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CREEP STRESSES IN A CIRCULAR CYLINDER SUBJECTED TO TORSION

NAPONI USLED PUZANJA U KRUŽNOM CILINDRU OPTEREĆENIM NA TORZIJU

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- cylinder
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Abstract

Creep stresses have been derived for a circular cylinder subjected to torsion by using Seth transition theory. Seth's transition theory is applied to problems of thickness variation parameter in a thin rotating disc by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield criterion. It has been observed that the value of maximum shear stress for a cylinder subjected to torsion of compressible material is greater than as compared to incompressible material.

Ključne reči
- naponi puzanja
- cilindar
- torzija
- smičući naponi
- stišljivost

Izvod

Naponi izazvani puzanjem su proračunati za kružni cilindar opterećen torzijom primenom Setove teorije prelaznih napona. Setova teorija prelaznih napona se primenjuje kod problema parametra promenljive debljine tankozidog rotirajućeg diska primenom konačne deformacije. U ovom slučaju se ne pretpostavlja niti kriterijum tečenja, a ni odgovarajući zakon tečenja. Dobijeni rezultati su primenljivi na stišljive materijale. Ako se uvede dodatni uslov nestišljivosti, izrazi za napon tada odgovaraju onima koji slede iz kriterijuma tečenja Treska. Uočeno je da je maksimalni smičući napon cilindra opterećenog na uvijanje veći za stišljive materijale u odnosu na nestišljive.

INTRODUCTION

Torsion testing of thin-walled tubes furnishes a number of advantages over tensile testing in the determination of creep properties. The most significant of these is that the measurement of strain in torsion tests is essentially unaffected by thermal expansions and contractions which accompany minor temperature fluctuations. Also, if the wall thickness is small compared to the diameter, the shear stress across the section may be regarded as uniform and, for a constant torque, this stress remains constant throughout the duration of the test. In spite of these advantages, most laboratory creep testing is carried out in tension. On the other hand, most structural members are subjected to multiaxial stress conditions. It is necessary, therefore, to relate creep behaviour under multi-axial stress states to the uniaxial creep behaviour that is usually obtained in the laboratory. Bailey, /1/, investigated creep of steel under simple and compound stresses and the use of high initial temperature in a steam power plant. Hovgaard, /2/, discusses torsion of rectangular tubes. Everett and Clark, /3/, have studied torsion creep tests for comparison with tension creep tests on a carbon-molybdenum steel. Tapsell and Johnson, /4/, investigated creep under combined tension and torsion, the behaviour of a 0.17% carbon steel at 455°C. Gubser et al., /5/, discuss creep torsion of prismatic bars. Banshchikova et al., /7/, analysed torsion of circular rods at anisotropic creep. Tsuan, /6/, discusses the problem of elastic-plastic torsion of a square bar and Seth, /8/, investigated elastic-plastic transition in torsion.

Incompressibility of the material in plasticity and creep problems is one of the most important assumptions which simplifies the problem. In fact, it is not possible to find a solution in closed form without this assumption. Seth's transition theory does not acquire any assumptions like a yield condition, incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of differential equations defining the field, and has been successfully applied to a large number of problems, /8-17/. Seth, /10/, has defined the generalized strain measure for the uni-axial case as

$$e = \frac{1}{n}\left[1 - \left(\frac{l_0}{l}\right)^n\right] \tag{1}$$

where: n is the measure; l_0 and l are initial and strained length of the rod, respectively; n = 0, 1, 2, –1, –2 gives the Hencky, Swaigner, Almansi, Cauchy and creep measures, in respect.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

Consider a circular cylinder of radius a subjected to finite twist. The displacement components in cylindrical coordinates are given by Seth, /8, 9/:

$$u = r(1-\beta),\ v = \alpha yz,\ w = dz \tag{2}$$

where: u, v, w (displacement components); $\beta = \beta(r)$ is the position function, depending on r only; α is the angle of twist per unit length; and d is a constant. The generalized components of strain from Eq.(1) are given by Seth /8/:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right],\quad e_{\theta\theta} = \frac{1}{n}[1-\beta^n],$$
$$e_{zz} = \left(\frac{D^n}{n}\right)\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right],\quad e_{\theta z} = \frac{1}{n}\left[\alpha^{\frac{n}{2}}r^{\frac{n}{2}}\beta^n\right] \tag{3}$$
$$e_{r\theta} = e_{zr} = 0$$

where: r, θ, z are polar coordinates; and $D_n = 1-(1-d)^n$ is any constant.

Stress-strain relation: the stress-strain relations for isotropic material are given by Sokolnikoff, /18/:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij}\ \ (i,j = 1,2,3) \tag{4}$$

where: T_{ij} and e_{ij} are the stress and strain components; λ and μ are Lame's constants; $I_1 = e_{kk}$ is the first strain invariant; δ_{ij} is the Kronecker's delta. Substituting Eq.(3) in Eq.(4), the stresses are obtained as:

$$T_{rr} = \lambda A + \left(\frac{2\mu}{n}\right)\left[1-(r\beta'+\beta)^n\right],$$
$$T_{\theta\theta} = \lambda A + \left(\frac{2\mu}{n}\right)[1-\beta^n],$$
$$T_{zz} = \lambda A + \left(\frac{2\mu D^n}{n}\right)\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right], \tag{5}$$
$$T_{\theta z} = \left(\frac{2\mu}{n}\right)\left[\alpha^{\frac{n}{2}}r^{\frac{n}{2}}\beta^n\right],$$
$$T_{r\theta} = T_{rz} = 0$$

where: $A = \frac{1}{n}\left\{\left[1-(r\beta'+\beta)^n\right] + (1-\beta^n) + D^n\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right]\right\}$

and $\beta' = d\beta/dr$.

Equation of equilibrium: the equations of motion are all satisfied except:

$$\frac{d}{dr}(T_{rr}) - \frac{T_{rr}-T_{\theta\theta}}{r} = 0 \tag{6}$$

where: T_{rr} and $T_{\theta\theta}$ are the radial and circumferential stresses.

Critical points or turning points: using Eqs.(5) and (6), we get a nonlinear differential equation in β as:

$$\frac{d\beta}{dP} = -\frac{\beta P(P+1)^n}{(P+1)^n\left(P+\frac{C}{n}\right)+(1-C)\left[P+(P+1)^n(\alpha r)^n\right]-\frac{C}{n}} \tag{7}$$

where: $r\beta' = \beta P$ (P is a function of β and β is the function of r). Transition points of β in Eq.(7) are $P \to -1$ and $\pm\infty$.

Boundary conditions: the boundary condition are:

$$T_{rr} = 0\ \text{ at }\ r = a\ \text{ and }\ \int_0^a rT_{zz}dr = 0 \tag{8}$$

SOLUTION THROUGH PRINCIPAL STRESS DIFFERENCE

For finding the creep stresses, the transition function is taken through principal stress difference, /8-17/, at the transition point $P \to -1$. We define the transition function $\Im_f$ as:

$$\Im_f = \tau_{rr} - \tau_{\theta\theta} = \frac{2\mu}{n}\left[1-(r\beta'+\beta)^n - (1-\beta^n)\right] \tag{9}$$

where: $\Im_f$ is a function of r only; and n is dimension.

Taking the logarithmic differentiation of Eq.(9) with respect to r and substituting Eq.(7) and taking asymptotic value $P \to -1$, we get:

$$\frac{d}{dr}(\ln\Im_f) = \frac{C}{\beta} + \frac{n\beta^{n-1}(2-C)}{\beta^n} \tag{10}$$

The asymptotic value of β as $P \to -1$ is D/r; D being a constant. Integrating Eq.(10) with respect to r, we get

$$\Im_f = \tau_{rr} - \tau_{\theta\theta} = A_1 r^{-2C+(n-1)C} \tag{11}$$

where: A_1 is a constant of integration, which can be determined by boundary condition; and C is a compressibility factor. From Eqs.(11) and (6), we have

$$T_{rr} = A_1\frac{r^{-2n+C(n-1)}}{2n-(n-1)C} + A_2 \tag{12}$$

where: A_2 is a constant of integration which can be determined by boundary condition. The asymptotic values of other stresses can be obtained by using Eq.(5), we have

$$T_{\theta\theta} = A_1 r^{-2n+C(n-1)}\left[\frac{1}{2n-C(n-1)}-1\right] + A_2 \tag{13}$$

$$T_{zz} = \left(\frac{1-C}{2-C}\right)A_1 r^{-2n+C(n-1)}\left[\frac{2}{2n-C(n-1)}-1\right] + $$
$$+2A_2 + 2\mu\left(\frac{3-2C}{2-C}\right)\frac{D^n}{n}\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right] \tag{14}$$

Using boundary condition Eq.(8) in Eq.(12), we get

$$A_1 = -A_2\left[2n-(n-1)C\right]a^{2n-(n-1)C}\ \text{ and}$$
$$A_2 = \frac{\mu\left[(n-2)-(n-1)C\right]}{n(\alpha a)^n\left[2n-(n-1)C\right]}. \tag{15}$$

Substituting Eq.(15) in Eqs.(12-14), we get

$$T_{rr} = \frac{\mu\left[(n-2)-(n-1)C\right]}{n(\alpha a)^n\left[2n-(n-1)C\right]}\left[\left(\frac{a}{r}\right)^{2n-(n-1)C}-1\right] \tag{16}$$

$$T_{\theta\theta} = \frac{\mu\left[(n-2)-(n-1)C\right]}{n(\alpha a)^n\left[2n-(n-1)C\right]}\left\{1-\left[1-2n+C(n-1)\right]\left(\frac{a}{r}\right)^{2n-(n-1)C}\right\} \tag{17}$$

$$T_{zz} = \frac{\mu\left[(n-2)-(n-1)C\right]}{n(\alpha a)^n\left[2n-(n-1)C\right]}\left[2-(1-C)(1-n)\left(\frac{a}{r}\right)^{2n-(n-1)C}\right]+ +2\mu\left(\frac{3-2C}{2-C}\right)\frac{D^n}{n}\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right] \quad (18)$$

Equations (16-18) give transitional creep stresses in the cylinder. The asymptotic value of β from Eqs.(9) and (11) is given by:

$$\beta^n = \left(\frac{2\mu}{nA_1}\right)^{\frac{1}{1-C}} r^{\frac{C}{1-C}} \quad (19)$$

Shearing stress: the shearing stress is given by:

$$T_{\theta z} = \frac{2\mu}{n}\left(\frac{2\mu}{nA_1}\right)^{\frac{1}{1-C}} r^{\frac{C}{1-C}} \alpha^{\frac{n}{2}} r^{\frac{n}{2}+\frac{C}{1-C}} \quad (20)$$

The twisting couple M is given by:

$$M = 2\pi\int_0^a r^2 T_{\theta z}\,dr = 4\pi\mu\left(\frac{2\mu}{nA_1}\right)^{\frac{1}{1-C}} \frac{\alpha^{\frac{n}{2}} a^{3+\frac{n}{2}+\frac{C}{1-C}}}{n\left(3+\frac{n}{2}+\frac{C}{1-C}\right)} \quad (21)$$

Eliminating μ between Eqs.(20) and (21), we get

$$T_{\theta z} = \frac{1}{4}\left(\frac{2M}{\pi a^3}\right)\left[3+\frac{n}{2}+\frac{C}{1-C}\right]\left(\frac{r}{a}\right)^{\frac{n}{2}+\frac{C}{1-C}} \quad (22)$$

The maximum shearing stress occurs at $r = a$, and denoting its value by τ, we have

$$\tau = \frac{1}{4}\left[3+\frac{n}{2}+\frac{C}{1-C}\right]\tau_e \quad (23)$$

where: τ_e is the elastic maximum shear stress equal to $2M/\pi a^3$. The incompressible case in creep deformation is characterized by $C \to 0$, Eq.(24) becomes:

$$\tau = \frac{1}{4}\left[3+\frac{n}{2}\right]\tau_e\,. \quad (24)$$

For the Hencky measure ($n \to 0$ and $C \to 0$), we get from Eq.(11) the Tresca yield condition

$$\tau_{rr} - \tau_{\theta\theta} = A_1\,. \quad (25)$$

NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating stresses and strain rates based on the above analysis, the following values have been taken: n = 0.2, 2, 5; and C = 0 (incompressible material), C = 0.5 and 0.75 (compressible materials). Curves are produced (Fig. 1) between shear stress ratio $\tau_{\theta z}/\tau_e$ and the relative distance from the centre for different combinations of C and measure n for incompressible and compressible materials. It has been observed form Fig. 1 that for n = 2 and C = 0, an elastic shear stress curve is obtained, and for n = 2; C = 0.5, C = 0.75, it gives the compressible effect for elastic shear stress distribution. The effect of compressibility is significant at the outer surface boundary. The value of the maximum shear stress for a cylinder of compressible material is greater than that for an incompressible material. It is maximum for a highly compressible material and increases with the measure n.

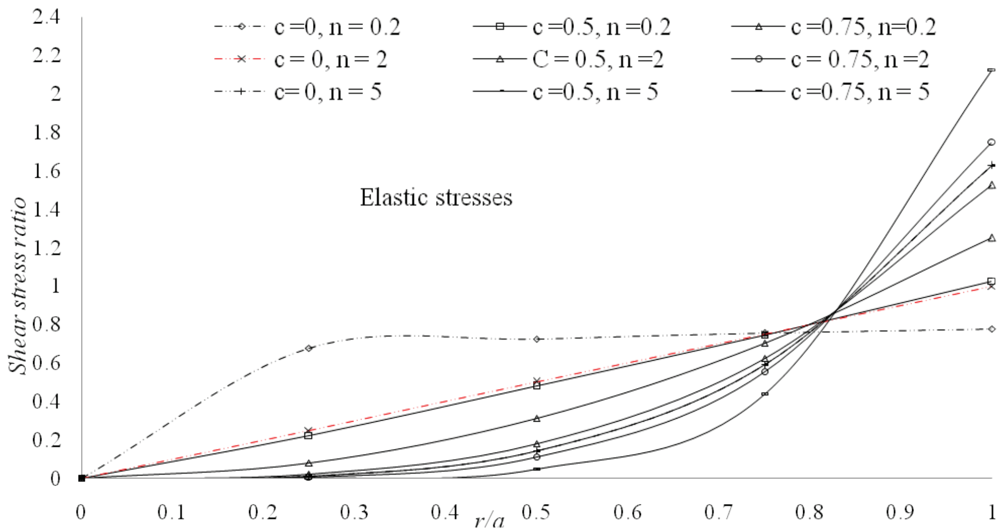

Figure 1. Creep stress in a circular cylinder subjected to torsion.

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