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CRACK KINKING AWAY FROM THE INTERFACE IN DYNAMIC GROWTH CONDITIONS

SKRETANJE PRSLINE SA INTERFEJSA U USLOVIMA DINAMIČKOG RASTA

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- crack kinking
- dynamic growth

Ključne reči
- interfejs
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Abstract

The behaviour of a crack is analysed, that lies between two elastic isotropic materials while it propagates dynamically. The question is whether the crack will continue to propagate along the interface or will it kink away from the interface. This 'competition' between crack kinking away from the interface and propagation along the interface in dynamic crack growth conditions can be estimated based on ratio of dynamic energy release rates for crack kinking away from the interface and for crack growth along the interface. By comparing this ratio to the ratio of interface toughness and material toughness without the interface, one can predict how the crack will behave, i.e. which way would it continue to propagate, as well as tip rate of the kinked crack.

Izvod

Analizirano je ponašanje prsline koja se širi neravnomerno, leži između dva elastična izotropna materijala, a pitanje je da li će nastaviti da se širi duž interfejsa ili će da skrene sa njega i nastavi da se širi u jednom od dva materijala. Ovo „takmičenje" između skretanja prsline sa interfejsa i širenja duž njega u uslovima dinamičkog rasta, može da se oceni na osnovu odnosa dinamičke brzine oslobađanja energije za skretanje prsline sa interfejsa i dinamičke brzine oslobađanja energije za prslinu na interfejsu. Poređenjem ovog odnosa sa odnosom žilavosti interfejsa i žilavosti materijala bez interfejsa, može da se predvidi kako će prslina da se ponaša, tj. da li će da skrene sa interfejsa, ili će dalje da se širi duž njega, kao i kolika je brzina vrha prsline koja je skrenula.

INTRODUCTION

Problem of a crack that lies at an interface between two different materials was first investigated by Williams, /1/, for geological investigations purposes. Scientific explanation of the initiation and growth of a crack at the bimaterial interface is fundamental for understanding fracture processes in materials like composites and ceramics.

Interfacial fracture resistance depends on the mixed mode of loading, which represents the ratio of the crack opening forces and the forces causing sliding ahead of the crack tip. The mixed concept was defined by Rice, /2/, for the interface in the isotropic solid. Recently, advances in investigating the static fractures at the interface are found in papers /3-5/.

The destruction process can also be dynamic, depending on the nature of the loading to which the composite structure is subjected. If the interface is already weakened by existence of flaws, they could serve as initiators of a crack, which could propagate in an unstable manner along the interface. This problem is dealt with in /6-11/.

The characteristic of the interfacial crack, in mixed mode loading conditions, is a great probability that the crack will kink away from the interface into one of the two materials. The kinking of a crack from the interface, in conditions of static loading is considered in /12-14/. He and Hutchinson /12/ were investigating the two possible criteria for determining the angle at which the crack is kinking away from the interface: (i) maximal opening stress criterion, based on the concept that the crack will propagate in the direction perpendicular to the maximal opening stress direction; and (ii) maximal energy release rate criterion, that predicts the crack to propagate in the direction where the maximal energy is released. Both criteria produce good results when the difference in characteristics of the two materials across the interface is small. In the case of the larger mismatch in material characteristics, the criterion that takes into account the energy release rate gives significantly better results. From a comparison of energy release rates for the crack that had kinked from the interface and for the crack that propagates along the interface, in order to determine which of the two events is more probable to occur, in the case of isotropic elastic - and orthotropic materials was the subject of investigations in /13-14/.

The objective of this paper is an analysis of the crack kinking from the interface in conditions of dynamic loading, i.e. to check whether it would be possible to apply criteria that are valid for the crack kinking in static loading conditions, which were considered in the mentioned papers, also for the crack kinking in dynamic loading conditions. The dynamic crack propagation criterion in homogeneous bodies is based on the static stress field with addition of the stress intensity factor that depends on time, /6/, and is used in this paper to extend the existing criterion for crack kinking in static loading conditions, to the dynamic case.

PROBLEM FORMULATION

The geometry of the considered problem is presented in Fig. 1. The main crack lies at the interface between the two semi-infinite isotropic elastic bodies with Young's moduli E_1 and E_2, shear moduli μ_1 and μ_2 and Poisson's ratios ν_1 and ν_2. Indices 1 and 2 denote characteristics of materials above and below the interface, respectively, Fig. 1.

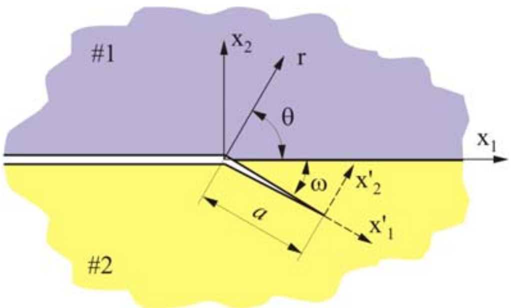

Figure 1. Geometry of a crack that is kinking from the interface

It is assumed that the length of the kinked portion of the crack a is small with respect to length of the main crack and that the problem considered is that of the semi-infinite main crack. The stress field prior to crack kinking ($a \to 0$) is a singular field of the interfacial crack, which is characterized by the complex stress intensity factor $K = K_1 + iK_2$. The stress intensity factor at the tip of the crack that has kinked from the interface is characterized by a combination of standard intensity factors for Mode I and II loading, $K_{\rm I}$ and $K_{\rm II}$, respectively.

The condition for the crack to kink away from the interface in static loading conditions is:

$$\frac{G_s}{G} > \frac{\Gamma}{\Gamma_i}, \tag{1}$$

where Γ is the fracture toughness of material 2 and Γ_i is the fracture toughness of the interface.

Analogously, the condition for the crack that propagates dynamically along the interface to kink into the material below it, material 2, is:

$$\frac{G_s^d}{G^d} > \frac{\Gamma^d}{\Gamma_i^d}, \tag{2}$$

where G_s^d is the dynamic energy release rate of the crack that has kinked from the interface, G^d is the dynamic energy release rate for the crack that propagates along the interface, Γ^d is the dynamic fracture toughness of material 2 and $\Gamma_\iota^{\,\delta}$ is the dynamic fracture toughness of the interface.

Dynamic energy release rate of the crack that has kinked from the interface into material 2 is, /6/:

$$G_s^d = \frac{1-\beta_2^2}{2\mu_2 D}(\beta_1 K_I^{d2} + \beta_2 K_{II}^{d2}), \tag{3}$$

where:

$$D \equiv D(\dot{a}) = 4\beta_1\beta_2 - \left(1+\beta_2^2\right)^2$$

and $\beta_{1/2} = \sqrt{1-(\dot{a}/c_{l(2)/s(2)})^2}$.

Variables $c_{l(i)}$ and $c_{s(i)}$ are the longitudinal and transversal wave speeds for materials above and below the interface, respectively, (i = 1, 2), defined by:

$$c_{l(i)} = \sqrt{\frac{\kappa_i+1}{\kappa_i-1}\cdot\frac{\mu_i}{\rho_i}}, \quad c_{s(i)} = \sqrt{\frac{\mu_i}{\rho_i}}, \tag{4}$$

where ρ_i are material densities, $\kappa_i = 3 - 4\nu_i$ for plane strain, $\kappa_i = (3-\nu_i)/(1+\nu_i)$ for plane stress conditions.

Dynamic stress intensity factors ahead of the crack that has kinked from the interface are calculated as, /15/:

$$\begin{aligned} K_I^d &\equiv K_I(\dot{a}) = B_1(\dot{a})\mu_2\sqrt{2\pi}\lim_{r\to 0}\frac{\delta_2}{\sqrt{r}} \\ K_{II}^d &\equiv K_{II}(\dot{a}) = B_2(\dot{a})\mu_2\sqrt{2\pi}\lim_{r\to 0}\frac{\delta_1}{\sqrt{r}} \end{aligned} \tag{5}$$

where $B_{1/2}(\dot{a}) = D/4\beta_{1/2}(1+\beta_2^2)^2$ and $\delta = \delta_1 + i\delta_2$ is the complex crack opening displacement (complex COD), where δ_1 represents shear behind the crack tip or tangential displacement of the crack surfaces (tangential COD) while δ_2 represents normal displacement of the crack surfaces (normal COD) which are being determined as, /7/:

$$\begin{aligned} \delta_1 \equiv \delta_1(r) &= \frac{H_{22}}{\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}\frac{\left|K^d\right|}{\sqrt{1+4\varepsilon^2}}\cdot \\ &\cdot\frac{1}{\eta}\sin(\varphi+\varepsilon\ln r-\arctan(2\varepsilon)) \\ \delta_2 \equiv \delta_2(r) &= \frac{H_{22}}{\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}\frac{\left|K^d\right|}{\sqrt{1+4\varepsilon^2}}\cdot \\ &\cdot\cos(\varphi+\varepsilon\ln r-\arctan(2\varepsilon)) \end{aligned} \tag{6}$$

where r is the distance from the crack tip, $\varphi = \varepsilon\ln(r/2a)$, η is the resolution factor, determined as $\eta = \sqrt{H_{22}/H_{11}}$, /16/, and ε is the oscillatory index, /2/:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right), \tag{7}$$

where β is one of the two Dundurs parameters, /16, 17/, being determined as:

$$\beta = -\frac{H_{12}}{\sqrt{H_{11}H_{22}}}, \tag{8}$$

where H_{ij} are components of matrix H:

$$\mathbf{H}=\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

$$H_{11}=\frac{\zeta_1(1-\zeta_1^2)}{\mu_1 D_1}+\frac{\zeta_2(1-\zeta_2^2)}{\mu_2 D_2}$$

$$H_{12}=i\left(\frac{1+\zeta_1^2-2\xi_1\zeta_1}{\mu_1 D_1}+\frac{1+\zeta_2^2-2\xi_2\zeta_2}{\mu_2 D_2}\right) \qquad (9)$$

$$H_{21}=-i\left(\frac{1+\zeta_1^2-2\xi_1\zeta_1}{\mu_1 D_1}+\frac{1+\zeta_2^2-2\xi_2\zeta_2}{\mu_2 D_2}\right)$$

$$H_{22}=\frac{\xi_1(1-\zeta_1^2)}{\mu_1 D_1}+\frac{\xi_2(1-\zeta_2^2)}{\mu_2 D_2}$$

where: $D_i \equiv D_i(\dot{a}) = 4\xi_i\zeta_i-(1+\zeta_i^2)^2$, $\xi_i=\sqrt{1-(\dot{a}/c_{l(i)})^2}$, and $\zeta_i=\sqrt{1-(\dot{a}/c_{s(i)})^2}$.

The intensity of the dynamic stress intensity factor can be obtained based on Eq.(6) as:

$$\left|K^d\right|=\frac{\cosh(\pi\varepsilon)}{H_{22}}\lim_{r\to 0}\sqrt{\frac{\pi\sqrt{1+4\varepsilon^2}}{2r}}\sqrt{\eta^2\delta_1^2+\delta_2^2}\,. \qquad (10)$$

The dynamic energy release rate for the crack that propagates along the interface is, /17/:

$$G^d=\frac{H_{22}}{4\cosh^2(\varepsilon\pi)}\left|K^d\right|^2. \qquad (11)$$

The load phase angle for the crack at the interface, in terms of distance from the crack tip is calculated as, /2/:

$$\psi(r)=\arctan\left\{\frac{\mathrm{Im}(K^d r^{i\varepsilon})}{\mathrm{Re}(K^d r^{i\varepsilon})}\right\}. \qquad (12)$$

For the load phase angle at the crack tip, for $r \to 0$, substituting Eq.(10) into Eq.(12), one obtains:

$$\psi=\arctan\left\{\lim_{r\to 0}\frac{1-\dfrac{\delta_2}{\delta_1}\dfrac{\tan(\varepsilon\ln(r/a))-2\varepsilon}{1+2\varepsilon\tan(\varepsilon\ln(r/a))}}{\dfrac{\delta_2}{\delta_1}+\dfrac{\tan(\varepsilon\ln(r/a))-2\varepsilon}{1+2\varepsilon\tan(\varepsilon\ln(r/a))}}\right\}. \qquad (13)$$

RESULTS AND DISCUSSION

Figure 2 shows the variation of the load phase angle ψ in terms of angle at which the crack kinks from the interface ω, based on Eq.(13) for five different bimaterial combinations and certain crack tip speed.

From Fig. 2 one can see that with increase of the load phase angle the crack kinking angle increases, as well. The solid line shows variation of ψ for the bimaterial combination $\varepsilon = 0$, which corresponds to the problem described in /12/, but for the case of quasi-static crack growth. Good agreement of those results and results shown in Fig. 2 point to the fact that it is justified to consider the crack kinking from the interface in the dynamic conditions analogously to the case of the static loading conditions.

Figure 3 shows the variation of the ratio of the energy release rate for the crack that has kinked from the interface and the energy release rate for the crack propagating along the interface in dynamic loading conditions G_s^d/G^d, in terms of the load phase angle ψ for five different bimaterial combinations and certain crack tip speed. From Fig. 3 one can see that the average energy needed for the crack kinking increases with load phase angle and then it drops.

Results for the crack that is kinking from the interface in the dynamic loading conditions can be used to estimate whether the interfacial crack will propagate along the interface or will it kink from it. If G_s^d is large enough with respect to G^d the crack will never kink into material 2.

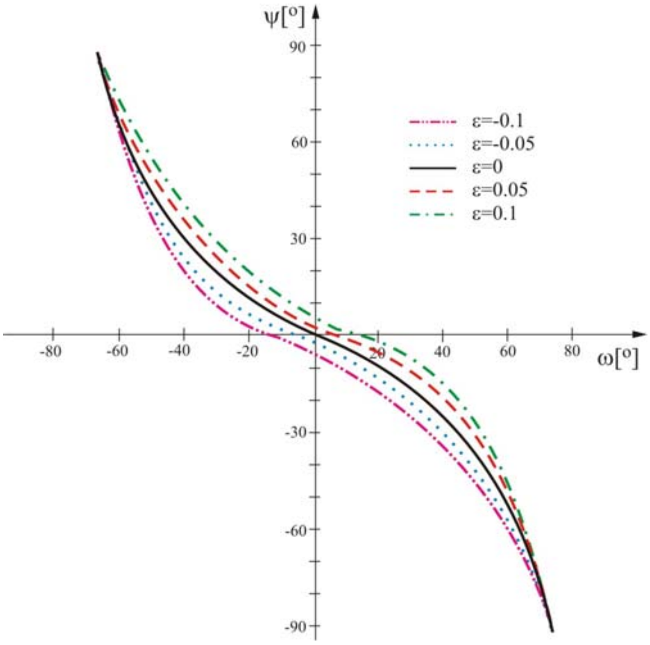

Figure 2. Load phase angle ψ variation in terms of the crack kinking angle ω.

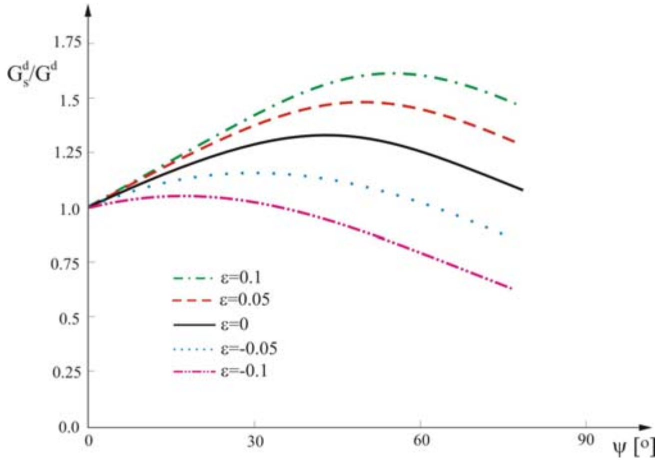

Figure 3. G_s^d/G^d variation in terms of the load phase angle ψ.

Increased elasticity of the material into which the crack is kinking increases the energy release rate, for all the other factors remaining the same and vice versa, if the material into which the crack is kinking is less elastic, the energy release rate decreases. When differences in the materials are relatively large, the energy release rate for the kinking crack can be smaller than for the interfacial crack, for all loading combinations. This implies that in conditions when a more elastic material is ductile and the less elastic material and interface are relatively brittle, the crack will tend to stop at the interface for all loading combinations. If the brittle material is even a little less ductile than the interface, the crack can leave the interface, and then again it does not have to kink.

In Fig. 4, the variation of the ratio of energy release rate for the crack that has kinked from the interface and the energy release rate for the crack that propagates at the interface, are presented in dynamic loading conditions, G_s^d/G^d, in terms of the crack speed for five different bimaterial combinations and certain load phase angle.

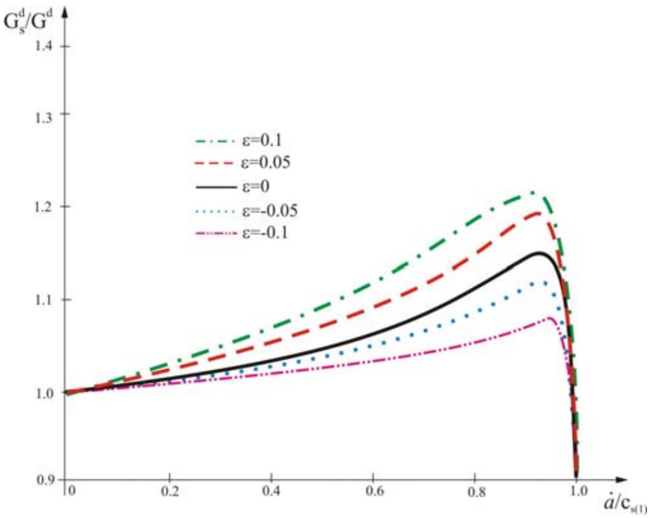

Figure 4. Ratio G_s^d/G^d in terms of crack speed $\dot{a}/c_{s(1)}$.

From Fig. 4 can be seen that as the crack speed increases, the ratio G_s^d/G^d is also increasing and after a certain value of the crack speed it drops abruptly. Drop of the ratio of the energy release rate for the crack that has kinked from the interface and the energy release rate for the crack that propagates at the interface in dynamic loading conditions means that there is a small possibility for the crack to propagate along the interface for large crack speeds, what complies well with experimental results of Tippur and Rosakis, /18/.

CONCLUSION

Interfacial crack kinking away from the interface in the dynamic loading conditions is analysed. Based on results presented here, it is proven that it is possible to extend the criteria, which are valid for the crack kinking case in static loading conditions, to the case of crack kinking in dynamic loading conditions.

The condition for a crack that propagates dynamically along the interface to kink into one of the materials, is that the ratio of energy release rates for a crack that has kinked from the interface and for a crack that propagates at the interface, in dynamic loading conditions, is larger that the ratio of the dynamic fracture toughness of the material into which the crack case kinked and the fracture toughness of the interface.

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