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Jelena M. Djoković[1], Ružica R. Nikolić[2,3], Dejan D. Ilić[4], Jozef Vičan[3]

DYNAMIC CRACK GROWTH ALONG THE ELASTIC-PLASTIC BIMATERIAL INTERFACE

DINAMIČKI RAST PRSLINE NA ELASTIČNO-PLASTIČNOM BIMATERIJALNOM INTERFEJSU

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Adresa autora / Author's address:
1) University of Belgrade, Technical Faculty of Bor
e-mail: jelenamdjokovic@gmail.com
2) University of Kragujevac, Faculty of Engineering
3) University of Žilina, Faculty of Civil Engineering
4) Pupin Telecom Company, Belgrade

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- interface
- crack
- dynamic growth
- elastic-plastic material

Ključne reči
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Abstract

The paper considers the behaviour of the stress field that surrounds the tip of a crack that propagates dynamically along the interface between two elastic-plastic materials. Behaviour of elastic-plastic materials is described by the J_2 - deformation theory.

Analysis shows that the stress field at the crack tip is mainly defined by the material characteristics and the crack propagating speed. The field in the vicinity of the crack tip is more influenced by the material with the smaller coefficient of deformation hardening. It is also noticed that the crack propagating speed affects only the stress field in the Mode I conditions, while the stress field in Mode II conditions is insensitive to the variation of the crack tip speed. With increase of the crack tip speed values of stresses in the Mode I conditions are decreasing, so the normal stresses at the crack tip have values significantly smaller than the von-Mises stress. This points to the fact that the interfacial crack in Mode I conditions possesses substantially larger toughness than in equilibrium growth conditions.

Izvod

U radu je razmatrano ponašanje naponskog polja koje okružuje vrh prsline koja se dinamički širi duž interfejsa između dva elastično-plastična materijala. Ponašanje elastično-plastičnih materijala opisano je J_2 – deformacionom teorijom.

Analiza pokazuje da je naponsko polje u vrhu prsline uglavnom određeno karakteristikama materijala i brzinom širenja prsline. Na polje u blizini vrha prsline više utiče materijal sa manjim koeficijentom deformacionog ojačanja. Uočeno je, takođe, da brzina širenja prsline utiče samo na naponsko polje u uslovima Moda I dok je naponsko polje u uslovima Moda II gotovo neosetljivo na promenu brzine vrha prsline. Sa porastom brzine vrha prsline, opadaju vrednosti napona u uslovima Moda I tako da u vrhu prsline svi normalni naponi imaju vrednosti znatno manje od vrednosti Von Mises-ovog napona. Ovo ukazuje na činjenicu da interfejsna prslina u uslovima Moda I ima značajno veću žilavost loma nego u uslovima ravnotežnog rasta.

INTRODUCTION

Interfacial fracture plays a very important role in determining the behaviour of multi-phase materials and coupled structures. The low interface toughness of those materials is a result of destruction between individual phases, what contributes to their limited application in the engineering practice.

The first papers about interfacial fracture were published by Williams /1/, who was investigating the local stress field in the vicinity of the semi-infinite crack, whose surfaces were force-free. Then Rice and Shih /2/ obtained explicit expressions for the stress field in the vicinity of the crack tip and, accordingly, the remote elastic stress fields for various problems. Additional contributions and advance in investigations of static fractures at the interface were provided by works of Rice /3/, Shih /4/, Hutchinson and Suo /5/, and Nikolić and Veljković /6/.

Elastic-plastic analysis of interfacial crack behaviour was the subject of investigations of Shih and Asaro /7, 8/, O′Dowd /9/ and Nikolić and Veljković /10/, as well.

Depending on the nature of the load to which the composite structure is subjected, the destruction process can be dynamic as well. If the interface is already weakened by existence of flaws, they could act as initiators of cracks which under certain circumstances could propagate unstably along the interface. Such situations lead to necessity of analysing the dynamic growth of interfacial cracks. Experimental investigation of the deformation field at the tip of an interfacial crack was conducted by Tippur and Rosakis /11/ by use of the optical method Coherent Gradient Sensing (CGS) and high speed photography. The bimaterial system that they used was the PMMA (polymethylmethacrylate/aluminium) combination. They considered speeds up to 90% of Rayleigh' wave speed for PMMA. Following their considerations, Yang et al. /12/ obtained structures of elastodynamic field in conditions of the uniform crack growth at bimaterial interface. Also, inspired by experiments of Tippur and Rosakis /11/, Lo et al. /13/ have conducted numerical analysis of the same bimaterial system that was used in their experiments. In paper by Liu, Lambros and Rosakis /14/ an asymptotic structure of the near tip field in the bimaterial system was given for the case of nonuniform crack growth. Nikolić and Djoković /15/ have analysed the structure of the field in conditions of nonuniform crack growth. All the mentioned analyses are related to elastic isotropic bodies.

Ponte Castaneda and Mataga /16/ and Drugan /17/ have dealt with a problem of dynamic crack growth at elastic-plastic interface.

The objective of this work is to analyse the stress field at the tip of a crack at the interface between the two elastic-plastic materials in conditions of dynamic crack growth within the framework defined by nonlinear fracture mechanics in homogeneous materials, Hutchinson /18/.

PROBLEM FORMULATION

A crack that is dynamically propagating along the interface between two elastic-plastic materials is shown in Fig. 1. Behaviour of elastic-plastic materials is described by the J_2 deformation theory with linear plastic hardening. The moving coordinate system (x, y) origin is at the crack tip, which propagates with speed v, while r and θ are the polar coordinates.

Considering the fact that the crack tip speed has more significant role in solving problems for the case of plane strain than for plane stress, only the former case is considered in this paper.

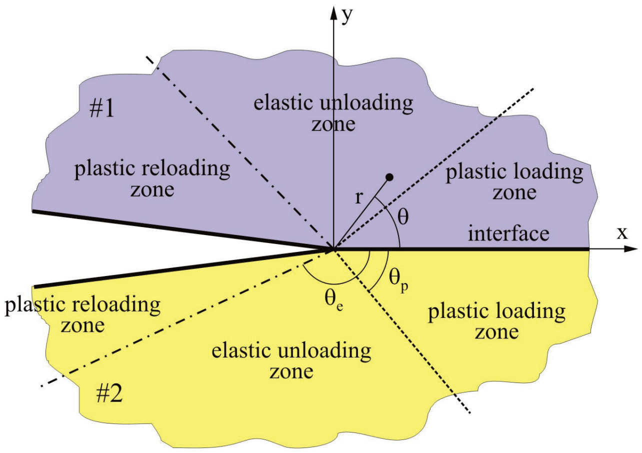

Figure 1. Schematic presentation of a crack propagating dynamically along the interface between two elastic-plastic materials.
Slika 1. Šematski prikaz prsline koja se širi dinamički duž interfejsa između dva elastično-plastična materijala.

Asymptotic analysis usually assumes that the field around the crack tip consists of the plastic loading zone, defined by angle θ_p and the zone of elastic unloading, defined by angle θ_e, Fig. 1, Ostlund and Gudmundson /19/. Elastic unloading occurs at θ_p when the effective stress rate of the material particle vanishes. In the zone of elastic unloading the material particle retains values of plastic strain which it possessed in the plastic loading zone. This is expected in the case when the point is deep within the unloading zone, where the effective stress at a point can reach some new value of yield stress, especially in materials with small strain hardening. Plastic reloading occurs at a certain critical value of angle θ_e, if material particle's effective stress again has the value that it had within the unloading zone.

In the polar-cylindrical coordinate system, Fig. 1, equilibrium equations can be written as:

$$\begin{aligned}(r\sigma_{rr})_{,r}+\sigma_{r\theta,r}-\sigma_{\theta\theta}&=2(1+\nu)(v/c_s)^2 r\dot{v}_r,\\ (r\sigma_{r\theta})_{,r}+\sigma_{\theta\theta,\theta}+\sigma_{r\theta}&=2(1+\nu)(v/c_s)^2 r\dot{v}_\theta.\end{aligned} \tag{1}$$

In Eq.(1) the considered stresses σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are normalised by average yield stress, $\sigma = (\sigma_{0(1)} + \sigma_{0(2)})/2$, while strain rate components v_r and v_θ are normalised by the corresponding yield strain $\varepsilon_0 = \sigma_0/E$ and the crack speed in the following way:

$$\begin{aligned}\sigma_{ij} &\sim \sigma_{ij}/\sigma_0,\\ v_i &\sim v_i/v\varepsilon_0,\end{aligned} \tag{2}$$

where Young's elasticity modulus E has value E_1 for material 1 and E_2 for material 2. Accordingly, other characteristics of materials 1 and 2, respectively, are: ν_1 and ν_2 – Poisson's ratio, ρ_1 and ρ_2 – materials' densities, μ_1 and μ_2 – shear moduli, α_1 and α_2 – hardening coefficients that represent ratios of tangent modulus E_t and respective elasticity moduli, i.e. $\alpha_I = E_t/E_i$, while $c_{s(i)}=\sqrt{\mu_i/\rho_i}$ are the transversal wave speeds, $i = 1, 2$.

Considering that the subject of this study is a problem of a crack that lies at the interface between two elastic-plastic materials whose behaviour is described by the J_2 deformation theory with linear plastic hardening, the constitutive equations are (Amazigo and Hutchinson /20/):

– in the plastic loading zone:

$$\dot{\varepsilon}_{ij}=(1+\nu)\dot{\sigma}_{ij}-\nu\dot{\sigma}_{kk}\delta_{ij}+\frac{3}{2}\cdot\frac{1-\alpha}{\alpha}\cdot\frac{s_{ij}\dot{\sigma}_e}{\sigma_e} \tag{3}$$

– in the elastic unloading zone:

$$\dot{\varepsilon}_{ij}=(1+\nu)\dot{\sigma}_{ij}-\nu\dot{\sigma}_{kk}\delta_{ij} \tag{4}$$

where: $\dot{\varepsilon}_{ij}$ - is the dimensionless strain rate tensor, $\dot{\sigma}_{ij}$ - is the dimensionless stress rate tensor, δ_{ij} - is Kronecker's delta symbol, $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$ is the stress deviator and $\sigma_e=\sqrt{3s_{ij}s_{ij}/2}$ is the von Mises effective stress.

Based on Eqs.(1), (3) and (4) it follows that for solving this problem there exist 6 differential equations for each of the two materials in the plane strain conditions, making a total of 12 partial differential equations for the whole field. Accordingly, the stress field at the crack tip has the following form, Shih /21/:

$$\sigma_{rr} = K_{pl} r^{-\frac{1}{2}+i\varepsilon} \tilde{\sigma}_{rr}(\theta),$$
$$\sigma_{\theta\theta} = K_{pl} r^{-\frac{1}{2}+i\varepsilon} \tilde{\sigma}_{\theta\theta}(\theta), \qquad (5)$$
$$\sigma_{r\theta} = K_{pl} r^{-\frac{1}{2}+i\varepsilon} \tilde{\sigma}_{r\theta}(\theta),$$

where K_{pl} is the plastic stress intensity factor. The mixed plasticity parameter m_{pl} is introduced due to the mixed loading mode at the crack tip, and is determined as (Shih /21/):

$$m_{pl} = \frac{2}{\pi} \arctan \left. \frac{\sigma_{\theta\theta}}{\sigma_{r\theta}} \right|_{r \to 0} \qquad (6)$$

Thus, $m_{pl} = 1$ for pure Mode I loading and $m_{pl} = 0$ for pure Mode II loading. Dimensionless angular functions $\tilde{\sigma}_{rr}$, $\tilde{\sigma}_{\theta\theta}$ and $\tilde{\sigma}_{r\theta}$ are defined in the Appendix. Parameter ε represents characteristics of the interfacial crack and is called the oscillatory index; it depends on crack tip speed in the manner described in Nikolić and Djoković /15/.

RESULTS AND DISCUSSION

Figure 2 shows the variation of mixed plasticity parameter m_{pl} defined by Eq.(6) in terms of variation of hardening coefficient for Mode I and Mode II loading conditions at the crack tip and four different tip speeds. The analysed bimaterial combination is characterized by $\varepsilon = 0.043$, $\alpha_2 = 0.1$, $\nu_1 = 0.33$, $\nu_2 = 0.5$, $E_1/E_2 = 1$ and $v/c_{s(1)} = v/c_{s(2)}$.

Figure 2 shows that the parameter of mixed plasticity is strongly influenced by higher values of the hardening coefficient, while for lower values it approaches zero. In Mode I conditions the change in crack tip speed practically has no influence on mixed plasticity parameter variation, while for Mode II loading there is a slight change with variation of crack tip speed.

Figure 3 shows the angular variation of stresses in Mode I loading conditions for four different crack tip speeds and following bimaterial combinations: $\varepsilon = 0.043$, $\alpha_2 = 0.05$, $\alpha_2 = 0.1$, $\nu_1 = 0.33$, $\nu_2 = 0.5$, $E_1/E_2 = 1$ and $v/c_{s(1)} = v/c_{s(2)}$.

From Figure 3 can be noticed that the radial stress component σ_{rr} has discontinuity across the interface, i.e. for $\theta = 0$, as well as that it tends to infinity for $\theta = \pm\pi$. Also, one can notice that with increase of crack tip speed, values of stresses are decreasing, thus all the stresses at the crack tip have values below the Von Mises effective stress. This observation points to the fact that the interfacial crack that propagates dynamically in the Mode I loading conditions has larger fracture toughness than in steady state conditions.

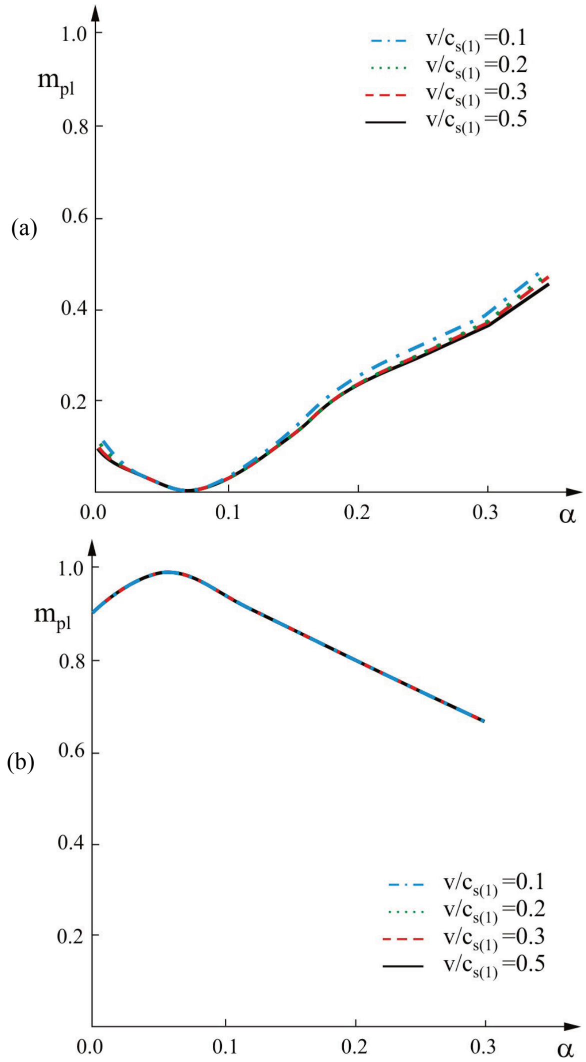

Figure 2. Influence of hardening coefficient α on the variation of mixed plasticity parameter m_{pl}: (a) in Mode I conditions and (b) in Mode II conditions.

Slika 2. Uticaj koeficijenta ojačanja α na promenu parametra mešovite plastičnosti m_{pl}: (a) u uslovima opterećenja Moda I i (b) u uslovima opterećenja Moda II

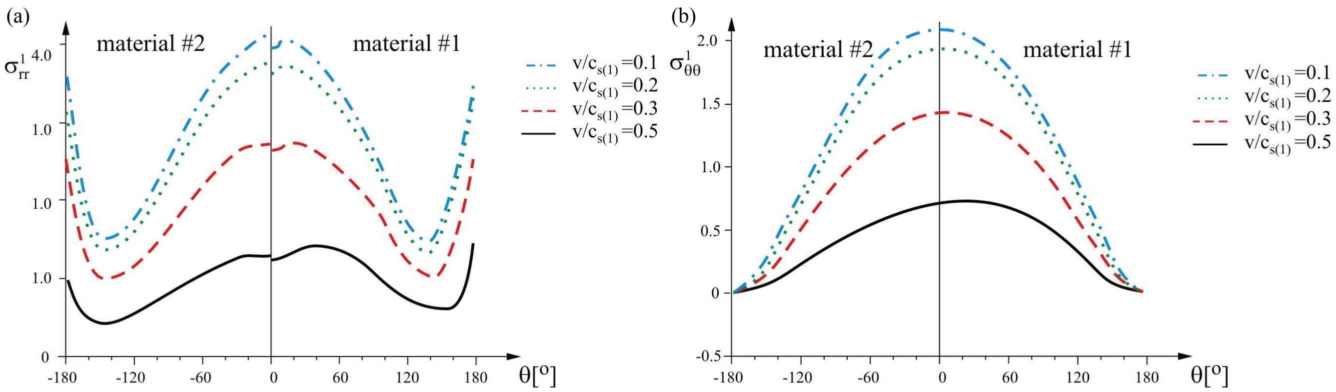

Figure 3. Angular distribution of stresses in the vicinity of crack tip at the interface between two elastic-plastic materials for different crack tip speeds in Mode I loading conditions: (a) σ_{rr}, (b) $\sigma_{\theta\theta}$ and (c) $\sigma_{r\theta}$.

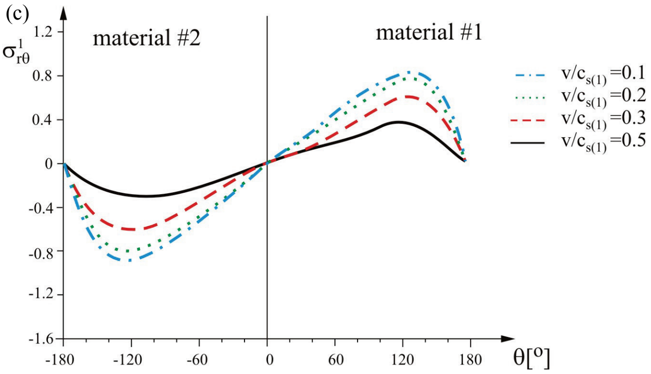

Figure 3. Angular distribution of stresses in the vicinity of crack tip at interface between two elastic-plastic materials for different crack tip speeds in Mode I loading conditions: (c) $\sigma_{r\theta}$.
Slika 3. Uglovna raspodela napona u blizini vrha prsline između dva elastoplastična materijala za različite brzine vrha prsline u uslovima opterećenja Moda I: (c) $\sigma_{r\theta}$.

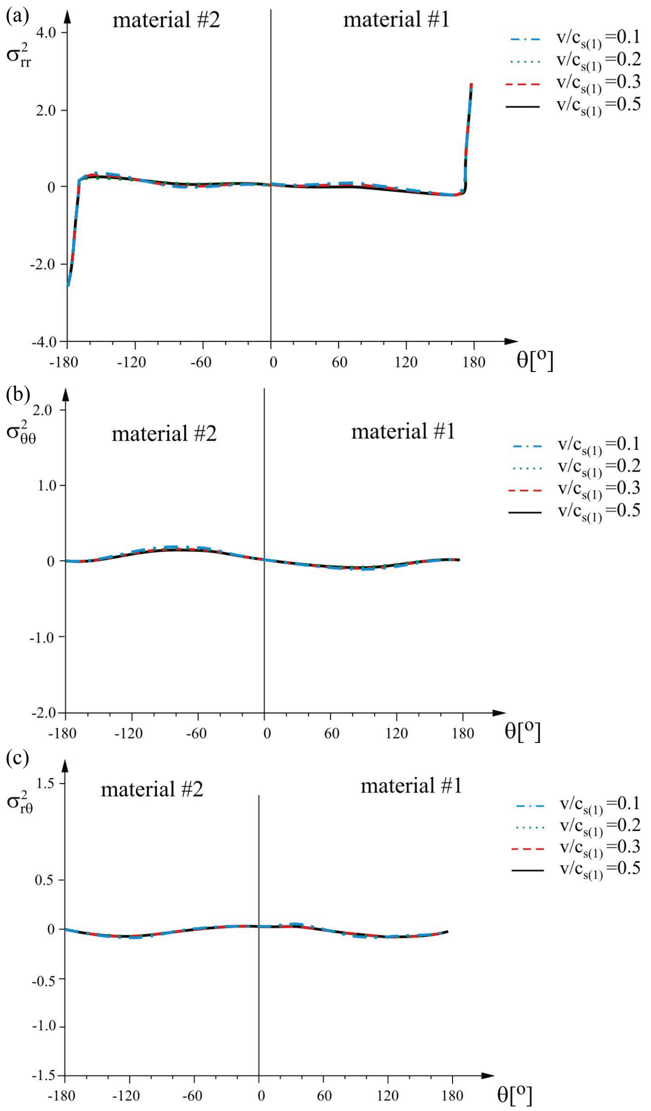

Figure 4. Angular distribution of stresses in the vicinity of crack tip at interface between two elastic-plastic materials for different crack tip speeds in Mode II loading conditions: (a) σ_{rr}, (b) $\sigma_{\theta\theta}$ and (c) $\sigma_{r\theta}$.
Slika 4. Uglovna raspodela napona u blizini vrha prsline između dva elastoplastična materijala za različite brzine vrha prsline u uslovima opterećenja Moda II: (a) σ_{rr}, (b) $\sigma_{\theta\theta}$ i (c) $\sigma_{r\theta}$

Figure 4 represents the angular variation of stresses in Mode II loading conditions for four different values of crack tip speed and the following bimaterial combination: $\varepsilon = 0.043$, $\alpha_2 = 0.05$, $\alpha_2 = 0.1$, $\nu_1 = 0.33$, $\nu_2 = 0.5$, $E_1/E_2 = 1$ and $v/c_{s(1)} = v/c_{s(2)}$.

Figure 4 shows that the angular distribution of stresses in Mode II loading conditions practically does not depend on the variation of the crack tip speed.

CONCLUSION

The behaviour of the stress field that surrounds the tip of a crack that is dynamically propagating along the interface between two elastic-plastic materials is analysed in this paper. Elastic-plastic material behaviour is described by J_2 deformation theory with linear plastic hardening.

Results presented here show that the parameter of mixed plasticity is strongly influenced by higher values of the hardening coefficient, while for lower values it approaches zero. At Mode I loading conditions the change of crack tip speed has almost no influence on the variation of mixed plasticity parameter, while for Mode II loading conditions there is a small change with variation of crack tip speed.

The analysis shows that the stress field at the crack tip is mainly governed by material characteristics and crack propagation speed. It is noticed that the crack speed affects the stress field only in Mode I loading conditions, while the stress field in Mode II is almost insensitive to variation of crack tip speed. With increase of crack tip speed, values of stresses in Mode I conditions are decreasing, so all normal stresses have values significantly smaller than Von-Mises effective stress. This shows that the dynamic interfacial crack in Mode I conditions has considerably higher fracture toughness than in steady state conditions.

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APPENDIX

Stress functions in Eq.(5) for $0 \le \theta \le \pi$ are:

$$\tilde{\sigma}_{rr}(\theta) = \frac{1}{D\cosh\pi\varepsilon}\left\{\begin{array}{l} \dfrac{(1+2\beta_1^2-\beta_2^2)}{\sqrt{1-\beta_1^2\sin^2\theta}}(S_{11}\cos\varepsilon\cos\dfrac{\theta}{2}+ \\ +S_{12}\sin\varepsilon\sin\dfrac{\theta}{2})+ \\ +\dfrac{2\beta_2}{\sqrt{1-\beta_2^2\sin^2\theta}}(S_{21}\cos\varepsilon\cos\dfrac{\theta}{2}+ \\ +S_{22}\sin\varepsilon\sin\dfrac{\theta}{2}) \end{array}\right.$$

$$\tilde{\sigma}_{r\theta}(\theta) = \frac{1}{D\cosh\pi\varepsilon}\left\{\begin{array}{l} \dfrac{-2\beta_1}{\sqrt{1-\beta_1^2\sin^2\theta}}(S_{12}\sin\varepsilon\cos\dfrac{\theta}{2} \\ -S_{11}\cos\varepsilon\sin\dfrac{\theta}{2})+ \\ +\dfrac{(1+\beta_2^2)}{\sqrt{1-\beta_2^2\sin^2\theta}}(S_{21}\cos\varepsilon\sin\dfrac{\theta}{2}- \\ -S_{22}\sin\varepsilon\cos\dfrac{\theta}{2}) \end{array}\right.$$

$$\tilde{\sigma}_{\theta\theta}(\theta) = \frac{1}{D\cosh\pi\varepsilon}\left\{\begin{array}{l} \dfrac{-(1+\beta_2^2)}{\sqrt{1-\beta_1^2\sin^2\theta}}(S_{11}\cos\varepsilon\cos\dfrac{\theta}{2}+ \\ +S_{12}\sin\varepsilon\sin\dfrac{\theta}{2})+ \\ +\dfrac{2\beta_2}{\sqrt{1-\beta_2^2\sin^2\theta}}(S_{21}\cos\varepsilon\cos\dfrac{\theta}{2}+ \\ +S_{22}\sin\varepsilon\sin\dfrac{\theta}{2}) \end{array}\right.$$

where: $D = 4\beta_1\beta_2 - (1+\beta_2^2)^2$

$$\beta_1^2 = 1-\frac{v^2}{c_\ell^2},\ \beta_2^2 = 1-\frac{v^2}{c_s^2},\ c_\ell = \sqrt{\frac{2(1-\nu)}{1-2\nu}\cdot\frac{\mu}{\rho}}$$

$$S_{11} = (1+\beta_2^2)\cosh\varepsilon(\pi-\theta) - 2\alpha_1\beta_2\sinh\varepsilon(\pi-\theta)$$

$$S_{12} = (1+\beta_2^2)\sinh\varepsilon(\pi-\theta) - 2\alpha_1\beta_2\cosh\varepsilon(\pi-\theta)$$

$$S_{21} = \alpha_1(1+\beta_2^2)\sinh\varepsilon(\pi-\theta) - 2\beta_1\cosh\varepsilon(\pi-\theta)$$

$$S_{22} = \alpha_1(1+\beta_2^2)\cosh\varepsilon(\pi-\theta) - 2\beta_1\sinh\varepsilon(\pi-\theta).$$