

THE ONSET OF THERMAL INSTABILITY OF VISCOELASTIC ROTATING FLUID PERMEATED WITH SUSPENDED PARTICLES IN POROUS MEDIUM

STANJE TERMIČKE NESTABILNOSTI VISOKELASTIČNOG ROTIRAJUĆEG FLUIDA SA ČESTICAMA U SUSPENZIJI U POROZNOJ SREDINI

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Adresa autora / Author's address:

¹⁾ Department of Mathematics, Govt. College, Hamirpur, Himachal Pradesh, India, e-mail: drgcrana15@gmail.com

²⁾ Department of Mathematics, Govt. College, Nurpur, Himachal Pradesh, India

³⁾ Department of Mathematics, NSCBM Govt. College, Hamirpur, Himachal Pradesh, India

Keywords

- Brinkman porous medium
- Rivlin-Ericksen fluid
- rotation
- suspended particles
- viscoelasticity

Abstract

In this paper, the effect of rotation on the onset of thermal instability in the viscoelastic fluid permeated with suspended (fine dust) particles in porous medium is investigated. For the porous medium, the Brinkman model is employed and for the viscoelastic fluid, the Rivlin-Ericksen fluid model is used. By applying linear stability theory and normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the rotation, suspended particles, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid and it is observed that the rotation has stabilizing effect while suspended particles have a destabilizing effect on the system, whereas Darcy number and medium permeability have stabilizing/destabilizing effects under certain conditions. These results are in good agreement with earlier published results.

INTRODUCTION

The problem of thermal instability in porous medium has attracted considerable interest during the last few decades, because it has various applications in geophysics, food processing and nuclear reactors. Many researchers have investigated thermal convection problems by taking different types of fluids. A detailed account of the thermal instability of a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar /1/. Scanlon and Segel /2/ have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

In recent years, considerable interest has been evinced in the study of Rivlin-Ericksen elastico-viscous fluid having

Ključne reči

- Brinkman porozna sredina
- Rivlin-Eriksen fluid
- rotacija
- čestice u suspenziji
- viskoelastičnost

Izvod

U ovom radu se istražuje uticaj rotacije na pojavu stanja termičke nestabilnosti kod viskoelastičnog fluida sa česticama u suspenziji (fina prašina) u poroznoj sredini. Kod porozne sredine se uvodi Brinkman model, a za viskoelastični fluid se primenjuje Rivlin-Eriksen model fluida. Primenom teorije linearne stabilnosti i metode analize normalnog moda, izvedena je i analitički rešena relacija disperzije. Primećuje se da rotacija, čestice u suspenziji, gravitaciono polje i viskoelastičnost izazivaju oscilatorna stanja. Kod stacionarne konvekcije, Rivlin-Eriksen viskoelastični fluid se ponaša kao običan Njutnovski fluid, a primećuje se da rotacija ima stabilizirajući efekat dok čestice u suspenziji imaju destabilizirajući efekat na sistem, gde pritom Darsijev broj i propustljivost sredine imaju stabilizirajuće/destabilizirajuće efekte pod određenim uslovima. Dobijeni rezultati se u zadovoljavajućoj meri slažu sa ranijim objavljenim rezultatima.

relevance in chemical technology and industry. The investigation in porous media has been started with the simple Darcy model and gradually was extended to the Darcy-Brinkman model. A good account of convection problems in a porous medium are given by Vafai and Hadim /3/, Ingham and Pop /4/, Srivastava and Singh /5/, Garg et al. /6/, Sharma /7/, Sharma and Rana /8, 9/ and Nield and Bejan /10/. Such and other polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, ropes, cushions, seats, foam, plastics, engineering equipment, adhesives, contact lenses etc. Recently, polymers are used in agriculture, communications appliances and in biomedical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Recently, Attia /11/ studied Hiemenz flow through a porous medium of a non-Newtonian Rivlin-Ericksen fluid with heat transfer whereas the onset of convection in the

Rivlin-Ericksen fluid in a Darcy-Brinkman porous medium is studied by Rana /12/, Rana and Sharma /13/, Rana and Jamwal /14/ and Kango and Rana /15/. In the present paper, the effect of suspended particles and rotation on the onset of thermal instability of the Rivlin-Ericksen elastico-viscous fluid saturating a Darcy-Brinkman porous medium is investigated.

MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Here, we consider an infinite, horizontal, incompressible Rivlin-Ericksen elastico-viscous fluid layer of depth d , bounded by planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1 , which is acted upon by gravity $\mathbf{g}(0,0,-g)$. This layer is heated from below such that a steady adverse temperature gradient $\beta \left(= \frac{dT}{dz}\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

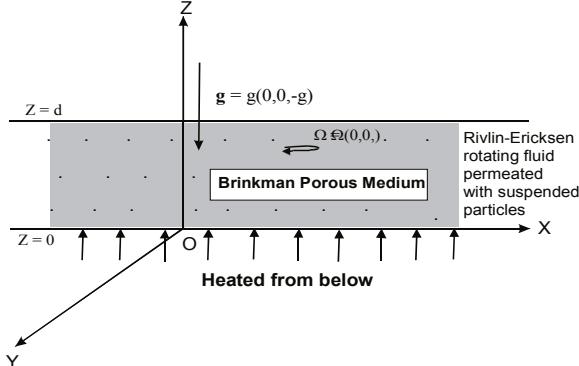


Figure 1. Schematic of the physical situation.
Slika 1. Shema fizičkog stanja

Let ρ , ν , ν' , p , ε , T , α and $\mathbf{v}(0,0,0)$, denote respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and equation of state for Rivlin-Ericksen elastico-viscous fluid (Chandrasekhar /1/, Sharma /7/, Garg et al. /6/, Sharma and Rana /8, 9/ and Rana /12/) in a Brinkman porous medium are

$$\frac{\rho}{\varepsilon} \left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\varepsilon} (\vec{v} \nabla) \vec{v} \right] = -\nabla p + \bar{g} \rho - \frac{1}{k_1} \left(\mu + \mu' \frac{\partial}{\partial t} \right) \vec{v} + \quad (1)$$

$$+ \frac{2}{\varepsilon} (\vec{v} \times \vec{\Omega}) + \tilde{\mu} \nabla^2 \vec{v} + \frac{K' N}{\varepsilon} (\vec{v}_d - \vec{v})$$

$$\nabla \cdot \vec{v} = 0 \quad (2)$$

$$\rho C_f \left(E \frac{\partial}{\partial t} + \vec{v} \nabla \right) T + m N C_{pt} \left[\varepsilon \frac{\partial}{\partial t} + \vec{v}_d \nabla \right] T = k_T \nabla^2 T \quad (3)$$

where $v_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively, $K' = 6\pi\rho\nu\eta$, where η is particle radius, is the Stokes drag coefficient, $v_d = (l, r, s)$ and $\bar{x} = (x, y, z)$, C_f , C_{pt} and k_T denote, respectively, the heat

capacity of the pure fluid, heat capacity of particles and 'effective thermal conductivity' of pure fluid. Here

$$E = \varepsilon + (1-\varepsilon) \left(\frac{\rho_s C_s}{\rho_0 C_f} \right)$$

which is constant, κ is the thermal diffusivity and $\tilde{\mu}$ is the effective viscosity of the porous medium; ρ_s , C_s ; ρ_0 , C_f denote the density and heat capacity of solid (porous) matrix and fluid respectively.

The equation of the state is

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \quad (4)$$

where α is the coefficient of thermal expansion, as the density variations arise mainly due to temperature variations.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN \left[\frac{\partial \vec{v}_d}{\partial t} + \frac{1}{\varepsilon} (\vec{v}_d \nabla) \vec{v}_d \right] = K' N (\vec{v} - \vec{v}_d) \quad (5)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla (N \vec{v}_d) = 0 \quad (6)$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion Eq.(1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles Eq.(5). The buoyancy force on the particles is neglected. Interparticle reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion Eq.(5) for the particles.

The initial state of the system is taken to be the quiescent layer (no settling) with a uniform particle distribution number N_0 . The initial state is

$$\vec{v} = (0, 0, 0), \quad \vec{q}_d = (0, 0, 0), \\ T = -\beta z + T_0, \quad \rho = \rho_0 (1 + \alpha \beta z), \\ N = N_0, \text{ a constant,} \quad (7)$$

is an exact solution to the governing equations.

Let $\mathbf{v}(u, v, w)$, θ , δp and $\delta \rho$ denote, respectively, the perturbations in fluid velocity $\mathbf{v}(0,0,0)$, temperature T , pressure p and density ρ .

Therefore, the change in density $\delta \rho$ caused by perturbation θ in temperature is given by

$$\delta \rho = -\alpha \rho_0 \theta. \quad (8)$$

The linearized perturbation equations governing the motion of the fluid are

$$\frac{1}{\varepsilon} \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p - g \alpha \theta - \frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) \vec{v} + \quad (9)$$

$$+ \frac{\tilde{\mu}}{\rho_0} \nabla^2 \vec{v} + \frac{K' N}{\rho_0 \varepsilon} (\vec{v}_d - \vec{v}) + \frac{2}{\varepsilon} (\vec{v} \times \vec{\Omega})$$

$$\nabla \cdot \vec{v} = 0 \quad (10)$$

$$\left(\frac{m}{K' \partial t} + 1\right) q_d = q \quad (11)$$

$$(E + b\varepsilon) \frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa \nabla^2 \theta \quad (12)$$

where $\nu = \frac{\mu}{\rho_0}$, $\nu' = \frac{\mu'}{\rho_0}$, $\kappa = \frac{k_T}{\rho_0 C_f}$ and w stand for kinematic viscosity, kinematic viscoelasticity, thermal diffusivity and vertical fluid velocity, respectively, $b = \frac{mN_0 C_{pt}}{\rho_0 C_f}$

and w, s are the vertical fluid and particles velocity.

In the Cartesian form, Eqs.(9-12) with the help of Eq.(8) can be expressed as

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial u}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial}{\partial x} (\delta p) - \frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) u + \frac{\tilde{\mu}}{\rho_0} \nabla^2 u + \\ &+ \frac{2}{\varepsilon} \Omega v - \frac{mN_0}{\varepsilon \left(\frac{m}{K' \partial t} + 1 \right) \rho_0} \frac{\partial u}{\partial t} \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial v}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial}{\partial y} (\delta p) - \frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) v + \frac{\tilde{\mu}}{\rho_0} \nabla^2 v + \\ &+ \frac{2}{\varepsilon} \Omega u - \frac{mN_0}{\varepsilon \left(\frac{m}{K' \partial t} + 1 \right) \rho_0} \frac{\partial v}{\partial t} \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial w}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial}{\partial z} (\delta p) - \frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) w + \frac{\tilde{\mu}}{\rho_0} \nabla^2 w + \\ &+ g\alpha\theta - \frac{mN_0}{\varepsilon \left(\frac{m}{K' \partial t} + 1 \right) \rho_0} \frac{\partial w}{\partial t} \end{aligned} \quad (15)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (16)$$

$$(E + b\varepsilon) \frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa \nabla^2 \theta \quad (17)$$

Operating Eqs.(13) and (14) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using Eq.(16), we get

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial z} \right) &= \frac{1}{\rho_0} \left(\nabla^2 - \frac{\partial^2}{\partial z^2} \right) \delta p - \frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) \left(\frac{\partial w}{\partial z} \right) + \\ &+ \frac{\tilde{\mu}}{\rho_0} \nabla^2 \left(\frac{\partial w}{\partial z} \right) + \frac{2}{\varepsilon} \Omega \zeta - \frac{mN_0}{\varepsilon \left(\frac{m}{K' \partial t} + 1 \right) \rho_0} \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial z} \right) \end{aligned} \quad (18)$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, is the z -component of vorticity.

Operating Eqs.(15) and (18) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2} \right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between Eqs.(15) and (18), we get

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial}{\partial t} (\nabla^2 w) &= -\frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) (\nabla^2 w) + \frac{\tilde{\mu}}{\rho_0} \nabla^4 w + \\ &+ g \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \alpha\theta + \frac{2}{\varepsilon} \Omega \frac{\partial \zeta}{\partial z} - \end{aligned} \quad (19)$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Operating Eqs.(13) and (14) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial \zeta}{\partial t} &= -\frac{1}{k_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) \zeta + \frac{\tilde{\mu}}{\rho_0} \nabla^2 \zeta + \frac{2}{\varepsilon} \Omega \frac{\partial \zeta}{\partial z} - \\ &- \frac{mN_0}{\varepsilon \left(\frac{m}{K' \partial t} + 1 \right) \rho_0} \frac{\partial \zeta}{\partial t} \end{aligned} \quad (20)$$

THE DISPERSION RELATION

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, s, \theta, \zeta] = [W(z), S(z), \Theta(z), Z(z)] \exp(ilx + imy + nt) \quad (21)$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is in general, a complex constant.

Using Eq.(21) in Eq.(19), Eqs.(20) and (17) become

$$\begin{aligned} \frac{n}{\varepsilon} \left(\frac{d^2}{dz^2} - k^2 \right) W &= -gk^2 \alpha\Theta - \frac{1}{k_1} (\nu + \nu' n) \left(\frac{d^2}{dz^2} - k^2 \right) W - \\ &- \frac{mN_0}{\varepsilon \rho_0 \left(\frac{m}{K' n} + 1 \right)} \left(\frac{d^2}{dz^2} - k^2 \right) W - \frac{2\Omega dZ}{\varepsilon dz} + \frac{\tilde{\mu}}{\rho_0} \left(\frac{d^2}{dz^2} - k^2 \right)^2 W \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{n}{\varepsilon} Z &= -\frac{1}{k_1} (\nu + \nu' n) Z - \frac{mN_0}{\varepsilon \rho_0 \left(\frac{m}{K' n} + 1 \right)} Z - \frac{2\Omega dW}{\varepsilon dz} + \\ &+ \frac{\tilde{\mu}}{\rho_0} \left(\frac{d^2}{dz^2} - k^2 \right) Z \end{aligned} \quad (23)$$

$$(E + b\varepsilon) n\Theta = \beta(W + bS) + \kappa \left(\frac{d^2}{dz^2} - k^2 \right) \Theta \quad (24)$$

Eqs.(22-24) in non-dimensional form, become

$$\begin{aligned} \left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1 + \tau_l \sigma)} + F \right) \sigma - D_A (D^2 - a^2) \right] \times \\ \times (D^2 - a^2) W + \frac{g\alpha a^2 d^2 P_l \Theta}{\nu} = 0 \end{aligned} \quad (25)$$

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F \right) \sigma - D_A(D^2 - a^2) \right] Z = \\ = \left(\frac{2\Omega d}{\varepsilon\nu} \right) P_l DW \quad (26)$$

$$(D^2 - a^2 - E'P_r\sigma)\Theta = -\left(\frac{\beta d^2}{\kappa} \right) W \quad (27)$$

where we have put $a = kd$, $\sigma = nd^2/\nu$, $E' = E + b\varepsilon$, $\tau = m/K'$, $\tau_1 = \tau\nu/d^2$, $M = mN_0/\rho_0$, $B = b + 1$. $P_l = k_l/d^2$ is the dimensionless medium permeability. $P_r = \nu/\kappa$ is the thermal Prandtl number, $F = \nu'/d^2$, is the dimensionless kinematic viscoelasticity, $D_A = \frac{\tilde{\mu}k_l}{\mu d^2}$, is the Darcy-Brinkman number and $D^* = d \frac{d}{dz}$ and the superscript * is suppressed.

Eliminating Θ and Z between Eqs.(25-27), we obtain

$$\left\{ 1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F \right) \sigma - D_A(D^2 - a^2) \right\} (D^2 - a^2) \times \\ \times (D^2 - a^2 - EP_r\sigma) \{W - Ra^2 P_l \left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma} \right) W + \\ + \frac{T_A P_l^2 (D^2 - a^2 - EP_r\sigma)}{1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l^2}{\varepsilon(1+\tau_1\sigma)} + F \right) \sigma - D_A(D^2 - a^2)} D^2 W = 0 \quad (28)$$

where $R = g\alpha\beta d^4/\nu\kappa$, is the thermal Rayleigh number, and $T_A = (2\Omega d^2/\varepsilon\nu)^2$, is the modified Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and the adjoining medium is electrically non-conducting. Boundary conditions appropriate to the problem are, /1/:

$$W = D^2 W = D^4 W = \theta = 0 \text{ at } z = 0 \text{ and } 1. \quad (29)$$

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using boundary conditions Eq.(29), we can show that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z; \quad W_0 \text{ is a constant.} \quad (30)$$

Substituting Eq.(29) in Eq.(27), we get

$$R_l x P = \left[1 + \left(\frac{P}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + \pi^2 F \right) i\sigma_1 + D_{A_l}(1+x) \right] \times \\ \times (1+x)(1+x+E'P_r i\sigma_1) \left(\frac{1+\tau_1 \pi^2 i\sigma_1}{B + \tau_1 \pi^2 i\sigma_1} \right) + \\ + \frac{T_A P (1+x+E'P_r i\sigma_1)}{1 + \left(\frac{P}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + \pi^2 F \right) i\sigma_1 + D_{A_l}(1+x)} \left(\frac{1+\tau_1 \pi^2 i\sigma_1}{B + \tau_1 \pi^2 i\sigma_1} \right) \quad (31)$$

where we have put

$$R_l = \frac{R}{\pi^4}, \quad T_A = \frac{T_A}{\pi^4}, \quad D_{A_l} = \frac{D_{A_l}}{\pi^2}, \quad x = \frac{a^2}{\pi^2}, \quad i\sigma_1 = \frac{\sigma}{\pi^2}, \quad P = \pi^2 P_l.$$

Eq.(31) is the required dispersion relation accounting for the onset of thermal convection in Rivlin-Ericksen rotating elasto-viscous fluid permeated with suspended particles in a Darcy-Brinkman porous medium.

STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here we examine the possibility of oscillatory modes if any, in the Rivlin-Ericksen elasto-viscous fluid due to the presence of rotation, suspended particles, viscoelasticity, medium permeability and gravity field. Multiplying Eq.(25) by W^* the complex conjugate of W , integrating over the range of z and making use of Eqs.(26) and (27) with the help of boundary conditions Eq.(29), we obtain

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F \right) \sigma \right] I_1 - \frac{\alpha a^2 g \kappa P_l}{\nu \beta} \left(\frac{1 + \tau_1 \sigma^*}{B + \tau_1 \sigma^*} \right) \times \\ \times (I_3 + EP_r \sigma^* I_4) - D_A (I_2 + d^2 I_5) + \\ + d^2 \left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F \right) \sigma^* \right] I_6 = 0 \quad (32)$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2 |W|^2) dz,$$

$$I_2 = \int_0^1 (|D^2 W|^2 + a^4 |W|^2 + 2a^2 |DW|^2) dz,$$

$$I_3 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta|^2) dz,$$

$$I_4 = \int_0^1 (|\Theta|^2) dz,$$

$$I_5 = \int_0^1 (|DZ|^2 + a^2 |Z|^2) dz,$$

$$I_6 = \int_0^1 (|Z|^2) dz.$$

The integral part I_1-I_6 are all positive definite. Putting $\sigma = i\sigma_i$ in Eq.(32), where σ_i is real and equating the imaginary parts, we obtain

$$\left[\left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1^2 \sigma_i^2)} + F \right) (I_1 - d^2 I_6) + \frac{\alpha a^2 g \kappa P_l}{\nu \beta} \times \right. \\ \left. \times \left(\frac{\tau_1(B-1)}{B^2 + \tau_1^2 \sigma_i^2} \right) (I_3 + E^4 P_l I_4) \right] \sigma_i = 0 \quad (33)$$

Eq.(33) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which means that modes may be non-oscillatory or oscillatory. The oscillatory modes introduced due to presence of viscoelasticity, suspended particles and rotation which were non-existent in their absence.

THE STATIONARY CONVECTION

For stationary convection, putting $\sigma = 0$ in Eqs.(31), we obtain

$$R_l = \frac{1+x}{xB} \left[\frac{1+x}{P} + \frac{(1+x)^2 D_{A_l}}{P} + \frac{T_{A_l} P}{1+(1+x) D_{A_l}} \right] \quad (34)$$

Eq.(34) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1} , B , D_{A_1} , P and Rivlin-Ericksen elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ .

To study the effects of suspended particles, Darcy number and medium permeability, we examine the behaviour of $\frac{\partial R_1}{\partial T_{A_1}}$, $\frac{\partial R_1}{\partial D_{A_1}}$, $\frac{\partial R_1}{\partial B}$ and $\frac{\partial R_1}{\partial P}$ analytically and numerically.

From Eq.(34), we get

$$\frac{\partial R_1}{\partial T_{A_1}} = \frac{1+x}{xB} \left(\frac{P}{1+(1+x)D_{A_1}} \right) \quad (35)$$

which is positive implying thereby the stabilizing effect of rotation on the thermal convection in Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles in a Brinkman porous medium which is an agreement with the result derived by Sharma /7/ and Sharma and Rana /8, 9/. Also in Fig. 2, R_1 increases with the increase in T_{A_1} . Thus rotation has a stabilizing effect on the system.

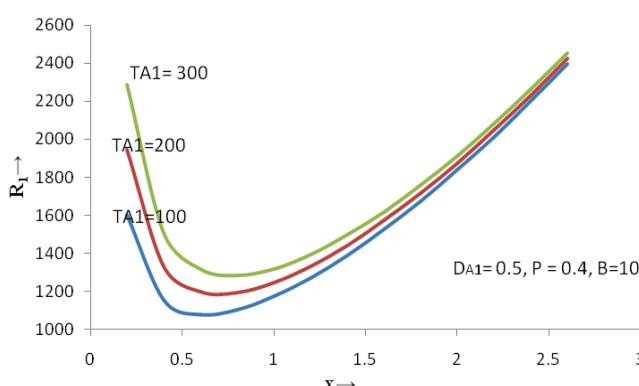


Figure 2. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Taylor number T_{A_1} .

Slika 2. Varijacija Rejlejevog broja R_1 sa bezdimenzionim talasnim brojem x za različite vrednosti Tejlorovog broja T_{A_1}

From Eq.(34), we get

$$\frac{\partial R_1}{\partial D_{A_1}} = \frac{(1+x)^2}{xB} \left(\frac{1+x}{P} - \frac{T_{A_1} P}{\{1+(1+x)D_{A_1}\}^2} \right) \quad (36)$$

From Eq.(36), we found that the modified Darcy number has a stabilizing effect if

$$\frac{1+x}{P} > \frac{T_{A_1} P}{\{1+(1+x)D_{A_1}\}^2} \quad (37)$$

and a destabilizing effect if

$$\frac{1+x}{P} < \frac{T_{A_1} P}{\{1+(1+x)D_{A_1}\}^2} \quad (38)$$

on the thermal convection in Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles in a Brinkman porous medium. Also in Fig. 3, R_1 increases/

decreases with the increase in D_{A_1} . Thus Darcy number has stabilizing/destabilizing effects, which clearly verify the result numerically. However, in the absence of rotation, the modified Darcy number has stabilizing effect which is an agreement with the result derived by Rana, /12/.

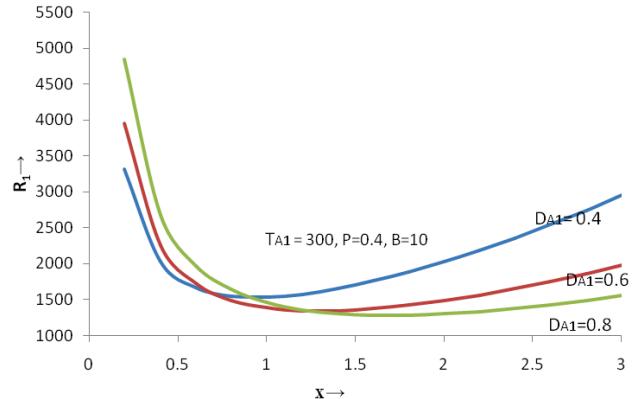


Figure 3. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Darcy number D_{A_1} .

From Eq.(33), we get

$$\frac{\partial R_1}{\partial B} = -\frac{1+x}{xB^2} \left[\frac{1+x}{P} + \frac{(1+x)^2 D_{A_1}}{P} + \frac{T_{A_1} P}{1+(1+x)D_{A_1}} \right] \quad (39)$$

which is negative. Hence, suspended particles have destabilizing effect on the thermal convection in Rivlin-Ericksen elastico-viscous fluid in a Brinkman porous medium. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel, /2/, Rana /11/ and Kango and Rana /15/. Also, in Fig. 4, R_1 decreases with the increase in suspended particles parameter B . Hence, suspended particles have a destabilizing effect.

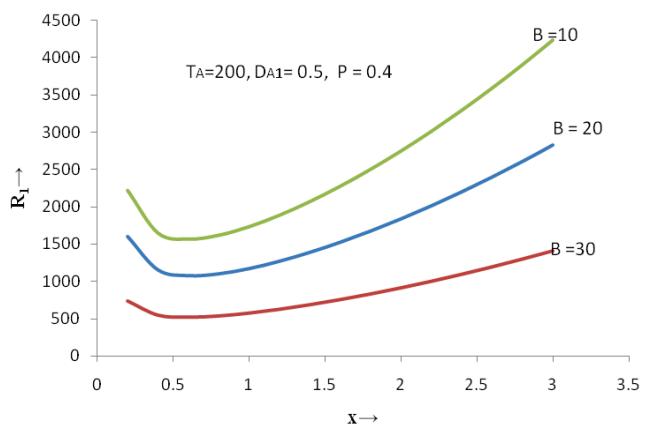


Figure 4. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of suspended particles parameter B .

Slika 4. Varijacija Rejlejevog broja R_1 sa bezdimenzionim talasnim brojem x za različite vrednosti čestica u suspenziji parametra B

It is evident from Eq.(34) that

$$\frac{\partial R_1}{\partial P} = -\frac{1+x}{xB} \left[\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} - \frac{T_{A_1} P}{1+(1+x)D_{A_1}} \right] \quad (40)$$

From Eq.(40) we found that the medium permeability has a stabilizing effect if

$$\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} < \frac{T_{A_1}}{1+(1+x)D_{A_1}} \quad (41)$$

and destabilizing effect if

$$\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} > \frac{T_{A_1}}{1+(1+x)D_{A_1}} \quad (42)$$

on the thermal convection in the Rivlin-Ericksen elasto-viscous fluid permeated with suspended particles in a Brinkman porous medium. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel /4/, Sharma and Rana /8, 9/, and Rana /12/. Also, in Fig. 5, R_1 increases/decreases with the increase in medium permeability parameter P . Hence, medium permeability has stabilizing/destabilizing effects.

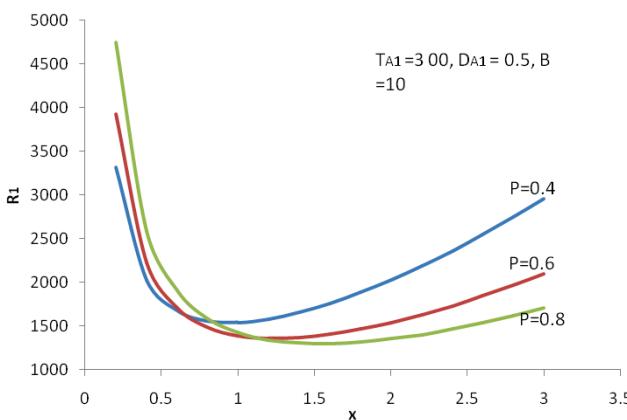


Figure 5. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of medium permeability P .

Slika 5. Varijacija Rejlejevog broja R_1 sa bezdimenzionim talasnim brojem x za različite vrednosti permeabilnosti sredine P

CONCLUSION

The effect of suspended particles and rotation on thermal convection in Rivlin-Ericksen elasto-viscous fluid in a Brinkman porous medium has been investigated. From the analysis, the main conclusions are as follows:

(i) For the case of stationary convection, Rivlin-Ericksen elasto-viscous fluid behaves like an ordinary Newtonian fluid as elasto-viscous parameter F vanishes with σ .

(ii) The expressions for $\frac{\partial R_1}{\partial T_{A_1}}$, $\frac{\partial R_1}{\partial D_{A_1}}$, $\frac{\partial R_1}{\partial B}$ and $\frac{\partial R_1}{\partial P}$ are

examined analytically and it has been found that the rotation has a stabilizing effect and suspended particles have a destabilizing effect on the system, whereas Darcy number has stabilizing/destabilizing effect on the system under conditions Eqs.(37) and (38), whereas medium permeability has a stabilizing/destabilizing effect on the system under conditions Eqs.(41) and (42).

(iii) The effects of suspended particles, Darcy number and medium permeability on thermal instability of Rivlin-Ericksen elasto-viscous fluid permeated with suspended particles in a Brinkman porous medium have also been shown graphically in Figs. 1, 2 and 3 respectively.

(iv) The oscillatory modes are introduced due to presence of rotation, viscoelasticity, suspended particles, and gravity field, which were non-existent in their absence.

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