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THERMAL INSTABILITY OF MAXWELL VISCO-ELASTIC NANOFLUID IN A POROUS MEDIUM WITH THERMAL CONDUCTIVITY AND VISCOSITY VARIATION

TERMIČKA NESTABILNOST MAKSVEL-VISKOELASTIČNOG NANOFLUIDA U POROZNOJ SREDINI SA TOPLOTNOM PROVODLJIVOŠĆU I PROMENLJIVE VISKOZNOSTI

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Ključne reči
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- Darsi model
- Luisov broj
- promena viskoznosti
- Galerkinova metoda težinskih ostataka
- porozna sredina

Abstract

Thermal instability of Maxwell visco-elastic nanofluid in a porous medium with thermal conductivity and viscosity variation for more realistic boundary conditions is investigated theoretically. A Darcy model is considered for the porous medium. The model used for nanofluid incorporates the effect of Brownian diffusion and thermophoresis. The eigen value problem is solved by employing the Galerkin weighted residuals method. The influence of the Lewis number, nanoparticle Rayleigh number, modified diffusivity ratio, the viscosity variation parameter, the thermal conductivity variation parameter and porosity parameter on the stationary convection is studied and it is found that the Lewis number, modified diffusivity ratio and nanoparticles Rayleigh number destabilizes while the viscosity variation parameter, the thermal conductivity variation parameter and porosity parameter stabilize the stationary convection.

Izvod

Teorijski je obrađena je termička nestabilnost Maksvel-viskoelastičnog nanofluida u poroznoj sredini sa toplotnom provodljivošću i promenljive viskoznosti sa realističnijim graničnim uslovima. Za poroznu sredinu je razmotren Darsijev model. Model nanofluida sadrži efekat Braunove difuzije i termoforeze. Problem sopstvene vrednosti je rešen uvođenjem Galerkinove metoda težinskih ostataka. Proučen je uticaj Luisovog broja, Rejlejevog broja nanočestica, modifikovanog odnosa difuzivnosti, parametra promene viskoznosti, parametra promene toplotne provodljivosti i parametra poroznosti na stacionarnu konvekciju. Otkriveno je da Luisov broj, modifikovani odnos difuzivnosti i Rejlejev broj nanočestica destabilizuju, a parametar promene viskoznosti, parametar promene toplotne provodljivosti i parametar poroznosti stabilizuju stacionarnu konvekciju.

INTRODUCTION

The onset of thermal instability in a horizontal layer of fluid heated from below is regarded as a classical problem due to its wide range of applications and a detailed account of the thermal instability under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar /1/. The convection problem in a porous medium was studied by Lapwood /2/, Wooding /3/, Nield /4/, Ingham and Pop /5/, Vafai and Hadim /6/, Nield and Bejan /7/.

A nanofluid is a colloidal mixture of nano-sized particles in base fluid and the term ‘nanofluid’ was first coined by Choi /8/. These fluids have unique properties that make them useful in heat transfer applications. For the last decade much research has been evinced on the study of nanofluids. The developments in the study of heat transfer using nanofluids have been reported by Wong and Leon /9/, Yu and Xie /10/, Taylor et al. /11/. Buongiorno /12/ proposed a mathematical model based on the effects of Brownian motion and thermophoresis of suspended nanoparticles, after analysing the effect of seven slips mechanism, he concluded that in the absence of turbulent eddies, Brownian diffusion and thermophoresis are the dominant slip mechanisms. The onset of convection in a horizontal layer heated from below for a nanofluid was studied by Tzou /13, 14/, Alloui et al. /15/, Kuznetsov and Nield /16-18/, Nield and Kuznetsov /19-21/, and Chand et al. /22-23/, Chand and Rana /24-29/, Chand /30-32/, Yadav /33/ and Yadav et al. /34-37/. These authors found that there is an optimum nanoparticle volume fraction, which depends on both the type of nanoparticle and the Rayleigh number, at which the heat transfer through the system is maximum. The above literature deals with the study of nanofluids as Newtonian fluids. The onset of convection in a horizontal layer of nanofluid as Newtonian fluids uniformly heated from below (Bénard convection) has been extensively investigated but a little attention has been made to study the thermal convection of non-Newtonian fluids. With the growing importance of non-Newtonian fluids in technology and industries, the investigations of such fluids are desirable. In the category of non-Newtonian fluids visco elastic fluids have distinct features and are well represented by the Oldroydian constitutive model. The Oldroydian constitutive model is adopted widely to examine the influence of elasticity on thermal convective instability. Thermal convection in a layer of visco-elastic fluid saturated by Brinkman-Darcy porous medium is investigated /38/ and found that Brinkman stabilize the fluid layer. Thermal instability problems in visco-elastic nanofluid were investigated theoretically /39-42/ by taking different non-Newtonian fluids as base fluid.

Recently Nield and Kuznetsov /43/ and thereafter authors /44-47/ studied the thermal instability of nanofluid by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. Zero-flux for nanoparticles means one could control the value of the nanoparticles fraction at the boundary in the same way as the temperature there could be controlled. Under the circumstances, it is desirable to investigate convective instability problems by utilizing these boundary conditions to get meaningful insight into the problems. Due to importance of Maxwell visco-elastic nanofluids in porous medium, an attempt has been made to study the thermal instability of a horizontal layer of Maxwell visco-elastic nanofluids in the presence of thermal conductivity and viscosity variation for more realistic boundary conditions in a porous medium.

MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of Maxwell visco-elastic nanofluid of thickness ‘d’ bounded by horizontal boundaries $z = 0$ and $z = d$ in porous medium of porosity ε and medium permeability k_1. Fluid layer is acted upon by a gravity force $\mathbf{g}(0,0,-g)$ and is heated from below in such a way that horizontal boundaries $z = 0$ and $z = d$ respectively maintained at a uniform temperature T_0 and T_1 ($T_0 > T_1$) as shown in Fig. 1. The normal component of the nanoparticles flux has to vanish at impermeable boundaries and the reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively.

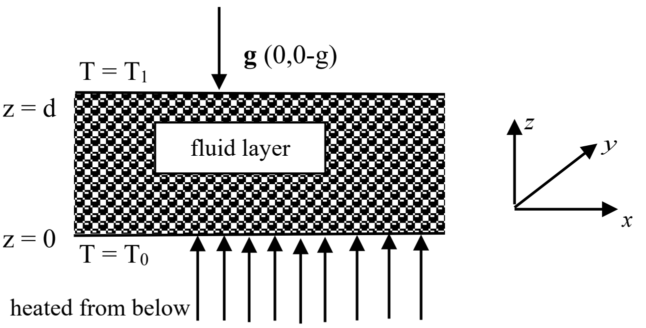

Figure 1. Physical configuration of the problem.

The equation of continuity and motion for Maxwell visco-elastic nanofluid in porous medium under the Boussinesq approximation (Chand /32/, Yadav et al. /37/) are

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$0 = \left(1 + \lambda \frac{\partial}{\partial t}\right) \times \left\{ -\nabla p + \left[\varphi\rho_p + (1-\varphi)\left\{\rho_f\left[1-\alpha(T-T_0)\right]\right\}\right]\mathbf{g}\right\} - \frac{\mu_{eff}}{k_1}\mathbf{q} \tag{2}$$

where $\mathbf{q(u,v,w)}$– Darcy velocity vector; p– hydrostatic pressure; $\mathbf{g}$– gravity; μ_{eff}– overall viscosity of porous medium saturated by nanofluid; α– coefficient of thermal expansion; ε– porosity; k_1– medium permeability; λ– relaxation time; T– temperature of nanofluid; φ– volume fraction of nanoparticles; ρ_p– density of nanoparticles; ρ_f– density of base fluid; and $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)$ stands for convection derivative.

The equation of energy for Maxwell visco-elastic nanofluid in porous medium is

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{q}\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p \left(D_B \nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T \right) \tag{3}$$

where: $(\rho c)_m$– effective heat capacity of fluid; $(\rho c)_p$– heat capacity of nanoparticles; and k_m– overall thermal conductivity of porous medium saturated by nanofluid. The overall thermal conductivity k_m is (Yadav et al. /37/) written as

$$k_m = \varepsilon k_{eff} + (1-\varepsilon)k_s \tag{4}$$

where: ε– porosity; k_{eff}– effective conductivity of nanofluid; and k_s– conductivity of solid material forming the matrix of the porous medium.

In the case of where volumetric fraction of nanoparticles φ is small as compared to unity, the viscosity and conductivity via linear relation are written as, /37/,

$$\mu_{eff} = \mu_f (1 + 2.5\phi) \tag{5}$$

$$k_{eff} = k_f (1 + \gamma\phi) \tag{6}$$

here: γ is a measure for the dependence of thermal conductivity on the concentration of nanoparticles; μ_f– viscosity and k_f– thermal conductivity of the nanofluid.

From Eqs.(4) and (6), the overall thermal conductivity, k_m, is given by

$$k_m = \varepsilon k_f (1 + \gamma\phi) + (1-\varepsilon)k_s \tag{7}$$

The continuity equation for the nanoparticles is

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi = D_B \nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T \tag{8}$$

where: D_B– Brownian diffusion coefficient, given by Einstein-Stokes equation; and D_T– thermophoretic diffusion coefficient of the nanoparticles.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus, boundary conditions /1, 43/ are

$$w = 0,\ T = T_0,\ D_B \frac{\partial\phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \ \text{ at } \ z = 0 \quad \text{and}$$
$$w = 0,\ T = T_1,\ D_B \frac{\partial\phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \ \text{ at } \ z = d \tag{9}$$

Introducing non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right),\ v'(u',v',w')=v\left(\frac{u,v,w}{\kappa}\right)d,$$

$$t'=\frac{t\kappa}{\sigma d^2},\ p'=\frac{pk_1}{\mu_c\kappa},\ T'=\frac{T-T_1}{T_0-T_1},\ \varphi'=\frac{\varphi-\varphi_0}{\varphi_0},$$

where: $\kappa=\frac{k_c}{(\rho c_p)_f}$; $\sigma=\frac{(\rho c_p)_m}{(\rho c_p)_f}$; $\mu_c=\mu_f(1+2.5\phi_0)$; and $k_m=\varepsilon k_f(1+\gamma\phi)+(1-\varepsilon)k_s$.

Equations (1)-(3) and (8) in non-dimensional form can be written as

$$\nabla'\cdot\mathbf{q}'=0 \tag{10}$$

$$\left(1+F\frac{\partial}{\partial t'}\right)\left(-\nabla'p'-Rm\hat{e}_z+RaT'\hat{e}_z-Rn\varphi'\hat{e}_z\right)-\\-\left(1+Q_1\phi'\right)\mathbf{q}'=0 \tag{11}$$

$$\frac{\partial T'}{\partial t'}+\mathbf{q}'\cdot\nabla'T'=\left(1+Q_2\phi'\right)\nabla'^2T'+\frac{N_B}{Le}\nabla'\varphi'\cdot\nabla'T'+\\+\frac{N_AN_B}{Le}\nabla'T'\cdot\nabla'T' \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial\varphi'}{\partial t'}+\frac{1}{\varepsilon}\mathbf{q}'\cdot\nabla'\varphi'=\frac{1}{Le}\nabla'^2\varphi'+\frac{N_A}{Le}\nabla'^2T' \tag{13}$$

Here the non-dimensional parameters are given as: $Le=\frac{\kappa}{D_B}$ is the Lewis number; $F=\frac{\lambda\kappa}{d^2}$ is the stress relaxation parameter; $Ra=\frac{\rho_0\alpha dk_1g(T_0-T_1)}{\mu_c\kappa}$ is the Rayleigh Darcy number; $Rm=\frac{\left[\rho_p\varphi_0+\rho(1-\varphi_0)\right]gdk_1}{\mu_c\kappa}$ is the density Rayleigh Darcy number; $Rn=\frac{(\rho_p-\rho)\varphi_0gdk_1}{\mu_c\kappa}$ is the nanoparticles Rayleigh Darcy number; $N_A=\frac{D_T(T_0-T_1)}{D_BT_1\varphi_0}$ is the modified diffusivity ratio; $N_B=\frac{(\rho c)_p\varphi_0}{(\rho c)_f}$ is the modified particle-density increment; $Q_1=\frac{2.5\varphi_0}{1+2.5\varphi_0}$ is the viscosity variation parameter; $Q_2=\frac{\gamma k_f\varepsilon}{(1+\gamma\varphi_0)k_f\varepsilon+(1-\varepsilon)k_s}\varphi_0$ is the thermal conductivity variation parameter; $\mathbf{e}_z$ is the unit vector along z-axis.

In the spirit of Oberbeck-Boussinesq approximation, Eq.(11) has been linearized by neglecting a term proportional to the product of φ_0 and T. This approximation is valid in the case of small temperature gradients in a dilute suspension of nanoparticles.

The dimensionless boundary conditions are

$$w'=0,\ T'=1,\ \frac{\partial\phi'}{\partial z'}+N_A\frac{\partial T'}{\partial z'}=0\ \text{at}\ z'=0\ \text{and}\\ w'=0,\ T'=0,\ \frac{\partial\phi'}{\partial z'}+N_A\frac{\partial T'}{\partial z'}=0\ \text{at}\ z'=1 \tag{14}$$

The basic state is assumed to be quiescent and is given by

$$\mathbf{q}'(u',v',w')=0,\ p'=p_b(z),\ T'=T_b(z),\ \varphi'=\varphi_b(z). \tag{15}$$

Equations (10)-(13) reduce to

$$0=-\frac{dp_b}{dz'}-Rm+RaT_b-Rn\varphi_b \tag{16}$$

$$\left(1+Q_2\phi_b\right)\frac{d^2T_b}{dz'^2}+\frac{N_B}{Le}\frac{d\varphi_b}{dz'}\frac{dT_b}{dz'}+\frac{N_AN_B}{Le}\left(\frac{dT_b}{dz'}\right)^2=0 \tag{17}$$

$$\frac{d^2\varphi_b}{dz'^2}+N_A\frac{d^2T_b}{dz'^2}=0. \tag{18}$$

Using boundary conditions in Eq.(14), Eq.(18) gives

$$\varphi_b=-N_AT_b+(1-N_A)z'+N_A \tag{19}$$

By substituting the value of φ_b from Eq.(19) in Eq.(17), we get

$$\left(1+Q_2\phi_b\right)\frac{d^2T_b}{dz'^2}+\frac{(1-N_A)N_B}{Le}\frac{dT_b}{dz'}+\frac{N_AN_B}{Le}\left(\frac{dT_b}{dz'}\right)^2=0 \tag{20}$$

By integrating Eq.(20) with respect to z and using boundary conditions Eq.(14), we get an approximate solution as $T_b = 1 - z'$, and $\varphi_b = \varphi_0 + N_Az'$.

PERTURBATION EQUATIONS

Let the initial basic state, described by Eq.(15), be slightly perturbed so that the perturbed state is given by

$$q'(u',v',w')=0+q''(u'',v'',w''),\ T'=T_b+T'',\\ \varphi'=\varphi_b+\varphi'',\ p'=p_b+p'' \tag{21}$$

where: $T_b = 1 - z'$, $\varphi_b = \varphi_0 + N_Az'$, and (u'',v'',w''), T'', φ'', and p'', respectively the perturbations in initial velocity, temperature, volume fraction of the nanoparticles and pressure.

By substituting Eq.(21) in Eqs.(10)-(13) and linearizing by neglecting the product of the prime quantities, we obtain the following equations

$$\nabla\cdot\mathbf{q}=0 \tag{22}$$

$$\left(1+F\frac{\partial}{\partial t}\right)\left(-\nabla p+RaT\hat{e}_z-Rn\varphi\hat{e}_z\right)-\left[1+Q_1(\phi_0+N_Az)\right]\mathbf{q}=0 \tag{23}$$

$$\frac{\partial T}{\partial t}-w=\left[1+Q_2(\phi_0+N_Az)\right]\nabla^2T-\frac{N_B}{Le}\left(N_A\frac{\partial T}{\partial z}+\frac{\partial\phi}{\partial z}\right)-\\-\frac{2N_AN_B}{Le}\frac{\partial T}{\partial z} \tag{24}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}wN_A=\frac{1}{Le}\nabla^2\varphi+\frac{N_A}{Le}\nabla^2T \tag{25}$$

Boundary conditions for the infinitesimal perturbations are given by

$$w=0,\ T=0,\ \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0\ \text{at}\ z=0,1 \tag{26}$$

[Dashes (″) have been suppressed for convenience.]

Applying the 'curl' operator twice to Eq.(23) under the assumption of linear theory, the resulting equations are given by

$$\left(1+F\frac{\partial}{\partial t}\right)\left(Ra\nabla_H^2 T - Rn\nabla_H^2\varphi\right)-\left[1+Q_1(\phi_0+N_A z)\right]\nabla^2 w=0 \quad (27)$$

where ∇_H^2 is the two-dimensional Laplacian operator.

NORMAL MODES ANALYSIS

Analysing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$\left[w,T,\varphi\right]=\left[W(z),\Theta(z),\Phi(z)\right]\exp(ik_x x+ik_y y+nt) \quad (28)$$

where: k_x, k_y are wave numbers in x and y direction, and n is growth rate of disturbances.

Using Eq.(28), Eqs.(27), (24) and (25) become

$$(1+Fn)(a^2Ra\Theta-a^2Rn\Phi)+\left[1+Q_1(\phi_0+N_A z)\right]\times \times(D^2-a^2)W=0 \quad (29)$$

$$\frac{1}{\varepsilon}N_A W-\frac{N_A}{Le}(D^2-a^2)\Theta-\left[\frac{1}{Le}(D^2-a^2)-\frac{n}{\sigma}\right]\Phi=0 \quad (30)$$

$$W+\left\{\left[1+Q_2(\phi_0+N_A z)\right](D^2-a^2)-n-\frac{N_A N_B}{Le}D\right\}\Theta+ +\frac{N_B}{Le}D\Phi=0 \quad (31)$$

where: $D\equiv\frac{d}{dz}$; and $a=\sqrt{k_x^2+k_y^2}$ is the dimensionless resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W=0,\ \Theta=0,\ D\Phi+N_A D\Theta=0 \ \text{at}\ z=0,1 \quad (32)$$

METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of Eqs.(29)-(31) with boundary conditions Eq.(32). In this method, the test functions are the same as the base (trial) functions. Accordingly, W, Θ and Φ are taken as

$$W=\sum_{p=1}^{N}A_pW_p,\ \Theta=\sum_{p=1}^{N}B_p\Theta_p,\ \Phi=\sum_{p=1}^{N}C_p\Phi_p \quad (33)$$

where: $W_p=\Theta_p=\sin p\pi z$; $\Phi_p=-N_A\sin p\pi z$; A_p, B_p and C_p are unknown coefficients; p = 1, 2, 3,…, N and the base functions W_p, Θ_p, and Φ_p satisfying the boundary conditions Eq.(32). Using expression for W, Θ and Φ in Eqs.(29)-(31) and multiplying the first equation by W_p, the second equation by Θ_p, and third equation by Φ_p and then integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations with 3N unknown A_p, B_p and C_p; p = 1, 2, 3,…, N. For an existing nontrivial solution, the vanishing of the determinant of coefficients produces the characteristic equation of the system in term of Rayleigh number Ra.

LINEAR STABILITY ANALYSIS

For the present formulation we have considered the system of Eqs.(29)-(31) together with the boundary conditions Eq.(32) constitute a linear eigen value problem for the system. Substituting Eq.(33) into the system of Eqs.(29)-(31) and multiplying the first equation by W_p, the second equation by Θ_p, and third equation by Φ_p and then integrating in the limits from zero to unity and performing some integration by parts, one obtains the following matrix equation

$$\begin{bmatrix} \dfrac{(\pi^2+a^2)\left[1+Q_1\left(\phi_0+\dfrac{N_A}{2}\right)\right]}{1+nF} & -a^2Ra & -a^2N_ARn \\ 1 & -\left\{(\pi^2+a^2)\left[1+Q_2\left(\phi_0+\dfrac{N_A}{2}\right)\right]+n\right\} & 0 \\ \dfrac{N_A}{\varepsilon} & \dfrac{N_A}{Le}(\pi^2+a^2) & -N_A\left(\dfrac{\pi^2+a^2}{Le}+\dfrac{n}{\sigma}\right) \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (34)$$

The non-trivial solution of the above matrix requires that

$$Ra=\frac{1}{a^2(1+nF)}\left\{(\pi^2+a^2)^2\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]+n(\pi^2+a^2)\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]\right\}- -\frac{\dfrac{\pi^2+a^2}{Le}+\dfrac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\dfrac{N_A}{2}\right)\right]+\dfrac{n}{\varepsilon}}{\dfrac{\pi^2+a^2}{Le}+\dfrac{n}{\sigma}}N_ARn \quad (35)$$

For neutral stability $n=i\omega$, (where ω is real and is a dimensionless frequency). Thus from Eq.(35), we have

$$Ra=\Delta_1+i\omega\Delta_2 \quad (36)$$

where

$$\Delta_1=\frac{\pi^2+a^2}{a^2}\frac{\left(\pi^2+a^2\right)\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]+\omega^2F\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]}{1+\omega^2F^2}-\\-\frac{\frac{\pi^2+a^2}{Le}\left\{\frac{\pi^2+a^2}{Le}+\frac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\right\}+\frac{\omega^2}{\sigma\omega}}{\frac{(\pi^2+a^2)^2}{Le^2}+\frac{\omega^2}{\sigma^2}}N_ARn \tag{37}$$

and

$$\Delta_2=\frac{\pi^2+a^2}{a^2}\frac{\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]\left\{1-F(\pi^2+a^2)\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\right\}}{1+\omega^2F^2}-\frac{\frac{\pi^2+a^2}{Le\varepsilon}-\frac{\frac{\pi^2+a^2}{Le}+\frac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]}{\sigma}}{\frac{(\pi^2+a^2)^2}{Le^2}+\frac{\omega^2}{\sigma^2}}N_ARn \tag{38}$$

Since Ra is a physical quantity, so it must be real. Hence, it follows from the Eq.(36) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset).

For the case of stationary convection $\omega = 0$, Eq.(36) reduces to

$$(Ra)_s=\frac{(\pi^2+a^2)^2\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]}{a^2}-\\-\left\{1+\frac{Le}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\right\}N_ARn \tag{39}$$

We find that for stationary convection, the stress relaxation time parameter, F, vanishes with n and the Maxwell visco-elastic nanofluid behaves like an ordinary Newtonian nanofluid.

If $Q_1 = Q_2 = 0$ then the critical value of the wave number is attained at $a_c = \pi$ and the corresponding critical Rayleigh number is given by

$$(Ra)_c=4\pi^2-\left(1+\frac{Le}{\varepsilon}\right)N_ARn \tag{40}$$

In the absence of nanoparticles ($Rn = Le = N_A = 0$) i.e. for ordinary Newtonian fluid, one recovers the well-known results that the critical Rayleigh-Darcy number is equal to $(Ra)_c = 4\pi^2$. This is good agreement of the result obtained by Nield and Kuznetsov, /21/.

For oscillatory convection $\omega \neq 0$, we must have $\Delta_2 = 0$ in Eq.(37), which gives an expression for frequency of oscillations as

$$\omega^2=\frac{\frac{(\pi^2+a^2)^3\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]}{Le^2}\left\{1-\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]F(\pi^2+a^2)\right\}-a^2\left\{\frac{\pi^2+a^2}{Le\varepsilon}-\frac{\frac{\pi^2+a^2}{Le}+\frac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]}{\sigma}\right\}N_ARn}{a^2F^2\left\{\frac{\pi^2+a^2}{Le}-\frac{\frac{\pi^2+a^2}{Le}+\frac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]}{\sigma}\right\}N_ARn-(\pi^2+a^2)\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]\frac{1-(\pi^2+a^2)F\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]}{\sigma^2}} \tag{41}$$

Then from Eqs.(37)-(38), with $\Delta_2 = 0$, the oscillatory Rayleigh number can be given as

$$Ra_{OSC}=\frac{\pi^2+a^2}{a^2}\frac{(\pi^2+a^2)\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]+\omega^2F\left[1+Q_1\left(\phi_0+\frac{N_A}{2}\right)\right]}{1+\omega^2F^2}-\\-\frac{\frac{\pi^2+a^2}{Le}\left\{\frac{\pi^2+a^2}{Le}+\frac{\pi^2+a^2}{\varepsilon}\left[1+Q_2\left(\phi_0+\frac{N_A}{2}\right)\right]\right\}+\frac{\omega^2}{\sigma\omega}}{\frac{(\pi^2+a^2)^2}{Le^2}+\frac{\omega^2}{\sigma^2}}N_ARn \tag{42}$$

The values of ω^2 are obtained from Eq.(41). If there is no positive value of ω^2 then oscillatory instability is not possible.

RESULTS AND DISCUSSION

Thermal instability in a Maxwell visco-elastic nanofluid in the presence of viscosity variation and thermal conductivity in a porous medium is investigated.

The expressions for both the stationary and oscillatory Rayleigh number, which characterize the stability of the system, are obtained. The stationary critical Rayleigh number is found to be independent of the visco-elastic parameter F, thus the Maxwell visco-elastic nanofluid behaves like an ordinary Newtonian nanofluid. The computations are carried out for different values of the Lewis number, Le, modified diffusivity ratio, N_A, nanoparticles Rayleigh number, Rn, viscosity variation parameter, Q_1, thermal conductivity variation parameter Q_2 and porosity parameter ε. The parameters considered are in the $10^2 \leq Ra \leq 10^5$ range (thermal Rayleigh number), $1 \leq N_A \leq 10$ (modified diffusivity ratio), $10^2 \leq Le \leq 10^4$ (Lewis number), $10^{-1} \leq Rn \leq 10$ (nanoparticles Rayleigh number), $0.1 \leq \varepsilon \leq 1$ (porosity parameter) (Chand and Rana /45/), and $0.01 \leq Q_1 \leq 0.1$ (viscosity variation parameter), $0.01 \leq Q_2 \leq 0.1$ (thermal conductivity variation parameter), Yadav et al. /37/.

Stability curves in (Ra, a) plane for Lewis number, Le, modified diffusivity ratio, N_A, nanoparticles Rayleigh number, Rn, viscosity variation parameter, Q_1, thermal conductivity variation parameter, Q_2, and porosity parameter are shown in Figs. 2-6.

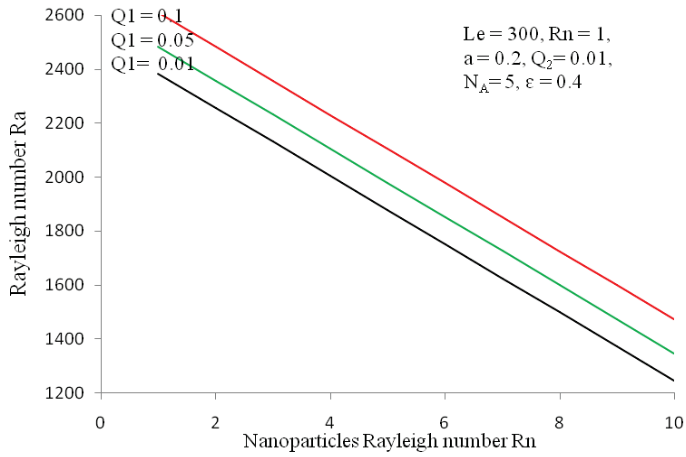

Figure 2. Variation of Rayleigh number, Ra, with nanoparticle Rayleigh number for different values of viscosity variation parameter Q_1.

Figure 2 shows the variation of Rayleigh number with the nanoparticle Rayleigh number for different values of the viscosity variation parameter, Q_1, and fixed values of other parameters. It is observed that the Rayleigh number decreases with an increase in the value of nanoparticle Rayleigh number. Also, Rayleigh number increases with an increase in the value of viscosity variation parameter, Q_1. This shows that nanoparticle Rayleigh number, Rn, advances while viscosity variation parameter, Q_1, delays the onset of convection.

Figure 3 shows the variation of Rayleigh number with thermal conductivity variation parameter, Q_2, for different values of the Lewis number, Le, and fixed values of other parameters. It is observed that the Rayleigh number decreases with an increase in the value of the Lewis number. Also, Rayleigh number increases with an increase in the value of thermal conductivity variation parameter, Q_2. Thus, Lewis number, Le, advances while thermal conductivity variation parameter, Q_2, delays the onset of convection.

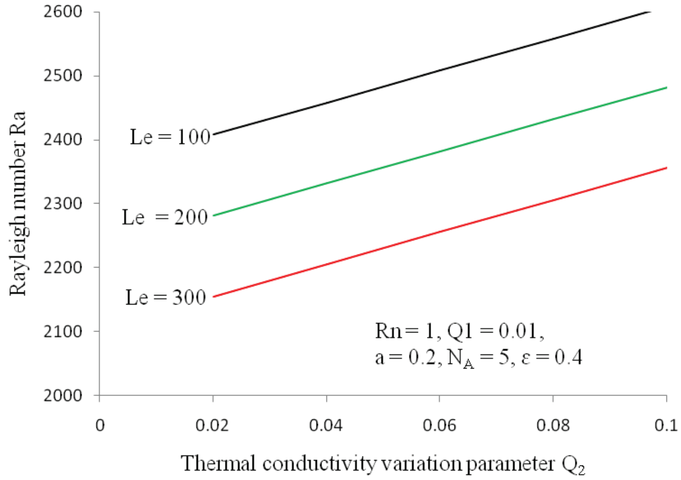

Figure 3. Variation of Rayleigh number, Ra, with thermal conductivity variation parameter, Q_2, for different values of Lewis number, Le.

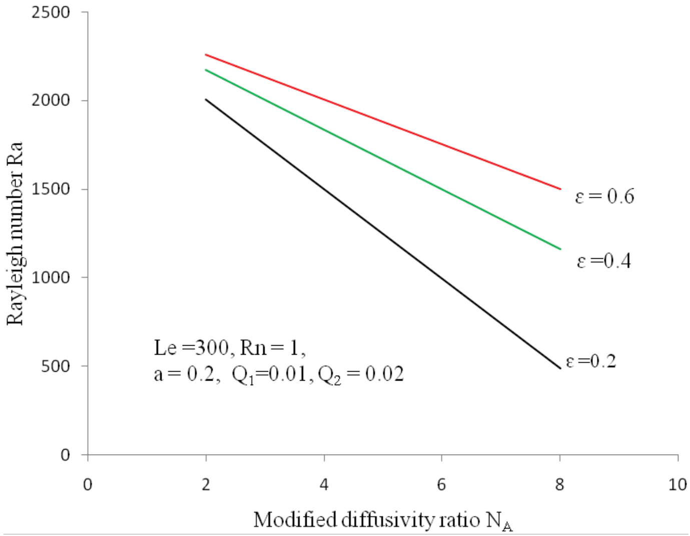

Figure 4. Variation of Rayleigh number, Ra, with modified diffusivity ratio, N_A, for different values of porosity parameter, ε.

Figure 4 shows the variation of the Rayleigh number with modified diffusivity ratio, N_A, for different values of the porosity parameter, ε, and fixed values of other parameters. It is observed that the Rayleigh number decreases with an increase in the value of modified diffusivity ratio, N_A. Also, the Rayleigh number increases with an increase in the value of porosity parameter. Thus, modified diffusivity ratio advances while porosity parameter delays the onset of convection.

CONCLUSIONS

Thermal instability in a horizontal layer of Maxwell visco-elastic nanofluid in a porous medium is investigated theoretically. The Darcy model is used for porous medium.

The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries and the eigen-value problem is solved using the Galerkin residual method. The results are shown graphically.

The main conclusions derived from the present analysis are as follows:

(i) At stationary convection, the Maxwell visco-elastic nanofluid behaves like an ordinary Newtonian nanofluid.

(ii) Lewis number, Le; modified diffusivity ratio, N_A; and nanoparticles Rayleigh number, Rn, advance while the viscosity variation parameter, Q_1; thermal conductivity variation parameter, Q_2; and porosity parameter, ε, delay the onset of stationary convection.

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ESIS ACTIVITIES and

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

January 14-16, 2018	ESIS TC-8 Meeting on Numerical Methods	Paris, France	flyer2018-1.pdf
March 14-16, 2018	XXXV Encuentro del Grupo Espanol de Fractura	Malaga, Spain	http://www.gef2018.es
April 12-13, 2018	ESIS TC-1 Meeting Workshop on 'Damage and Damage Tolerance of Welded Structures'	Prague, Czech Republic	Invitation ESIS TC 1 Spring 2018-1.pdf
May 10-11, 2018	ESIS TC-9 Meeting Innov. in Cement and Concrete Techn. Materials, Methods and Applications	Torino, Italy	info TC09-1.pdf
June 4-6, 2018	IGF Workshop Fracture and Structural Integrity: ten years of 'Frattura ed integrità Strutturale'	Cassino, Italy	link
June 17-20, 2018	1st International Conference on Theoretical, Applied and Experimental Mechanics (ICTAEM 1)	Paphos, Cyprus	https://www.ictaem.org/
July 1-5, 2018	18th International Conference on Experimental Mechanics (ICEM 2018)	Brussels, Belgium	http://www.icem18.org/
July 2-6, 2018	10th European Solid Mechanics Conference (ESMC 2018)	Bologna, Italy	http://www.esmc2018.org
July 5-6, 2018	2nd International Conference on Materials Design and Applications	Porto, Portugal	https://web.fe.up.pt/~mda2018/
July 8-11, 2018	8th Internat. Conference on Engineering Failure Analysis (ICEFA VIII)	Budapest, Hungary	link
August 25-26, 2018	ESIS Summer School in the scope of ECF22	Belgrade, Serbia	link
August 26-31, 2018	22nd European Conference of Fracture (ECF22)	Belgrade, Serbia	http://www.ecf22.rs
September 19-21, 2018	CP 2018- 6th International Conference on 'Crack Paths'	Verona, Italy	http://www.cp2018.unipr.it/
June 24-26, 2019	12th International Conference on Multiaxial Fatigue and Fracture (ICMFF12)	Bordeaux, France	link
March 30 - April 3, 2020	VAL4, 4th International Conference on Material and Component Performance under Variable Amplitude Loading	Darmstad, Germany	First Announcement