*Sanjeev Sharma*

# STRESS ANALYSIS OF ELASTIC-PLASTIC THICK-WALLED CYLINDRICAL PRESSURE VESSELS SUBJECTED TO TEMPERATURE

# ANALIZA NAPONA U ELASTOPLASTIČNIM DEBELOZIDNIM CILINDRIČNIM POSUDAMA POD PRITISKOM SA TEMPERATUROM

Originalni naučni rad / Original scientific paper
UDK /UDC: 62-988:539.319
Rad primljen / Paper received: 22.08.2017.

Adresa autora / Author's address:
Jaypee Institute of Information Technology, Department of Mathematics, Noida, India, sanjiit12@rediffmail.com

**Keywords**
- thermal loading
- elastic-plastic
- cylinder pressure vessel
- stresses

**Ključne reči**
- toplotna opterećenja
- elastoplastično
- cilindrična posuda pod pritiskom
- napon

*Abstract*

*In this paper, an attempt is made to analyse the thermal stresses in cylindrical vessel made up of functionally graded material subjected to internal and external pressure. The main purpose of this study is to assess the design guidelines of pressure vessels in order to overcome the fracture conditions. The methodology based on transition theory is applied to evaluate the stresses. Stresses for fully plastic state have been discussed with two-zone theory. The thick-walled circular cylinder made up of functionally graded material becomes fully plastic at internal and external surface. This is because initial yielding starts at any radius that lies between the internal and external surface. On the basis of analysis, it can be concluded that the circular cylinder of functionally graded material with thermal effects is on the safer side of the design as compared to the cylinder without thermal effects and also to the homogeneous cylinder with pressure, which leads to the idea of 'stress saving' that minimizes the possibility of fracture of the cylinder.*

*Izvod*

*U radu je opisan pokušaj analize termičkih napona u cilindričnoj posudi izrađenoj od funkcionalnog kompozitnog materijala koji je izložen unutrašnjem i spoljnom pritisku. Osnovni cilj rada je procena projektnih preporuka za posude pod pritiskom radi prevazilaženja uslova za pojavu loma. Metodologija zasnovana na teoriji prelaznih napona je primenjena za proračun napona. Naponi u uslovima potpune plastičnosti su obrađeni teorijom dve zone. Debelozidni kružni cilindar od funkcionalnog kompozitnog materijala postaje potpuno plastičan na unutrašnjoj i spoljnoj površini. Ovo se dešava zbog toga što iniciranje tečenja započinje na nekom radijusu između unutrašnje i spoljne površine. Na bazi analize, zaključuje se da je ponašanje kružnog cilindra od funkcionalnog kompozitnog materijala sa termičkim uticajima bezbednije od cilindra bez termičkih uticaja, kao i od homogenog cilindra pod pritiskom, što navodi na ideju „uštede napona" koji smanjuje verovatnoću loma cilindra.*

## INTRODUCTION

Pressure vessels are extensively used in thermal and nuclear power plants, chemical industry, space and food supply systems. Analytical solutions of problems of thick-walled hollow cylinders in the elastic stress state are discussed by many authors /1, 2/. Mukhopadhyay /3/ studied the effect of non-homogeneity on yield stress in a thick-walled cylindrical tube subjected to pressure by allowing the modulus of rigidity to obey some cosine law of its radial distance. The analytical solution for the stress, strain and displacement in a thick-walled cylinder of strain-hardening plastic material under the influence of pressure at inner surface was given by Gao /4/. Yoo et. al. /5/ evaluated the collapse pressure in cylinders with intermediate thickness and applied external pressure, and suggested that yield strength is most effective in estimation of collapse pressure. Reghunath and Korah /6/ calculated the stress intensity factor at different crack orientations and predicted which crack fails faster. Sobhaniaragh et. al. /7/ discussed thermal stresses in cylindrical shells made up of ceramic matrix composite and noticed the impact of aggregation factor on circumferential stresses. Pydah and Batra /8/ determined interfacial bending stress, peak interfacial shear stress and interfacial peeling stress in a thick-walled circular beam of functionally graded material. All the above authors applied concepts of classical theory i.e. the assumptions of strain laws and yield criteria etc. in order to calculate the stresses. Transition theory /9, 10/ does not require any of the assumptions of classical theory and thus it provides the solution using the concept of generalized strain measure /9/. This theory has been applied to many problems /11-21/, for example, Gupta and Sharma /11/ studied thermal elastic-plastic transition of non-homogeneous circular cylinder under internal pressure. Sharma /12/ determined elastic-plastic stresses for non-homogeneous thick-walled circular cylinder under internal pressure, while Borah /13/ discussed on the transition theory in detail with thermal stresses. Sharma and Panchal /20/ evaluated creep stresses in pressurized thick-walled rotating spherical shell made of functionally graded material and deduced that both rotation and non-homogeneity affect the creep stresses. Sharma et al /21/

analysed the thermal creep stresses for functionally graded thick-walled cylinder subjected to torsion and internal and external pressure and it was found that in creep torsion cylinder made up of less functionally graded material under pressure is a better choice for the designing point of view as compared to the homogeneous cylinder.

The factor of non-homogeneity is taken in terms of variable compressibility of the material as

$C = C_0 r^k$ , $a \le r \le b$; $C_0$ and $k$ ($\ge 0$) are constants (1)

The generalized principal strain measures /8/ are given by the following equation

$$e_{ii} = \int_0^{e_{ii}^A} \left[1-2e_{ii}^A\right]^{\frac{n}{2}-1} de_{ii}^A = \frac{1}{n}\left[1-\left(1-2e_{ii}^A\right)^{\frac{n}{2}}\right] \quad (2)$$

MATHEMATICAL FORMULATION OF THE PROBLEM

For the creep stress analysis, we have considered the functionally graded thick cylinder of internal and external radii $a$ and $b$, respectively, subjected to pressure on both inner and outer surface and temperature on the inner surface only.

The components of displacement are taken as /9-21/,

$$u = r(1-Q),\ v = 0 \text{ and } w = \phi z, \quad (3)$$

where $Q$ is a function of $r$ only and $\phi$ is a constant.

The strain components are expressed as follows

$$\begin{aligned} e_{rr} &= \frac{1}{n}\left[1-(rQ'+Q)^n\right], \\ e_{\theta\theta} &= \frac{1}{n}\left[1-Q^n\right], \\ e_{zz} &= \frac{1}{n}\left[1-(1-\phi)^n\right], \\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0 \end{aligned} \quad (4)$$

where, $n$ is the measure and $Q' = dQ/dr$.

The stress-strain relation for isotropic material with temperature is

$$\tau_{ij} = \lambda\delta_{ij}I_1 + 2\mu\varepsilon_{ij} - \xi T \quad (i,j = 1,2,3), \quad (5)$$

where $\tau_{ij}$, $\varepsilon_{ij}$ are stress and strain tensors respectively, $I_1 = \varepsilon_{kk}$ are strain invariants, $\lambda$, $\mu$ are Lame's constants, $\delta_{ij}$ is Kronecker's delta, $T$ is temperature and $\xi = \alpha(3\lambda + 2\mu)$, $\alpha$ is the coefficient of thermal expansion.

We have calculated the temperature $T = \left(T_0 \log\frac{r}{b}\right)\Big/\left(\log\frac{a}{b}\right)$ by solving the equation $T_{,ii} = 0$, with boundary conditions $T = T_0$ at $r = a$; $T = 0$ at $r = b$, where $T_0$ is constant.

The equilibrium equation of the axially symmetric cylinder, in the absence of body forces, is given by

$$\frac{d}{dr}(\tau_{rr}) + \frac{(\tau_{rr}-\tau_{\theta\theta})}{r} = 0 \quad (6)$$

With the help of Eqs.(4), (5) and (6), the following equation is obtained

$$\begin{aligned} n\psi Q(\psi+1)^{n-1}\frac{d\psi}{dQ} = \Bigg[ & r\left(\frac{-3C'}{(3-2C)C}\right)[\{(3-2C)-(1-C)(1-\phi)^n\}\times \\ & \times\frac{1}{Q^n}-(1-C)-(\psi+1)^n]+C[1-(\psi+1)^n]+rC'[1-\{2-(1-\phi)^n\}\times \\ & \times\frac{1}{Q^n}]-n\psi[(1-C)+(\psi+1)^n]-\frac{Cn\overline{T}_0}{2\mu Q^n}\left(\xi+r\xi'\log\frac{r}{b}\right)\Bigg] \end{aligned} \quad (7)$$

where

$$C = \frac{2\mu}{\lambda+2\mu},\ rQ' = Q\psi \text{ and } \overline{T}_0 = \frac{T_0}{\log\frac{a}{b}}.$$

The critical points of $Q$ in the above equation are $\psi \to -1$ and $\psi \to \pm\infty$.

The boundary conditions which are used to simplify the problem are given by

$$\begin{cases} \tau_{rr} = -p_1 & \text{at } r = a \\ \tau_{rr} = -p_2 & \text{at } r = b \end{cases} \quad (8)$$

In the cylinder, the resultant axial force is given by

$$2\pi\int_a^b rT_{zz}dr = 0 \quad (9)$$

ANALYTICAL SOLUTION THROUGH PRINCIPAL STRESS

As principal stresses are considered for elastic-plastic transition and therefore the transition functions can be taken as functions of radial stress, hoop stress or their difference /11-21/. We define the transition function $TR$ in terms of radial stresses i.e. $\tau_{rr}$ as

$$\begin{aligned} TR = \tau_{rr} - B + \alpha(3-2C)T \equiv & \frac{3}{Cn(3-2C)}\times \\ & \times\left[C-Q^n\left\{(1-C)+(\psi+1)^n\right\}\right]-\alpha(3-2C)T\left(\frac{3}{C(3-2C)}-1\right) \end{aligned} \quad (10)$$

Substituting Eq.(7) in Eq.(10) and applying the critical value $\psi \to \pm\infty$, the following results are obtained

$$TR = A\exp f(r) \quad (11)$$

where $f(r) = -\int Cr^{-1}dr$ and $A$ is a constant of integration.

By applying the boundary conditions Eq.(8) in Eq.(11), we get

$$\tau_{rr} = A[\exp f(r) - \exp f(b)] - p_2 - \alpha T(3-2C) \quad (12)$$

Substituting Eq.(12) in Eq.(6), we get

$$\begin{aligned} \tau_{\theta\theta} = & A\left[(1-C)\exp f(r) - \exp f(b)\right] - \\ & -\alpha\overline{T}_0\left[(3-2C)\left(1+\log\frac{r}{b}\right)-2rC'\log\frac{r}{b}\right]-p_2 \end{aligned} \quad (13)$$

Substitution of Eqs.(12) and (13) into Eq.(5) yields,

$$\tau_{zz} = \left(\frac{1-C}{2-C}\right)(\tau_{rr}+\tau_{\theta\theta}) + \frac{3}{(2-C)}(\varepsilon_{zz}-\alpha T) \quad (14)$$

where

$$\varepsilon_{zz} = \frac{\left(\frac{p_2 - p_1}{2\pi}\right) + \alpha \int_a^b \frac{3rT}{2-C} dr - \int_a^b r \frac{C(1-C)}{2-C} (\tau_{rr} + \tau_{\theta\theta}) dr}{\int_a^b \frac{3r}{2-C} dr} .$$

By applying variable compressibility Eqs.(12)-(14) become

$$\tau_{rr} = A_1 \left[ \exp\left(\frac{-C_0 r^k}{k}\right) - \exp\left(\frac{-C_0 b^k}{k}\right) \right] - p_2 - \alpha T (3 - 2C_0 r^k) \quad (15)$$

$$\tau_{\theta\theta} = A_1 \left[ (1 - C_0 r^k) \exp\left(\frac{-C_0 r^k}{k}\right) - \exp\left(\frac{-C_0 b^k}{k}\right) \right] - \\ -\alpha \overline{T}_0 [(3 - 2C_0 r^k)(1 + \log\frac{r}{b}) - 2C_0 k r^k \log\frac{r}{b} - p_2 \quad (16)$$

$$\tau_{zz} = \left(\frac{1 - C_0 r^k}{2 - C_0 r^k}\right)(\tau_{rr} + \tau_{\theta\theta}) + \left(\frac{3}{2 - C_0 r^k}\right)(\varepsilon_{zz} - \alpha T) . \quad (17)$$

where $A_1 = \dfrac{\alpha T_0 (3 - 2C_0 a^k) + (p_1 - p_2)}{\exp\left(\frac{-C_0\, a^k}{k}\right) - \exp\left(\frac{-C_0\, b^k}{k}\right)}$ and

$$\varepsilon_{zz} = \frac{\left(\frac{p_2 - p_1}{2\pi}\right) + \alpha \int_a^b \frac{3rT}{2 - C_0 r^k} dr - \int_a^b r \frac{C_0 r^k (1 - C_0 r^k)}{2 - C_0 r^k} (\tau_{rr} + \tau_{\theta\theta}) dr}{\int_a^b \frac{3r}{2 - C_0 r^k} dr} .$$

Equations (15), (16) and (17) give radial, hoop and axial stresses respectively in transition state.

If the concept of classical theory is applied, then one has to assume some yield conditions to join the two spectrums, i.e. elastic and plastic regions, while in transition theory the yield condition has been calculated from the constitutive equations in the transition state. Thus, from Eqs.(15) and (16), we have

$$\tau_{\theta\theta} - \tau_{rr} = -A_1 C_0 r^k \exp\left(\frac{-C_0 r^k}{k}\right) + 2kC_0 \alpha \overline{T}_0 r^k - \alpha \overline{T}_0 (3 - 2C_0 r^k) . \quad (18)$$

The first derivative of Eq.(18)

$$\frac{d}{dr}(T_{\theta\theta} - T_{rr}) = A_1 C_0 (k + C_0 r^{-k}) r^{-k-1} \exp\left(\frac{C_0 r^{-k}}{k}\right) - \\ -2\alpha \overline{\theta}_0 k C_0 r^{-k-1} \left[ 2 - k \log\frac{r}{b} \right]$$ is zero at $r = (e^2 b^{-k})^{-\frac{1}{k}} = r_1$,

$C_0 = kb^{-k} e^2$ and $k > 0$, where $e$ = exponential. The second derivative of Eq.(18)

$$\frac{d^2}{dr^2}(T_{\theta\theta} - T_{rr}) = -A_1 C_0 r^{-k-2} \exp\left(\frac{C_0 r^{-k}}{k}\right) \left[ k(k+1) + kC_0 r^{-k} + \right. \\ \left. +(2k+1) C_0 r^{-k} + C_0^2 r^{-2k} \right] - \alpha \overline{\theta}_0 \left[ -4k(k+1) C_0 + \right. \\ \left. +2k^2 (k+1) C_0 \log\frac{r}{b} - 2k^2 C_0 \right] r^{-k-2}$$

is negative at $r = (e^2 b^{-k})^{-\frac{1}{k}} = r_1$, $C_0 = kb^{-k} e^2$ and $k > 0$, if

$$\alpha \theta_0 \left[ (3 - 2C_0 a^{-k}) - \frac{2e \left\{ \exp\left(\frac{C_0 a^{-k}}{k}\right) - \exp\left(\frac{C_0 b^{-k}}{k}\right) \right\}}{\log\frac{b}{a}} \right] < p \quad (18a)$$

Hence $|\tau_{\theta\theta} - \tau_{rr}|$ is maximum at $r = (e^2 b^{-k})^{-\frac{1}{k}} = r_1$, $C_0 = kb^{-k}e^2$ and $k > 0$, provided it satisfies the condition (18a). Therefore yielding of a non-homogeneous rotating cylinder will takes place at $r = (e^2 b^{-k})^{-\frac{1}{k}}$, depending on values of $C_0$ and $k$.

$$|\tau_{\theta\theta} - \tau_{rr}|_{r=r_1} = \left| -A_1 C_0 r^k \exp\left(\frac{-C_0 r^k}{k}\right) + 2kC_0 \alpha \overline{T}_0 r^k - \right. \\ \left. -\alpha \overline{T}_0 (3 - 2C_0 r^k) \right|_{r=r_1} \equiv Y(\text{say}) \quad (19)$$

where $Y$ is yield stress and $A_1 = \left[ \dfrac{\alpha T_0 (3 - 2C_0 a^k) + (p_1 - p_2)}{\exp\left(\frac{-C_0 a^k}{k}\right) - \exp\left(\frac{-C_0 b^k}{k}\right)} \right]_{r=r_1}$

Thus, from Eq.(19), the effective pressure required for initial yielding is given by

$$\left| \frac{p_1}{Y} - \frac{p_2}{Y} \right| = \left| \frac{1}{A_2} \right| - T_1 \left| \frac{A_3}{A_2} \right| \quad (20)$$

where $T_1 = \dfrac{\alpha T_0}{Y}$, $A_2 = \dfrac{-k}{e\left\{ \exp\left(\frac{-C_0 a^k}{k}\right) - \exp\left(\frac{-C_0 b^k}{k}\right) \right\}}$ and

$$A_3 = \frac{3 - 2k}{\log\frac{a}{b}} - \frac{(3 - 2C_0 a^k) k}{e\left\{ \exp\left(\frac{-C_0 a^k}{k}\right) - \exp\left(\frac{-C_0 b^k}{k}\right) \right\}} - 2k^2 \log\frac{r_1}{b} .$$

Now we introduce the following non-dimensional components as: $R = r/b$, $R_0 = a/b$, $\sigma_{rr} = \tau_{rr}/Y$, $\sigma_{\theta\theta} = \tau_{\theta\theta}/Y$ and $\sigma_{zz} = \tau_{zz}/Y$.

Equation (20) of effective pressure required for initial yielding can be written in non-dimensional form as

$$|P_i| = \frac{1}{|A_8|} - T_1 \frac{|A_9|}{|A_8|} \quad (21)$$

where

$P_i = \dfrac{p_1}{Y} - \dfrac{p_2}{Y} = P_{i1} - P_{i2}$, $A_8 = \dfrac{-k}{e\left[ \exp(-e^2 R_0^k) - \exp(-e^2) \right]}$ and

$$A_9 = \frac{3 - 2k}{\log R_0} + \frac{(3 - 2ke^2 R_0^k) k}{e\left[ \exp(-e^2 R_0^k) - \exp(-e^2) \right]} + 4k .$$

The radial, circumferential and axial stresses in transition state from Eqs.(15)-(17) in non-dimensional form can be written as

$$\sigma_{rr} = \left\{T_1\left[3-2C_0(bR_0)^k\right]-P_i\right\}\frac{\exp\left[\frac{-C_0b^k}{k}(R^k-1)\right]-1}{\exp\left[\frac{-C_0b^k}{k}(R_0^k-1)\right]-1}-$$

$$-P_{i2}-T_1\frac{\log R}{\log R_0}\left[3-2C_0(bR)^k\right] \quad (22)$$

$$\sigma_{\theta\theta} = \frac{T_1\left[3-2C_0(bR_0)^k\right]-P_i}{\exp\left[\frac{-C_0b^k}{k}(R_0^k-1)\right]-1}\left[(1-C_0b^kR^k)\times\right.$$

$$\left.\times\exp\left\{\frac{-C_0b^k}{k}(R_0^k-1)\right\}-1\right]-\frac{T_1}{\log R_0}\left\{\left[3-2C_0(bR)^k\right]\times\right.$$

$$\left.\times(1+\log R)-2C_0k(bR)^k\log R\right\}-P_{i2} \quad (23)$$

$$\sigma_{zz} = \frac{1-C_0b^kR^k}{2-C_0b^kR^k}(\sigma_{rr}+\sigma_{\theta\theta})+\frac{3}{2-C_0(bR)^k}\left(\varepsilon_{zz}-\frac{T_1\log R}{\log R_0}\right) \quad (24)$$

where $\varepsilon_{zz} =$

$$= \frac{-\frac{P_i}{2\pi}+\frac{T_1}{\log R_0}\int_{R_0}^{1}\frac{3\log R}{2-C_0(bR)^k}dR-\int_{R_0}^{1}b^2R\frac{1-C_0(bR)^k}{2-C_0(bR)^k}(\sigma_{rr}+\sigma_{\theta\theta})dR}{\int_{R_0}^{1}\frac{3}{b\left[2-C_0(bR)^k\right]}dR}$$

**Fully plastic state:**

There are two plastic zones:

(i) inner plastic region: $a \le r \le r_1$

(ii) outer plastic region: $r_1 \le r \le b$.

**(i) Inner plastic region:** $a \le r \le r_1$

For fully plastic state /6-10/ ($C_0 \to 0$), Eq.(18) can be expressed in non-dimensional form as

$$\left|P_{fa}\right| = \frac{1}{\left|A_{10}\right|}+T_1\frac{\left|A_{11}\right|}{\left|A_{10}\right|}, \quad (25)$$

where $P_{fa} = \frac{p_1}{Y_1^*}-\frac{p_2}{Y_1^*} = P_{fa1}-P_{fa2}$, $A_{10} = \frac{-k}{1-R_0^{-k}}$ and $A_{11} = 3\left[\frac{-k}{1-R_0^{-k}}+\frac{1}{\log R_0}\right]$.

Fully plastic stresses in Eqs.(22)-(24) as ($C_0 \to 0$) in non-dimensional form are as follows:

$$\sigma_{rr}^a = (3T_1-P_{fa})\frac{R^k-1}{R_0^k-1}-P_{fa2}-3T_1\frac{\log R}{\log R_0}, \quad (26)$$

$$\sigma_{\theta\theta}^a = \sigma_{rr}^a-\frac{3T_1}{\log R_0}-\frac{kR^k(3T_1-P_{fa})}{R_0^k-1}, \quad (27)$$

$$\sigma_{zz}^a = \frac{3(1-C_0b^kR^k)k\left[\frac{P_{fa}}{2\pi}+\frac{1}{2}\int_{R_0}^{1}Rb^2(\sigma_{rr}^a+\sigma_{\theta\theta}^a)dR\right]}{C_0b^k(3-2C_0b^kR^k)(R_0^k-1)}. \quad (28)$$

**(ii) Outer plastic region:**

For fully plastic state /7-11/ ($C_0 \to 0$), Eq.(19) can be expressed in non-dimensional form as

$$\left|P_{fb}\right| = \frac{1}{\left|A_{12}\right|}+\theta_1\frac{\left|A_{13}\right|}{\left|A_{12}\right|} \quad (29)$$

where $P_{fb} = \frac{p_1}{Y_1^{**}}-\frac{p_2}{Y_1^{**}} = P_{fb1}-P_{fb2}$, $A_{12} = \frac{-k}{R_0^k-1}$ and $A_{13} = 3\left[\frac{-k}{R_0^k-1}+\frac{1}{\log R_0}\right]$.

Fully plastic radial, circumferential and axial stresses from Eqs.(23)-(25) in non-dimensional form can be written as

$$\sigma_{rr}^b = (3T_1-P_{fb})\frac{R^k-1}{R_0^k-1}-P_{fb2}-3T_1\frac{\log R}{\log R_0}, \quad (30)$$

$$\sigma_{\theta\theta}^b = \sigma_{rr}^b-\frac{3T_1}{\log R_0}-\frac{kR^k(3T_1-P_{fb})}{R_0^k-1}, \quad (31)$$

$$\sigma_{zz}^b = \frac{3(1-C_0b^kR^k)k\left[\frac{P_{fb}}{2\pi}+\frac{1}{2}\int_{R_0}^{1}Rb^2(\sigma_{rr}^b+\sigma_{\theta\theta}^b)dR\right]}{C_0b^k(3-2C_0b^kR^k)(R_0^k-1)} \quad (32)$$

**Particular case: cylinder with internal pressure**

Pressure required for fully plastic state without thermal effects and external pressure is given from Eq.(29) as

$$\left|P_{f1}\right| = \left|\frac{R_0^k-1}{k}\right|. \quad (33)$$

Equations (30)-(32) are radial, circumferential and axial stresses for fully plastic state without temperature and external pressure, in non-dimensional form can be expressed as

$$\sigma_{rr} = -P_{f1}\left(\frac{R^k-1}{R_0^k-1}\right), \quad (34)$$

$$\sigma_{\theta\theta} = \sigma_{rr}-\frac{3T_1}{\log R_0}-\frac{kR^k(3T_1-P_f)}{R_0^k-1}, \quad (35)$$

$$\sigma_{zz} = \frac{3(1-C_0b^kR^k)k\left[\frac{P_f}{2\pi}+\frac{1}{2}\int_{R_0}^{1}Rb^2(\sigma_{rr}+\sigma_{\theta\theta})dR\right]}{C_0b^k(3-2C_0b^kR^k)(R_0^k-1)}. \quad (36)$$

NUMERICAL DISCUSSION

A model of pressurized thick-walled cylinder is formed for different radii ratios. Radial and hoop stresses for different radii ratios are determined under different pressure ratios and temperature. In case of functionally graded cylinder ($k > 0$, non-homogeneity increases radially) yielding begins at any radius $r$ where $a < r < b$ at different temperatures. In the absence of temperature, circular cylinder of high compressibility needs high effective pressure to yield as compared to the circular cylinder of less compressibility. The effective pressure needed for initial yielding is less for a cylinder with temperature. This pressure keeps on decreasing with the increasing value of temperature as can be identified from

Fig. 1. It is clear from Fig. 2 that effective pressure required for plastic state is maximum at the inner surface of the cylinder and this pressure keeps on increasing with temperature. Also, circular cylinder of high compressibility needs very less effective pressure to become fully plastic than the cylinder of less compressibility.

In Fig. 3, the external pressure needed for initial yielding with pressure (internal) (= 5, say) is maximum at the inner surface. Also cylinder of high compressibility requires very high external pressure to yield and this pressure decreases with temperature. It can be seen from Fig. 4 that external pressure required for fully plastic state is very high for circular cylinder of less compressibility as compared to circular cylinder of high compressibility. Also, pressure decreases remarkably with temperature. As internal pressure increases, external pressure needed for initial yielding and fully plastic state increases as can be identified from Figs. 5 and 6.

In Fig. 7, it is clear that in absence of temperature, transitional hoop stresses are maximum at outer surface and are compressive in cylinder whose internal pressure is less than that of external pressure. It is also noticed that as pressure on the outer surface increases, compressive hoop stresses also increase. In Fig. 8, notable changes have been observed in the stresses due to increase in temperature. Without thermal effects, transitional stresses are compressive when pressure on inner surface is higher than the pressure at outer surface as can be seen in Fig. 9. With the increase of temperature and pressure, stresses change remarkably (Fig. 10).

From Fig. 11 it is found that without temperature, hoop stresses have maximum value at the outer surface and are compressive for the cylinder in which pressure on the inner surface is less than the pressure on the outer surface. Also, with the increase in pressure, there is a remarkable increase in hoop stress. Also hoop stresses are high for the circular cylinder of less compressibility as compared to the circular cylinder of high compressibility. From Fig. 12, it is seen that with the increase in temperature, circumferential stresses increase significantly. From Fig. 13, it is observed that when the internal pressure is higher, hoop stresses have maximum value at the inner surface. Also, circular cylinder of high compressibility has high hoop stresses as compared to the circular cylinder of less compressibility. With the increase in pressure and temperature, circumferential stresses further increase (Fig. 14).

## CONCLUSION

The cylinder was subjected to high internal and external pressures and temperatures. On the basis of analysis of effective pressure, it is found that the functionally graded circular cylinder of high compressibility with temperature is better than the functionally graded cylinder of less compressibility because the cylinder of high compressibility needs very high effective pressure for yielding. It is also concluded that functionally graded circular cylinder of high compressibility with thermal effects is safer as compared to the circular cylinder of less compressibility because the cylinder of high compressibility requires very high external pressure to yield and then to become fully plastic.

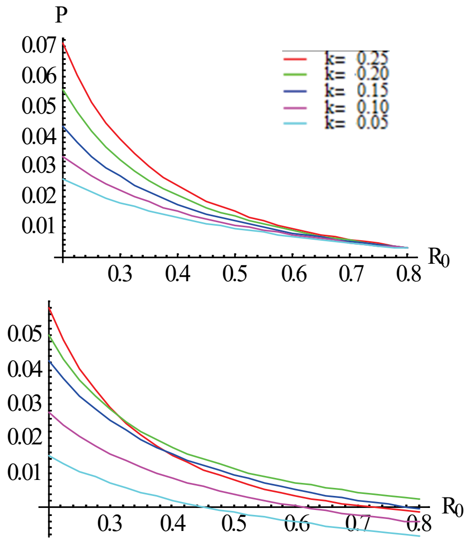


Figure 1. Effective pressure for initial yielding at $T = 0$ and 0.005.

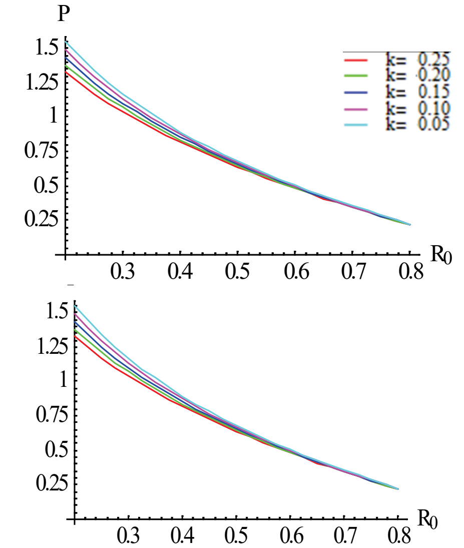


Figure 2. Effective pressure for fully plastic state at $T = 0$ and 0.005.

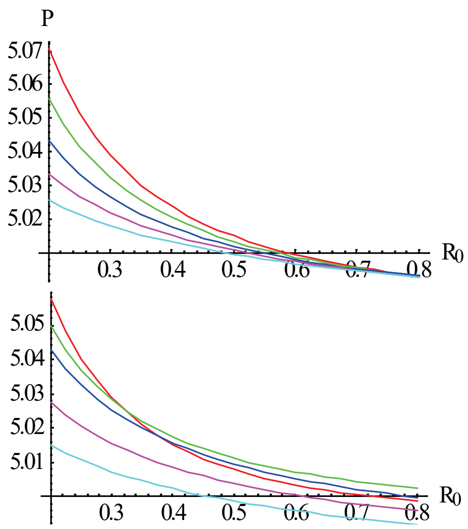


Figure 3. Ext. pressure for initial yielding at $T = 0$ and 0.005 when internal pressure = 5.

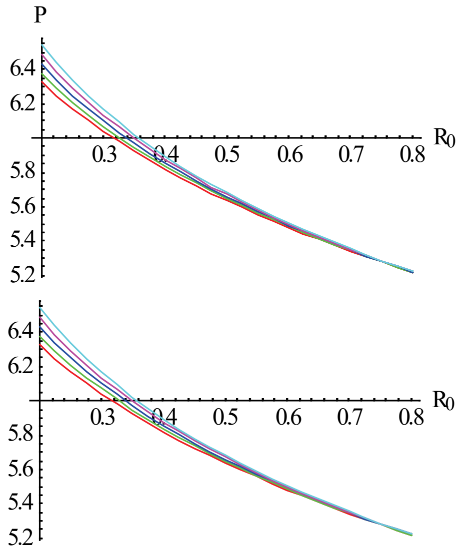


Figure 4. Ext. pressure for fully plastic state at $T = 0$ and 0.005 when internal pressure = 5.

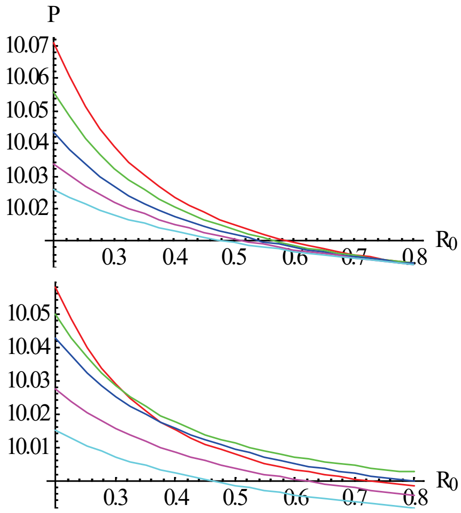


Figure 5. Ext. pressure for initial yielding at $T = 0$ and 0.005 when internal pressure is 10.

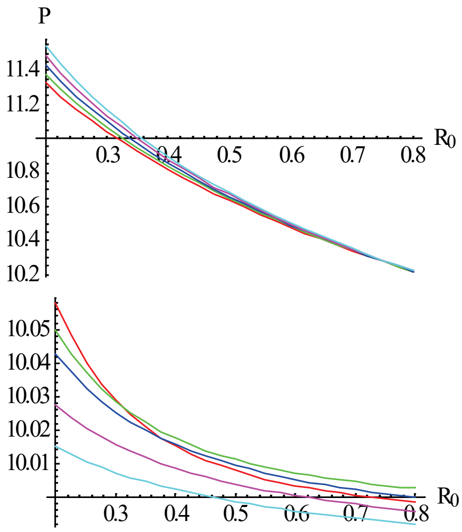


Figure 6. Ext. pressure for fully plastic state at $T = 0$ and 0.005 when internal pressure is 10.

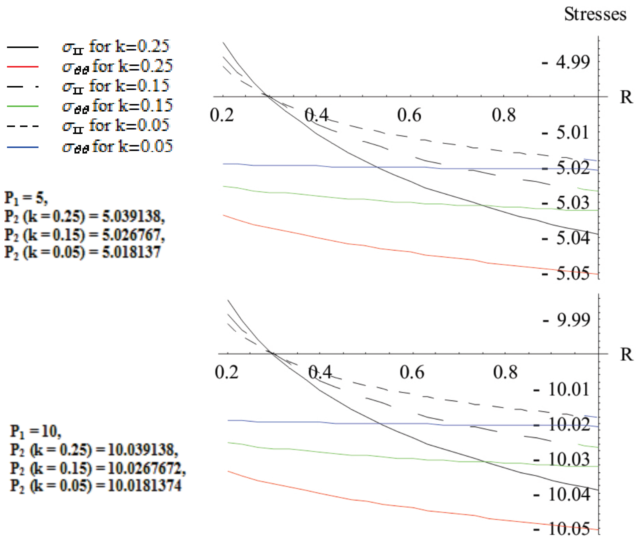


Figure 7. Transitional stresses for a cylinder under internal and external pressure.

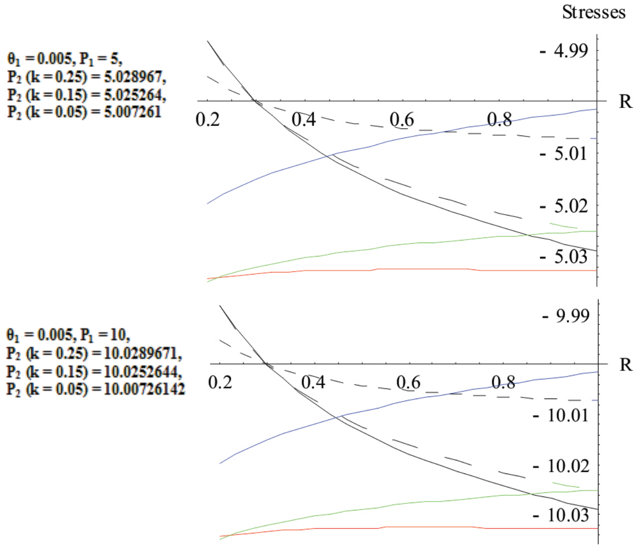


Figure 8. Thermal ($T_1 = 0.005$) transitional stresses for a cylinder under internal and external pressure.

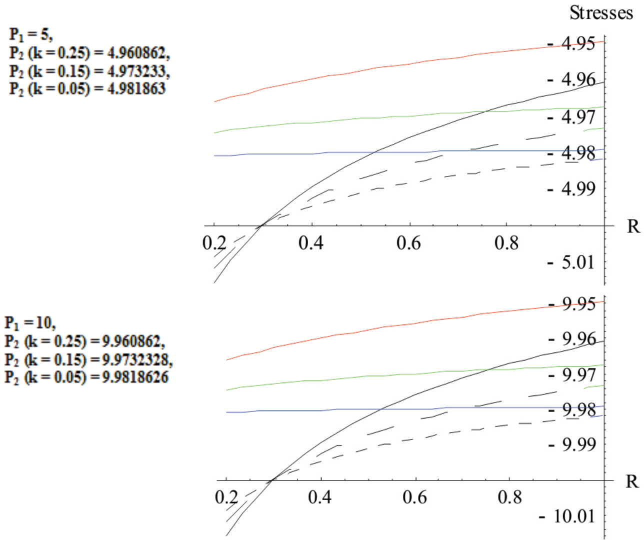


Figure 9. Transitional stresses for a cylinder under internal and external pressure.

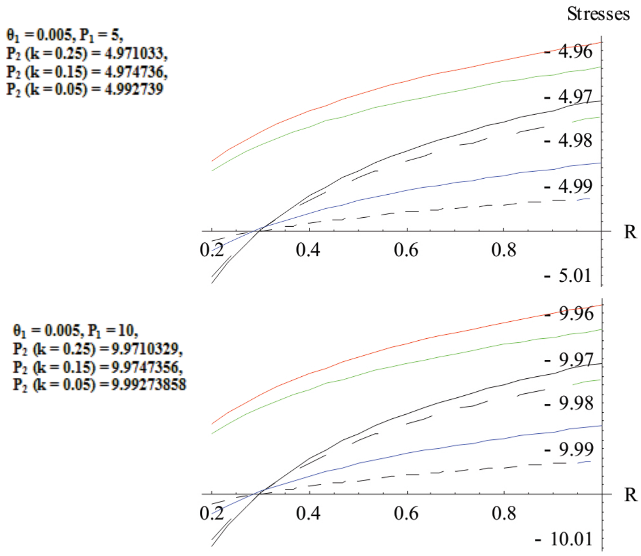


Figure 10. Thermal ($T_1$ = 0.005) transitional stresses for a cylinder under internal and external pressure.

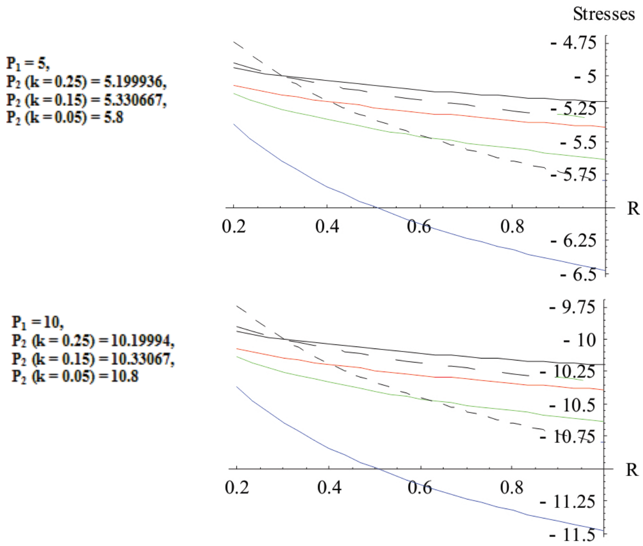


Figure 11. Fully plastic stresses for a cylinder under internal and external pressure.

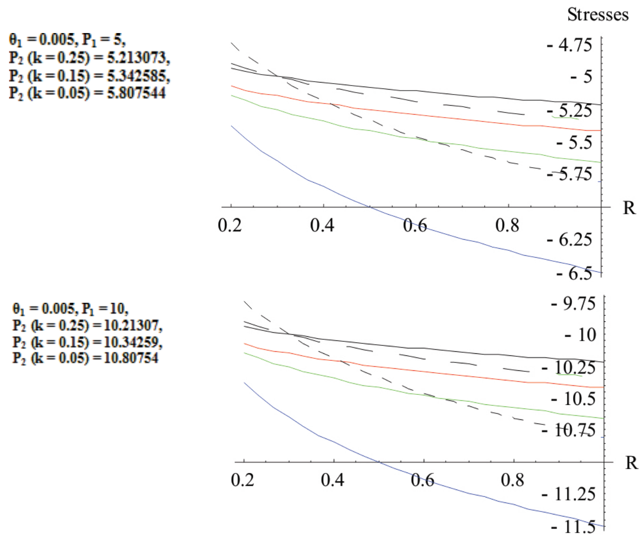


Figure 12. Thermal ($T_1$ = 0.005) fully plastic stresses for a cylinder under internal and external pressure.

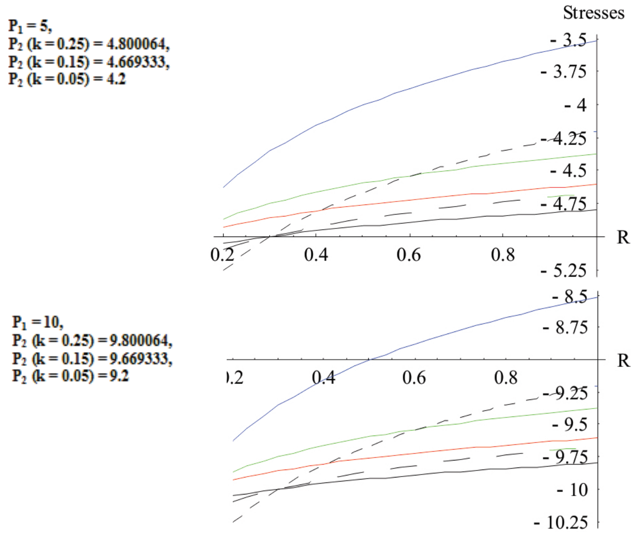


Figure 13. Fully plastic stresses for a cylinder under internal and external pressure.

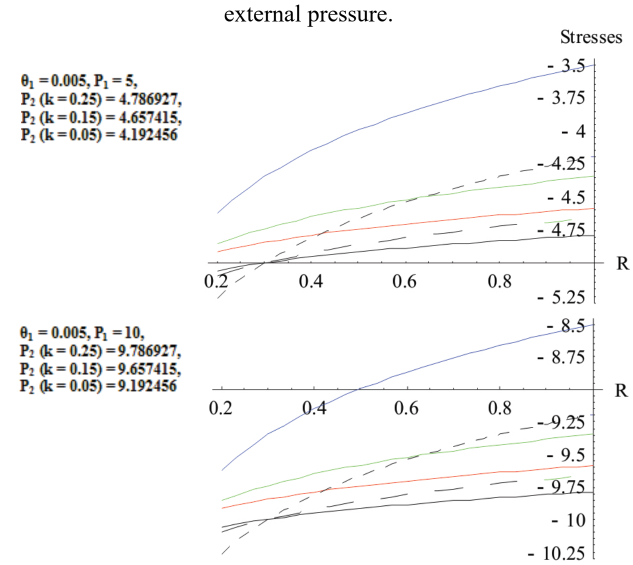


Figure 14. Thermal ($T_1$ = 0.005) fully plastic stresses for a cylinder under internal and external pressure.

## REFERENCES


1. Sokolinokoff, S., Mathematical Theory of Elasticity, 2nd Ed., McGraw-Hill, New York, 1956.
2. Chakrabarty, J., Applied Plasticity, Verlag Springer, Berlin, 2000.
3. Mukhopadhyay, J. (1982), *Effect of non-homogeneity on yield stress in a thick-walled cylindrical tube under internal pressure*, Lett. J Appl. Engng. Sci., 20: 963-968.
4. Gao, X.L. (2003), *Elasto-plastic analysis of an internally pressurized thick-walled cylinder using a strain gradient plasticity theory*, Int. J Solids and Struc., 40(23): 6445-6455.
5. Yoo, Y.-S., Huh, N.-S., Choi, S., et. al. (2010), *Collapse pressure estimates and the application of a partial safety factor to cylinders subjected to external pressure*, Nuclear Engng. & Techn., 42(4): 450-459.
6. Reghunath, R., Korah, S. (2014), *Analysis of internally pressurized thick-walled cylinders*, J Basic & Appl. Engng. Res. 1(2): 88-93.
7. Sobhaniaragh, B., Batra, R.C., Mansur, W.J., Peters, F.C. (2017), *Thermal response of ceramic matrix nanocomposite cylindrical shells using Eshelby-Mori-Tanaka homogenization scheme*, Composites Part B 118: 41-53.

8. Pydah, A., Batra, R.C. (2017), *Shear deformation theory using logarithmic function for thick circular beams and analytical solution for bi-directional functionally graded circular beams*, Composite Struc. 172, 45-60.
9. Seth, B.R. (1966), *Measure concept in mechanics*, Int. J Non-Linear Mechanics, 1(1): 35-40.
10. Seth, B.R. (1970), *Transition conditions: the yield condition*, Int. J Non-Linear Mech., 5(2): 279-285.
11. Gupta, S.K., Sharma, S. (1997), *Thermo elastic-plastic transition of non-homogeneous thick-walled circular cylinder under internal pressure*, Ind. J Pure & Appl. Math., 28(12), 1621-1633.
12. Sharma, S. (2004), *Elastic-plastic transition of non-homogeneous thick-walled circular cylinder under internal pressure*, Defence Sci. J, 54(2): 135-141.
13. Borah, B.N. (2005), *Thermo elastic-plastic transition*, Contemporary Math., 379: 93-111.
14. Sharma, S. (2009), *Thermo creep transition in non-homogeneous thick-walled rotating cylinders*, Defence Sci. J, 59(1): 30-36.
15. Sharma, S., Yadav, S. (2013), *Thermo elastic-plastic analysis of rotating functionally graded stainless steel composite cylinder under internal and external pressure using finite difference method*, Advances in Mater. Sci. and Engng., doi:10.1155/2013/810508: 1-10.
16. Sharma, S., Aggarwal, A.K., Sharma, R. (2013), *Safety analysis of thermal creep non-homogeneous thick-walled circular cylinder under internal and external pressure using Lebesgue strain measure*, Multidisc. Model. in Mater. & Struc., 9(4): 499-513.
17. Aggarwal, A.K., Sharma, R., Sharma, S. (2013), *Safety analysis using Lebesgue strain measure of thick-walled cylinder for functionally graded material under internal and external pressure*, The Sci. World J, doi:10.1155/2013/676190: 1-10.
18. Sharma, S., Sahai, I., Kumar, R. (2014), *Thermo elastic-plastic transition of transversely isotropic thick-walled circular cylinder under internal and external pressure*, Multidis. Model. in Mater. & Struc., 10(2): 211-227.
19. Aggarwal, A.K., Sharma, R., Sharma, S. (2014), *Collapse pressure analysis of transversely isotropic thick-walled cylinder using Lebesgue strain measure and transition theory*, The Sci. World J, doi:10.1155/2014/240954: 1-10.
20. Sharma, S., Rekha, P. (2017), *Thermal creep deformation in pressurized thick-walled functionally graded rotating spherical shell*, Int. J Pure & Appl. Math., 114(3): 435-444.
21. Sharma, S., Yadav, S., Sharma, R. (2017), *Thermal creep analysis of functionally graded thick-walled cylinder subjected to torsion and internal and external pressure*, J Solid Mech., 9(2): 302-318.