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WAVELETS - AN OVERVIEW OF VIBRATION-RELATED APPLICATIONS

VEJVLETI (TALASIĆI) – PREGLED PRIMENE VIBRACIJA VEJVLETA

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Abstract

In this paper an overview of wavelets vibration-related applications for evolutionary spectrum estimation, random field simulation, system identification, damage detection, and material characterization is given.

Izvod

U ovom radu je dat pregled vibracije talasića - njihove primene u razvojnoj proceni gustine spektra, simulacije slučajnog polja, identifikacije sistema, detekcije oštećenja i određivanja osobina materijala.

INTRODUCTION

Wavelet-based approaches are significant tools for joint time-frequency analysis of problems related to vibrations of mechanical and structural systems. This applies to the characterization of the system excitation, the system identification, and system response determination. Several examples exist in nature of stochastic phenomena with a time-dependent frequency content. The frequency content of earthquake records, for instance, evolves in time due to the dispersion of propagating seismic waves /1, 2/. Further, sudden changes in the wave frequency at a given location of the sea surface are often induced by fast moving of meteorological fronts /3/. Also, a rapid change in the frequency content is generally associated with waves at the breaking stage. Similarly, turbulent gusts of time-varying frequency content are often embedded in wind fields. Appropriate description of such phenomena is obviously crucial for design and reliability assessments. In an early attempt, concepts of traditional Fourier spectral theory were generalized to provide spectral estimates, such as the Wigner-Ville method /4, 5/. However, it soon became clear that the extension of the traditional concept of a spectrum is not unique, and proposed time-varying spectra could have contradictory properties /6, 7/.

TIME-DEPENDENT SPECTRA ESTIMATION OF STOCHASTIC PROCESSES

The first steps in time-frequency analysis trace back to the work of Gabor /8/, who applied for signal analysis a fundamental concept developed in quantum mechanics by Wigner /4/, a decade earlier. Gabor functions are shown in Fig. 1 for three different values of ω. Wavelet analysis is readily applicable for estimating time-varying spectral properties, and significant effort has been devoted to formulating 'wavelet energy principles' that work as alternatives to classical Fourier methods. Measures of a time-varying frequency content were first obtained by *sectioning*, at different time instants, the wavelet coefficients mean square map /9-12/. Developing consistent spectral estimates from such sections, however, is not straightforward. From a theoretical point of view, either it requires an appropriate wavelet-based definition of time-varying spectra or it must relate to well-established notions of time-varying spectra. From a numerical point of view, it involves certain difficulties in converting the scale axis to a frequency axis, especially when the frequency content of wavelet functions at adjacent scales do overlap.

Investigations on wavelet-based spectral estimates may be found in references such as /13-18/, where wavelet analysis has been applied in the context of earthquake engineering problems. In a particular approach, a modified Littlewood-Paley (MLP) wavelet basis can be introduced, whose mother wavelet is defined in the frequency domain by

$$\hat{\Psi}(\omega)=\begin{cases}\dfrac{1}{\sqrt{2(\sigma-1)\pi}}, & \pi\le|\omega|\le\sigma\pi\\ 0, & \text{elsewhere}\end{cases} \tag{1}$$

where symbol σ denotes a scalar factor, to be adjusted depending on the desired frequency resolution.

In /19/, Spanos and Failla have applied wavelet analysis to estimate the evolutionary power spectral density (EPSD) of non-stationary oscillatory processes defined as, /20/,

$$f(t)=\int_{-\infty}^{+\infty}A(\omega,t)e^{i\omega t}dZ(\omega) \tag{2}$$

where symbol $A(\omega,t)$ denotes a slowly varying time- and frequency-dependent modulating function, and $Z(\omega)$ is a complex random process with orthogonal increments. The wavelets transform of $f(t)$, Eq.(2), may be approximated as an oscillatory stochastic process.

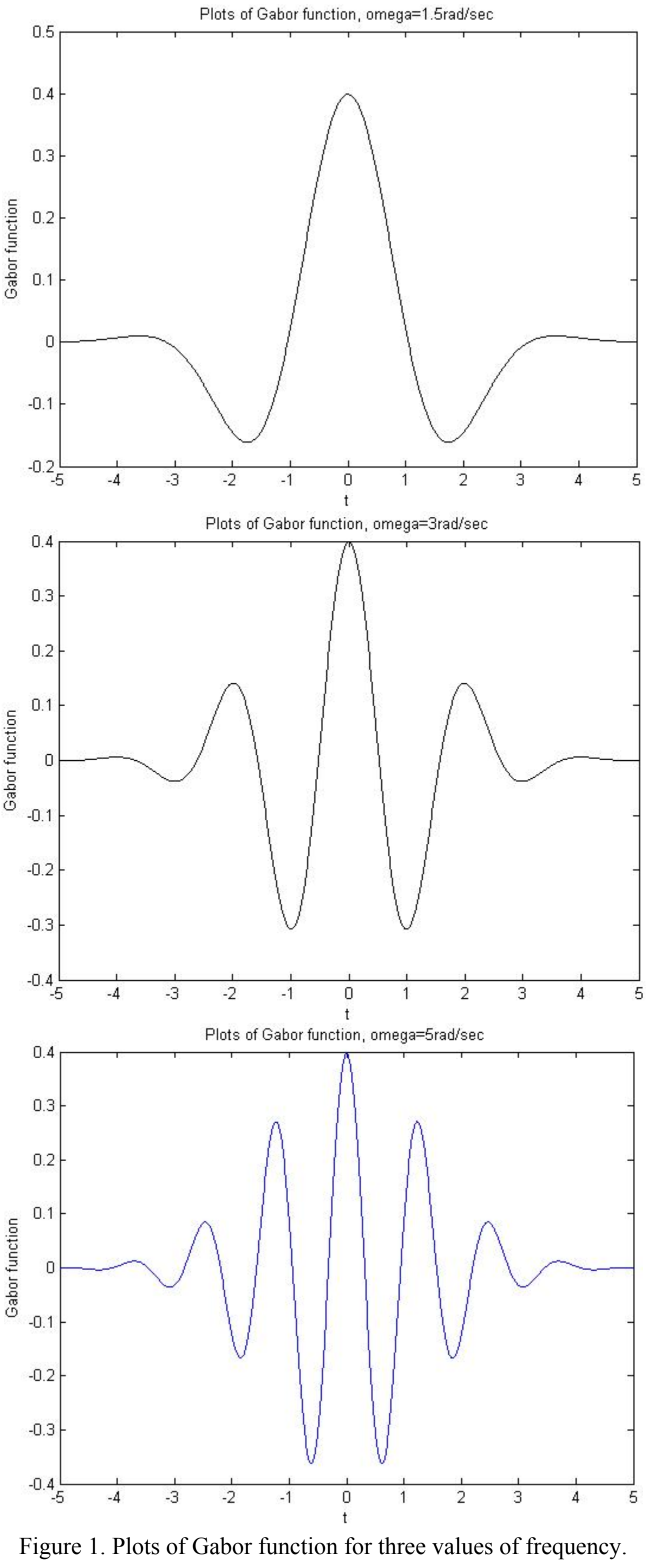

Figure 1. Plots of Gabor function for three values of frequency.
Slika 1. Krive funkcije Gabora za tri vrednosti frekvencije

Figure 1 shows plots of the Gabor function:

$$g_{(\omega,t_0)}(t) = e^{i\omega(t-t_0)} g(t-t_0) \qquad (3)$$

versus the independent variable t for three values of the frequency ω, where for $t_0 = 0$,

$$g(t-t_0) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} \exp\left(-\frac{t^2}{4\sigma^2}\right), \quad \sigma = 1. \qquad (4)$$

RANDOM FIELD SIMULATION

The use of wavelets for random field synthesis can be examined within the more general framework of scale-type methods. The latter have been developed to improve the computational performances of Monte Carlo simulations. Classical methods such as the spectral approach /21/ or the autoregressive moving average (ARMA) /22/, are not readily applicable for this purpose, especially when using non-uniform meshes or when enhancement of local resolution is desirable. To address these shortcomings, Fournier et al. /23/ have proposed a 'random mid-point method' to synthesize fractional Brownian motion. That is, a scale-type method where values of the random field for points within a coarser scale are generated first, and then the generated samples are used to determine values for a finer scale. This approach has been extended by Lewis /24/ into a *generalized stochastic subdivision method*, suitable for a broad class of stationary processes, and by Fenton and Vanmarcke /25/ into a *local average subdivision method*, which includes a random field smoothing procedure producing averages of the field for an increasingly finer scale. Further studies on the role of wavelet analysis in stochastic mechanics applications may be found in /26/.

SYSTEM IDENTIFICATION

Wavelet analysis lends itself to system identification applications. For instance, frequency localization properties allow detection and decoupling of individual vibration modes of multi-degrees-of-freedom (MDOF) linear systems. The wavelet representation of the system response can be truncated to an appropriate scale parameter, in order to filter measurement noise. Also, the wavelet transform coefficients can be related directly to the system parameters, as long as specific wavelet families are used. Early investigations trace back to the work by Robertson et al. /27/, who have used the DWT for the estimation of the impulse response function of MDOF systems. Compared to alternative time-domain techniques, the DWT-based extraction procedure offers significant advantages. It is robust, since singularities in the procedure related matrices can generally be avoided by selecting orthonormal wavelet functions. Further, the reconstructed impulse response function captures the low-frequency components, referred to as static modes and mode shape errors, which ordinarily are difficult to estimate. An important application of wavelet analysis to structural identification is due from Staszewski /28/, who has used complex Morlet wavelets for modal damping estimation. Specifically, Staszewski has interpreted in terms of the wavelet transform some concepts already used in well-established methods, where the Hilbert transform has been applied to a free-vibration linear response, /29/. Staszewski /28/ has also proposed an alternative damping estimation method based on the *ridge* and *skeletons* of the wavelet transform. A ridge is a curve of local maxima in the mean-square wavelet map and the corresponding skeleton is given by the values of the wavelet transform restricted to the ridge. Due to the localization properties of the wavelet transform, the ridges and skeletons of the wavelet transform can be detected separately for each mode. Specifically, the real part of the skeleton of the wavelet transform gives the impulse response function for each single mode, from which a straightforward estimate of the damping ratio is obtained from a logarithmic equation,

$$\ln\left|W_{x_j}(a_j,b)\right| = -\zeta_j\,\omega_j b + \ln\left(A_j\left|\hat{\Psi}\left(\pm i\,a_j\,\omega_j\sqrt{1-\zeta_j^2}\right)\right|\right). \quad (5)$$

A generalization of the method for non-linear systems can also be formulated, /30/. Ruzzene et al. /31/ have also presented a damping estimation algorithm based on the same concepts and leading to analogous results. Certain issues have been addressed in detail, concerning the frequency resolution of the adopted wavelet families, crucial for detecting coupled modes, and appropriate algorithms for ridge extraction, /32/. Lardies and Gouttebroze /33/ have estimated modal parameters via ambient records, without input measurements. To this end, the random decrement method (see /34/, and references therein) has been used to convert ambient vibration response into a free vibration response. Also, a modified Morlet wavelet has been developed with enhanced properties for modal parameter estimation. The method devised by Staszewski and Ruzzene et al. has also been implemented by Slavic et al. /35/, by replacing Morlet wavelets by Gabor wavelets, whose time and frequency resolutions may be adjusted by an appropriate parameter. Explicit conditions have been given on the frequency bandwidths of the Gabor wavelet transform, in order to estimate the instantaneous frequencies of two adjacent modes. Damping coefficients have been estimated using a logarithmic decrement formula, where the ratio of the wavelet transform at two subsequent extremes of the pseudo-period $T_j = 2\pi/\omega_j$ of the response in each mode is involved, for a selected wavelet transform scale, /36/. For the procedure to estimate the damping coefficient associated with the fundamental mode, it is sufficient to adapt the analysing scale so that the higher-frequency modes are filtered. For an arbitrary mode j, low-pass filtering is used to cancel the fundamental and the first $j - 1$ modes. Ghanem and Romeo /37/ have formulated a wavelet-Galerkin method for time-varying systems, where both damping and stiffness parameters are computed by solving a matrix equation. The latter is built by a standard Galerkin method, by projecting the solution of the differential equation of motion onto a subspace described by the wavelet scaling functions of a compactly supported Daubechies wavelet basis. The method is accurate for both free and forced vibration responses. A formulation for non-linear systems has also been proposed /38/. Another application is due to Yu et al., who have used wavelet transform to identify the parameters of a Preisach model of hysteresis; see /39, 40/, and references therein. The output function of the Preisach model is expanded in terms of the scaling functions of a given wavelet family. Then, the coefficients of such an expansion are determined by fitting a number of experimental data points with a minimum energy method. From the output function, the so-called Preisach function can be determined in a closed form. An interesting use of wavelet analysis for detecting non-linear behaviours in structures has been proposed by Argoul and Le, /41/. From the Cauchy wavelet transform of the transient response, four instantaneous indicators have been singled out, based on which an appropriate analytical model may be constructed for the structure. Specific applications have involved a non-linear beam excited by an impact hammer. In this context, note that further applications of wavelet analysis to non-linear vibrating systems may be found in the extensive work Pernot and Lamarque /42, 43/.

DAMAGE DETECTION

Properties of the wavelet transform are also quite appealing for damage detection purposes. Early investigations in this field /44, 45/ used wavelet analysis to detect local faults in machineries. Visual inspection of the modulus and phase of the wavelet transform has been used to localize the fault /44/. Further, it has been shown that transient vibrations due to developing damage are disclosed by the local maxima of the mean-square wavelet map, /45/. Additional results have been then proposed by Boulahbal et al. /46/. Specifically, the latter suggested a combined use of amplitude and phase map to distinguish the nature of damage, such as a cracked tooth. Also, they have pointed out that a Morlet CWT amplitude map performs better if applied on an ‘overall residual’ signal, obtained by filtering out the gear meshing frequency from the time synchronous averaged signal. A confirmation in this sense has been given by Dalpiaz et al. /47/ and Wang et al. /48/, for a variety of types of damage. Applications in machinery fault diagnostics have been also proposed by Adewusi and Al-Bedoor /49/, who used Daubechies wavelets to monitor startup and steady-state vibrations of an overhang rotor with a propagating crack. Results in terms of amplitude map have shown how the crack propagation may reduce the critical speed of the rotor and determine continuous changes in the amplitude of the vibration harmonics, unlike imbalance or misalignment which generally show constant amplitude, /50/. For this, a new family of wavelets reflecting the boundary conditions has been introduced. Then, Wang and Deng /51/ used Haar and Gabor wavelet transforms on the numerical displacement response of statically loaded cracked beams and plates. The method has proved robust for various boundary conditions and damage characteristics, such as crack length, embedment, orientation and width, with a relatively low spatial resolution of measurement data, /52/. However, no investigation has been performed on the feasibility of the method in the presence of noise and no relation has been found between the characteristic values of the wavelet transform and the damage degree. A first attempt to estimate the damage degree was made by Okafor and Dutta, /53/. Specifically, Daubechies wavelets were used to wavelet transform the mode shapes of a damaged cantilever beam, and a regression analysis by a least-squares method was conducted to correlate the peaks of the wavelet coefficients with the corresponding damage degree.

Wavelet analysis has also yielded encouraging results for global structural health monitoring. In this regard, interesting results have been presented by Hera and Hou /54/, who applied Daubechies wavelets to American Society of Civil Engineers (ASCE) benchmark study data. Specifically, a four-story, two-bay by two-bay prototype steel building subjected to stochastic wind loading has been considered. It has been shown that the occurrence of damage, due to a sudden breakage of interstory braces, is revealed by a spike in the high-resolution wavelet details of the acceleration response data. Further, the location of the damage region may be determined by the spatial distribution pattern of the spikes in the acceleration responses at some representative points in the structure. An attempt to extend the method to damage events of finite time duration has been also proposed by Hou et al. /55/, based on experimental data from a shaking table test of a full-size two-story wooden frame.

Wavelet analysis has been also used for damage detection in composite structures. In a first attempt, Zhu et al. /56/ presented wavelet transformed experimental data of delaminated carbon reinforced composite plates, but no quantitative determination of the location and amplitude of the defect was given. Then, Staszewski et al. /57/ used a cross-wavelet analysis to improve the interpretation of Lamb wave data related to defects in a carbon fibre composite plate. The Lamb waves, in fact, prove quite effective since they can propagate over long distances in the composite material and can interfere with damage. Sung et al. /58/ applied the Daubechies wavelet transform on the acoustic emission waves generated by low-velocity impact loads to determine damage modes and size in composite laminates. Specifically, they found a relation between levels of detail of the wavelet transform and damage modes such as matrix cracks and delamination. In order to detect small and incipient damage, Yam et al. /59/ have devised a method based on the energy variation of the vibration response due to the occurrence of damage. The method is implemented in two steps. The first involves the construction of damage feature proxy vectors using the energy at various scales of the wavelet transformed vibration response. Then, classification and identification of the structural damage status is pursued by using artificial neural networks (ANNs), which offer significant advantages compared to genetic algorithms (GAs), developed by Moslem and Nafaspour /60/ for damage identification purposes. GA-based damage detection requires repeatedly searching among numerous damage parameters to find the optimal solution of the objective function. Yet another approach for applications of wavelet analysis for damage detection in composite plates has been discussed by Paget et al., /61/. It is based on Lamb waves generated and received by embedded piezoceramic transducers. To characterize the damage, the Lamb waves are wavelet transformed using an original wavelet family, devised from the recurrent waveforms of the Lamb waves. The changes in the Lamb waves interacting due to the occurrence of damage are captured by the amplitude change of the wavelet coefficients. From this effect, an estimate of the impact energy and the damage level is obtained based on experimental results.

MATERIAL CHARACTERIZATION

The description of material properties is another application for wavelet analysis. Intuition suggests that multiscale analysis is a natural way for describing microstructure or material heterogeneity. Various, in fact, are the examples of multiscale microstructures, such as porosity distributions in ceramics, defects, dislocations, grain boundaries, and pores. It is important, however, to understand how information at different scales is related, and whether large or small scales affect macroscopic material properties such as deformation, toughness, and electrical conductance. Additional interest towards a multiscale description of material properties is motivated by the need for alternatives to the standard finite element method (FEM). The latter, although capable in principle, cannot simulate efficiently the actual behaviour of materials such as aluminium alloys, where pores may attain a size up to 500 μm, and inclusions may attain sizes up to 3-6 μm in diameter. Further, in FEM-based methods the constitutive response of the material at increasing scales is not the result of microstructural analysis at smaller scales, but it is rather assumed on the basis of macroscopic experiments.

Willam et al. /62/ have performed multiresolution homogenization based on a recursive Schur reduction method, in conjunction with the Haar wavelet transform. The method allows coarse grained parameters, such as Young's modulus of elasticity, to be extracted from fine grained properties at the mesoscale and microscale. Also, progressive elastic degradation can be modelled, which initiates at a quite fine scale and evolves into a macroscopic zero stiffness at the continuum level.

Frantzikonis /63/ has focused on stationary and isotropic porous media. The geometry of porous media is generally described in terms of a fundamental function, defined as unity for spatial locations in the matrix and as zero for locations in the pores or flaws. At a solid-flaw interface the porous medium is represented mathematically through a local jump in the fundamental function. It has been found that such a jump can be captured by a wavelet transform, as long as the finest scale is small enough relative to the size of the pores. From this fact, a relationship between the energy of the wavelet transform of the porous medium, and the variance and the correlation distance of the solid phase can be derived. In the presence of heterogeneous materials, with multiscale porosity, the role of porosity at each scale has been identified through the variation of the energy of the wavelet transform as a function of scale. Peaks of the energy reveal the dominant scale in determining macroscopic properties of the materials, such as mechanical failure. Specifically, a biorthogonal spline with four vanishing moments has been employed as a wavelet family. The results obtained have been subsequently extended in a second study, addressing the crack formation in an aluminium alloy with distributed pores and inclusions /64/. The problem, implemented for a one-dimensional solid, is tackled by wavelet transforming the flexibility function, assumed to vary along the longitudinal axis of the one-dimensional solid. The relationship between the energy of the wavelet transform and the variance of the flexibility is used to detect the dominant scale in the crack formation process. Note that an application of a two-dimensional wavelet transform has been described in /65/ for porosity classification on carbon fibre reinforced plastics.

CONCLUDING REMARKS

Concepts of wavelet-based continuous and discrete representations of signals have been reviewed. Further, included is an overview of vibration-related applications for evolutionary spectrum estimation, random field simulation, system identification, damage detection, and material characterization. The list of references cannot be exhaustive and, thus, other perhaps relevant applications of the wavelet transform have been omitted for succinctness; for instance, we mention the wavelet-based analysis of elastic waves in solids for which interesting contributions can be found in references such as /66, 67/. Nevertheless, it is believed that the references cited in this paper can serve as readily available resources for canvassing the multitude of concepts and applications of wavelet analysis, this remarkable tool for capturing and representing localization features of many physical phenomena. Wavelet-based algorithms and commercial codes are indeed an indispensable family of tools for vibration analysis, and offer, in many cases, a potent improvement over the classical Fourier transform based approaches.

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REFERENCES

1. Trifunac, M.D., *Response Envelope Spectrum and Interpretation of Strong Earthquake Ground Motion*, Bulletin of the Seismological Society of America, Vol.61, 343 (1971).
2. Spanos, P.D., Donley, M., Roesset, J., *Evolutionary Power Spectrum Estimation of the September 19, 1985, Mexico Earthquake Accelerograms*, Stochastic Approaches in Earthquake Engineering, US-Japan Joint Seminar on Stochastic Approaches in Earthquake Engineering, Boca Raton, FL, May 6-7, 1987, Springer-Verlag, Berlin, Vol.32, 322.
3. Massel, S.R., Ocean Surface Waves: Their Physics and Prediction, World Scientific, River Edge, NJ, 1996.
4. Wigner, E.P., *On the Quantum Correction for Thermodynamic Equilibrium*, Physics Reviews, Vol.40, 749 (1932).
5. Ville, J., *Théorie et Applications de la Notion de Signal Analytique*, Cables et Transmission, Vol.2, 61 (1948).
6. Loynes, R.M., *On the Concept of the Spectrum for Non-stationary Processes*, J of the Royal Statistical Society, Series B (Methodological), Vol.30, 1 (1968).
7. Cohen, L., Time–Frequency Analysis, Prentice-Hall, Englewood Cliffs, NJ (1995).
8. Gabor, D., *Theory of Communication*, J of the Institution of Electrical Engineers, Vol.93, 429 (1946).
9. Gurley, K., Kareem, A., *Applications of Wavelet Transforms in Earthquake, Wind, and Ocean Engineering*, Engineering Structures, Vol.21, 149 (1999).
10. Newland, D.E., Butler, G.D., *Time-varying Cross-spectra by Harmonic Wavelets for Soil Motion Damage*, Proc. of ASME DETC: 17th ASME Biennial Conf. on Mechanical Vibration and Noise, Las Vegas, NV, 1335 (1999).
11. Kareem, A., Kijewski, T., *Time-Frequency Analysis of Wind Effects on Structures*, J of Wind Engineering and Industrial Aerodynamics, Vol.90, 1435 (2002).
12. Gurley, K., Kijewski, T., Kareem, A., *First- and Higher-order Correlation Detection Using Wavelet Transforms*, J Engineering Mechanics, Vol.129, 188 (2003).
13. Spanos, P.D., Tezcan, J., Tratskas, P.N., *Stochastic Processes Evolutionary Spectrum Estimation via Harmonic Wavelets*, J Computer Methods in Applied Mechanics and Engng, Vol.194, 1367 (2005).
14. Basu, B., Gupta, V.K., *Non-stationary Seismic Response of MDOF Systems by Wavelet Transform*, Earthquake Engng. and Struc. Dynamics, Vol.26, 1243 (1997).
15. Basu, B., Gupta, V.K., *Seismic Response of SDOF Systems by Wavelet Modeling of Non-stationary processes*, J Engng. Mechanics, Vol.124, 1142 (1998).
16. Basu, B., Gupta, V.K., *Stochastic Seismic Response of Single-degree-of-freedom Systems Through Wavelets*, Engng. Structures, Vol.22, 1714 (2000).
17. Basu, B., Gupta, V.K., *Wavelet-based Non-stationary Response Analysis of a Friction Base-isolated Structure*, Earthquake Engng. and Struc. Dynamics, Vol.29, 1659 (2000).
18. Tratskas, P., Spanos, P.D., *Linear Multi-degree-of-freedom System Stochastic Response by Using the Harmonic Wavelet Transform*, J Applied Mechanics, Vol.70, 724 (2003).
19. Spanos, P.D., Failla, G., *Evolutionary Spectra Estimation Using Wavelets*, J Engng. Mechanics, Vol.130, 952 (2004).
20. Priestley, M.B., Spectral Analysis and Time Series, Academic Press, New York, 1981.
21. Shinozuka, M., Deodatis, G., *Stochastic Process Models for Earthquake Ground Motion*, Probabilistic Engng. Mechanics, Vol.3, 114 (1988).
22. Spanos, P.D., Mignolet, M.P., *Simulation of Homogeneous Two-dimensional Random Fields: Part II – MA and ARMA Models*, J Applied Mechanics, Vol.114, 270 (1992).
23. Fournier, A., Fussell, D., Carpenter, L., *Computer Rendering of Stochastic Models*, Communications of the ACM, Vol.25, 371 (1982).
24. Lewis, J.P., *Generalized Stochastic Subdivision*, ACM Transactions on Graphics, Vol.6, 167 (1987).
25. Fenton, G.A., Vanmarcke, E.H., *Simulation of Random Fields via Local Average Subdivision*, J Engng. Mechanics, Vol.116, 1733 (1990).
26. Walter, G.G., Wavelets and Other Orthogonal Systems with Applications, CRC Press, Boca Raton, FL, 1994.
27. Robertson, A.N., Park, K.C., Alvin, K.F., *Extraction of Impulse Response Data via Wavelet Transform for Structural System Identification*, Proc. of Design Engineering Technology Conf., DE-84.1 ASME-95, 1323 (1995).
28. Staszewski, W.J., *Identification of Damping in MDOF Systems Using Time-scale Decomposition*, J Sound and Vibration, Vol. 203, 283 (1997).
29. Jones, D.I.G., *Application of Damping Treatments*, Shock and Vibration Handbook, 3rd Ed., Harris, C.M., ed., McGraw-Hill, New York, 1988.
30. Staszewski, W.J., *Identification of Non-linear Systems Using Multi-scale Ridges and Skeletons of the Wavelet Transform*, Shock and Vibration Digest, Vol.214, 639 (1998).
31. Ruzzene, M., Fasana, A., Garibaldi, L., Piombo, B., *Natural Frequencies and Dampings Identification Using Wavelet Transform: Application to Real Data*, Mechanical Systems and Signal Processing, Vol.11, 207 (1997).
32. Haase, M., Widjajakusuma, J., *Damage Identification Based on Ridges and Maxima Lines of the Wavelet Transform*, Inter. J Engng. Science, Vol.41, 1423 (2003).

33. Lardies, J., Gouttebroze, S., *Identification of Modal Parameters Using the Wavelet Transform*, Inter. J Mech. Sciences, Vol.44, 2263 (2002).
34. Spanos, P.D., Zeldin, B.A., *Generalized Random Decrement Method for Analysis of Vibration Data*, J Vibration and Acoustics, Transactions of the ASME, Vol.120, 806 (1998).
35. Slavic, J., Simonovski, I., Boltezar, M., *Damping Identification Using a Continuous Wavelet Transform: Application to Real Data*, J Sound and Vibration, Vol.262, 291 (2003).
36. Lamarque, C.-H., Pernot, S., Cuer, A., *Damping Identification in Multi-degree-of-freedom Systems via a Wavelet-logarithmic Decrement. Part 1: Theory*, J Sound and Vibration, Vol.235, 361 (2000).
37. Ghanem, R., Romeo, F., *A Wavelet-based Approach for the Identification of Linear Time-varying Dynamical Systems*, J Sound and Vibration, Vol.234, 555 (2000).
38. Ghanem, R., Romeo, F., *Wavelet-based Approach for Model and Parameter Identification of Non-linear Systems*, Inter. J Non-Linear Mechanics, Vol.36, 835 (2001).
39. Spanos, P.D., Cacciola, P., Redhorse, J., *Random Vibration of SMA Systems via Preisach Formalism*, Nonlinear Dynamics, Vol.36, 405 (2004).
40. Mayergoyz, I.D., Mathematical Models of Hysteresis and Their Applications, 1st Ed., Academic Press, Boston, MA, 2003
41. Argoul, P., Le, T.-P., *Instantaneous Indicators of Structural Behavior Based on the Continuous Cauchy Wavelet Analysis*, Mechanical Systems and Signal Processing, Vol.17, 243 (2003)
42. Pernot, S., Lamarque, C.-H., *A Wavelet–Galerkin Procedure to Investigate Time-Periodic Systems: Transient Vibration and Stability Analysis*, J Sound and Vibration, Vol.245 (2001).
43. Pernot, S., Lamarque, C.-H., *A Wavelet-balance Method to Investigate the Vibrations of Non-linear Dynamical Systems*, Nonlinear Dynamics, Vol.32, 33 (2003).
44. Staszewski, W.J., Tomlinson, G.R., *Application of the Wavelet Transform to Fault Detection in a Spur Gear*, Mechanical Systems and Signal Processing, Vol.8, 289 (1994).
45. Wang, W.J., McFadden, P.D., *Application of Orthogonal Wavelets to Early Gear Damage Detection*, Mech. Systems and Signal Processing, Vol.9, 497 (1995).
46. Boulahbal, D., Farid, G.M., Ismail, F., *Amplitude and Phase Wavelet Maps for the Detection of Cracks in Geared Systems*, Mech. Systems and Signal Processing, Vol.13, 423 (1999).
47. Dalpiaz, G., Rivola, A., Rubini, R., *Effectiveness and Sensitivity of Vibration Processing Techniques for Local Fault Detection in Gears*, Mech. Systems and Signal Processing, Vol.14, 387 (2000).
48. Wang, W.Q., Ismail, F., Golnaraghi, M.F., *Assessment of Gear Damage Monitoring Techniques Using Vibration Measurements*, Mech. Systems and Signal Proc., Vol.15, 905 (2001).
49. Adewusi, S.A., Al-Bedoor, B.O., *Wavelet Analysis of Vibration Signals of an Overhang Rotor with a Propagating Transverse Crack*, J Sound and Vibration, Vol.246, 777 (2001).
50. Liew, K.M., Wang, Q., *Application of Wavelet Theory for Crack Identification in Structures*, J Engng. Mechanics, Vol. 124, 152 (1998).
51. Wang, Q., Deng, X., *Damage Detection with Spatial Wavelets*, Inter. J Solids and Structures, Vol.36, 3443 (1999).
52. Quek, S.-T., Wang, Q., Zhang, L., Ang, K.-K., *Sensitivity Analysis of Crack Detection in Beams by Wavelet Technique*, Inter. J Mechanical Sciences, Vol.43, 2899 (2001).
53. Okafor, A.C., Dutta, A., *Structural Damage Detection in Beams by Wavelet Transform*, Smart Materials and Structures, 9, 906 (2000).
54. Hera, A., Hou, Z.K., *Application of Wavelet Approach for ASCE Structural Health Monitoring Benchmark Studies*, J Engng. Mechanics, Vol.130, 96 (2004).
55. Hou, Z.K., Suzuki, Y., Shimizu, H., Hera, A., *Damage Detection of a Wooden House During Shaking Table Testing Using Wavelet-based Approach*, Proc. of 3rd World Conf. on Struc. Control, Como, Italy, 2002.
56. Zhu, X.R., Baoqi, T., Shenfang, Y., *Study on Delamination Detection in CFRP Using Wavelet Signal Singularity Analyses*, Proc. of 2nd Int. Workshop on Structural Health Monitoring, Stanford University, CA, 1999.
57. Staszewski, W.J., Pierce, S.G., Worden, K., Culshaw, B., *Cross-wavelet Analysis for Lamb Wave Damage Detection in Composite Materials Using Optical Fibers*, Key Engineering Materials, Vol.167, 373 (1999).
58. Sung, D.-U., Kim, C.-G., Hong, C.-S., *Monitoring of Impact Damages in Composite Laminates Using Wavelet Transform*, Composites B, Vol.33, 35 (2002).
59. Yam, L.H., Yan, Y.J., Jiang, J.S., *Vibration-based Damage Detection for Composite Structures Using Wavelet Transform and Neural Network Identification*, Composite Structures, Vol. 60, 403 (2003).
60. Moslem, K., Nafaspour, R., *Structural Damage Detection by Genetic Algorithms*, AIAA Journal, Vol.40, 1395 (2002).
61. Paget, C.A., Grondel, S., Levin, K., Delebarre, C., *Damage Assessment in Composites by Lamb Waves and Wavelet Coefficients*, Smart Materials and Struc. 05/2003; 12(3):393.
62. Willam, K., Rhee, I., Beylkin, G., *Multiresolution Analysis of Elastic Degradation in Heterogeneous Materials*, Meccanica, Vol.36, 131 (2001).
63. Frantziskonis, G., *Wavelet-based Analysis of Multiscale Phenomena: Application to Material Porosity and Identification of Dominant Scales*, Probab. Engng. Mech., Vol.17, 349 (2002).
64. Frantziskonis, G., *Multiscale Characterization of Materials with Distributed Pores and Inclusions and Application to Crack Formation in an Aluminum Alloy*, Probabilistic Engng. Mechanics, Vol.17, 359 (2002).
65. Ciliberto, A., Cavaccini, G., Salvetti, O., Chimenti, M., Azzarelli, L., Bison, P.G., Marinetti, S., Freda, A., Grinzato, E., *Porosity Detection in Composite Aeronautical Structures*, Infrared Physics and Technology, Vol.43, 139 (2002).
66. Kishimoto, K., Inoue, H.H.M., Shibuya, T., *Time Frequency Analysis of Dispersive Waves by Means of Wavelet Transform,* ASME Trans. J Applied Mechanics, Vol.62 (1995).
67. Park, H.C., Kim, D.S., *Evaluation of the Dispersive Phase and Group Velocities Using Harmonic Wavelet Transform*, NDT & E International, Vol.34, No.7, pp.457-467.