This paper was presented at the Fourteenth Meeting "New Trends in Fatigue and Fracture" (NT2F14) Belgrade, Serbia, 15–18 September, 2014

Marko D. Katinić[1], Dražan V. Kozak[2], Pejo I. Konjatić[2], Darko Z. Damjanović[2]

A NUMERICAL ELASTIC ANALYSIS ON THE INTERACTION OF TWIN EDGE CRACKS IN A FINITE PLATE UNDER TENSION

NUMERIČKA ANALIZA ELASTIČNOSTI INTERAKCIJE DVOJNIH IVIČNIH PRSLINA U KONAĆNOJ ZATEGNUTOJ PLOČI

Originalni naučni rad / Original scientific paper
UDK /UDC: 539.375
Rad primljen / Paper received: 14.09.2014

Adresa autora / Author's address:
[1] Petrochemical Kutina, Kutina, Croatia,
contact: marko.katinic@petrokemija.hr
[2] Mechanical Engineering Faculty, Slavonski Brod, Croatia

Keywords
- crack interaction
- twin edge cracks
- stress intensity factor
- finite element analysis

Ključne reči
- interakcija prslina
- dvojne ivične prsline
- faktor intenziteta napona
- analiza konačnim elementima

Abstract

Multiple cracks frequently appear in actual components such as ageing aircrafts, pressure vessels, piping systems and others. A good understanding of the behaviour of crack interaction and coalescence is essential for reliable structural integrity assessment. In this paper, the interaction of twin edge cracks in a finite plate subjected to the remote tension is investigated. The stress intensity factors (SIFs) for these cracks are calculated using two-dimensional linear finite element analysis. The results of the calculation show that crack interaction intensity and the crack interaction limit at elastic condition are influenced by crack distance ratio and crack width ratio.

Izvod

Višestruke prsline se često pojavljuju u stvarnim komponentama kao što su vremešne letelice, posude pod pritiskom, sistemi cevovoda i drugi. Dobro razumevanje ponašanja interakcije prslina i njihova koalescencija su bitni za pouzdanu procenu integriteta konstrukcije. U ovom radu je istražena interakcija dvojnih ivičnih prslina u konačnoj ploči pod dejstvom udaljenog zatezanja. Faktori intenziteta napona (SIF) ovih prslina su sračunati primenom analize dvodimenzionalnim linearnim konačnim elementima. Rezultati proračuna pokazuju da na intenzitet interakcije prslina i na granični uslov elastičnosti pri interakciji prslina utiču odnos rastojanja i odnos širine prslina.

INTRODUCTION

In many practical situations, different structural components contain more than one crack-like defect. Such cracking often occurs in localized patches or colonies owing to various types of material failure, such as stress-corrosion cracking /1/, fatigue /2/, and corrosion fatigue /3/. The different geometrical parameters of multiple cracks produce various elastic crack interactions. The interaction of multiple cracks changes the stress field around the crack tip and hence changes the stress intensity factor (SIF) at multiple crack tips. Depending on the geometrical parameters of multiple cracks, the interaction of these cracks contributes to stress shielding or the stress amplification effect. A good understanding of the crack interaction is therefore essential for integrity assessment of cracked components.

Assessment of the structural integrity of components containing crack-like defects in a linear elastic condition should be done using the SIF. An analytical SIF solution based on linear elastic fracture mechanics for single crack in cracked body is only applicable when there is no elastic crack interaction. In the elastic crack interaction range, the above analytical SIF formulation cannot be applied and should be modified. The boundary between the existence and non-existence of elastic crack interaction and analytical formulation of SIF to determine the stress crack interaction are still in research, /4-9/.

In this study, the interaction of twin parallel edge cracks in a finite plate subjected to remote tension is investigated. SIFs for these cracks are calculated using two-dimensional linear finite element (FE) analysis. These simulations are carried out to investigate the effect of crack distance ratio and crack width ratio on the determination of elastic crack interaction intensity and crack interaction limit.

FINITE ELEMENT ANALYSIS

Figure 1 depicts twin parallel edge cracks in a plate under remote tension σ, with relevant dimensions considered in the present work. The size of the plate considered herein is $H/W = 5$. To reduce the work-load, H and W are fixed in the construction of the FE model. Crack width ratio a/W and crack distance ratio d/a are varied in the following ranges: $0.05 \leq a/W \leq 0.5$ and $0.5 \leq d/a \leq 3.0$.

Elastic analysis of the FE model for this case is performed using the general-purpose FE program ABAQUS,

/10/. It is assumed that the material of the plate is homogeneous and isotropic. The selected material properties are the elastic modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.3$. Thus a total of 60 numerical calculations are performed in the presented work.

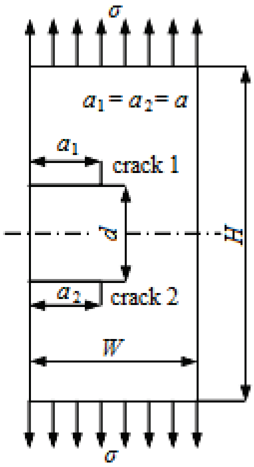

Figure 1. Twin parallel edge cracks in plate under tension.
Slika 1. Dvojne paralelne ivične prsline u zategnutoj ploči

Although there is a plane of symmetry, the 2D model of the plate was done completely. Therefore, two seam cracks are included in this model. The crack front is chosen to be equivalent to the crack tip. The direction of virtual crack extension at the crack tip is specified by the virtual crack extension direction.

A small geometry change continuum FE model is employed. Eight-node full integration elements for two-dimensional, plain-strain problems (CPE8) are used. The total meshes are varied in different models, but the same number of elements in the mesh is used for the crack tip in all of the cases considered here. To obtain the square root singularity, the elements of the innermost ring at the crack tip are degenerated into triangles. The three nodes along one side of the eight-node element are defined so that they share the same geometrical place. The three collapsed nodes are constrained to move together. The mid-side nodes on the sides connected to the crack tip are moved to the 1/4 point nearest the crack tip. Figure 2 shows the typical FE mesh for $a/W = 0.5$ and $d/a = 1.0$.

The domain integral method is used to evaluate SIF in ABAQUS. The method is quite robust in the sense that accurate contour integral estimates are usually obtained even with quite coarse meshes. The method is robust because the integral is taken over a domain of elements surrounding the crack and because errors in local solution parameters have less effect on the evaluated quantity of SIF, /10/.

To gain confidence in the present FE analysis, the results of FE elastic SIF solutions for single edge crack are compared with well-known solutions, /11/, in Fig. 3, indicating small differences. The results are given using the dimensionless shape factor for a single edge crack F_S.

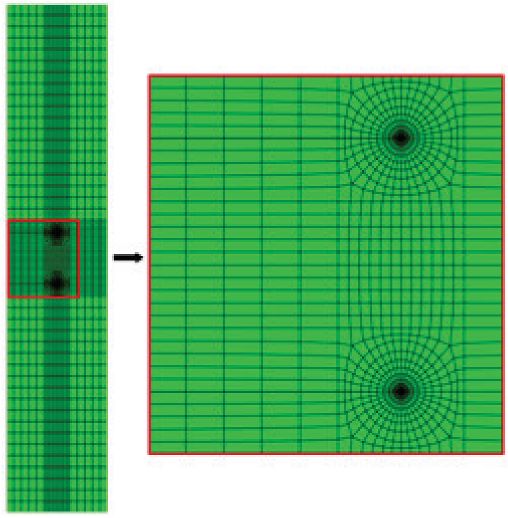

Figure 2. Typical FE mesh for $a/W = 0.5$ and $d/a = 1.0$.
Slika 2. Tipična mreža za $a/W = 0,5$ i $d/a = 1,0$

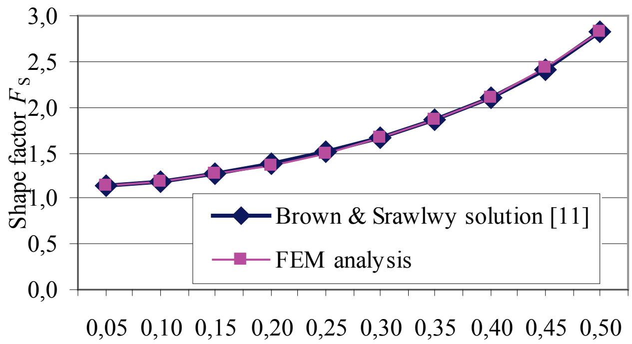

Figure 3. Comparison of shape factor F_S obtained using literature /11/ and using FEM.
Slika 3. Poređenje faktora oblika F_S dobijenog korišćenjem literature /11/ i primenom MKE.

RESULTS AND DISCUSSION

The elastic interaction analysis of twin edge cracks is performed using the so-called shape factor for twin edge cracks F_D. The analytical formulation of SIF for a single edge crack in a pure Mode I loading condition can be expressed as follows:

$$K_I^S = \sigma\sqrt{\pi a} \cdot F_S(a/W) \quad (1)$$

where the shape factor F_S depends only on the crack width ratio a/W.

Analogously one can express the Mode I SIF of twin edge cracks in the following way:

$$K_I^D = \sigma\sqrt{\pi a} \cdot F_D(a/W,\ d/a) \quad (2)$$

where the shape factor F_D depends not only on the crack width ratio a/W but on the crack distance ratio d/a also. The values of the shape factor F_D are obtained by using Eq.(2) based on SIF values calculated by FEM.

Figure 4 depicts the variation of the shape factor F_D depending on the crack distance ratio d/a for crack width ratios $a/W = 0.5$ and 0.45. The values of the shape factor F_S at $a/W = 0.5$ and 0.45 are also listed. It can be seen that interaction of the cracks is in shielding mode in both cases because the shape factor F_D is lower than the shape factor F_S. The most intensive crack interaction is at $d/a = 0.5$ because of the strongest shielding effect. The effect of crack interaction represented by the shape factor F_D shows an ascending trend from $d/a = 0.5$ to 3.0. It means that the shielding effect becomes weaker with the increasing ratio d/a, and the effect of crack interaction slowly diminishes. This ascending trend of the shape factor F_D coincides with the crack interaction theory as defined by /12/. The shape factor F_D converges to the shape factor F_S at $d/a = 2.5$–3.0 reaching in that way the crack interaction limit.

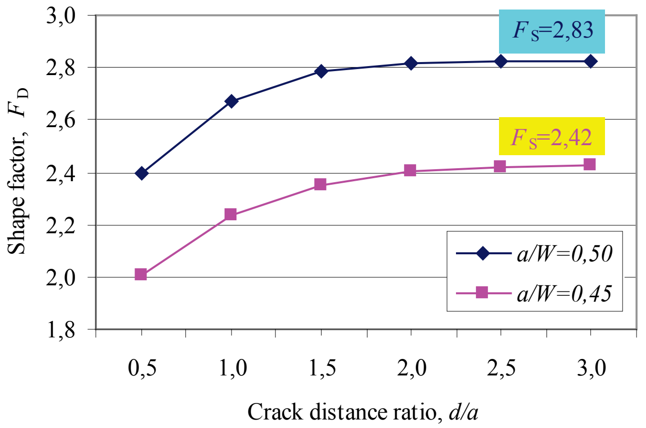

Figure 4. Variation of F_D versus ratio d/a for $a/W = 0.5$ and 0.45.
Slika 4. Promena F_D sa d/a za $a/W = 0.5$ i 0.45

The variation of the shape factor F_D versus the crack distance ratio d/a for crack width ratios of $a/W = 0.4$ and 0.35 is shown in Fig. 5. The values of the shape factor F_S at $a/W = 0.4$ and 0.35 are given as well. It can be seen that the crack interaction in the shielding mode exists also in these cases. The most intensive crack interaction is also at $d/a = 0.5$. The shape factor F_D shows again the ascending trend from $d/a = 0.5$ to 3.0 because the effect of crack interaction slowly diminishes with the increasing ratio d/a. The crack interaction limit for both ratios $a/W = 0.4$ and 0.35 is approximately at $d/a = 3.0$.

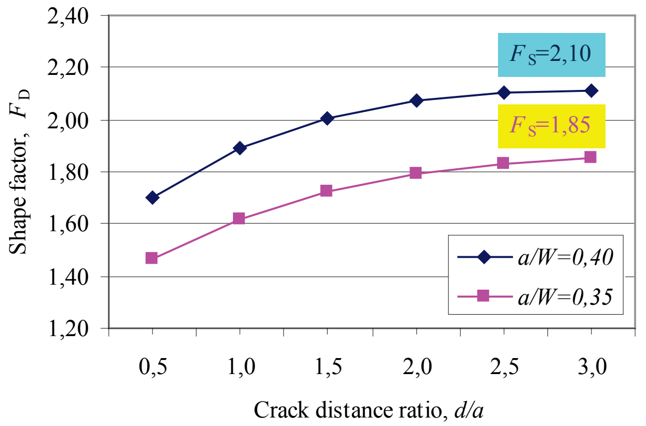

Figure 5. Variation of F_D versus ratio d/a for $a/W = 0.40$ and 0.35.
Slika 5. Promena F_D sa odnosom d/a za $a/W = 0{,}40$ i 0,35

Figure 6 shows the variation of the shape factor F_D depending on the crack distance ratio d/a for crack width ratios $a/W = 0.3$ and 0.25. The values of the shape factor F_S at $a/W = 0.3$ and 0.25 are also given. As it can be seen again the crack interaction is in shielding mode. The strongest shielding effect also belongs to the ratio $d/a = 0.5$. The shape factor F_D has again the ascending trend from $d/a = 0.5$ to 3.0. However, in this range of the d/a ratio, the factor F_D does not reach the corresponding value of factor F_S. It means that the crack interaction limit for both ratios $a/W = 0.3$ and 0.25 should be at $d/a > 3.0$.

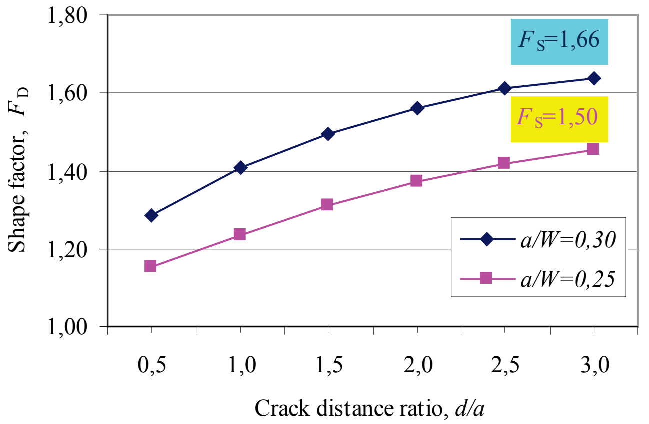

Figure 6. Variation of F_D versus ratio d/a for $a/W = 0.3$ and 0.25.
Slika 6. Promena F_D sa odnosom d/a za $a/W = 0{,}3$ i 0,25

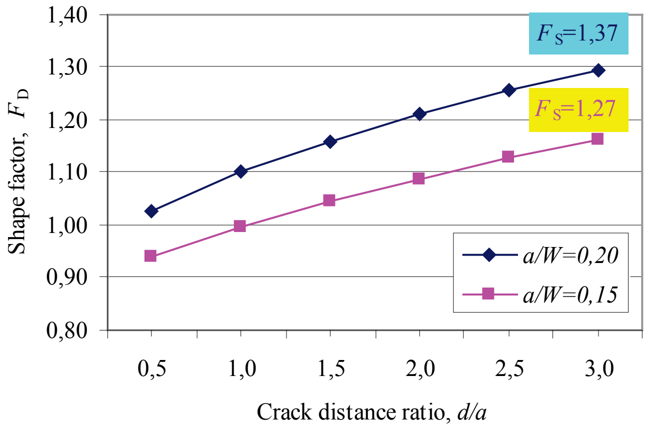

Figure 7. Variation of F_D versus ratio d/a for $a/W = 0.2$ and 0.15.
Slika 7. Promena F_D sa odnosom d/a za $a/W = 0{,}2$ i 0,15

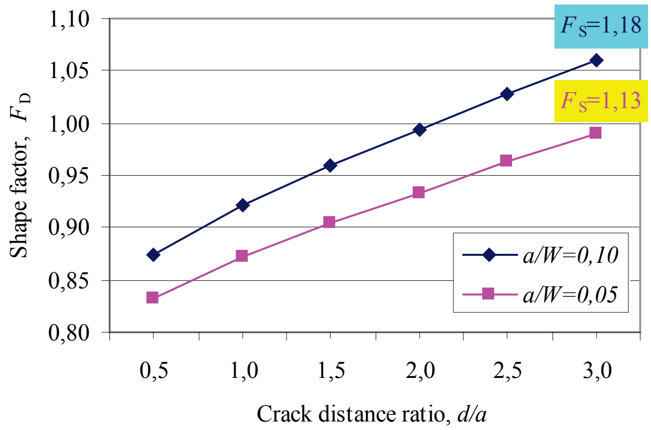

Figure 8. Variation of F_D versus ratio d/a for $a/W = 0.1$ and 0.05.
Slika 8. Promena F_D sa odnosom d/a za $a/W = 0{,}1$ i 0,05

Figures 7 an 8 depict the dependence of the shape factor F_D on the crack distance ratio d/a for ratios $a/W = 0.2$ and 0.15, respectively for ratios $a/W = 0.1$ and 0.05. The corresponding values of the shape factor F_S for each considered ratio a/W are also given. In all of these cases, the interaction of cracks shows the existence of the shielding effect which is the strongest at $d/a = 0.5$. The ascending trend of the shape factor F_D in range from $d/a = 0.5$ to 3.0 exists as in the previous cases. The crack interaction limit for ratios $a/W = 0.2$, 0.15, 0.1 and 0.05 is not reached in the considered range of ratios d/a. Based on the trends of the factor F_D it can be assumed that the crack interaction limit for each of these ratios a/W is at $d/a > 3.0$.

Figure 9 depicts the variation of the shape factor F_D versus the crack width ratio a/W at $d/a = 3.0$. Variations of the shape factor F_S versus the ratio a/W are also shown. It can be seen that the difference between factors F_D and F_S is higher at lower values of the ratio a/W. Due to this it can be concluded that the crack interaction limit for lower values of the ratio a/W will be reached at a higher value of the ratio d/a. The biggest difference between factors F_D and F_S is at $a/W = 0.05$. It means that the crack interaction limit for ratio $a/W = 0.05$ will be reached at the highest ratio d/a compared to the other considered a/W ratios. The crack interaction limit values for ratios $a/W = 0.05$, 0.1, 0.15, 0.2, 0.25 and 0.3 are at $d/a > 3.0$. In order to determine these values it is necessary to make some additional FEM calculations. However, it is not within the scope of this paper.

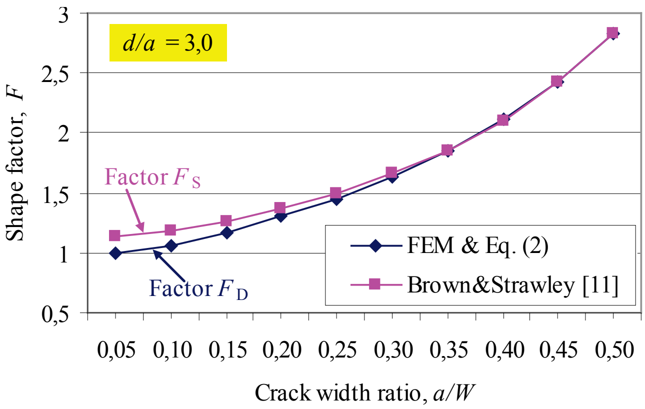

Figure 9. Variation of F_D versus ratio a/W for $d/a = 3.0$.
Slika 9. Promena F_D sa odnosom a/W za $d/a = 3,0$.

CONCLUSIONS

The interaction effect on the shape factor F_D of twin parallel edge cracks in a plate under tension has been analysed on the basis of comprehensive FE elastic calculations. The main conclusions are listed as follows:

- The interaction effect of these cracks in the elastic regime is influenced by the crack distance ratio d/a and the crack width ratio a/W.
- The interaction of these cracks is in the shielding mode which is the strongest at ratio $d/a = 0.5$.
- The shielding effect is advantageous from the standpoint of structural failure, because it reduces the stress intensity factor K_I.
- Based on a known value of the factor F_D, one can calculate the stress intensity factor K_I which is required for structural integrity assessment.
- The present FE calculations have proven the abilities to define the crack interaction limit in elastic regime, which also depends on the values of the crack distance ratio d/a and the crack width ratio a/W.
- The crack interaction limit for lower values of the ratio a/W is at higher values of the ratio d/a.
- The crack interaction limit values for ratios $a/W = 0.35$, 0.4, 0.45 and 0.5 are at $d/a \leq 3.0$. For other considered ratios a/W, the crack interaction limit values are at $d/a >$ 3.0.

REFERENCES

1. Leis, B.N., Mohan, R., *Coalescence conditions for stress-corrosion cracking*, Proceedings of the Seventh ISOPE, Vol.4, Quebec, Canada, 1997, pp.607-613.
2. Soboyejo, W.O., Knott, J.F., *Fatigue crack propagation of coplanar semielliptical cracks in pure bending*, Eng. Fracture Mech. 1990, 37, pp.323-340.
3. Wang, Y.Z., Atkinson, J.D., Akid, R., Parkins, R.N., *Crack interaction, coalescence and mixed mode fracture mechanics,* Fatigue Fracture Eng. Mater. Struct., 1996, 19, pp.427-439.
4. Kamaya, M., *Growth evaluation of multiple interacting surface cracks. Part I: Experiments and simulation of coalesced crack*, Engineering Fracture Mechanics, 2008, 75, pp.1336-1349.
5. Kamaya, M., *Growth evaluation of multiple interacting surface cracks. Part II: Growth evaluation of parallel cracks*, Engng. Fracture Mechanics, 2008, 75, pp.1350-1366.
6. Kamaya, M., Miyokawa, E., Kikuchi, M., *Growth prediction of two interacting surface cracks of dissimilar sizes*, Engineering Fracture Mechanics, 2010, 77, pp.3120-3131.
7. Milwater, H.R., *A simple and accurate method for computing stress intensity factors of collinear interacting cracks*, Aerospace Science and Technology, Vol.14 (8), 2010, pp.542-550.
8. Daud, R., Ariffin, A.K., Abdullah, S., Ismail, A.E., *Interacting cracks analysis using finite element method*, Applied Fracture Mechanics, Chapter 13, Ed. Alexander Belov, Published by InTech, 2012, pp.357-378.
9. Daud, R., Rojan, M.A., Ariffin, A.K., Abdullah, S., *Elastic crack interaction limit of two interacting edge cracks in finite body*, Int. Conf. on Applications and Design in Mech. Engng. (ICADME), 2012, Penang, Malaysia, CD-ROM, AM17.
10. ABAQUS User's Manual. Version 6.10, Hibbitt, Karlsson and Sorensen Inc. 2010.
11. Brown, W.F.Jr., Srawley, J.E., *Plane Strain Crack Toughness Testing of High Strength Metallic Materials*, ASTM STP 410, ASTM 1966, p.13.
12. Kachanov, M., *Elastic solids with many cracks and related problems*, Advances in Applied Mechanics, Eds. J. Hutchinson and T. Wu, vol.30, Academic Press, New York, 1993, pp.259-445.