Thakur Pankaj

ANALYSIS OF THERMAL CREEP STRESSES IN TRANSVERSELY THICK-WALLED CYLINDER SUBJECTED TO PRESSURE

ANALIZA TERMIČKIH NAPONA PUZANJA KOD POPREČNOG DEBELOZIDOG CILINDRA OPTEREĆENOG NA PRITISAK

Originalni naučni rad / Original scientific paper
UDK /UDC: 539.3/.4
Rad primljen / Paper received: 17.06.2014

Adresa autora / Author's address:
IEC University Baddi, Solan, Himachal Pradesh, India

Keywords
- elastic
- plastic
- temperature
- pressure
- cylinder
- transversely
- isotropic

Abstract

Creep stresses for a transversely isotropic thick-walled cylinder subjected to internal pressure under steady state temperature have been obtained by using Seth's transition theory. Results obtained have been discussed numerically and depicted graphically. It is seen that the rotating circular cylinder under internal pressure of transversely isotropic material is on the safer side of design as compared to the rotating circular cylinder under internal pressure of isotropic material.

Ključne reči
- elastičnost
- plastičnost
- temperatura
- pritisak
- cilindar
- poprečni
- izotropan

Izvod

Naponi puzanja kod transverzalnog izotropnog debelozidog cilindra opterećenog unutrašnjim pritiskom i stacionarnim uticajem temperature su određeni primenom Setove teorije prelaznog stanja. Dobijeni rezultati su diskutovani sa numeričkog aspekta i predstavljeni grafički. Uočava se da je rotirajući kružni cilindar pod dejstvom unutrašnjeg pritiska na poprečni izotropni materijal, sigurniji sa aspekta konstruisanja u poređenju sa rotirajućim kružnim diskom pod dejstvom unutrašnjeg pritiska od izotropnog materijala.

INTRODUCTION

Thick-walled circular cylinders are used commonly either as pressure vessels intended for storage of industrial gases or for a media transport of high pressurised fluids. Many authors /2, 3, 4/ have discussed creep of thick-walled cylinder under internal pressure. Rimrott analysed the above problem by considering large strains. These authors made the following assumptions:

1. The volume of the material remains constant, or
$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 .$$
2. The ratios of principal shear strain rates to the principle shear stresses are equal, i.e.
$$\frac{\dot{\varepsilon}_{\theta\theta} - \dot{\varepsilon}_{rr}}{\sigma_{\theta\theta} - \sigma_{rr}} = \frac{\dot{\varepsilon}_{rr} - \dot{\varepsilon}_{zz}}{\sigma_{rr} - \sigma_{zz}} = \frac{\dot{\varepsilon}_{zz} - \dot{\varepsilon}_{\theta\theta}}{\sigma_{zz} - \sigma_{\theta\theta}} .$$
3. The axial strain rate is zero, i.e. $\dot{\varepsilon}_z = 0$.
4. There is a significant stress–rate of true strain relationship which coincides with the true stress–creep rate relationship in simple tension, for example Norton's Law.
5. The creep deformation is infinitesimally small.

Seth's transition /1/ does not require any assumptions stated above and thus poses and solves a more general problem from which cases pertaining to these assumptions can be worked out. It utilises the concept of generalised strain measure and the asymptotic solution at turning points or transition points of the governing equation defining the deformed field. It has successfully been applied to a number of creep problems.

Seth, /19/, has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)\right], \; i,j = 1,2,3 \qquad (1)$$

where *'n'* is the measure and $\overset{A}{e}_{ii}$ are Almansi finite strain components. For $n = -2, -1, 0, 1, 2$ it gives Cauchy, Green Hencky, Swainger and Almansi measures, respectively.

In this paper, we calculated creep stresses in a transversely isotropic thick-walled cylinder under internal pressure under steady-state temperature by using Seth's transition theory.

GOVERNING EQUATIONS

Consider a thick-walled circular cylinder of transversely isotropic material of internal and external radii *a* and *b*, respectively, under the combined effect of pressure *p* and temperature T_0 applied at the internal surface.

The displacement component in cylindrical polar co-ordinates are given as:

$$u = r(1-\beta); \; v = 0 \; \text{ and } \; w = dz \qquad (2)$$

where β is a function of $r = \sqrt{x^2 + y^2}$ only, and *d* is a constant.

The finite strain components are given by Seth /19/ as:

$$\begin{aligned}\overset{A}{e}_{rr} &= \frac{1}{2}\left[1-(\beta+r\beta')^2\right]\\ \overset{A}{e}_{\theta\theta} &= \frac{1}{2}\left[1-\beta^2\right]\\ \overset{A}{e}_{zz} &= \frac{1}{2}\left[1-(1-d)^2\right]\\ \overset{A}{e}_{r\theta} &= \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0\end{aligned} \tag{3}$$

where $\beta' = d\beta/dr$ and the meaning of superscripts 'A' is Almansi.

By substituting Eq.(3) into Eq.(1), the generalized components of strain are:

$$\begin{aligned}e_{rr} &= \frac{1}{n}\left[1-(\beta+r\beta')^n\right]\\ e_{\theta\theta} &= \frac{1}{n}\left[1-\beta^n\right]\\ e_{zz} &= \frac{1}{n}\left[1-(1-d)^n\right]\\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0\end{aligned} \tag{4}$$

The stress-strain relations for transversely isotropic material are given by /10/:

$$\begin{aligned}T_{rr} &= C_{11}e_{rr} + (C_{11}-2C_{66})e_{\theta\theta} + C_{13}e_{zz} - \beta_1 T\\ T_{\theta\theta} &= (C_{11}-2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz} - \beta_2 T\\ T_{zz} &= C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz} - \beta_2 T\\ T_{zr} &= T_{\theta z} = T_{r\theta} = 0\end{aligned} \tag{5}$$

where $\beta_1 = C_{11}\alpha_1 + 2C_{12}\alpha_2$, $\beta_2 = C_{12}\alpha_1 + (C_{22} + C_{33})\alpha_2$, C_{ij} are elastic parameters; T – temperature change; α_1 – coefficient of linear thermal expansion along the axis of symmetry; and α_2 – corresponding quantities orthogonal to axis of symmetry.

By substituting Eqs.(4) in Eqs.(5), one gets:

$$\begin{aligned}T_{rr} &= \frac{C_{11}}{n}\left[1-(\beta+r\beta')^n\right] + \left(\frac{C_{11}-2C_{66}}{n}\right)\left[1-\beta^n\right] + C_{13}e_{zz} - \beta_1 T\\ T_{\theta\theta} &= \left(\frac{C_{11}-2C_{66}}{n}\right)\left[1-(\beta+r\beta')^n\right] + \frac{C_{11}}{n}\left[1-\beta^n\right] + C_{13}e_{zz} - \beta_2 T\\ T_{zz} &= \frac{C_{13}}{n}\left[1-(\beta+r\beta')^n\right] + \frac{C_{13}}{n}\left[1-\beta^n\right] + C_{33}e_{zz} - \beta_2 T\\ T_{r\theta} &= T_{\theta z} = T_{zr} = 0\end{aligned} \tag{6}$$

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr}(T_{rr}) + \frac{T_{rr}-T_{\theta\theta}}{r} = 0 \tag{7}$$

The temperature field satisfying the Fourier heat equation $\nabla^2 T = 0$ and $T = T_0$ at $r = a$, $T = 0$ at $r = b$, where T_0 is a constant, is given by /9/:

$$T = \frac{T_0}{\log\frac{a}{b}}\log\frac{r}{b} \tag{8}$$

By substituting Eqs.(6) and (8) into Eq.(7), one gets a non-linear differential equation with respect to β as:

$$\begin{aligned}&nPC_{11}\beta^{n+1}(1+P)^{n-1}\frac{dP}{d\beta} = -nPC_{11}\beta^n(1+P)^n -\\ &-(C_{11}-2C_{66})nP\beta^n + 2C_{66}\left[1-\beta^n(1+P)^n\right] -\\ &-2C_{66}(1-\beta^n) - n\beta_1\overline{T}_0 + (\beta_2-\beta_1)n\log(r/b)\overline{T}_0\end{aligned} \tag{9}$$

where $\overline{T}_0 = T_0/\log(a/b)$ and $r\beta' = \beta P$ (P is function of β, and β is function of r).

The transitional points of β in Eq.(9) are $P \to -1$ and $P \to \pm\infty$. Boundary conditions are given by:

$$\begin{aligned}T_{rr} &= -p \quad \text{at} \quad r = a\\ T_{rr} &= 0 \quad \text{at} \quad r = b\end{aligned} \tag{10}$$

The resultant force normally applied to the ends of the cylinder is:

$$2\pi\int_a^b rT_{zz}dr = \pi a^2 p \tag{11}$$

SOLUTION THROUGH PRINCIPAL STRESS DIFFERENCE

It has been shown that the transition function through the stress difference /5-18/ at the transition point $P \to -1$ gives the creep stresses. For finding creep stresses at the transition point $P \to -1$, we define the transition function R as:

$$R = T_{rr} - T_{\theta\theta} = \frac{2C_{66}}{n}\beta^n\left[1-(1+P)^n\right] + (\beta_2-\beta_1)T \tag{12}$$

and by taking the logarithmic differentiation of Eq.(12) with respect to r, one gets:

$$\begin{aligned}\frac{d}{dr}(\log R) = \frac{1}{rR}\Big[&2C_{66}P\beta^n\left[1-(1+P)^n\right] -\\ &-2C_{66}P\beta^{n+1}(1+P)^{n-1}\frac{dP}{d\beta} + \overline{T}_0(\beta_2-\beta_1)\Big]\end{aligned} \tag{13}$$

By substituting the value of $dP/d\beta$ from Eq.(9) into Eq.(13), one gets:

$$\begin{aligned}\frac{d}{dr}(\log R) = \frac{1}{rR_6}\Big[&2C_{66}P\beta^n\left[1-(1+P)^n\right] + \overline{T}_0(\beta_2-\beta_1) +\\ &+2C_{66}P\beta^n(1+P)^n + C_1\beta_1\overline{T}_0 + 2C_{66}P\beta^n(1-C_1) -\\ &-\frac{2C_{66}}{n}\beta^n C_1\left[1-(1+P)^n\right] - C_1 T_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\Big]\end{aligned} \tag{14}$$

where $C_1 = 2C_{66}/C_{11}$.

Asymptotic value of Eq.(14) as $P \to -1$ is:

$$\begin{aligned}\frac{d}{dr}(\log R) &=\\ &= \frac{-4C_{66}\beta^n + \overline{T}_0(\beta_2-\beta_1) + 2C_1C_{66}\beta^n +}{r\left[\frac{2C_{66}}{n}\beta^n + \overline{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]}\\ &\quad \frac{+C_1\beta_1\overline{T}_0 - \frac{2C_{66}}{n}C_1\beta^n - C_1\overline{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)}{r\left[\frac{2C_{66}}{n}\beta^n + \overline{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]}\end{aligned} \tag{15}$$

where asymptotic value of β as $P \to 1$ is D/r, D being a constant.

By integrating Eq.(15) with respect to r, one gets:

$$R = A_1 r^{-C_1}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr \tag{16}$$

where A_1 is a constant of integration which can be determined by boundary condition and $C_1 = 2C_{66}/C_{11}$, and

$$f = \int \frac{\dfrac{2C_{66}D^n C_1}{r^{n+1}} + \dfrac{\bar{T}_0\{\beta_1(1+C_1)-\beta_2\}}{r}}{\dfrac{2C_{66}}{n}\beta^n + \bar{T}_0(\beta_2-\beta_1)\log\left(\dfrac{r}{b}\right)} dr \ .$$

By substituting Eq.(16) into Eq.(12), one gets:

$$R = T_{rr} - T_{\theta\theta} = = -A_1 \int r^{-C_1}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr \tag{17}$$

By substituting Eq.(17) into Eq.(7), one gets:

$$T_{rr} = -A_1\,\mathrm{I} + A_2 \tag{18}$$

where I is the integral:

$$\mathrm{I} = \int r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr$$

and where A_2 is a constant of integration that can be determined by the boundary condition.

By substituting Eq.(17) into Eq.(18), one gets:

$$T_{\theta\theta} = T_{rr} - A_1 r^{-C_1}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f \tag{19}$$

Constants A_1 and A_2 are obtained by using boundary conditions given by Eq.(10) into Eq.(18), thus one gets:

$$A_1 = \frac{-p}{\int_a^b r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr}$$

and

$$A_2 = \frac{-p\left[\int r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr\right]_{r=b}}{\int_a^b r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr}$$

By substituting values of A_1 and A_2 from Eqs.(18) and (19), one gets:

$$T_{rr} = = \frac{-p\left[\int_b^r r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr\right]}{\int_a^b r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr} \tag{20}$$

$$T_{\theta\theta} = = T_{rr} + \frac{-p r^{-C_1}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f}{\int_a^b r^{-(1+C_1)}\left[\frac{2C_{66}}{n}\frac{D^n}{r^n} + \bar{T}_0(\beta_2-\beta_1)\log\left(\frac{r}{b}\right)\right]^2 \exp f dr} \tag{21}$$

The axial stress is obtained from Eq.(6) as:

$$T_{zz} = \frac{C_{13}}{2(C_{11}-C_{66})}\left[T_{rr}+T_{\theta\theta}\right] + \left[\frac{C_{33}(C_{11}-C_{66})-C_{13}^2}{C_{11}-C_{66}}\right]e_{zz} + + \frac{T}{2(C_{11}-C_{66})}\left[C_{13}(\beta_1+\beta_2) - 2\beta_2(C_{11}-C_{66})\right] \tag{22}$$

By applying condition Eq.(11) into Eq.(22), the axial strain is given by:

$$e_{zz} = \frac{(C_{11}-C_{66})}{\left[C_{33}(C_{11}-C_{66})-C_{13}^2\right]}\left[\frac{a^2 p\left[C_{11}-C_{66}-C_{13}\right]}{(b^2-a^2)(C_{11}-C_{66})} + +\beta_0\left\{C_{13}\left(1+\frac{\beta_2}{\beta_1}\right) - \frac{2\beta_2}{\beta_1}(C_{11}-C_{66})\right\}\left\{\frac{a^2\log(a/b)}{b^2-a^2}+\frac{1}{2}\right\}\right] \tag{23}$$

Eqs.(20)–(22) give thermal creep stresses for a thick-walled cylinder under internal pressure. Now we introduce the following non-dimensional quantities:

$$R = \frac{r}{b},\ R_0 = \frac{a}{b},\ \sigma_r = \frac{T_{rr}}{p},\ \sigma_\theta = \frac{T_{\theta\theta}}{p},\ \sigma_z = \frac{T_{zz}}{p},\ p_1 = \frac{p}{C_{66}}$$

Creep stresses, Eqs.(20)–(22), in non-dimensional form become:

$$\sigma_r = \frac{-\int_R^1 R^{-(1+C_1)}\left[\frac{2C_{66}}{nR^n}\left[\frac{D}{b}\right]^n + \beta_0\left(\frac{\beta_2}{\beta_1}-1\right)\log R\right]^2 \exp f_1 dR}{\int_{R_0}^1 r^{-(1+C_1)}\left[\frac{2C_{66}}{nR^n}\left[\frac{D}{b}\right]^n + \beta_0\left(\frac{\beta_2}{\beta_1}-1\right)\log R\right]^2 \exp f_1 dR} \tag{24}$$

$$\sigma_\theta = \sigma_r + + \frac{-R^{-C_1}\left[\left[\frac{2C_{66}}{nR^n}\left(\frac{D}{b}\right)^n + \beta_0\left(\frac{\beta_2}{\beta_1}-1\right)\log R\right]^2 \exp f_1\right]}{\int_{R_0}^1 r^{-(1+C_1)}\left[\frac{2C_{66}}{nR^n}\left(\frac{D}{b}\right)^n + \beta_0\left(\frac{\beta_2}{\beta_1}-1\right)\log R\right]^2 \exp f_1 dR} \tag{25}$$

$$\sigma_z = \frac{1-C_1}{2-C_1}(\sigma_r+\sigma_\theta) - \frac{C_1 R_0^2}{(2-C_1)(1-R_0^2)} + +\beta_0\left[\left(\frac{1-C_1}{2-C_1}\right)\left(1+\frac{\beta_2}{\beta_1}\right) - \frac{2\beta_2}{\beta_1}\right]\left[\frac{R_0^2}{(1-R_0^2)}\log R_0 + \frac{1}{2} + \log R\right] \tag{26}$$

where $f_1 = \int \dfrac{\dfrac{2C_{66}C_1}{R^{n+1}}\left(\dfrac{D}{b}\right)^n + \dfrac{\beta_0}{R}\left[1+C_1-\dfrac{\beta_2}{\beta_1}\right]}{\dfrac{2C_{66}}{nR^n}\left(\dfrac{D}{b}\right)^n + \beta_0\left[\dfrac{\beta_2}{\beta_1}-1\right]\log R} dR$,

and $\beta_0 = \beta_1 \bar{T}_0$.

Case with negligible temperature

Creep stresses for thick-walled cylinder without thermal effect are obtained by placing $\overline{T}_0 = 0$ into Eqs.(24)–(26):

$$\sigma_r = -\frac{R^{-2n+C_1(n-1)}-1}{R_0^{-2n+C_1(n-1)}-1} \tag{27}$$

$$\sigma_\theta = \sigma_r - \frac{\left[-2n+C_1(n-1)\right]R^{-2n+C_1(n-1)}}{R_0^{-2n+C_1(n-1)}-1} \tag{28}$$

$$\sigma_z = \left(\frac{1-C_1}{2-C_1}\right)(\sigma_r+\sigma_\theta) - \frac{C_1R_0^2}{(2-C_1)(1-R_0^2)} \tag{29}$$

Isotropic case

For isotropic materials, the material constants reduce to two only, i.e., $C_{11} = C_{22} = C_{33}$, $C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} - 2C_{66})$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. In term of constants λ and μ, these can be written as: $C_{12} = \lambda$, $C_{66} = ½(C_{11} - C_{12}) \equiv \mu$ and $C_{11} = \lambda + 2\mu$.

For isotropic materials Eqs.(24–26) become:

$$\sigma_r = \frac{-\int_R^1 R^{-2n+C(n-1)-1}\exp f_2\,dR}{\int_{R_o}^1 R^{-2n+C(n-1)-1}\exp f_2\,dR} \tag{30}$$

$$\sigma_\theta = \sigma_r + \frac{-R^{-2n+C(n-1)}\exp f_2}{\int_{R_0}^1 R^{-2n+C(n-1)-1}\exp f_2\,dR} \tag{31}$$

$$\sigma_z = \left(\frac{1-C}{2-C}\right)(\sigma_r+\sigma_\theta) - \frac{CR_0^2}{(2-C)(1-R_0^2)} \tag{32}$$

where $f_2 = (\overline{T}_0\alpha)\left(\frac{D}{b}\right)^n R^n(3-2C)$.

Case with negligible temperature (isotropic case)

Creep stresses for thick-walled cylinder without thermal effect are obtained by placing $\overline{T}_0 = 0$ in Eqs.(30–32):

$$\sigma_r = -\frac{R^{-2n+C_1(n-1)}-1}{R_0^{-2n+C(n-1)}-1} \tag{33}$$

$$\sigma_\theta = \sigma_r - \frac{\left[-2n+C(n-1)\right]R^{-2n+C(n-1)}}{R_0^{-2n+C(n-1)}-1} \tag{34}$$

$$\sigma_z = \left(\frac{1-C}{2-C}\right)(\sigma_r+\sigma_\theta) - \frac{CR_0^2}{(2-C)(1-R_0^2)} \tag{35}$$

Incompressible material

Creep stresses for thick-walled cylinder under internal pressure for incompressible material are obtained from Eqs.(33–35):

$$\sigma_r = -\frac{R^{-2n}-1}{R_0^{-2n}-1} \tag{36}$$

$$\sigma_\theta = \sigma_r + 2n\frac{R^{-2n}}{R_0^{-2n}-1} \tag{37}$$

$$\sigma_z = \frac{\sigma_r+\sigma_\theta}{2} \tag{38}$$

These equations are the same as obtained by Gupta, /11/.

Strain rates

The stress-strain rate relationship can be given as:

$$\begin{aligned}
\dot{e}_{rr} &= -\frac{(A-2C_{66})}{H}\Theta + 2\frac{(A-2C_{66})}{H}T_{rr} + \frac{2C_{66}\beta_2 T}{H}\left(1-\frac{C_{13}}{C_{33}}\right) + \\
&\quad + \frac{AT\beta_1}{H}\left(1-\frac{\beta_2}{\beta_1}\right) + \frac{T_{zz}}{H}\left(A-2C_{66}-\frac{2C_{13}C_{66}}{C_{33}}\right) \\
\dot{e}_{\theta\theta} &= \frac{2C_{66}\beta_1 T}{H}\left(1-\frac{\beta_2}{\beta_1}\frac{C_{13}}{C_{33}}\right) - \frac{AT\beta_1}{H}\left(1-\frac{\beta_2}{\beta_1}\right) + \frac{2(A-2C_{66})}{H}T_{\theta\theta} - \\
&\quad - \frac{(A-2C_{66})}{H}\Theta + \frac{T_{zz}}{H}\left(A-2C_{66}-\frac{2C_{13}C_{66}}{C_{33}}\right) \\
\dot{e}_{zz} &= -\frac{2C_{13}C_{66}}{HC_{33}}\Theta + \frac{\beta_2 T}{C_{33}} - \frac{2C_{13}C_{66}\beta_1 T}{HC_{33}} \times \\
&\quad \times\left[1+\frac{\beta_2}{\beta_1} - 2\frac{\beta_2 C_{13}}{\beta_1 C_{33}}\right] + \frac{T_{zz}}{C_{33}}\left[\frac{C_{11}-C_{66}+2C_{13}}{4(A-C_{66})}\right]
\end{aligned} \tag{39}$$

where $\dot{e}_{rr}$, $\dot{e}_{\theta\theta}$, $\dot{e}_{zz}$ is the strain rate tensor with respect to flow parameter t and $\Theta = T_{rr} + T_{\theta\theta} + T_{zz}$, $H = 4C_{66}(A - C_{66})$, $A = C_{11} - (C_{13}^2/C_{33})$.

Differentiating Eq.(4) with respect to time t, one gets:

$$\dot{e}_{\theta\theta} = -\beta^{n-1}\dot{\beta} \tag{40}$$

For Swainger measure ($n = 1$), we have from Eq.(40):

$$\dot{\varepsilon}_{\theta\theta} = -\dot{\beta} \tag{41}$$

The transition value of Eq.(12) as $P \to -1$ gives:

$$\beta = \left\{\frac{n}{2C_{66}}\left[T_{rr} - T_{\theta\theta} + \beta_1 T\left(1-\frac{\beta_2}{\beta_1}\right)\right]\right\}^{\frac{1}{n}} \tag{42}$$

Using Eqs.(40), (41) and (42) in Eq.(39), one gets:

$$\begin{aligned}
\dot{\varepsilon}_{rr} &= \chi\left[\frac{2\beta_2\overline{T}_0\log R}{\eta}\left(1-\frac{C_{13}}{C_{33}}\right) - \frac{\alpha}{\eta}(\sigma_r+\sigma_\theta+\sigma_z) + \right. \\
&\quad \left. + \frac{\sigma_r}{2} + \frac{\beta\beta_0\log R}{\eta C_{66}}\left(1-\frac{\beta_2}{\beta_1}\right) + \frac{\sigma_z}{\eta}\left(\alpha - \frac{2C_{13}C_{66}}{C_{11}C_{33}}\right)\right] \\
\dot{\varepsilon}_{\theta\theta} &= \dot{\varepsilon}_{rr} = \chi\left[\frac{2\beta_0\log R}{C_{11}\eta}\left(1-\frac{\beta_2}{\beta_1}\frac{C_{13}}{C_{33}}\right) - \frac{\beta\beta_0\log R}{\eta C_{66}}\left(1-\frac{\beta_2}{\beta_1}\right) + \right. \\
&\quad \left. + \frac{\sigma_\theta}{2} - \frac{\alpha}{\eta}(\sigma_r+\sigma_\theta+\sigma_z) + \frac{\sigma_z}{\eta}\left(\alpha - \frac{2C_{13}C_{66}}{C_{11}C_{33}}\right)\right] \\
\dot{\varepsilon}_{zz} &= \dot{\varepsilon}_{rr} = \chi\left[-\frac{2C_{13}C_{66}}{C_{11}C_{33}\eta}(\sigma_r+\sigma_\theta+\sigma_z) + \frac{\beta_2 T}{C_{33}} - \right. \\
&\quad \left. - \frac{2C_{13}\beta_0\log R}{C_{11}C_{33}\eta}\left(1+\frac{\beta_2}{\beta_1} - 2\frac{C_{13}}{C_{33}}\frac{\beta_2}{\beta_1}\right) + \frac{\sigma_z C_{66}}{\eta C_{33}}\left(1-\frac{C_{66}}{C_{11}}+\frac{2C_{13}}{C_{11}}\right)\right]
\end{aligned} \tag{43}$$

where

$$\eta = 4\left(1 - \frac{C_{13}^2}{C_{11}C_{33}} - \frac{C_{66}}{C_{11}}\right),$$

$$\chi = \left\{\frac{n}{2}\left[\sigma_r - \sigma_\theta + \beta_0 \log(R)\left(1 - \frac{\beta_2}{\beta_1}\right)\right]\right\}^{\frac{1}{n}-1},$$

$$\beta = 1 - \frac{C_{13}^2}{C_{11}C_{33}}, \text{ and } \alpha = 1 - \frac{C_{13}^2}{C_{11}C_{33}} - \frac{2C_{66}}{C_{11}}.$$

NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating the stress distribution based on the above analysis, the following values of measure n, D and temperature β_0 have been taken as: n = 1, 1/3 and 1/7 (i.e. N = 1, 3 and 7); β_0 = 0, 0.50, 0.75 and D = 1. The elastic constants C_{ij} for transversely isotropic material (magnesium) and isotropic material (brass) are given in Table 1.

Table 1. Elastic constants C_{ij} (in terms of 10^{10} N/m^2).

	C_{44}	C_{11}	C_{12}	C_{13}	C_{33}
Transversely isotropic material (Mg)	1.64	5.97	2.62	2.17	6.17
Isotropic material (Brass)	1.0	3.0	1.0	1.0	3.0

Curves have been drawn in Figs. 1–3 as creep stress vs. radii ratio $R = r/b$ for transversely isotropic material/isotropic material with and without thermal effects and different measure. It can be seen from Fig. 1 that without temperature, circumferential stress is maximum at the internal surface for transversely isotropic/isotropic circular cylinder under internal pressure for measure n = 1 and n = 1/3 (i.e. N = 1, 3) while for measure n = 1/7 (i.e. N = 7), the circumferential stress is maximum at the external surface. It can be seen from Figs. 2 and 3 that with the introduction of thermal effects, the circumferential stress again is maximum at the internal surface for transversely isotropic circular cylinder under internal pressure. With increased value of temperature, it can be seen that the circumferential stress is increasing at the internal surface for transversely isotropic circular cylinder under internal pressure as compared to the isotropic circular cylinder. With increase in measure, the circumferential stress is decreasing at the internal surface for transversely isotropic/isotropic circular cylinder under internal pressure. With the introduction of thermal effects, it can be seen from Figs. 4–6 that the stress at the internal surface is decreasing with the increase of measure n.

Nomenclature

a,b – internal and external radii of cylinder (m)
C_1 – compressibility factor (-)
u, v, w – displacement components (m)
T_{ij}, e_{ij} – stress and strain rate tensor
A_1, A_2 – constants of integration
P – internal pressure (Pa)
σ_r – radial stress component (T_{rr}/P) (-)
σ_θ – circumferential stress ($T_{\theta\theta}/P$) (-)
σ_z – axial stress component (T_{zz}/P) (-)
T – temperature (°F)
$R = r/b$ – radius ratio (-)
$R_0 = a/b$ – radii ratio (-)

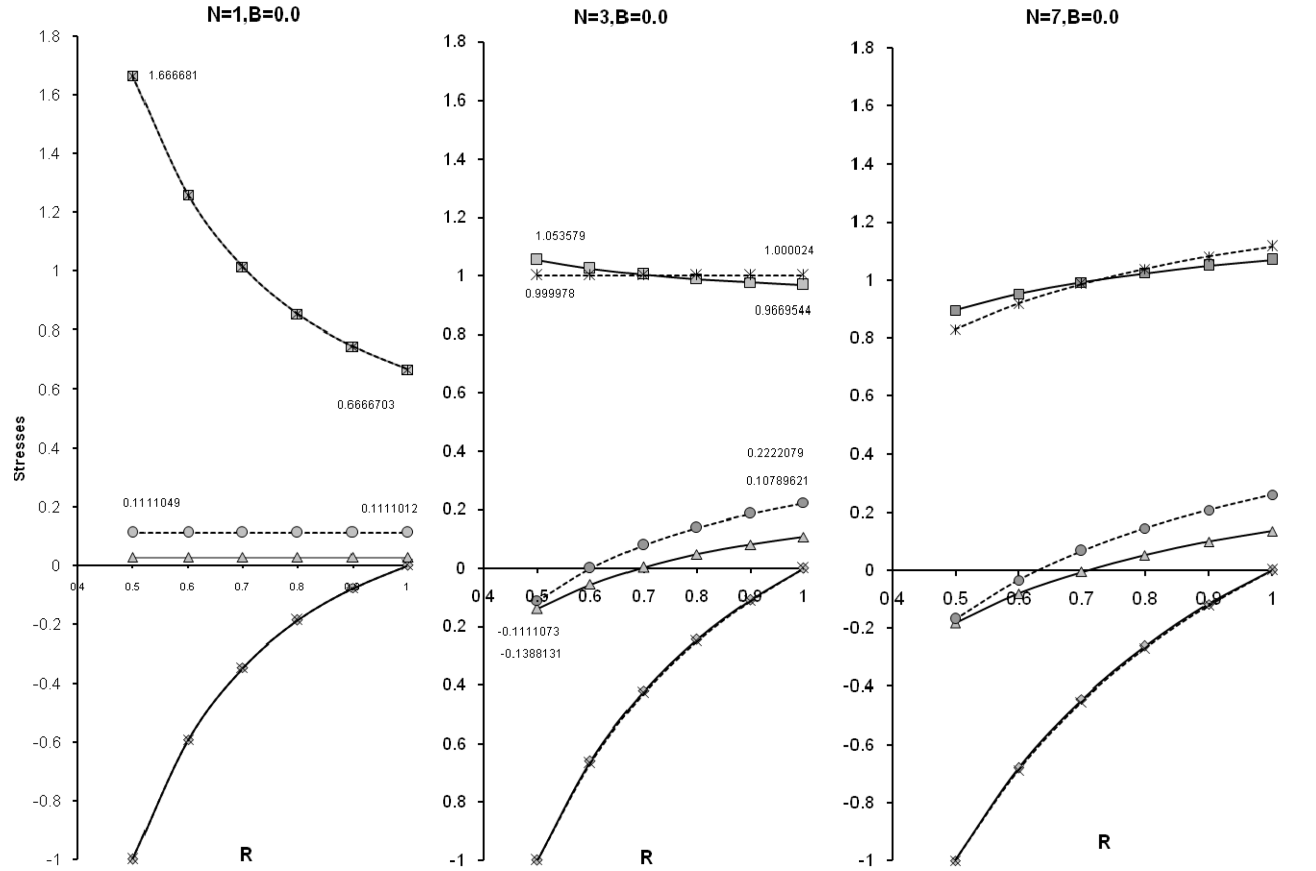

Figure 1 Creep stresses in transv. isotropic thick-walled cyl. under int. pressure along R for different N (= $1/n$) and without temp. (β_0 = 0).

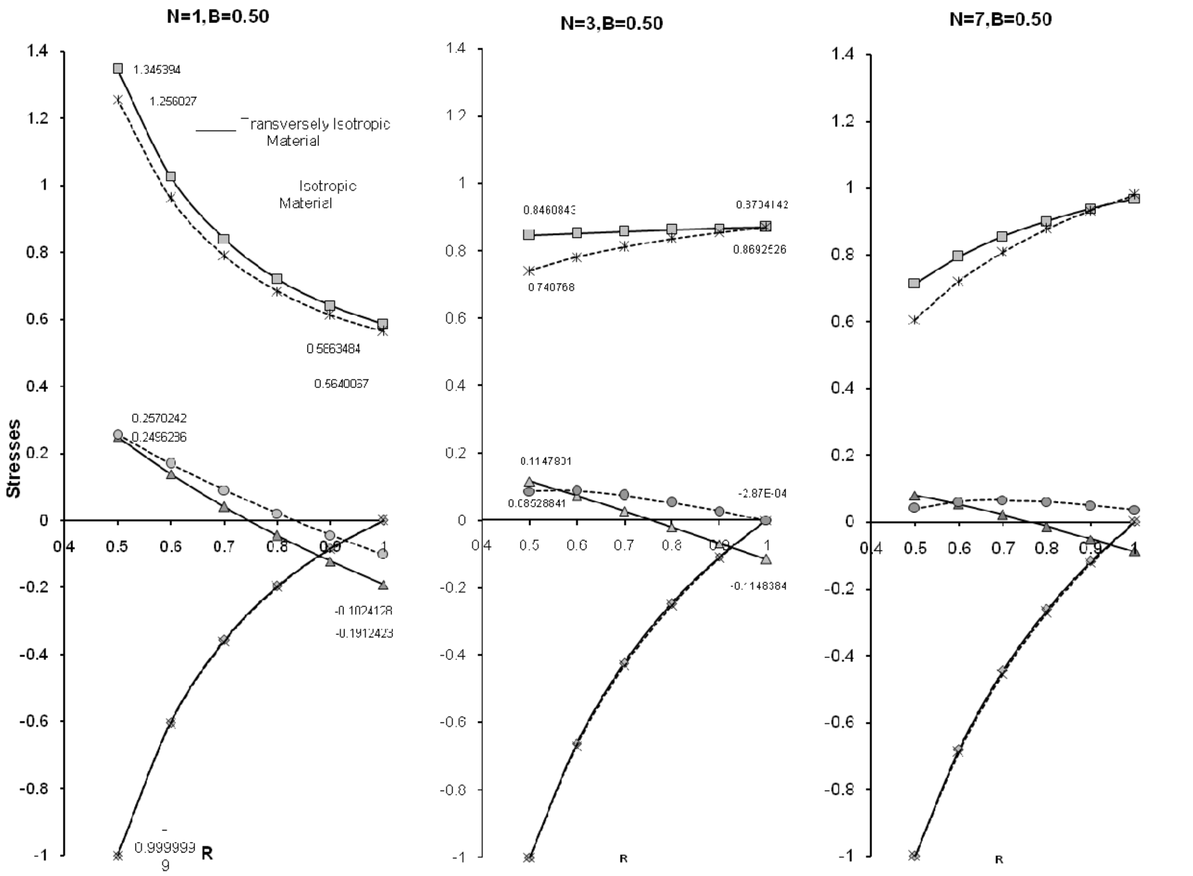

Figure 2. Creep stresses in transv. isotropic thick-walled cyl. under int. pressure along R for different N and temperature $\beta_0 = 0.50$.

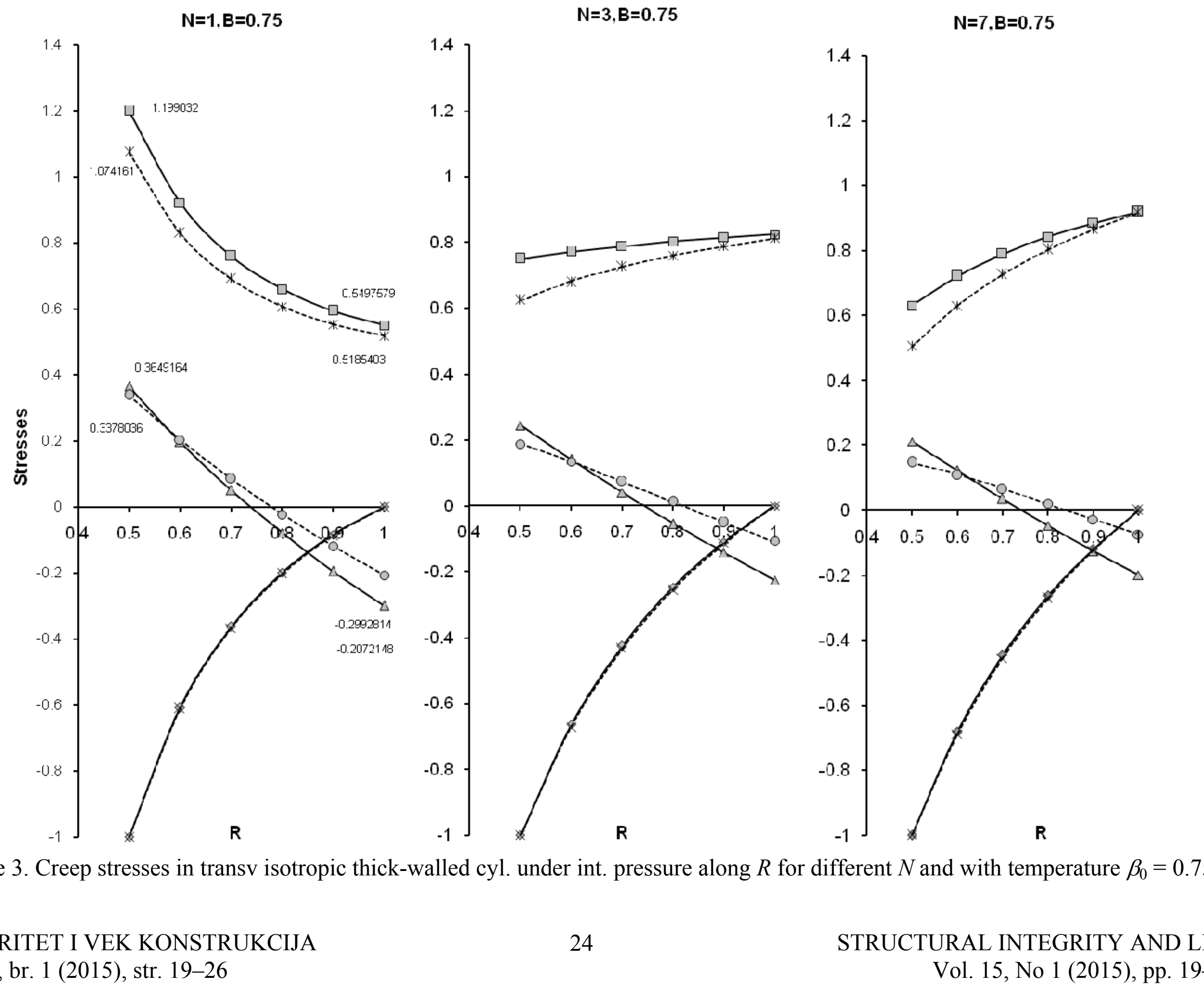

Figure 3. Creep stresses in transv isotropic thick-walled cyl. under int. pressure along R for different N and with temperature $\beta_0 = 0.75$.

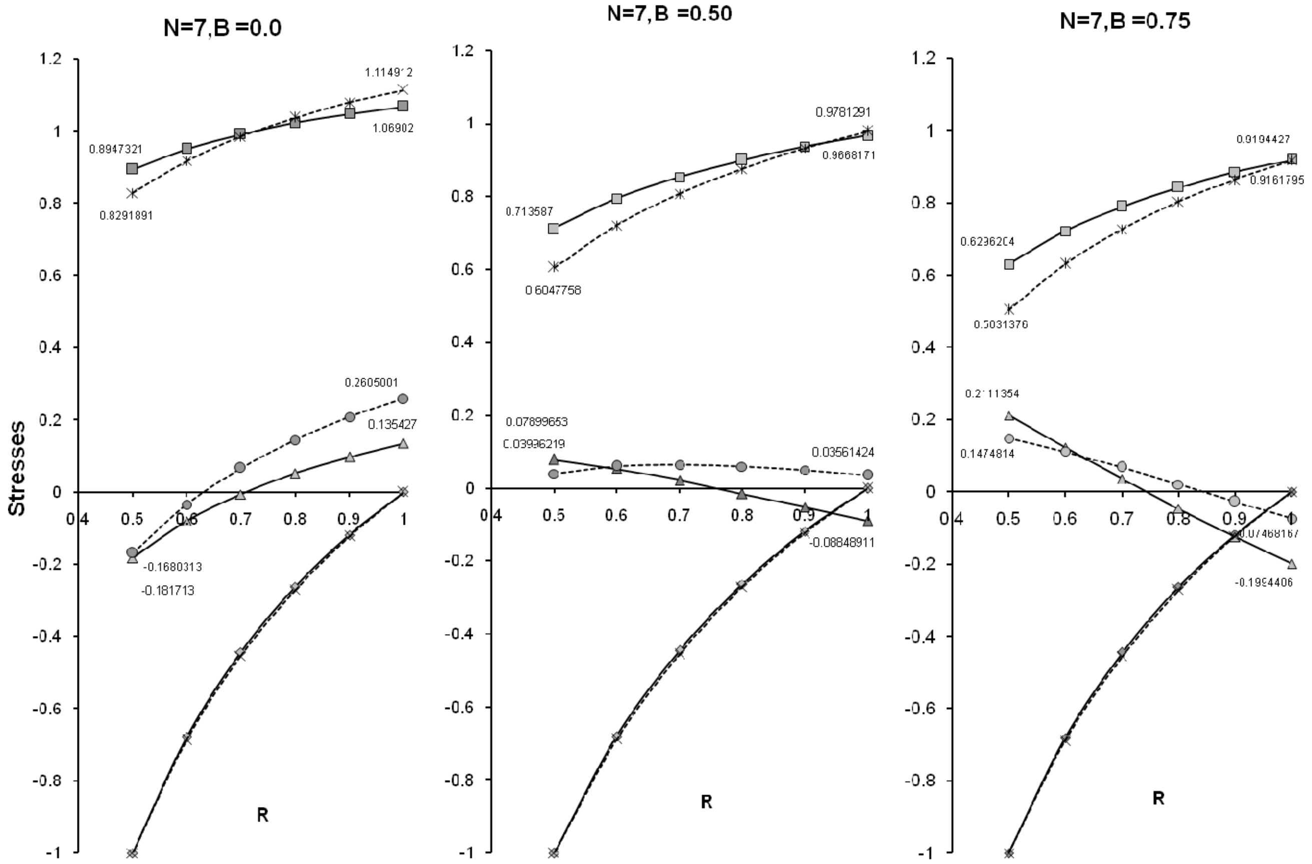

Figure 4. Creep stresses in a transv. isotropic thick-walled cyl. with int. pressure along R for N (=7) and temperature β_0 = 0; 0.50; 0.75.

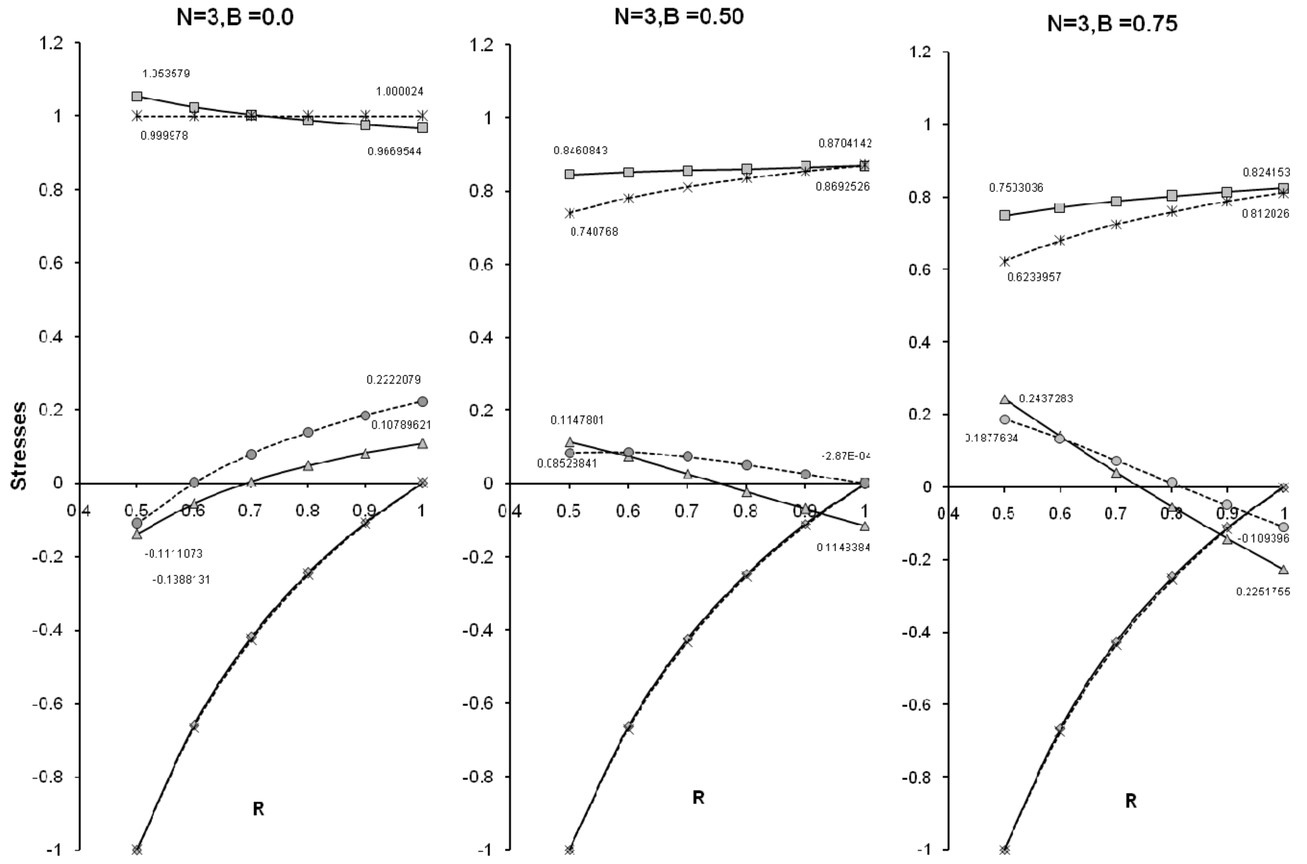

Figure 5. Creep stresses in transv. isotropic thick-walled cyl. with int. pressure along R for N (=3) and temperature β_0 = 0, 0.50, 0.75.

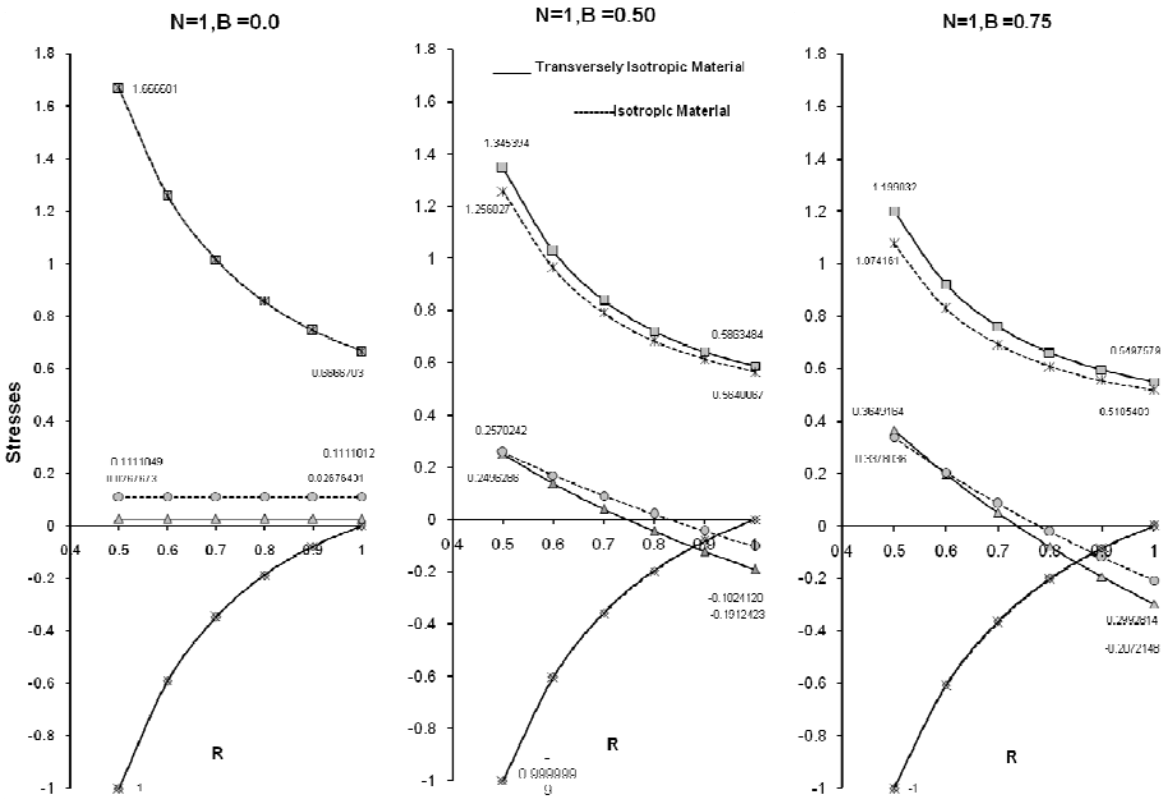

Figure 6. Creep stresses in transv. isotropic thick-walled cyl. with int. pressure along R for N (=1) and temperature β_0 = 0, 0.50, 0.75.

REFERENCES

1. Seth, B.R., *Creep Transition*, J Math. Phys. Sci., (1972), 6.
2. Nabarro, F.R.N., de Villers, H.L., Physics of Creep, Taylor & Francis, PA, 1995.
3. Odqvist, F.K.G., Mathematical Theory of Creep and Creep Structure, Claredon Press, Oxford 1974.
4. Kraus, H., Creep Analysis, Wiley, New York, 1980.
5. Sharma, S., *Elastic-plastic Transition of Non-Homogeneous Thick walled Circular Cylinder under internal Pressure*, Def. Sc. Journal, 54(2), (2004).
6. Gupta, S.K., Pankaj, T., *Creep transition in a thin rotating disc with rigid inclusion*, Def. Sc. Journal, 57(2), (2007): 185-195.
7. Gupta, S.K., *Thermo Elastic-Plastic Transition of Thick-walled Rotating Cylinder*, Proc. 1st Int. Symp. on Thermal Stresses and Related Topics, June 5-7, (1995), Japan.
8. Seth, B.R., *Transition Analysis of Collapse of Thick-walled Cylinder*, ZAMM, 50 (2), (1970): 617-621.
9. Parkus, H., Thermo-elasticity, Springer-Verlag, NY USA, 1976.
10. Gupta, S.K., *Thermo elastic-plastic transition in a transversely isotropic thick-walled cylinder under internal pressure*, PhD Thesis, Himachal Pradesh University, Shimla (2006).
11. Gupta, S.K., *Elastic-plastic and Creep Transition of Thick-Walled Cylinder under Uniform Pressure*, Proc. Int. Symp. on Non-linear Mech., Kharagpur, 1980, pp.169-175.
12. Odquist, F.K.G., Mathematical Theory of Creep and Creep Rupture, Clarendo Press, Oxford, 1974.
13. Gupta, S.K., Dharmani, R.L., Creep Transition in Thick-walled Cylinder Under Internal Pressure, ZAMM Mech., 59 (1979): 517-521.
14. Pankaj, T., Gupta, S.K., *Thermo Elastic-plastic Transition in a Thin Rotating Disc with Inclusion*, Thermal Science 11(1) (2007): 103-118.
15. Pankaj, T., *Elastic-plastic transition stresses in rotating cylinder by finite deformation under steady-state temperature*, Thermal Science, 15(2) (2011): 537-543.
16. Pankaj, T., *Elastic-plastic transition stresses in a thin rotating disc with rigid inclusion by infinitesimal deformation under steady state temperature*, Thermal Science, 14(1) (2010): 209-219.
17. Pankaj, T., *Creep transition stresses in a thin rotating disc with shaft by finite deformation under steady state temperature*, Thermal Science, 14(2) (2010): 425-436.
18. Pankaj, T., *Creep transition stresses of a thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure*, Thermal Science, 15 (suppl. 2) (2011): S157-S165.
19. Pankaj, T., *Stresses in a thin rotating disc of variable thickness with rigid shaft*, J Technology of Plasticity, 37(1) (2012): 1-14.
20. Pankaj, T., *Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature*, Thermal Science, (2012) doi: 10.2298/TSCI110315080P.
21. Pankaj, T., *Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature*, Thermal Science, (2012) doi: 10.2298/TSCI110315080P.
22. Pankaj, T., *Deformation in a thin rotating disc of variable thickness and edge load with inclusion at the elastic-plastic transitional stresses*, Structural Integrity and Life 12(1) (2012): 65-70.
23. Pankaj, T., *Thermo creep transition stresses in a thick-walled cylinder subjected to internal pressure by finitesimal deformation*, Structural Integrity and Life 12(3) (2012): 165-173.
24. Pankaj, T., Singh, S.B., Kaur, J., *Thickness variation parameter in a thin rotating disc by finite deformation*, FME Transactions 41(2) (2013): 96-102.
25. Pankaj, T., Singh, S.B., Kaur, J., *Effect of stresses in a thin rotating disk loading edge for different materials*, accepted for publication, Int. J Technology of Plasticity, 38(1) (2013).
26. Pankaj, T., *Stresses in thick-walled circular cylinder having pressure by using concept of generalized strain measure*, accepted for publ., Kragujevac J of Science, Serbia, Vol. 35, (2013).
27. Pankaj, T., Singh, S.B., Kaur, J., *Elastic-plastic stresses in a thin rotating disk with shaft having density variation parameter under steady-state temp.*, Kragujevac J of Sci. 36 (2014): 5-17.