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ELASTIC-PLASTIC STRESS ANALYSIS IN A SPHERICAL SHELL UNDER INTERNAL PRESSURE AND STEADY STATE TEMPERATURE

ELASTOPLASTIČNA ANALIZA NAPONA SFERNE LJUSKE PRI DEJSTVU UNUTRAŠNJEG PRITISKA I RAVNOMERNE TEMPERATURE

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Keywords
- elastic-plastic
- temperature
- pressure
- shell
- stresses

Abstract

The purpose of the paper is to present the study of elastic -plastic stress analysis in a spherical shell under the combined effect of pressure and temperature. The solution of the problem is obtained by using the concept of generalized strain measures and Seth's transition theory. It has been seen that circumferential stress has a maximum at the external surface of the spherical shell made of incompressible- as well as compressible materials. The thermal effect increases the values of stresses.

Ključne reči
- elastoplastičan
- temperatura
- pritisak
- ljuska
- naponi

Izvod

Cilj ovog rada je prikaz istraživanja elastoplastične analize napona sferne ljuske pri istovremenom dejstvu unutrašnjeg pritiska i temperature. Rešenje problema je dobijeno korišćenjem koncepta generalisane mere deformacija i teorija prelaznih napona Seta. Uočeno je da napon po obimu ima maksimum na spoljašnjoj površini sferne ljuske od nestišljivih ili stišljivih materijala. Toplotni uticaj povećava vrednosti napona.

INTRODUCTION

These days, thermal investigation of the elastic-plastic stresses found in spherical shells under pressure has been a point of interest of engineers because of advanced technical developments in the areas such as high speed and high power space engines, nuclear industry for power generation, etc. In various cases of steam turbine design, high-speed centrifugal separators, gas turbines and diesel engines, the thermal elastic-plastic stresses are of great practical significance. The occurrence of a combined effect of high temperature and pressure, particularly in the mechanical and chemical industries, has led to new advancements in the manufacture of spherical shells. The elastic properties of various metals are highly dependent on temperature and the presence of pressure. Metals exhibit all types of distortion in behaviours such as plastic, creep, fracture, buckling, etc., as materials attain plastic behaviour easily in the presence of high pressure and temperature, due to which this problem is important in the design of oil and chemical plants, steam and gas turbines, high speed centrifugal structures and so on. The problem of elastic-plastic- and creep analysis in pipes, shells, cylinders, under internal pressure with thermal effect have been discussed, /1/. Johnson et al. /2/ have discussed the effect of temperature on the thick spherical shell subjected to internal pressure. Rimrott /3/ discussed the creep of thick-walled tube under internal pressure considering large strains under the assumptions of constant density, zero axial strain and distortion energy law to calculate creep stresses and strain rates in the thick-walled closed end hollow cylinder made of isotropic material under uniform pressure. Eberlein et al. /4/ used the finite element concept to find elastoplastic strains and the isotropic stress response in shells and has discussed the three parameterization strategies for calculating stresses in arbitrary shells. Civalek et al. /5/ have done the free vibrational analysis of rotating shells by using discrete singular convolution technique. All these authors mentioned above have analysed the problems considering the assumptions: (i) incompressibility condition; (ii) creep-strain laws, like Norton; (iii) yield condition, like that of Tresca; (iv) associated flow rule. Thakur /7/ has analysed creep transition stresses in a thick isotropic spherical shell by finitesimal deformation under steady state temperature and internal pressure by using the Seth transition theory. The necessity of use of these ad-hoc semi-empirical laws in the classical theory of elastic-plastic transition is based on the approach that the transition is a linear phenomenon, which is not possible. Under elastic-plastic and creep transition, the fundamental structure of an object undergoes a change and rearranges such to cause nonlinear effects. Therefore, it suggests that at transition behaviour, nonlinear terms are significant and cannot be ignored. The classical theory of elasticity, plasticity and creep makes use of the linear strain measure. But we have shown that transition fields are sub-harmonic (super-harmonic) fields and that they are nonlinear and non-conservative in character and hence, it is very important that a nonlinear strain measure such as the Almansi measure should be used in the constitutive equation. The recognition of transition state or mid-zone as a separate state necessitates showing the existence of the constitutive equation for that state. In this context, we have used Seth's transition theory to obtain the stresses and strains in the transition state, and the same may be obtained from the plastic state when a certain parameter $C = 1 - 2\nu/1 - \nu$; where ν is the Poisson's ratio of the material, is made to approach zero. From these solutions, the constitutive equations for both transition and plastic states are obtained, the latter takes the form of the Levy-von-Mises equation. In nature, transitions do occur frequently and the existing classical theory fails to explain them successfully. Thus the transition theory, as it stands, now can be fruitfully exploited to explain a variety of physical phenomena and hence, has a very wide application in all applied sciences. Seth's transition theory does not acquire any assumptions as a yield condition, incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of differential equations defining the deformed field, and has been successfully applied to a large number of problems /7-19/.

FORMULATION OF THE PROBLEM AND IDENTIFICATION OF TRANSITION POINTS

We consider a thick-walled spherical shell, whose internal and external radii are a and b, respectively, is subjected to uniform internal pressure p and a temperature Θ applied to the internal surface of the shell. It is convenient to use spherical polar coordinates (r, θ, ϕ), where θ is the angle made by the radius vector with a fixed axis, and ϕ is the angle measured around this axis. By virtue of the spherical symmetry $\sigma_\theta = \sigma_\phi$ everywhere in the shell, due to spherical symmetry of the structure, the components of displacement in spherical coordinates (r, θ, ϕ) are given by:

$$u = r(1-\beta);\ v = 0;\ w = 0 \tag{1}$$

where u, v, w (displacement components); β is the position function. The generalized components of strain are given by Seth's /17, 18/:

$$\begin{aligned} e_{rr} &= \frac{1}{n}\left[1-\left(\beta + r\beta'\right)^n\right], \\ e_{\theta\theta} &= \frac{1}{n}\left[1-\beta^n\right] = e_{\phi\phi}, \\ e_{r\theta} &= e_{\theta z} = e_{\phi r} = 0. \end{aligned} \tag{2}$$

where $\beta' = d\beta/dr$.

Stress-strain relation: the constitutive equation for stress-strain relations for the thermo-elastic isotropic material are given as /6/:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\Theta\delta_{ij},\ (i,j = 1,2,3) \tag{3}$$

where: T_{ij} are the stress components; λ and μ are Lame's constants; $I_1 = e_{kk}$ is the first strain invariant; δ_{ij} is the Kronecker's delta, $\xi = \alpha(3\lambda + 2\mu)$; α being the coefficient of thermal expansion; and Θ is the temperature. The temperature of any part of a spherical shell under elastic-plastic stress, by virtue of the first and second law of thermodynamics, has to satisfy:

$$\nabla^2\Theta = 0 \tag{4}$$

By using Eq.(2) in Eq.(3), the stresses are obtained as:

$$\begin{aligned} T_{rr} &= \frac{\lambda}{n}\left[3-2\beta^n-\left(r\beta'+\beta\right)^n\right]+\frac{2\mu}{n}\left[1-\left(r\beta'+\beta\right)^n\right]-\xi\Theta, \\ T_{\theta\theta} &= \frac{\lambda}{n}\left[3-2\beta^n-\left(r\beta'+\beta^n\right)\right]+\frac{2\mu}{n}\left[1-\beta^n\right]-\xi\Theta = T_{\phi\phi}, \\ T_{r\theta} &= T_{\theta\phi} = T_{\phi r} = 0. \end{aligned} \tag{5}$$

Equation of equilibrium: the radial equilibrium of an element of the spherical shell requires:

$$\frac{\partial T_{rr}}{\partial r} = \frac{2}{r}\left(T_{\theta\theta}-T_{rr}\right) \tag{6}$$

where T_{rr} and $T_{\theta\theta}$ are the radial and hoop stresses.

Boundary conditions: the temperature field satisfying Laplace Eq.(4) with boundary condition:

$$\begin{aligned} &\Theta = \Theta_0 \text{ and } T_{rr} = -p \text{ at } r = a \\ &\Theta = 0 \text{ and } T_{rr} = 0 \text{ at } r = b, \end{aligned} \tag{7}$$

where Θ_0 is constant, given by /13/:

$$\Theta = \frac{\Theta_0 a}{(b-a)}\left(\frac{b}{r}-1\right) \tag{8}$$

Critical points or turning points: using Eqs.(5) and (8) in Eq.(6), we get a nonlinear differential equation in β as:

$$\begin{aligned} &P(P+1)^{n-1}\beta\frac{dP}{d\beta}+P(P+1)^n+2(1-c)P+ \\ &+\frac{c\xi\bar{\Theta}_0}{2\mu r\beta^n}-\frac{2c}{n\beta^n}\left[\left\{1-\beta^n(P+1)^n\right\}-(1-\beta^n)\right]=0 \end{aligned} \tag{9}$$

where $\bar{\Theta}_0 = -\Theta_0 ab/(b-a)$, $c = 2\mu/\lambda + 2\mu$ and $r\beta' = \beta P$ (P is a function of β and β is a function of r). The transition points of β in Eq.(9) are: $P = 0$, $P \to -1$ and $P \to \pm\infty$.

DETERMINATION OF STRESSES IN ELASTIC-PLASTIC TRANSITION:

In order to calculate elastic-plastic stresses, we define the transition function γ by taking the principal stress /7-19/ at the transition point $P \to \pm\infty$. The transition function γ is given as:

$$\begin{aligned} \gamma &= 1-\frac{n}{3\lambda+2\mu}T_{rr}-\frac{n\beta_0}{3\lambda+2\mu}\left(\frac{b}{r}-1\right)= \\ &= \frac{\beta^n}{3-2c}\left[(P+1)^n+2(1-c)\right] \end{aligned} \tag{10}$$

where $\beta_0 = \xi\Theta_0 a/(b-a)$.

Taking the logarithmic differentiation of Eq.(10) with respect to r and substituting the value of $dP/d\beta$ from Eq.(9) and taking the asymptotic value $P \to \infty$, after integration we get:

$$\gamma = A_0 r^{-2c} \tag{11}$$

where: A_0 is a constant of integration.

By using Eqs.(10) and (11), we have the transition value T_{rr} as:

$$T_{rr} = \frac{2u(3-2c)}{nc}\left[1 - A_0 r^{-2c}\right] - \beta_0\left(\frac{b}{r} - 1\right) \tag{12}$$

The value of material constant E in the transition range is given by Seth.

$$Y = \frac{E}{n} = \frac{2\mu(3-2c)}{n(2-c)} \tag{13}$$

where Y is the yield stress in tension.

$$T_{rr} = \frac{Y(2-c)}{c}\left[1 - A_0 r^{-2c}\right] - \beta_0\left(\frac{b}{r} - 1\right) \tag{14}$$

By using Eq.(7) in (14), we get

$$T_{rr} = \frac{Y(2-c)}{c}\left[1 - \left(\frac{b}{r}\right)^{2c}\right] - \beta_0\left(\frac{b}{r} - 1\right). \tag{15}$$

Substituting Eq.(15) in Eq.(9), we get:

$$T_{\theta\theta} - T_{rr} = Y(2-c)\left(\frac{b}{r}\right)^{2c} + \frac{\beta_0}{2}\left(\frac{b}{r}\right) \tag{16}$$

Therefore

$$T_{\theta\theta} = T_{\phi\phi} = Y(2-c)\left(\frac{b}{r}\right)^{2c} + \\ +Y\left(\frac{2-c}{c}\right)\left[1 - \left(\frac{b}{r}\right)^{2c}\right] - \beta_0\left(\frac{b}{2r} - 1\right) \tag{17}$$

Initial yielding: it is found that the value of $|T_{\theta\theta} - T_{rr}|$ is maximum at the internal surface (i.e. $r = a$), which means that the yielding of the spherical shell will take place at the internal surface of the shell and Eq.(17) becomes:

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=a} = Y(2-c)\left(\frac{b}{a}\right)^{2c} + \frac{\beta_0}{2}\left(\frac{b}{a}\right) \tag{18}$$

The pressure required for initial yielding at the internal surface is given as:

$$p_i = \frac{Y(2-c)}{c}\left[\left(\frac{b}{a}\right)^{2c} - 1\right] + \beta_0\left(\frac{b}{a} - 1\right). \tag{19}$$

Fully-plastic state: the stresses for fully plastic state are obtained by taking ($c \to 0$) in Eqs.(15)-(17), and

$$T_{rr} = 4Y\log\left(\frac{r}{b}\right) - \beta_0^*\left(\frac{b}{r} - 1\right) \tag{20}$$

$$Y^* = \left|T_{\theta\theta} - T_{rr}\right| = 2Y + \frac{\beta_0^*}{2}\left(\frac{b}{r}\right) \tag{21}$$

where: $\beta_0^* = \lim\limits_{c\to 0}\dfrac{\alpha(3\lambda + 2\mu)\theta_0 a}{b-a}$.

Pressure required for the fully plastic state is given as:

$$p_f = 4Y\log\frac{b}{a} + \beta_0\left(\frac{b}{r} - 1\right) \tag{22}$$

The Eq.(21) shows that presence of the combined load of pressure and temperature gradient on the same surface of the spherical shell slows down the yield. Because in the presence of temperature gradient, the yield stress in compression is found to be more than $2Y$. The above result shows that Y^* depends on both Θ_0, and on the ratio of radii and yield stress in compression $2Y$. Hence a thick-walled spherical shell requires more heat to yield than a thin-walled spherical shell, so long as the pressure remains constant in both the cases.

Particular cases

Case (i): spherical shell under internal pressure only: elastic-plastic transitional stresses for the spherical shell under internal pressure only are obtained by putting $\Theta_0 = 0$ in Eqs.(15), (17) and (19), we get

$$T_{rr} = \frac{Y(2-c)}{c}\left[1 - \left(\frac{b}{r}\right)^{2c}\right] \\ T_{\theta\theta} = Y(2-c)\left(\frac{b}{r}\right)^{2c} + Y\left(\frac{2-c}{c}\right)\left[1 - \left(\frac{b}{r}\right)^{2c}\right] \tag{23}$$

$$p_i = \frac{Y(2-c)}{c}\left[\left(\frac{b}{a}\right)^{2c} - 1\right] \tag{24}$$

For fully plastic state i.e. $c \to 0$, we have

$$T_{rr} = 4Y\log\left(\frac{r}{b}\right) \tag{25}$$

$$Y^* = \left|T_{\theta\theta} - T_{rr}\right| = 2Y \tag{26}$$

The pressure required for fully plastic state is

$$p_f = 4Y\log\frac{b}{a}.$$

Case (ii): spherical shell under steady temperature: elastic-plastic transitional stresses for the spherical shell under steady temperature only are obtained from Eqs.(15)-(17) by using boundary condition $T_{rr} = 0$ at $r = a$, we get

$$T_{rr} = \beta_0\left[\left(\frac{b}{a} - 1\right)\frac{1 - \left(\frac{b}{r}\right)^{2c}}{1 - \left(\frac{b}{a}\right)^{2c}} - \left(\frac{b}{r} - 1\right)\right], \\ T_{\theta\theta} = T_{rr} + \beta_0\left[\frac{c\left(\frac{b}{a} - 1\right)\left(\frac{b}{r}\right)^{2c}}{1 - \left(\frac{b}{a}\right)^{2c}} + \left(\frac{b}{2r}\right)\right] \tag{27}$$

where $\beta_0 = \dfrac{\alpha(3\lambda + 2\mu)\Theta_0 a}{(b-a)}$.

For fully plastic state (i.e. $c \to 0$), Eq.(27) becomes:

$$T_{rr}=\beta_0\left[\left(\frac{b}{a}-1\right)\frac{\log(b/r)}{\log(b/a)}-\left(\frac{b}{r}-1\right)\right],$$
$$T_{\theta\theta}=T_{rr}+\beta_0\left[\frac{\left(1-\dfrac{b}{a}\right)}{2\log(b/a)}+\left(\frac{b}{2r}\right)\right] \tag{28}$$

NUMERICAL DISCUSSION ON ELASTIC-PLASTIC STRESSES

For calculating elastic-plastic stresses based on the above analysis, the following values have been taken: c = 0.25; 0.50; 0.75, and $B = \beta_0/Y$ = 0; 1.5. Curves are drawn between stresses along the radii ratio $R = r/b$ (see Fig. 1) for the spherical shell made of compressible materials.

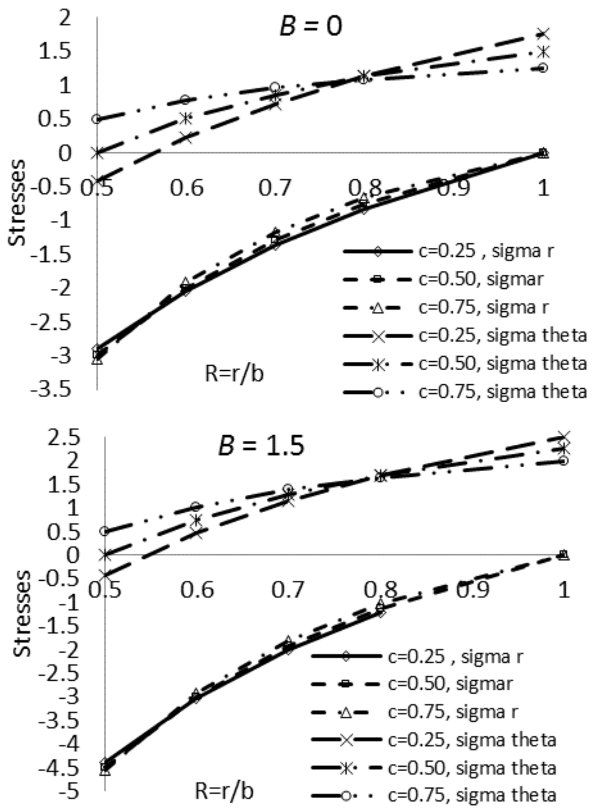

Figure 1. Distribution of elastic-plastic stresses in the spherical shell under pressure and steady state temperature, $B = \beta_0/Y$.

It is observed from Fig. 1, that the value of circumferential stress is maximum at the external surface for compressible material. Compressibility and the thermal gradient increase the value of circumferential stress at the outer surface of the spherical shell.

In Fig. 2, the curves are drawn depicting the stresses and radii ratio $R = r/b$ for the fully plastic state. It is observed that the circumferential stress has a maximum at the external surface. The thermal effect increases the value of circumferential stresses at the outer surface. The spherical shell made of incompressible material requires a higher value of circumferential stresses as compared to compressible materials.

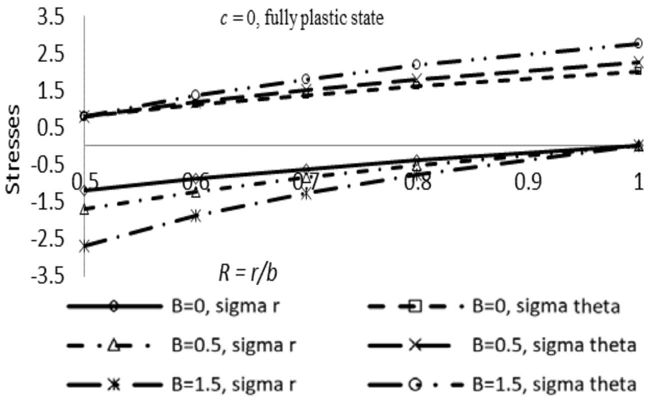

Figure 2. Distribution of elastic-plastic stresses in the spherical shell for fully plastic state.

REFERENCES

1. Bland, D.R. (1956), *Elasto-plastic thick walled tubes of work hardening materials subjected to internal and external pressures and to temperature gradients*, J Mech. Phys. Solids, 4(4): 209-229.
2. Johnson, W., Derrington, M.G. (1958), *The onset of yield in a thick spherical shell subject to internal pressure and uniform heat flow*, Appl. Scient. Res. Sec. A, 7(6): 408-420.
3. Rimrott, F.P.J. (1959), *Creep of thick-walled tubes under internal pressure considering large strains*, J Appl. Mech.-Trans. ASME, 26: 271–275.
4. Eberlein, R., Wriggers, P. (1999), *Finite element concepts for finite elasto-plastic strains and isotropic stress response in shells: theoretical and computational analysis*. Comp. Meth. Appl. Mech. Engng. 171: 243-279.
5. Civalek, Ö., Gürses, M. (2009), *Free vibration analysis of rotating cylindrical shells using discrete singular convolution technique*, Int. J Pres. Ves. & Piping, 86(10): 677-683.
6. Parkus, H., Thermoelasticity, 2nd ed. Springer-Verlag, Wien-New York, USA, 1976.
7. Thakur Pankaj (2011), *Creep transition stresses of a thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure*, Therm. Sci., 15, Suppl. issue 2: S157-S165.
8. Thakur P. (2011), *Elastic-plastic transition stresses in rotating cylinder by finite deformation under steady-state temperature*, Therm. Sci., 15(2): 537-543.
9. Thakur P. (2015), *Analysis of thermal creep stresses in transversely thick-walled cylinder subjected to pressure*, Struc. Integr. and Life, 15(1): 19-26.
10. Thakur P., Singh, S.B., Sawhney, S. (2015), *Elastic-plastic infinitesimal deformation in a solid disk under heat effect by using Seth theory*, Int. J Appl. Comput. Math., Springer, doi 10.1007/s40819-015-0116-9.
11. Thakur P., Singh, S.B., Kaur, J. (2016), *Thermal creep stresses and strain rates in a circular disc with shaft having variable density*, Engng. Comp., 33(3): 698-712.
12. Thakur, P., Gupta, N., Singh, S.B. (2017), *Creep strain rates analysis in cylinder under temperature gradient materials by using Seth's theory*, Engng. Comp., 34(3): doi: 10.1108/EC-05-2016-0159.
13. Thakur, P., Kumar, S., Singh, J., Singh, S.B., (2016), *Effect of density variation parameter in a solid disk*, Struc. Integ. and Life, 16(3): 143-148.
14. Thakur, P., Kumar, S. (2016), *Stress evaluation in a transversely isotropic circular disk with an inclusion*, Struc. Integ. and Life, 16(3): 155-160.

15. Thakur, P., Singh, S.B., Šajić Lozanović, J. (2015), *Thermo elastic-plastic deformation in a solid disk with heat generation subjected to pressure*, Struc. Integ. and Life, 15(3): 135-142.
16. Thakur, P. (2015), *Analysis of thermal creep stresses in transversely thick-walled cylinder subjected to pressure*, Struc. Integ. and Life, 15(1): 19-26.
17. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897, doi:10. 1038/195896a0.
18. Seth, B.R. (1966), *Measure-concept in mechanics*, Int. J Non-linear Mech., 1(1): 35-40.
19. Seth, B.R. (1963), *Elastic-plastic transition in shells and tubes under pressure*, ZAMM, 43(7-8): 345-351.

ESIS ACTIVITIES

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

June 12-14, 2017	European Commission funded Enlargement Workshop Material resistant to extreme conditions for future energy systems	Kyiv, Ukraine	http://www.eera-jpnm.eu/meetings/EnlWSUkrMaterials2017/
June 18-23, 2017	ICF14-Fourteenth International Conference on Fracture	Rhodes, Greece	http://www.icf14.org/
June 27-29, 2017	LCF8-Eighth International Conference on Low Cycle Fatigue	Dresden, Germany	http://www.lcf8.de/
July 3-5, 2017	VHCF7-Seventh International Conference on Very High Cycle Fatigue	Dresden, Germany	http://www.vhcf7.de/
July 3-5, 2017	Fatigue 2017	Cambridge, UK	http://www.fatigue2017.com/
August 12-19, 2017	Summer School and International workshop – symposium	Dubrovnik, Croatia	Zeljko.Bozic@fsb.hr
September 4-7, 2017	2nd Int. Conf. on Structural Integrity (ICSI2)	Madeira, Portugal	http://icsi.inegi.up.pt/
September 10-14, 2017	ESIS TC-4 Meeting 8th International Conference on Fracture of Polymers, Composites and Adhesives	Les Diablerets, Switzerland	http://www.esistc4conference.com/
September 19-22, 2017	3rd Int. Symp. on Fatigue Design and Material Defects	Lecco, Italy	http://www.fdmd3.polimi.it
September 25-27, 2017	ESIS TC-5 Meeting XXVII International Conference 'Mathematical and Computer Simulation in Mechanics of Solids and Structures'	St. Petersburg, Russia	http://www.onlinereg.ru/mcm2017
November 29-30, 2017	Fatigue Design 2017	Senlis, France	http://www.fatiguedesign.org/
August 26-31, 2018	22nd European Conference of Fracture (ECF22)	Belgrade, Serbia	http://www.ecf22.rs
August 24-26, 2018	Summer School in the scope of ECF22	Belgrade, Serbia	asedmak@mas.bg.ac.rs
September 19-21, 2018	CP 2018- 6th International Conference on 'Crack Paths'	Verona, Italy	
March 30 - April 3, 2020	VAL4, 4th International Conference on Material and Component Performance under Variable Amplitude Loading	Darmstad, Germany	First Announcement