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INFLUENCE OF RESIDUAL STRESSES ON THE CRACK DEFLECTION INTO THE INTERFACE BETWEEN THE TWO MATERIALS

UTICAJ ZAOSTALIH NAPONA NA SKRETANJE PRSLINE U MEĐUPOVRŠ DVA MATERIJALA

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Keywords
- interface
- approaching crack
- crack deflection
- crack penetration
- residual stresses

Ključne reči
- međupovrš
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- skretanje prsline
- prodiranje prsline
- zaostali naponi

Abstract

The paper considers the problem of a crack approaching an interface between two materials at a right angle and the influence of residual stresses on its behaviour. The crack attacking the interface can behave in three ways: (a) it can disappear, i.e. it can stop at the contact with the interface; (b) it can penetrate the interface and continue to propagate into the material across it; and (c) it can deflect into the interface and continue to propagate along it. This so-called 'competition' between the latter two cases depends on whether the ratio of energy release rates for the crack deflecting into the interface and the crack penetrating the interface is larger or smaller than the ratio of the fracture toughness of the interface and the fracture toughness of the material across it. Residual stresses, as the consequence of the difference in thermal expansion coefficients of the two materials constituting the interface, affect the above two energy release rates. Results confirm the significant influence of difference in thermal expansion coefficients.

Izvod

U radu je razmatran problem prsline koja se približava međupovrši dva materijala pod pravim uglom i uticaj zaostalih napona na njeno ponašanje. Prslina koja se približava međupovrši može da se ponaša na tri načina: (a) može da nestane, odnosno, da se zaustavi pri kontaktu sa međupovrši; (b) može da pređe preko međupovrši i nastavi da propagira u materijal preko nje; i (c) može da skrene u međupovrš i da nastavi da se širi duž nje. Ovo takozvano „takmičenje" između dva poslednja slučaja zavisi od toga da li je odnos brzine oslobađanja energije za skretanje prsline u međupovrš i brzine oslobađanja energije za prelaženje preko međupovrši veći ili manji od odnosa žilavosti loma međupovrši i žilavosti loma materijala preko međupovrši. Zaostali naponi, koji su rezultat razlike u koeficijentima toplotnog širenja dva materijala koji čine međupovrš, utiču na dve pomenute brzine oslobađanja energije. Rezultati prikazani u ovom radu potvrđuju značajan uticaj razlike u koeficijentima toplotnog širenja.

INTRODUCTION

An important problem in considering interfacial cracks is how the interface reacts to the approaching crack. When the crack is attacking an interface it could penetrate it and continue to propagate in the material across it, or it could deflect into the interface and continue to propagate along it. Such a question is important in designing the interface between matrix and reinforcing fibres in composite materials and between layers in laminar materials, /1/. There, the objective is for the crack to deflect into the interface and leave the fibre undamaged and not to penetrate it, since the latter would mean that the fibre is broken, as well as in laminar materials to prevent delamination between layers.

In paper by He and Hutchinson /2/ the behaviour of a crack approaching an interface between the two different materials at a right angle is analysed. The approaching crack behaviour is analysed based on the ratio between the energy release rate for the crack that is deflecting into the interface G_d and the energy release rate for the crack that is crossing the interface G_p. The competition between the crack penetrating or deflecting into the interface is being determined by comparing the ratio G_d /G_p with the ratio of fracture toughnesses of the interface and the material across the interface G_{Ic} /G_c. Results of He and Hutchinson /2/ are combined with the linear elastic fracture mechanics concept for the interfacial crack in a paper by Djokovic, /3/. However, in papers He and Hutchinson /2/ and Djokovic /3/ the residual stresses are not considered, though in some cases they could impose a significant influence on interfacial fracture. The influence of residual stresses on the crack approaching an interface at the right angle is considered in the work by He et al., /4/.

In this paper results presented by He et al., /4/, are analysed by application of the LEFM concept for interfacial fracture of Rice, /5/, and with application of the symbolic programming routine Mathematica®.

PROBLEM FORMULATION

The problem of a crack approaching the interface is presented in Fig. 1; the semi-infinite crack attacking the interface at the right angle.

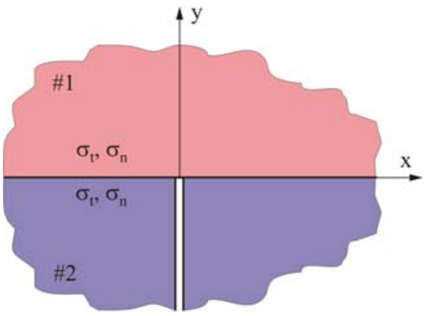

Figure 1. Semi-infinite crack attacking interface at the right angle.

For the case presented in Fig. 1 the stresses ahead of the crack tip are:

$$\sigma_{xx}(0,y)=k_{\mathrm{I}}(2\pi y)^{-\lambda}\,, \tag{1}$$

where λ is the real variable, called the exponent (power) of the stress singularity and it depends on Dundurs parameters α and β according to He and Hutchinson, /2/:

$$\cos\lambda\pi=\frac{2(\beta-\alpha)}{1+\beta}(1-\lambda)^2+\frac{\alpha+\beta^2}{1-\beta^2}\,. \tag{2}$$

Dundurs parameters, /6/, for plane strain conditions are, /5/:

$$\alpha=\frac{\bar{E}_1-\bar{E}_2}{\bar{E}_1+\bar{E}_2}\,,\quad \beta=\frac{\mu_1(1-2\nu_2)-\mu_2(1-2\nu_1)}{2(\mu_1(1-\nu_2)+\mu_2(1-\nu_1))}\,, \tag{3}$$

where: E, μ and ν are Young's modulus of elasticity, shear modulus and Poisson's ratio, respectively, and $\bar{E}=E/(1-\nu^2)$. Indices 1, 2 refer to the properties of the materials above and below the interface, in respect (Fig. 1). Oscillatory index ε depends on β and is defined as, Rice /6/:

$$\varepsilon=\frac{1}{2\pi}\left(\frac{1-\beta}{1+\beta}\right). \tag{4}$$

Factor k_{I} is proportional to load and not necessarily known explicitly. Due to the existence of residual stresses, two new dimensionless parameters are introduced, /4/:

$$\eta_n=\frac{\sigma_n a^{\lambda}}{k_{\mathrm{I}}}\quad\text{and}\quad \eta_t=\frac{\sigma_t a^{\lambda}}{k_{\mathrm{I}}}\,, \tag{5}$$

where σ_n and σ_t are normal and transversal residual stresses; a is the length of a crack deflecting into the interface or penetrating it, for which it is assumed that it is small in comparison to the length of the main approaching crack. Whether residual stresses are going to be compressive or tensile depends on the elasticity modulus and on the thermal expansion coefficient. If the elasticity modulus is large (large positive α) and the thermal expansion coefficient is small, the residual stresses would be compressive.

In Figs. 2(a), (b) and (c) presented are three cases important for practical applications. Figure 2(a) presents the crack that has penetrated the interface and continued to propagate into the material above it. Figures 2(b) and (c) show a crack that has deflected into the interface as single-sided and double-sided, respectively.

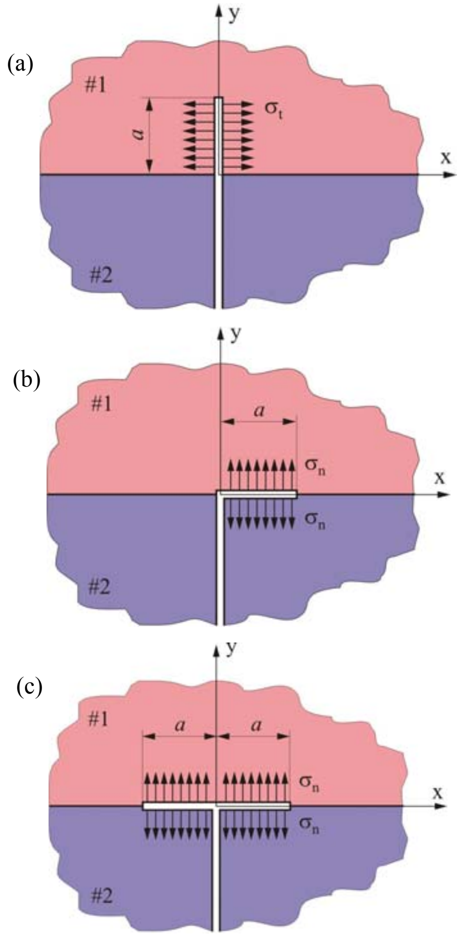

Figure 2. Crack penetrating the interface (a); crack deflecting into the interface – single-sided (b); double-sided (c).

Whether the deflection would be single-sided or double-sided depends on the loading conditions at the crack tip. The more probable and frequent is double-sided deflection which occurs when the load phase angle ψ is less than 45°. Single-sided deflection occurs in cases when Mode II loading conditions prevail at the crack tip. In those cases the load phase angle is larger than 45°.

In case of the crack crossing the interface, Fig. 2(a), the stress field ahead of the crack tip corresponds to pure Mode I loading conditions. Based on dimensional analysis, the stress intensity factor is, /2/:

$$K_{\mathrm{I}}=c(\alpha,\beta)k_{\mathrm{I}}a^{1/2-\lambda}+h(\alpha,\beta)\sigma_t a^{1/2}\,, \tag{6}$$

where $c(\alpha, \beta)$ and $h(\alpha, \beta)$ are dimensionless functions of α and β. Taking into account that the influence of parameter β is negligible, /7/, only the influence of parameter α is considered, it is assumed that $\beta = 0$. The energy release rate for the crack crossing the interface is, /4/:

$$G_p = \frac{1-\nu_1}{2\mu_1} K_\mathrm{I}^2 = \frac{1-\nu_1}{2\mu_1}\left(c^2 k_\mathrm{I}^2 a^{1-2\lambda} + 2chk_\mathrm{I}\sigma_t a^{1-\lambda} + h^2\sigma_t^2 a\right) \quad (7)$$

The stress field at the interface, ahead of crack tip that has deflected into it, Figs. 2(b) and (c), is described with:

$$\sigma_{yy}(x,0) + i\sigma_{xy}(x,0) = (K_1 + iK_2)(2\pi r)^{-1/2} r^{i\varepsilon}, \quad (8)$$

where $r = x - a$. Taking into account that $\beta = 0$, K_1 and K_2 can be considered as stress intensity factors for Mode I and II, respectively; thus based on dimensional analysis one obtains:

$$K_1 + iK_2 = k_1 a^{1/2-\lambda} d(\alpha) + \sigma_n a^{1/2} e(\alpha), \quad (9)$$

where $d(\alpha)$ and $e(\alpha)$ are dimensionless functions of α and β, defined in /3/:

$$d = \frac{\alpha + 2\beta}{\alpha + \beta}, \quad e = \frac{\alpha^2 + \alpha + 2\beta}{\alpha - \beta}. \quad (10)$$

The energy release rate for the crack deflecting into the interface is:

$$G_d = \frac{(1-\alpha)\bar{E}_1}{1-\beta^2}\Big\{k_1^2 a^{1-2\lambda}|d|^2 + k_1\sigma_n a^{1-\lambda}(d\bar{e} + \bar{d}e) + \sigma_n^2 a|e|^2\Big\}, \quad (11)$$

where (‾) denotes the complex conjugate function. Ratio G_d/G_p does not depend on a and k_I, i.e.:

$$\frac{G_d}{G_p} = \frac{1}{1-\alpha}\,\frac{|d|^2 + \eta_n(d\bar{e} + \bar{d}e) + \eta_n^2|e|^2}{c^2 + 2\eta_t ch + \eta_t^2 h^2}. \quad (12)$$

The relative tendency of a crack to deflect into the interface, or continue to propagate across it, can be determined using Eq.(12). It depends on the Dundurs parameter α and on dimensionless parameters η_t and η_n introduced in Eq.(5).

The load phase angle which measures the relative value of Mode II with respect to Mode I, for the crack that is deflecting into the interface, is:

$$\psi = \tan^{-1}\left(\frac{K_2}{K_1}\right) = \tan^{-1}\left(\frac{\mathrm{Im}(d) + \eta_n \mathrm{Im}(e)}{\mathrm{Re}(d) + \eta_n \mathrm{Re}(e)}\right). \quad (13)$$

If the fracture toughness of the interface is denoted as $G_{\mathrm{I}c}$, and the fracture toughness of material 1 into which the crack is crossing is denoted as G_c, then the crack attacking the interface at the right angle would deflect into the interface if the following inequality holds:

$$\frac{G_{\mathrm{I}c}}{G_c} < \frac{G_d}{G_p}. \quad (14)$$

If the sign in inequality, Eq.(14), is reversed, the crack would penetrate the interface and continue to propagate into material 1 above it.

RESULTS AND DISCUSSION

The ratio of energy release rates G_d/G_p is presented in Fig. 3 in terms of dimensionless parameters α and η_t, where the value of parameter $\eta_n = 0$, and for the crack deflecting into the interface, single-sided. Diagrams are obtained using Eq.(12) and by fitting the tabular results of functions c, h, d and e by symbolic programming routine Mathematica®.

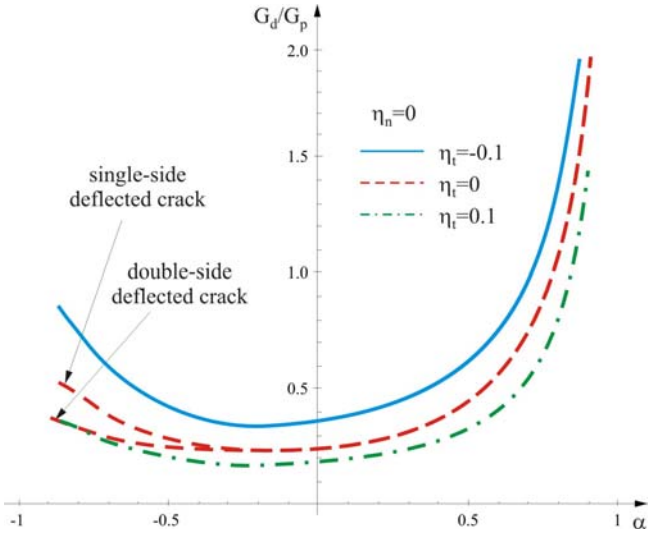

Figure 3. Energy release rate in terms of parameter α for different values of parameter η_t and for $\eta_n = 0$.

Figure 3 shows that the ratio G_d/G_p decreases with the increase of parameter η_t. Figure 2 also shows the ratio G_d/G_p for the case of the double-sided crack deflecting into the interface for $\eta_t = 0$ and $\eta_n = 0$ (lower portion of the red – dashed line). The same dependence can be noticed as in the case of single-sided deflection, with a small deviation.

The corresponding diagrams of ratio G_d/G_p dependence on α and η_t for two values of parameter η_n (0.1; –0.1) are shown in Figs. 4 and 5, respectively, for a single-sided deflecting crack. From both figures can be seen that with the increase of parameter η_t, the ratio G_d/G_p are decreasing, meaning that significant values of residual stress σ_t have a strong influence on the G_d/G_p ratio, for both values $\eta_t = 0.1$ and $\eta_t = -0.1$.

From a comparison of Figs. 3-5 one can notice that the ratio G_d/G_p increases with increase of parameter η_n, as a consequence of the influence of residual stresses, perpendicular to the interface, changing from positive to negative values.

Figure 6 shows the variation of the load phase angle with parameter α for the crack that is single-sided deflecting into the interface for three different values of parameter η_n. Also shown in the same Figure is the variation of the load phase angle for the double-sided deflecting crack, for $\eta_n = 0$.

Figures 3 to 6 could be used in combination with Eq. (14) for predicting whether the incoming crack would penetrate the interface or deflect into it. When the interface is exposed to the tensile residual stress, two factors are influencing, whether the interfacial crack would become unstable: parameter η_n increases with the increase of crack length a, and when η_n increases the load phase angle ψ decreases, causing a reduction in the interface fracture toughness $G_{\mathrm{I}c}$. Compressive residual stresses, on the other hand, lead towards stable crack growth, regardless of the fact that the crack is propagating along the interface or that it has penetrated the interface, because either η_n or η_t become more negative as the crack length extends.

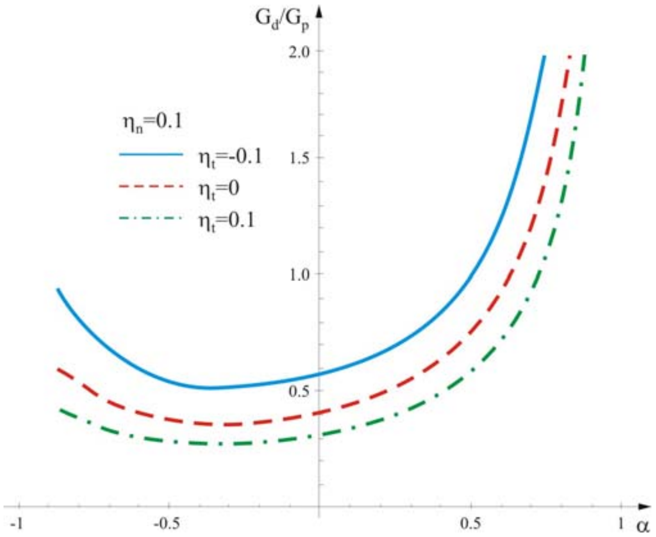

Figure 4. Energy release rate in terms of parameter α for different values of parameter η_t and for $\eta_n = 0.1$.

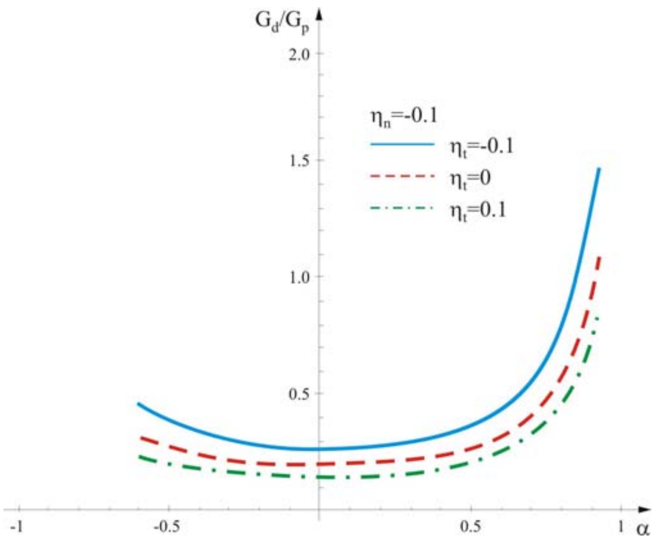

Figure 5. Energy release rate in terms of parameter α for different values of parameter η_t and for $\eta_n = -0.1$.

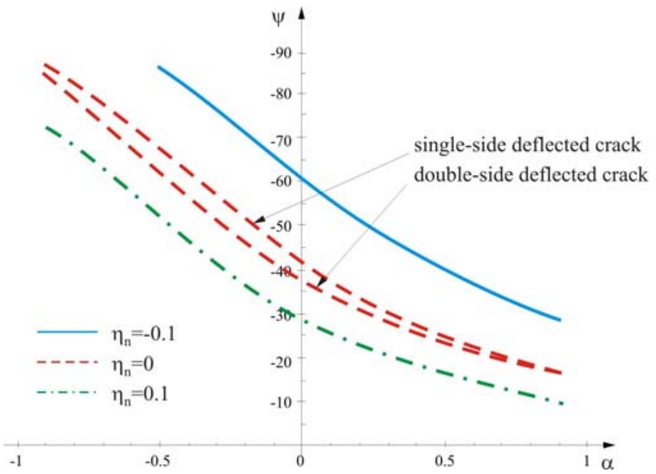

Figure 6. Load phase angle in terms of parameter α for three different values of parameter η_n.

CONCLUSIONS

Residual stresses as the consequence of the difference in thermal expansion coefficients of two materials constituting the interface, have a significant influence on which of the two phenomena should occur when the crack attacks the interface: crack penetration of the interface or crack deflection into it.

The paper shows that the ratio of energy release rates for the crack deflecting into the interface and energy release rates for the crack penetrating it, G_d/G_p, decreases with the increase of η_t while it increases with the increase of η_n, as a consequence of the influence of the change in values of residual stresses perpendicular to the interface, from positive to negative values.

When the interface is subjected to residual tensile stress, the interfacial crack becomes unstable if the parameter η_n increases as the crack length extends and due to its increase the load phase angle ψ decreases, causing the decrease of the interface fracture toughness G_{Ic}. Compressive residual stresses, on the other hand, are causing stable crack growth; either along the interface or across it, since either η_n or η_t becomes more negative as the crack propagates.

This influence of residual stresses, caused by difference in thermal coefficients of the two materials, could be used in designing the interface in layered materials and fibre-reinforced composites. The objective is to prevent undesirable behaviour, either delamination of layers in laminar materials or failure of the fibre in composites, i.e. fracture.

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