27	2.908	0.819
Magnesium (transversely isotropic)	5.97	2.62	2.17	6.17	1.64
Beryl (transversely isotropic)	2.746	0.98	0.67	4.69	0.883

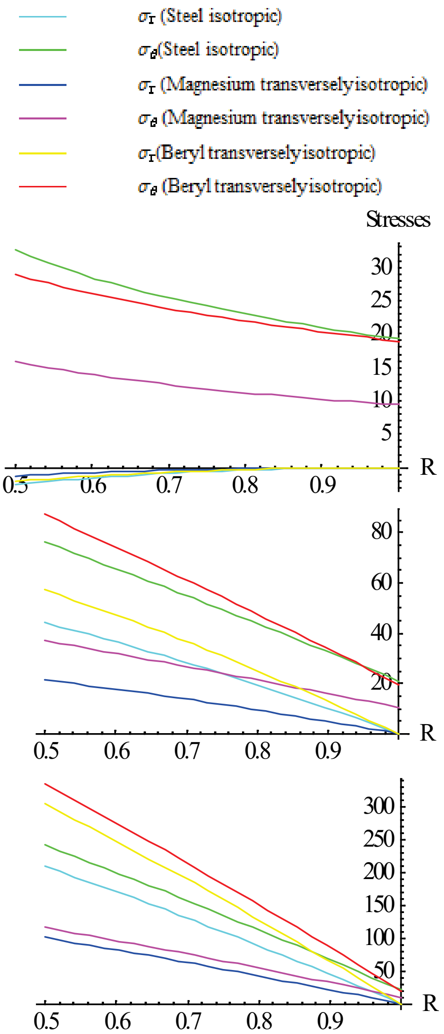

Figure 1. Creep stresses for steel, magnesium and beryl with $P = 5$, $N = 1, 3, 5$ and $k = 0$.

It is identified from Fig. 1 that hoop stresses are tensile in nature and have maximum value at internal surface for the cylinder of isotropic and transversely isotropic material with linear measure under the influence of internal pressure. These creep stresses have maximum values in case of a steel cylinder as compared to the cylinder of beryl and magnesium. As the measure changes from linear to nonlinear, there is significant increase in circumferential creep stresses. Also, circumferential creep stresses are lower for the magnesium cylinder (transversely isotropic) as compared to cylinders of steel (isotropic) and beryl (transversely isotropic), as noticed in Fig. 1. In Fig. 2, with the introduction of non-homogeneity, the hoop stresses are tensile and maximum at the inner surface of the cylinder. Also, these hoop creep stresses increase for the cylinder with linear measure while they decrease for cylinder with nonlinear measure at the

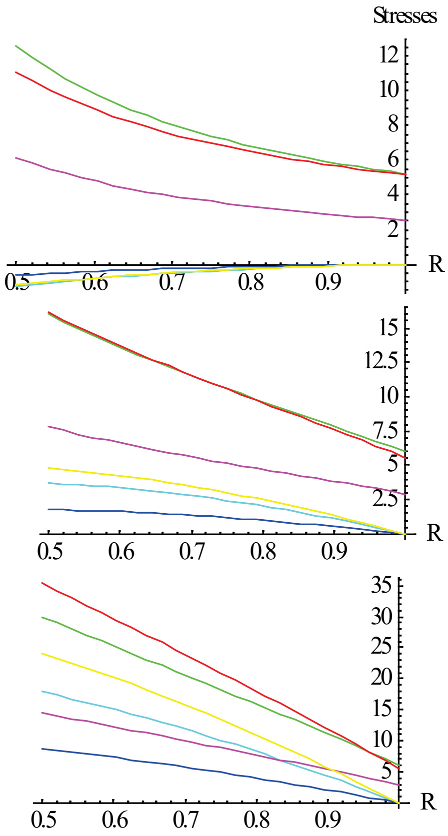

Figure 2. Creep stresses for steel, magnesium and beryl with $P = 5$, $N = 1, 3, 5$ and $k = 0.125$.

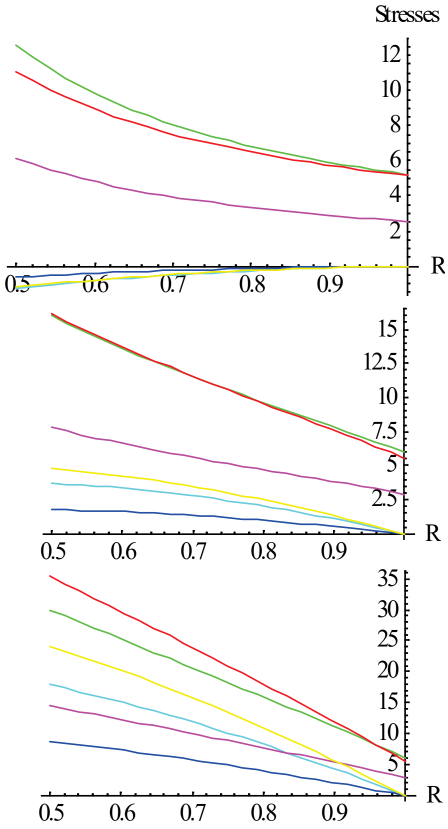

Figure 3. Creep stresses for steel, magnesium and beryl with $P = 5$, $N = 1, 3, 5$ and $k = 0.25$.

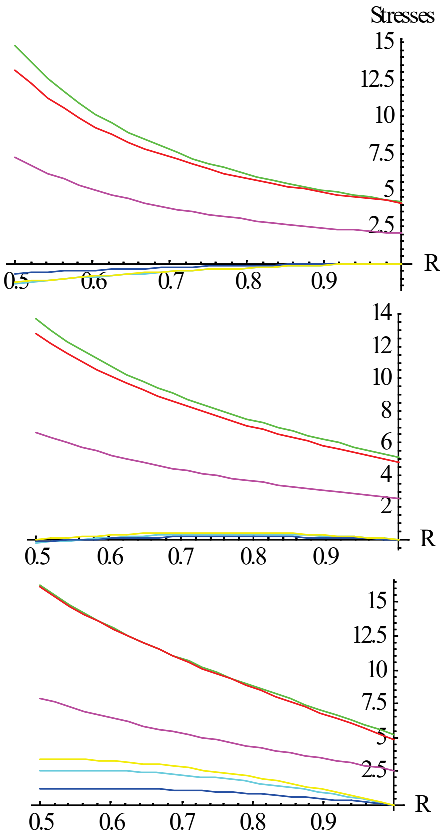

Figure 4. Creep stresses for steel, magnesium and beryl with $P = 5$, $N = 1, 3, 5$ and $k = 0.5$.

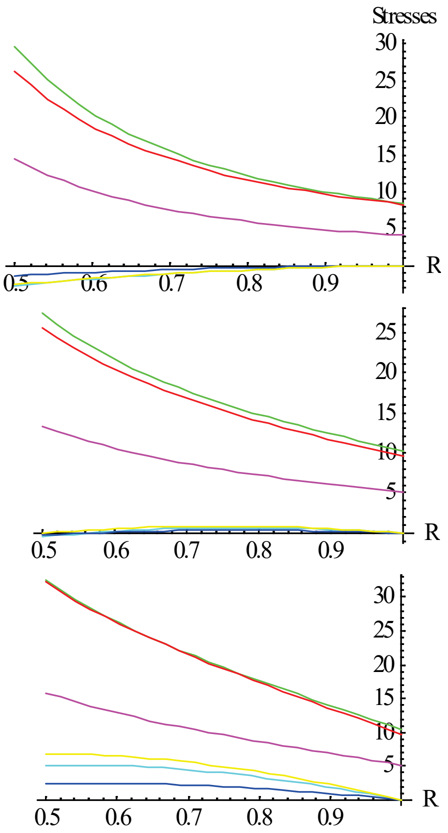

Figure 5. Creep stresses for steel, magnesium and beryl with $P = 10$, $N = 1, 3, 5$ and $k = 0$.

internal surface with introduction of the non-homogeneity factor. With increase in factor of non-homogeneity, these hoop creep stresses further increase for cylinder of isotropic and transversely isotropic material under the influence of internal pressure with linear measure, while they decrease for the cylinder of isotropic and transversely isotropic material under internal pressure with nonlinear measure, as can be identified from Figs. 3 and 4. Also, it is noticed that circumferential creep stresses are again lower for the functionally graded transversely isotropic cylinder (magnesium) as compared to the isotropic (steel) and transversely isotropic cylinder material (beryl) for nonlinear measure. With increase in internal pressure, hoop creep stresses increase at the internal surface for the isotropic and transversely isotropic cylinder, as can be seen in Fig. 5. It has been noticed from Fig. 6 that with the introduction of non-homogeneity, these circumferential creep stresses increase for linear measure while they decrease for nonlinear measure. Also, these stresses increase significantly with increase in internal pressure in cylinders of functionally graded- isotropic and transversely isotropic material. These circumferential stresses increase for linear measure while they decrease for nonlinear measure with an increase in non-homogeneity as can be seen in Figs. 7 and 8. Also, these stresses increase significantly with increase in pressure.

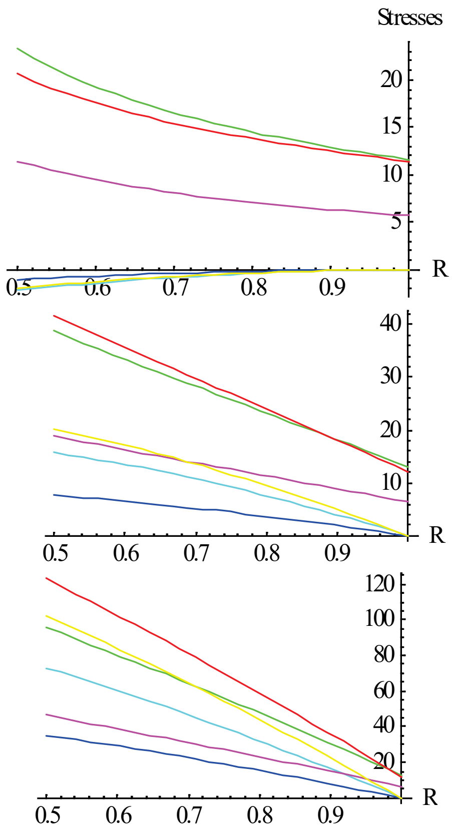

Figure 6. Creep stresses for steel, magnesium and beryl with $P = 10$, $N = 1, 3, 5$ and $k = 0.125$.

CONCLUSION

Based on the above discussion, it is concluded that the circular cylinder of highly functionally graded transversely isotropic material (magnesium) with nonlinear measure under

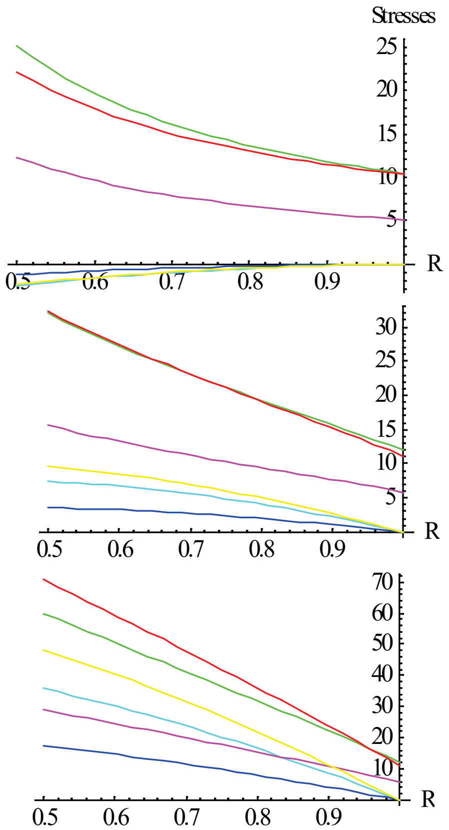

Figure 7. Creep stresses for steel, magnesium and beryl with $P = 10$, $N = 1, 3, 5$ and $k = 0.25$.

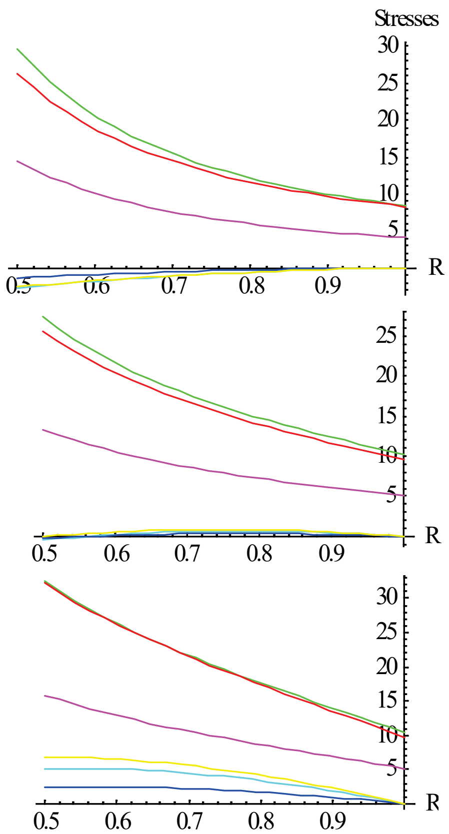

Figure 8. Creep stresses for steel, magnesium and beryl with $P = 10$, $N = 1, 3, 5$ and $k = 0.5$.

internal pressure is a better option for design than the cylinder of other functionally graded materials: beryl (transversely isotropic) and steel (isotropic material). The reason is that hoop stresses are lower in the case of magnesium as compared to steel and beryl. This conclusion helps in minimising the possibility of fracture in cylinders due to external factors such as pressure and temperature.

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