

STRESS EVALUATION IN A TRANSVERSELY ISOTROPIC CIRCULAR DISK WITH AN INCLUSION

PRORAČUN NAPONA U TRANSVERZALNO IZOTROPNOM KRUŽNOM DISKU SA UKLJUČKOM

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Keywords

- inclusion
- isotropic
- stresses
- yielding
- disk

Abstract

It has been observed that that a rotating disk made of transversely isotropic material requires a low percentage increase in angular speed to become fully plastic as compared to a rotating disk made of isotropic material. Circumferential stresses are maximum at the external surface for the fully-plastic state.

INTRODUCTION

Disks are an essential part of the rotating machinery structure, e.g. rotors, turbines, compressors, flywheel and computer disk drive. The analytical procedures presently available are restricted to problems with simplest configurations. In the design of modern structures, increasing use is being made of materials which are transversely isotropic. The analysis of stress distribution in the circular rotating disk is important for a better understanding of the behaviour and optimal design of structures. The solution for the thin isotropic disk can be found in most of the elasticity- and plasticity standard textbooks [3-8, 14, 15]. Güven [13] found the elastic-plastic rotating disk with a rigid inclusion under the assumption of the Tresca yield condition, its associated flow rule, and linear strain hardening. To obtain the stress distribution, Guven matched the elastic-plastic stresses at the same radius $r = z$ of the disk. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of an ad-hoc rule as the yield condition amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, the transition takes place. Since this transition is non-linear in character and difficult to investigate, researchers have taken certain *ad hoc* assumptions like the yield condition, incompressibility condition, and a strain law, which may or may not be valid for this problem. Sharma et al. [9] solved problems in the elastic-plastic transition of a

Ključne reči

- uključak
- izotropan
- naponi
- tečenje
- disk

Izvod

Izvedeno je da je za rotirajući disk od transversalno izotropnog materijala potreban manji procentualni porast ugaone brzine kako bi postao potpuno plastičan, u poređenju sa rotirajućim diskom od izotropnog materijala. Tangencijalni obimski naponi imaju maksimum na spoljnoj površini pri stanju totalne plastičnosti.

transversely isotropic thin rotating disk by using Seth's transition theory. Seth's transition theory [1] does not require these assumptions and thus poses a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points or turning points of the differential equations, defining the deformed field, and has been successfully applied to a large number of problems. In this research paper, we investigate a transversely isotropic thin rotating disk with a rigid inclusion by using Seth's transition theory. Results have been discussed numerically and are depicted graphically.

GOVERNING EQUATION

Consider a thin disk of constant density with the central bore of inner radius a and outer radius b . The annular disk is mounted on a shaft. The disk is rotating with angular speed ω about an axis perpendicular to its plane, passing through the centre. The thickness of the disk is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress τ_{zz} is zero. The disk considered in the present study is with constant density and containing an inclusion.

$$u = r(1 - \beta); v = 0; w = dz \quad (1)$$

where β is position function, depending on $r = \sqrt{x^2 + y^2}$ only, and d is a constant.

The finite components of strain are given by as:

$$\begin{aligned}\varepsilon_{rr}^A &= \frac{1}{2} \left[1 - (\beta + r\beta')^2 \right], \\ \varepsilon_{\theta\theta}^A &= \frac{1}{2} \left[1 - \beta^2 \right], \\ \varepsilon_{zz}^A &= \frac{1}{2} \left[1 - (1-d)^2 \right], \\ \varepsilon_{r\theta}^A &= \varepsilon_{\theta z}^A = \varepsilon_{zr}^A = 0.\end{aligned}\quad (2)$$

where $\beta' = d\beta/dr$.

The generalized components of strain are:

$$\begin{aligned}\varepsilon_{rr} &= \frac{1}{n} \left[1 - (\beta + r\beta')^n \right], \\ \varepsilon_{\theta\theta} &= \frac{1}{n} \left[1 - \beta^n \right], \\ \varepsilon_{zz} &= \frac{1}{n} \left[1 - (1-d)^n \right], \\ \varepsilon_{r\theta} &= \varepsilon_{\theta z} = \varepsilon_{zr} = 0.\end{aligned}\quad (3)$$

The stress-strain relations for transversely isotropic material are given [3/:

$$\begin{aligned}\tau_{rr} &= c_{11}\varepsilon_{rr} + (c_{11} - 2c_{66})\varepsilon_{\theta\theta} + c_{13}\varepsilon_{zz}, \\ \tau_{\theta\theta} &= (c_{11} - 2c_{66})\varepsilon_{rr} + c_{11}\varepsilon_{\theta\theta} + c_{13}\varepsilon_{zz}, \\ \tau_{zz} &= c_{13}\varepsilon_{rr} + c_{13}\varepsilon_{\theta\theta} + c_{33}\varepsilon_{zz} = 0, \\ \tau_{zr} &= \tau_{\theta z} = \tau_{r\theta} = 0.\end{aligned}\quad (4)$$

Using Eq.(2) in Eq.(4), the strain components in terms of stresses are obtained as:

$$\begin{aligned}\varepsilon_{rr} &\equiv \frac{\partial u}{\partial r} - \frac{1}{2} \left(\frac{\partial u}{\partial r} \right)^2 = \frac{1}{2} \left[1 - (r\beta' + \beta)^2 \right] = \\ &= \frac{1}{E} \left[\tau_{rr} - \left(\frac{c_{11}c_{33} - c_{13}^2 - 2c_{66}c_{33}}{c_{11}c_{33} - c_{13}^2} \right) \tau_{\theta\theta} \right], \\ \varepsilon_{\theta\theta} &\equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2} \left[1 - \beta^2 \right] = \\ &= \frac{1}{E} \left[\tau_{\theta\theta} - \left(\frac{c_{11}c_{33} - c_{13}^2 - 2c_{66}c_{33}}{c_{11}c_{33} - c_{13}^2} \right) \tau_{rr} \right], \\ \varepsilon_{zz} &\equiv \frac{\partial w}{\partial z} - \frac{1}{2} \left(\frac{\partial w}{\partial z} \right)^2 = \frac{1}{2} \left[1 - (1-d)^2 \right] = \\ &= -\frac{1}{E} \left(\frac{c_{11}c_{33} - c_{13}^2 - 2c_{66}c_{33}}{c_{11}c_{33} - c_{13}^2} \right) [\tau_{rr} - \tau_{\theta\theta}], \\ \varepsilon_{r\theta} &= \varepsilon_{\theta z} = \varepsilon_{zr} = 0,\end{aligned}\quad (5)$$

where: $E = 4c_{66} \left(\frac{c_{11}c_{33} - c_{13}^2 - c_{66}c_{33}}{c_{11}c_{33} - c_{13}^2} \right)$ is Young's modulus.

By substituting Eqs.(3) into Eqs.(1), one gets:

$$\begin{aligned}\tau_{rr} &= \frac{A}{n} \left[2 - \beta^n \left\{ 1 + (1+P)^n \right\} \right] - 2 \frac{c_{66}}{n} \left[1 - \beta^n \right], \\ \tau_{\theta\theta} &= \frac{A}{n} \left[2 - \beta^n \left\{ 1 + (1+P)^n \right\} \right] - 2 \frac{c_{66}}{n} \left[1 - \beta^n (1+P)^n \right], \\ \tau_{r\theta} &= \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0,\end{aligned}\quad (6)$$

where: $A = c_{11} - (c_{13}^2/c_{33})$.

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr} (r\tau_{rr}) - \tau_{\theta\theta} + \rho\omega^2 r^2 = 0 \quad (7)$$

where ρ is the density of the material.

By substituting Eqs.(6) into Eq.(7), one gets a non-linear differential equation with respect to β :

$$\begin{aligned}\beta^{n+1} (1+P)^{n-1} \frac{dP}{d\beta} &= \left[\frac{\rho\omega^2 r^2}{A} + \beta^n \times \right. \\ &\left. \times \left\{ \frac{2c_{66}}{nA} \left[1 + nP - (1+P)^n - P \left\{ 1 + (1+P)^n \right\} \right] \right\} \right]\end{aligned}\quad (8)$$

where $r\beta' = \beta P$ (P is a function of β and β is a function of r). The transition points of β in Eq.(8) are $P \rightarrow -1$ and $P \rightarrow \pm\infty$.

The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk has an applied mechanical loading. Thus, the boundary conditions of the problem are given by:

$$\begin{aligned}r = a, \quad u &= 0, \quad \text{and} \\ r = b, \quad \tau_{rr} &= 0\end{aligned}\quad (9)$$

where u and τ_{rr} denote displacement and stress along the radial direction.

Solution of the problem

The asymptotic solution through the principal stress /1, 2, 9-12/ leads from elastic to plastic state at the transition point $P \rightarrow \pm\infty$. If the transition function Υ is defined as:

$$\Upsilon \equiv \tau_{\theta\theta} = \frac{A}{n} \left[2 - \beta^n \left\{ 1 + (1+P)^n \right\} \right] - 2 \frac{c_{66}}{n} \left[1 - \beta^n (1+P)^n \right] \quad (10)$$

Taking the logarithmic differentiation and substituting the value of $dP/d\beta$ from Eq.(10) in Eq.(10), one gets:

$$\begin{aligned}\frac{d}{dr} (\log \Upsilon) &= -\frac{A}{rR} \left[-\beta^n P \left\{ 1 + (1+P)^n \right\} + \frac{2c_{66}}{nA} \beta^n - \frac{2c_{66}}{nA} \beta^n (1+P)^n + \frac{4c_{66}}{A} P\beta^n + \right. \\ &\left. + \frac{\rho\omega^2 r^2}{A} - \frac{4c_{66}^2}{nA^2} \beta^n \left\{ 1 - (1+P)^n \right\} - \frac{4c_{66}^2}{A^2} P\beta^n - \frac{2c_{66}}{A^2} \rho\omega^2 r^2 \right]\end{aligned}\quad (11)$$

Taking the asymptotic value of Eq.(11) as $P \rightarrow \pm\infty$ and integrating, we get:

$$\Upsilon = A_1 r^{-C_2} \quad (12)$$

where: $c_2 = 2c_{66}/A$; $A = c_{11} - (c_{13}^2/c_{33})$, and A_1 is a constant of integration which can be determined by boundary condition.

Using Eq.(12) in Eq.(10), we have

$$\tau_{\theta\theta} = A_1 r^{-C_2} \quad (13)$$

By substituting Eq.(14) into Eq.(8), one gets:

$$\tau_{rr} = \frac{B_a}{r} + A_1 \frac{r^{-C_2}}{1-C_2} - \frac{\rho\omega^2 r^2}{3} \quad (14)$$

where: B_1 is a constant of integration which can be determined by the boundary condition.

Substituting Eq.(13) and (14) in the second Eq.(6), we get:

$$\beta = \sqrt{1 - \frac{2(1-C_2)}{E} \left[\frac{\rho\omega^2 r^2}{3} - \frac{B_1}{r} \right]}.$$

Substituting the value of β in Eq.(1), one gets

$$u = r - r \sqrt{1 - \frac{2(1-C_2)}{E} \left[\frac{\rho\omega^2 r^2}{3} - \frac{B_1}{r} \right]}, \quad (15)$$

where: $1-C_2 = \left(\frac{c_{11}c_{33} - c_{13}^2 - 2c_{66}c_{33}}{c_{11}c_{33} - c_{13}^2} \right)$; $E = 2C_{66}(2-C_2)$

is the Young's modulus.

By applying boundary conditions Eq.(9) in Eqs.(14) and (15), we get:

$$B_1 = \rho\omega^2 a^3/3, \\ A_1 = \frac{\rho\omega^2 (b^3 - a^3)(1-C_2)}{3b \cdot b^{-C_2}}.$$

Substituting values of constants A_1 and B_1 in Eqs.(13)-(15) respectively, one gets the transitional stresses and displacement as:

$$\tau_{\theta\theta} = \frac{\rho\omega^2 (b^3 - a^3)(1-C_2)}{3b} \left(\frac{r}{b} \right)^{-C_2}, \quad (16)$$

$$\tau_{rr} = \frac{\rho\omega^2}{3r} \left[(b^3 - a^3) \left(\frac{r}{b} \right)^{1-C_2} + a^3 - r^3 \right], \quad (17)$$

$$u = r - r \sqrt{1 - \frac{2(1-C_2)\rho\omega^2 (r^3 - a^3)}{3Er}}. \quad (18)$$

From Eqs.(16) and (17), one gets

$$\tau_{rr} - \tau_{\theta\theta} = \left(\frac{\rho\omega^2}{3} \right) \left[\frac{(b^3 - a^3)}{r} \left(\frac{r}{b} \right)^{1-C_2} C_2 - r^2 + \frac{a^3}{r} \right]. \quad (19)$$

Initial yielding: from Eq.(19), it is clear that $|\tau_{rr} - \tau_{\theta\theta}|$ is maximum at the internal surface (that is at $r = a$), therefore yielding will take place at the internal surface of the disk and Eq.(19) gives:

$$|\tau_{rr} - \tau_{\theta\theta}|_{r=a} = \left| \frac{\rho\omega^2 (b^3 - a^3) C_2}{3a} \left(\frac{a}{b} \right)^{1-C_2} \right| \equiv y(\text{say})$$

where y is the yielding stress. Angular velocity ω_i required for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{y} = \left| \frac{3ab^2}{(b^3 - a^3) C_2 (a/b)^{1-C_2}} \right|. \quad (20)$$

and $\omega_i = \frac{\Omega_i}{b} \sqrt{\frac{y}{\rho}}$.

Fully-plastic state: the disk becomes fully plastic ($C_2 \rightarrow 1/2$; $C \rightarrow 0$) at the external surface and Eqs.(19) become:

$$|\tau_{rr} - \tau_{\theta\theta}|_{r=b} = \left| \frac{\rho\omega^2 (a^3 - b^3)}{3b} \right| \equiv y^*,$$

where: y^* is the yielding stress. The angular velocity ω_f for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{y^*} = \left| \frac{3b^3}{(a^3 - b^3)} \right|, \quad (21)$$

where: $\omega_f = \frac{\Omega_f}{b} \sqrt{\frac{y^*}{\rho}}$.

We introduce the following non-dimensional components as: $R = r/b$, $R_0 = a/b$, $\sigma_r = \tau_{rr}/y$, $\sigma_\theta = \tau_{\theta\theta}/y$, $H^* = y^*/E$, $H = y/E$, and $U = u/b$. Elastic-plastic transitional stresses, displacement and angular speed from Eqs.(16)-(18) and Eq.(21) in non-dimensional form become:

$$\omega_f = \frac{\Omega_f}{b} \sqrt{\frac{y^*}{\rho}} \sigma_\theta = \frac{\Omega_i^2 (1-R_0^3)(1-C_2)}{3} R^{-C_2},$$

$$\sigma_r = \frac{\Omega_i^2}{3R} \left[(1-R_0^3) R^{1-C_2} + R_0^3 - R^3 \right], \quad (22)$$

$$u = r - r \sqrt{1 - \frac{2(1-C_2)\Omega_i^2 H (R^3 - R_0^3)}{3R}},$$

and $\Omega_f^2 = \left| \frac{3R_0}{(1-R_0^3) C_2 R_0^{1-C_2}} \right|$. (23)

Stresses, displacement and angular speed for fully-plastic state are obtained from Eqs.(22) and (23) and become:

$$\sigma_\theta = \frac{\Omega_f^2 (1-R_0^3)}{3},$$

$$\sigma_r = \frac{\Omega_f^2}{3R} \left[(1-R_0^3) R + R_0^3 - R^3 \right],$$

$$u = r - r \sqrt{1 - \frac{2\Omega_f^2 H^* (R^3 - R_0^3)}{3R}}, \quad (24)$$

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{y^*} = \left| \frac{3}{(1-R_0^3)} \right|.$$

Isotropic case: for isotropic materials, the material constants reduce to two only, i.e. $c_{11} = c_{22} = c_{33}$, $c_{12} = c_{21} =$

$c_{13} = c_{31} = c_{23} = c_{32} = (c_{11} - 2c_{66})$, and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. In terms of constants λ and μ , these can be written as:

$$\begin{aligned} c_{12} &= \lambda, \\ c_{11} &= \lambda + 2\mu, \\ c_{66} &= \frac{1}{2}(c_{11} - c_{12}) \equiv \mu. \end{aligned} \tag{25}$$

Elastic-plastic transitional stresses are obtained by using Eq.(25) in Eqs.(16)-(18), and (20) as:

$$\begin{aligned} \sigma_\theta &= \frac{\Omega_i^2(1-R_0^3)(1-C)}{3(2-C)} R^{-\frac{1}{2-C}}, \\ \sigma_r &= \frac{\Omega_i^2}{3R} \left[(1-R_0^3)R^{\frac{1-C}{2-C}} + R_0^3 - R^3 \right], \end{aligned} \tag{26}$$

$$\begin{aligned} u &= r - r_0 \sqrt{1 - \frac{2(1-C)\Omega_i^2 H(R^3 - R_0^3)}{3(2-C)R}}, \\ \Omega_i^2 &= \left| \frac{3(2-C)R_0^{\frac{1-C}{2-C}}}{(1-R_0^3)} \right| \end{aligned} \tag{27}$$

where: $C = 2\mu/(\lambda + 2\mu)$, $1 - C_2 = (1 - C/2 - C)$.

Fully-plastic state (isotropic case): for fully plastic state ($C \rightarrow 0$), Eq.(26) becomes:

$$\begin{aligned} \sigma_\theta &= \frac{\Omega_f^2}{6R} \left[(1-R_0^3)R^{1/2} \right], \\ \sigma_r &= \frac{\Omega_f^2}{3R} \left[(1-R_0^3)R^{1/2} + R_0^3 - R^3 \right], \end{aligned} \tag{28}$$

$$U_f = R - R_0 \sqrt{1 - H^* \left[\frac{\Omega_f^2}{3R} (R^3 - R_0^3) \right]}.$$

The disk becomes of fully plastic state ($C_2 \rightarrow 1/2$ or $C \rightarrow 0$) at the external surface and Eq.(19) becomes:

$$\left| \tau_{rr} - \tau_{\theta\theta} \right|_{r=b} = \left| \frac{\rho\omega_f^2(b^3 - a^3)}{6b} \right| \equiv y^*$$

where y^* is the yielding stress. The angular velocity ω_f for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{y^*} = \left| \frac{6}{(1-R_0^3)} \right| \tag{29}$$

where: $R_0 = a/b$, and $\omega_f = \frac{\Omega_f}{b} \sqrt{\frac{y^*}{\rho}}$.

NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating stresses and displacement based on the above analysis, the following values c_{ij} have been taken as shown in Table 1 for transversely isotropic materials and isotropic material.

From Table 2, it is clear that the rotating disk made of transversely isotropic materials requires lower values of angular speed to become fully plastic as compared to the rotating disk made of isotropic materials. Curves are drawn in Fig. 1, between angular speed Ω_i^2 required for initial yielding in the rotating disk along the radii ratios $R_0 = a/b$. It has been seen that the rotating disk made of isotropic material requires higher angular speed to yield at the internal surface as compared to the disk made of transversely isotropic materials. In Figs. 2 and 4, curves are drawn between the stress distribution and displacement for initial yielding and fully plastic state along the radius ratio $R = r/b$. Form Fig. 2, it has been observed that isotropic material requires maximum stresses and displacement as compared to the transversely isotropic materials. The radial stress is maximum at the internal surface of both isotropic and transversely isotropic material. In Fig. 3, it is seen that radial stresses for the isotropic material are maximum at the internal surface whereas for transversely isotropic material circumferential stress is maximal at the external surface.

Table 1. Elastic constants c_{ij} (in units of 10^{10} N/m²).

Materials	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}
Transversely isotropic ($C_2 = 0.69$, beryl)	2.746	0.980	0.674	5	0.883
Isotropic ($C_2 = 0.50$, brass)	3.0	1.0	1.0	3.0	1
Transversely isotropic ($C_2 = 0.64$, magnesium)	5.9	2.6	2.2	6.2	1.7

Table 2. Angular speed required for initial yielding and fully plastic state.

$0.5 \leq R \leq 1$	Material		C_2	Angular speed required for initial yielding Ω_i^2	Angular speed required for fully-plastic state Ω_f^2	Percentage increase in angular speed $\left(\sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1 \right) \times 100$
	magnesium	transversely	0.64	3.08	3.42	0.2 %
beryl	isotropic	0.69	3.42	3.42	6 %	
brass	isotropic	0.5	4.8	7	19 %	

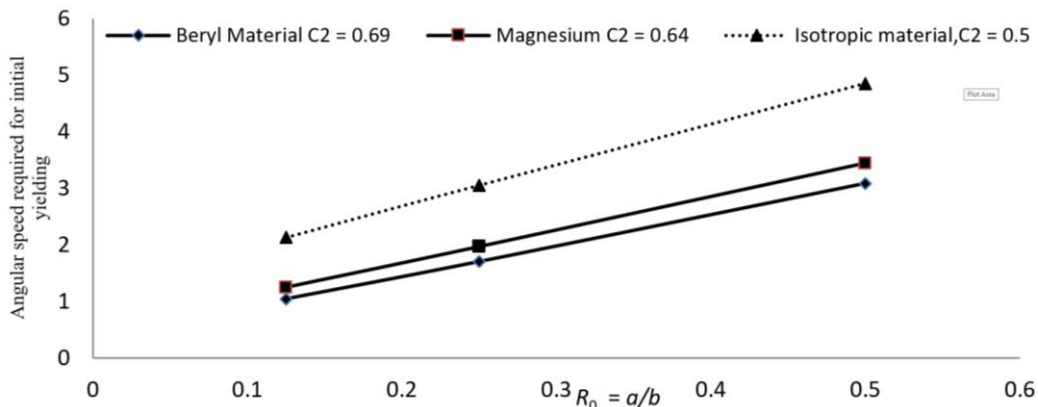


Figure 1. Angular speed required for initial yielding for a rotating disk along the radii ratio $R_0 = a/b$.

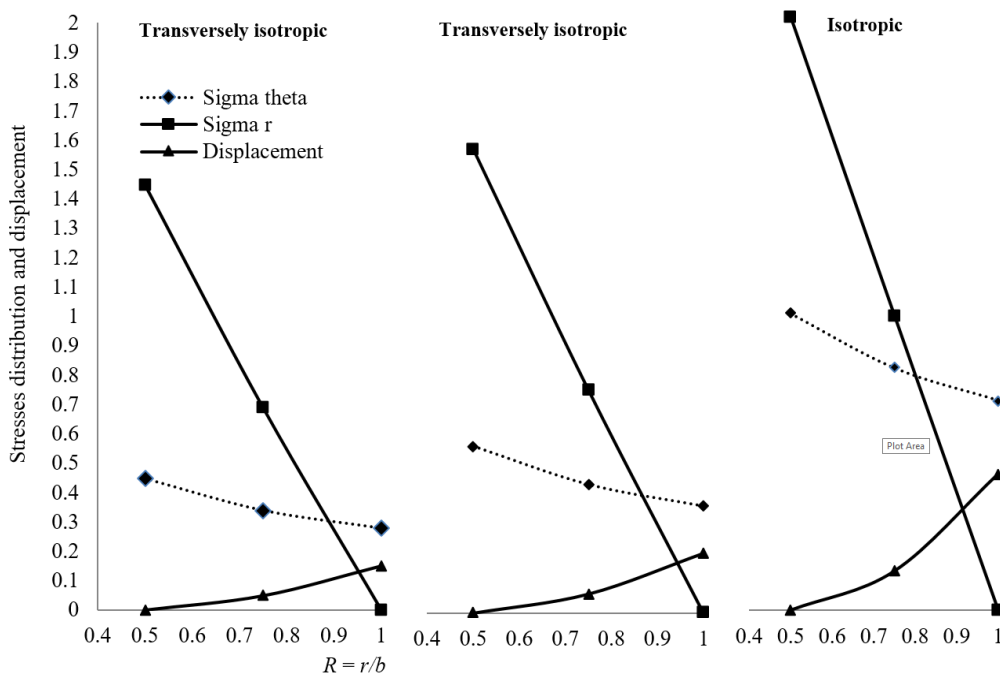


Figure 2. Angular speed required for initial yielding for a rotating disk along the radii ratio $R_0 = a/b$.

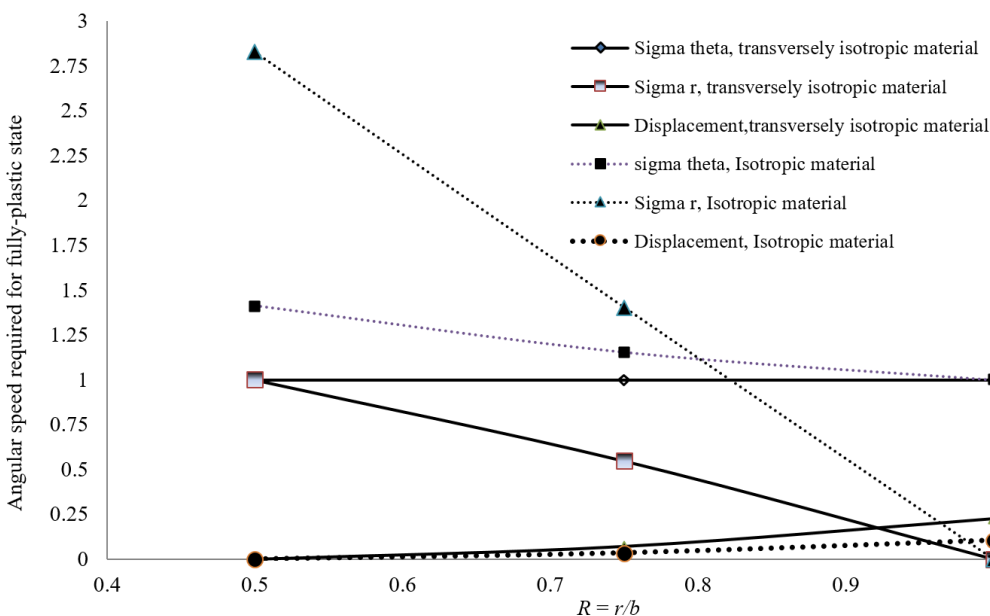


Figure 3. Stress distribution and displacement in rotating disk for fully-plastic state along the radius ratio $R = r/b$.

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ESIS ACTIVITIES

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

June 18-23, 2017	ICF14-Fourteenth International Conference on Fracture	Rhodes, Greece	http://www.icf14.org/
June 27-29, 2017	LCF8-Eighth International Conference on Low Cycle Fatigue	Dresden, Germany	http://www.lcf8.de/
July 3-5, 2017	VHCF7-Seventh International Conference on Very High Cycle Fatigue	Dresden, Germany	http://www.vhcf7.de/
September 4-7, 2017	2 nd Int. Conf. on Structural Integrity (ICSI2)	Madeira, Portugal	http://icsi.inegi.up.pt/
September 10-14, 2017	ESIS TC-4 Meeting	Les Diablerets, Switzerland	
September 19-22, 2017	3 rd Int. Symp. on Fatigue Design and Material Defects	Lecco, Italy	http://www.fdm3.polimi.it
September 25-27, 2017	ESIS TC-5 Meeting	St. Petersburg, Russia	
August 26-31, 2018	22 nd European Conference of Fracture (ECF22)	Belgrade, Serbia	
September 19-21, 2018	CP 2018- 6 th International Conference on 'Crack Paths'	Verona, Italy	