ELASTIC-PLASTIC TRANSITIONAL STRESSES IN A THIN ROTATING DISC WITH SHAFT HAVING VARIABLE THICKNESS UNDER STEADY STATE TEMPERATURE

ELASTOPLASTIČNI PRELAZNI NAPONI U TANKOM ROTIRAJUĆEM DISKU SA VRATILOM SA PROMENLJIVOM DEBLJINOM POD RAVNOMERNIM UTICAJEM TEMPERATURE

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Abstract

Stresses for the elastic-plastic transition and fully plastic state have been derived for a thin rotating disc having variable thickness with shaft at different temperatures. Results have been discussed and depicted graphically. It has been observed that in the absence of thickness, the rotating disc made of incompressible material e.g. rubber with inclusion require a higher angular speed to yield at the internal surface as compared to a disc of compressible material e.g. copper, brass and steel and a much higher angular speed is required to yield with the increase in radii ratio. With the effect of variation thickness, reverse results are for the internal surface. When thickness varies (say k = 2.5, 5), reverse in angular speed is required to yield with the increase in radii ratio. Their solution for the problem of fully plastic state have been derived for a thin rotating disc having variable thickness with shaft at different temperatures.

INTRODUCTION

Rotating discs form an essential part of the design of rotating machinery, namely rotors, turbines, compressors, flywheel and computer disc drive etc. The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier /1/ in the elastic range and by Chakrabarty /2/ and Heyman /3/ for the plastic range. Their solution for the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e. \( T_{xx} = 0 \)). Gupta and Shukla /4/ obtained a different solution for the fully plastic state by using Seth’s transition theory and plane stress condition. This theory, /5/, does not require any assumptions as a yield condition, incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems /4, 5, 10-31/.

Seth, /6/, has defined the generalized principal strain measure as:

\[
e_{ij} = \int_0^1 \left[ 1 - 2e_{ij}^d \right] n^{-1} d\eta = \frac{1}{n} \left[ 1 - (1 - 2e_{ij}^d)^\frac{1}{n} \right], \quad (i=1,2,3) \quad (1)
\]

where ‘\( n \)’ is the measure, and \( e_{ij}^d \) are Almansi finite strain components. For \( n = -2, -1, 0, 1, 2 \) it gives Cauchy, Green Hencky, Swainger and Almansi measures, respectively.
Here, elastic-plastic transition stresses in a thin rotating disc of variable thickness with shaft, under steady state temperature are investigated by Seth’s transition theory. The thickness of disc is assumed to vary along the radius in the form:

\[ h = h_0 (r/b)^k \]

Where \( h_0 \) is the thickness at \( r = b \) and \( k \) is the thickness parameter. Results are numerically obtained and depicted graphically.

MATHEMATICAL MODEL

We consider a thin disc of variable thickness with central bore of radius \( a \) and external radius \( b \). The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed \( \omega \) of gradually increasing magnitude about an axis perpendicular to its plane and passing through the centre as shown in Fig. 1. The disc is so thin that it is effectively in a state of plane stress \( (T_{zz} = 0) \) and variation of thickness is radial and symmetric with respect to the mid plane. The temperature applied for central bore of the disc is \( \theta \).

Figure 1. Geometry and scheme of rotating disc with concentric circular hole.

**BOUNDARY CONDITIONS**

The disk considered in the present study is of variable thickness and is subjected to a temperature gradient field. The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk is free from mechanical load. Thus, the boundary conditions of the problem are given by:

(i) \( r = a \), \( u = 0 \)

(ii) \( r = b \), \( T_{rr} = 0 \)

where \( u \) and \( T_{rr} \) denote displacement and stress along the radial direction.

**FORMULATION OF THE PROBLEM**

The displacement components in cylindrical polar coordinate are given by Seth /6/:

\[ u = r(1 - \beta), \quad v = 0, \quad w = dz \]

where \( \beta \) is function of \( r = \sqrt{x^2 + y^2} \) only and \( d \) is a constant.

The finite strain components are given by Seth /6/ as:

\[ e_{rr}^A = -\frac{1}{2} \left( \frac{\partial u}{\partial r} \right)^2 = \frac{1}{2} \left[ 1 - \left( r \beta' + \beta \right)^2 \right], \]
\[ e_{\theta\theta}^A = -\frac{r}{2\beta} \frac{u^2}{r^2} = \frac{1}{2} \left[ 1 - \beta^2 \right], \]
\[ e_{zz}^A = -\frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 = \frac{1}{2} \left[ 1 - \left( 1 - d \right)^2 \right], \]
\[ e_{r\theta}^A = e_{\theta r}^A = e_{rr}^A = 0 \]
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\[ \nabla^2 \theta = 0 \]
\[ \frac{d^2 \theta}{dr^2} + \frac{1}{r} \frac{d \theta}{dr} = \frac{1}{r} \left( \frac{d \theta}{dr} \right) = 0 \]
or
\[ \frac{d \theta}{dr} = \frac{D_1}{r} \]

which has solutions
\[ \theta = D_1 (\log r + D_2) \] (8)

where \( D_1 \) and \( D_2 \) are constants of integration and can be determined from the boundary condition.

Equations (7) for this problem become:
\[ T_{rr} = \frac{2 \lambda \mu}{\lambda + 2 \mu} \left[ e_{rr} + e_{\theta \theta} \right] + 2 \mu e_{rr} - \frac{2 \mu \xi \theta}{(\lambda + 2 \mu)} \]
\[ T_{\theta \theta} = \frac{2 \lambda \mu}{\lambda + 2 \mu} \left[ e_{rr} + e_{\theta \theta} \right] + 2 \mu e_{\theta \theta} - \frac{2 \mu \xi \theta}{(\lambda + 2 \mu)} \]
\[ T_{\theta \varphi} = T_{\varphi \theta} = T_{\varphi r} = T_{rr} = 0. \] (9)

Substituting Eq.(5) in Eq.(6), the strain components in terms of stresses are obtained as /10/:
\[ e_{rr} = \frac{\ddot{u}}{E} \left( \frac{1}{1 - (r^\beta)^2} \right) = \frac{1}{2} \left[ 1 - (r^\beta)^2 \right] \]
\[ e_{\theta \theta} = \frac{\ddot{u}}{E} \left( \frac{1}{1 - (r^\beta)^2} \right) = \frac{1}{2} \left[ 1 - (r^\beta)^2 \right] \]
\[ e_{\varphi \varphi} = \frac{\ddot{w}}{E} \left( \frac{1}{1 - (r^\beta)^2} \right) = \frac{1}{2} \left[ 1 - (r^\beta)^2 \right] \]
\[ e_{r \varphi} = e_{\varphi r} = e_{\varphi \theta} = e_{\theta r} = 0 \] (10)

where \( E \) is the Young’s modulus and \( C \) is compressibility factor of the material in terms of Lame’s constant, given by
\[ E = \frac{\mu (3 \lambda + 2 \mu)}{\lambda + \mu} \]
\[ C = \frac{\mu}{\lambda + 2 \mu} \]

Substituting Eq.(6) in Eq.(9), one gets
\[ T_{rr} = \frac{2 \mu}{n} \left[ 3 - 2 C - \beta^2 + (1 - C) \right] \frac{1}{2} \left[ 1 - (1 - d^2)^2 \right] \]
\[ T_{\theta \theta} = \frac{2 \mu}{n} \left[ 3 - 2 C - \beta^2 + (1 - C) \right] \frac{1}{2} \left[ 1 - (1 - d^2)^2 \right] \]
\[ T_{r \varphi} = T_{\varphi r} = T_{\varphi \theta} = T_{r \theta} = 0 \] (11)

where
\[ r^\beta = \beta P \]

Equations of equilibrium are all satisfied except:
\[ \frac{d}{dr} (r \theta T_{rr}) - h T_{r \theta} + \rho \omega^2 r^2 h = 0 \] (12)

where \( \rho \) is the density of the rotating disc material.

The temperature satisfying Laplace Eq.(8) with boundary condition
\[ \theta = \theta_0 \text{ at } r = a \]
\[ \theta = 0 \text{ at } r = b \]

where \( \theta_0 \) is a constant, given by /27/:
\[ D_1 = \frac{\theta}{\log(a/b)} \text{ and } D_2 = -\frac{\theta}{\log(a/b)} \]

Substituting \( D_1 \) and \( D_2 \) from Eq.(8), one gets:
\[ \theta = \frac{\theta_0}{\log(a/b)} (r/b) \]

Using Eqs.(11) and (12), one gets a non-linear differential equation for \( \beta \) as:
\[ (2 - C)n \beta^{n+1} P (P+1)^{n-1} \frac{dP}{d \beta} = \]
\[ = \frac{r h'}{n} \left[ 3 - 2C - \beta^2 \left[ 1 - (1 - C) \right] \right] + n \frac{\rho \omega^2 r^2}{2 \mu} \]
\[ + \beta^n \left[ 1 - (1 + P) \right] - np \left[ 1 - (1 + C) \right] \]
\[ - \frac{n C \xi \theta_0}{2 \mu} \left( \frac{r h'}{n} \right)^{n} \left( \ln(r/b) + 1 \right) \] (13)

where \( \theta_0 = \theta_0 / \log(a/b) \) and \( r^\beta = \beta P \) (\( P \) is function of \( \beta \) and \( \beta \) is function of \( r \) and \( \beta^2 = d \beta / dr \) (\( P \) is function of \( \beta \) and \( \beta \) is function of \( r \) only).

SOLUTION THROUGH THE PRINCIPAL STRESSES

For finding the plastic stress, the transition function is taken through the principal stress (see Seth’s and Hulsurkar /5-9/, Gupta /11-13, 17/, Pankaj /10, 14-31/) at the transition point \( P \to \pm \infty \). The transition function \( R \) is defined as:
\[ R = \frac{n}{2 \mu} \left[ T_{\theta \theta} - C \xi \theta \right] = \]
\[ = \left[ 3 - (2 - C) - \beta^2 \left[ 2C + (1 - C) (P+1)^n \right] - \frac{n C \xi \theta}{\mu} \right] \] (14)

Taking the logarithmic differentiation of Eq.(14) with respect to \( r \), one gets:
\[ \frac{d}{dr} \left( \frac{\log R}{n} \right) \]
\[ = \frac{\beta^n P}{r} \]
\[ \times \frac{2 - C + (1 - C) (P+1)^n - (P+1) \beta \frac{dP}{d \beta}}{(2 - C) - \beta^2 (2 - C + (1 - C) (P+1)^n) - \frac{n C \xi \theta}{\mu}} \] (15)

Substituting the value \( dP/d \beta \) from Eq.(13) in Eq.(15) and taking the asymptotic value \( P \to \pm \infty \) and integrating, one gets:
\[ R = \frac{A_1 r^{-1}}{h} \] (15)

where \( \nu = (1 - C)/(2 - C) \) and \( A_1 \) is a constant of integration.
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From Eqs.(13) and (15) and using Eq.(2), we have

\[
T_\omega = \frac{2\mu}{n} r^{n-1} - \frac{C \xi \theta_0 \log(r/b)}{h_0} + C \xi \theta_0 \log(r/b) \log(a/b) \tag{16}
\]

Substituting Eq.(16) in Eq.(12) using Eq.(2) and integrating, one gets:

\[
T_r = \frac{2\mu b^{-k}}{n h_0} A r^{n-1} - \frac{B b^{-k}}{3-k} \frac{b^{-k} \log(r/b)}{h_0 r^{-k}} + \frac{C \xi \theta_0 \log(r/b)}{h_0 r^{-k}} - \frac{C \xi \theta_0 \log(r/b)}{h_0 r^{-k}} \log(a/b) \log(a/b) + \frac{2(2-C)\alpha \xi \theta_0 \log(r/b)}{h_0 r^{-k}} \log(a/b) \log(a/b) \tag{17}
\]

where \(B\) is a constant of integration.

Substituting Eqs.(16) and (17) in the second equation of Eq.(10), one gets:

\[
\beta = \sqrt{\frac{2\mu b^{-k}}{n h_0} A r^{n-1} - \frac{B b^{-k}}{3-k} \frac{b^{-k} \log(r/b)}{h_0 r^{-k}} + \frac{C \xi \theta_0 \log(r/b)}{h_0 r^{-k}} - \frac{C \xi \theta_0 \log(r/b)}{h_0 r^{-k}} \log(a/b) \log(a/b) + \frac{2(2-C)\alpha \xi \theta_0 \log(r/b)}{h_0 r^{-k}} \log(a/b) \log(a/b)} \tag{18}
\]

where \(C \xi = \alpha E(2-C)\).

Substituting Eq.(18) in Eq.(4), one gets Eq.(19):

\[
T_\omega = \left( \frac{\rho \omega^2 v r^k (b^{3-k} - a^{3-k})}{3-k} \right) + \alpha \xi \theta_0 (2-C) \left[ \frac{\log(r/b)}{\log(a/b)} + \frac{2a}{(2-C) r \log(a/b) \log(a/b)(2-C) \log(a/b)} \left( \frac{r}{a} \right)^{1-C/2-C} \right] + \alpha \xi \theta_0 (2-C) \left( \frac{(b-a)}{r} \frac{r^{1-C/2-C}}{a\log(a/b)} \log(a/b) \log(a/b) \right) \tag{22}
\]

and

\[
T_r - T_\omega = \left( \frac{\rho \omega^2 r^k}{3-k} \right) \left( 1-\nu \right) (b^{3-k} - a^{3-k}) \left( \frac{r}{b} \right)^{1-C/2-C} \left( \frac{r}{a} \right)^{1-C/2-C} + \alpha \xi \theta_0 \left[ \frac{b-a}{r} \frac{r^{1-C/2-C}}{a \log(a/b)} \log(a/b) \log(a/b) \right] \tag{23}
\]

From Eq.(25) it is seen that \(|T_r - T_\omega|\) is maximum at the internal surface (at \(r = a\)), therefore yielding of the disc that takes place at the internal surface of the disc and Eq.(25)

can be written as:

\[
|T_r - T_\omega|_{r=a} = \frac{\rho \omega^2 (1-\nu)(b^{3-k} - a^{3-k})}{3-k} \left( \frac{a}{b} \right) + \alpha \xi \theta_0 \left[ \frac{(b-a)}{a \log(a/b)} \frac{a^{1-C/2-C}}{(1-C) \left( \frac{a}{b} \right)} + \frac{2(2-C)}{(1-C) \left( \frac{a}{b} \right)} \left( \frac{a}{b} \right) \right] = Y(\text{say})
\]

The angular speed necessary for initial yielding is given by:

\[
\Omega^2 = \frac{\rho \omega^2 b^2}{Y} = \left( \frac{(3-k)ab^2}{(1-\nu)a^2 b^{3-k}} \left( \frac{b}{a} \right)^{1-C/2-C} \left( \frac{b}{a} \right)^{1-C/2-C} \right) \frac{3(2-C)}{(1-C) b^{1-C/2-C} \left( \frac{a}{b} \right)^{1-C/2-C} - \frac{2(2-C)}{(1-C) \left( \frac{a}{b} \right)} \left( \frac{a}{b} \right) \right] \tag{26}
\]

and

\[
\omega = \frac{\Omega}{\sqrt{\frac{Y}{\rho}}}.
\]
Elastic-plastic transitional stresses in a thin rotating disc with velocity from equations (32) to (35) become:

\[ \tau' - \tau_0 = \frac{\rho_0 \gamma (b^{3-k} - a^{3-k})}{2(3-k)b^{3-k}} + \alpha E \theta_0 \left( \frac{b-a}{b \log(a/b)} \right) \left( \frac{a}{b} \right) = \gamma'(\text{say}) \]

The angular speed required for fully plastic state is given by:

\[ \Omega^2 = \frac{\rho_0 \gamma \theta^2}{Y'} \left( \frac{2(3-k)b^{3-k}}{(b^{3-k} - a^{3-k})} \right) \left( \frac{6 \alpha E \theta_0}{Y} \right) \left[ \left( \frac{1}{a/b} \right) \log(a/b) - 2 \left( \frac{a}{b} \right) \right] \]

(27)

where

\[ \omega_f = \frac{Y'}{b} \sqrt{\frac{\rho}{\gamma}}. \]

We introduce the following non-dimensional components:

\[ \sigma_f = \frac{\Omega^2}{(3-k)} \left[ \frac{(1-R_0^{3-k})R^{k-1} + R_0^{3-k}}{1 - (R_0^{3-k})^2} \right] \theta(2-C) \left[ \frac{(1-R_0^{2})(1-C)}{(2-C)R \log R_0} \right] R^{1-C/2-C} \]

(28)

\[ \sigma_r = \left[ \frac{\Omega^2 \rho v(1-R_0^{3-k})R^{k-1}}{(3-k)} \right] \left[ \frac{(1-R_0^{3-k})R^{k-1} + R_0^{3-k}}{1 - (R_0^{3-k})^2} \right] \theta(2-C) \left[ \frac{(1-R_0^2)(1-C)}{(2-C)R} \log R_0 \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(29)

\[ \Omega^2 = \frac{\rho_0 \gamma \theta^2}{Y'} \left( \frac{2(3-k)b^{3-k}}{(b^{3-k} - a^{3-k})} \right) \left[ \frac{6 \alpha E \theta_0}{Y} \right] \left( \frac{1}{a/b} \right) \log(a/b) - 2 \left( \frac{a}{b} \right) \]

(30)

\[ \bar{u} = R - R \sqrt{1 - 2vH} \left[ \frac{\Omega^2}{(3-k)} \right] R^{k-1} \left[ R^{3-k} - R_0^{3-k} \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(31)

Stresses, displacement and angular speed for fully plastic state, are obtained from Eqs.(28), (29), (31) and (27) as:

\[ \sigma_f = \frac{\Omega^2 \rho v(1-R_0^{3-k})R^{k-1}}{(3-k)} \left[ \frac{(1-R_0^{3-k})R^{k-1} + R_0^{3-k}}{1 - (R_0^{3-k})^2} \right] \theta(2-C) \left[ \frac{(1-R_0^2)(1-C)}{(2-C)R} \log R_0 \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(32)

\[ \sigma_r = \left[ \frac{\Omega^2 \rho v(1-R_0^{3-k})R^{k-1}}{(3-k)} \right] \left[ \frac{(1-R_0^{3-k})R^{k-1} + R_0^{3-k}}{1 - (R_0^{3-k})^2} \right] \theta(2-C) \left[ \frac{(1-R_0^2)(1-C)}{(2-C)R} \log R_0 \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(33)

\[ \bar{u} = R - R \sqrt{1 - 2vH} \left[ \frac{\Omega^2}{(3-k)} \right] R^{k-1} \left[ R^{3-k} - R_0^{3-k} \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(34)

and

\[ \Omega^2 = \frac{2(3-k)}{(1-R_0^{3-k})} \left[ \frac{6 \theta_0}{(1-R_0^{2}) \log R} - 2R_0 \right] \]

(35)

\[ \frac{\rho_0 \gamma \theta^2}{Y'} \left( \frac{2(3-k)b^{3-k}}{(b^{3-k} - a^{3-k})} \right) \left[ \frac{6 \alpha E \theta_0}{Y} \right] \left( \frac{1}{a/b} \right) \log(a/b) - 2 \left( \frac{a}{b} \right) \]

Particular case:

When there is (k = 0), the transitional stresses from Eqs.(28) to (31) become:

\[ \sigma_f = \left[ \frac{\Omega^2 \rho v(1-R_0^{3})R^{k-1}}{3} \right] \theta(2-C) \left[ \frac{(1-R_0^2)(1-C)}{(2-C)R} \log R_0 \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(36)

\[ \sigma_r = \left[ \frac{\Omega^2 \rho v(1-R_0^{3})R^{k-1} + R_0^{3}}{3R} \right] \theta(2-C) \left[ \frac{(1-R_0^2)(1-C)}{(2-C)R} \log R_0 \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(37)

\[ \Omega^2 = \frac{\rho_0 \gamma \theta^2}{Y'} \left( \frac{3R_0^3}{(1-R_0^{3})} \right) \left[ \frac{3 \theta(2-C)}{(1-R_0^{2})} \right] \left[ \frac{(1-R_0^2) + 2(R_0^{3})}{1-C} \right] \log \frac{R}{R_0} - \frac{2}{1-C} \]

(38)

\[ \bar{u} = R - R \sqrt{1 - 2vH} \left[ \frac{\Omega^2}{3R} \right] R^{k-1} \left[ R^{3-k} - R_0^{3-k} \right] + \frac{2\theta(2-C)(R-R_0)}{(1-C)} \log \frac{R}{R_0} \]

(39)

For fully plastic state stresses, displacement and angular velocity from equations (32) to (35) become:

\[ \sigma_f = \frac{\Omega^2 \rho v(1-R_0^{3})R^{k-1}}{6} + \frac{2\theta}{2 \log R_0 \log R} \left[ \frac{1}{R} \right] \log \frac{R}{R_0} \]

(40)
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\[
\sigma_r = \left( \frac{\Omega^2}{3R} \right) \left( 1 - R_0^2 \right) \left( 1 - \frac{R}{R_0} \right)^2 + \frac{2\theta_l}{\log R_0} \left( \log R + \frac{1}{R} \log \left( \frac{R_0}{R} \right) \right) + 4\theta_l \left( \frac{R_0 - R}{R} \right)
\]

\[
\sigma_t = \frac{\Omega^2}{3R} \left[ 1 - R_0^2 \right] \left( 1 - \frac{R}{R_0} \right)^2 + \frac{2\theta_l}{\log R_0} \left( \log R + \frac{1}{R} \log \left( \frac{R_0}{R} \right) \right) + 4\theta_l \left( \frac{R_0 - R}{R} \right)
\]

Equations (36) to (43) are the same as given by Pankaj /10/.

For calculating the stresses, angular speed based on the above analysis, the following values have been taken as \( \nu = 0.5 \) (rubber or incompressible material), \( \nu = 0.333 \) (copper and brass or compressible material), \( \nu = 0.29 \) (steel or compressible material); \( C = 0.00, 0.25, 0.5, 0.75; E/Y = 1/2 \) and 2; \( \theta_l = 0^\circ, 0.125, 0.25 \); and \( \theta_l = 0^\circ, 0.125, 0.25 \) for different values of temperature \( \theta_l = 0, 0.25, 0.5 \). It has been observed that in the absence of thickness, the rotating disc made of incompressible material, e.g. rubber, with an inclusion requires higher angular speed to yield at the internal surface as compare to the disc of compressible material, e.g. copper, brass and steel, and a much higher angular speed is required to yield with the increase in radii ratio. With the effect of varied thickness, reverse results and a required lesser angular speed to yield at the internal surface is noted. With the introduction of thermal effects for \( k = 0 \), lesser angular speed is required to yield at the internal surface. When thickness varies (say \( k = 2.5, 5 \)), results are reversed.

In Figs. 3 and 4, curves have been drawn for stresses and with respect to radii ratio \( R = r/b \) for elastic-plastic transition and fully plastic state respectively. It has been seen that radial stresses are maximum at the internal surface with no thickness, when thickness varies the circumferential stress is maximum at the internal surface. With the introduction of thermal effect it decrease the value of radial and circumferential stress at the internal surface and external surface for transitional state, whereas from Fig. 4, it can been seen that the thermal effect increases the values of radial and circumferential stress at the internal surface and external surface for fully-plastic state.
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CONCLUSION

It has been observed that in the absence of thickness the rotating disc of incompressible material, e.g. rubber, with an inclusion require higher angular speed to yield at the internal surface as compare to disc made of compressible material, e.g. copper, brass and steel and a much higher angular speed is required to yield with the increase in radii ratio. With the effect of thickness variation, results are reversed and a lesser angular speed is required to yield at the internal surface. With the introduction of thermal effects for \( k = 0 \), lesser angular speed is required to yield at the internal surface. When thickness varies (say \( k = 2.5, 5 \)), reversed are the results. The radial stresses are maximum at the internal surface with no thickness, when thickness varies, the circumferential stress is maximum at the internal surface.

Nomenclature

- \( a, b \) – internal and external radii of the disc, (m)
- \( u, v, w \) – displacement components
- \( r, \theta, z \) – radial, circumferential, axial direction
- \( \omega \) – angular velocity of rotation
- \( \rho \) – density of material
- \( C \) – compressibility
- \( T_{ij}, e_{ij} \) – stress and strain rate tensors
- \( Y \) – yield stress
- \( K_1, K_2 \) – constants of integration
- \( \Omega^2 = \rho \omega^2 b^2 / E \) (speed factor)
- \( \sigma_r \) – radial stress component \( (T_{rr}/Y) \)
- \( \sigma_{\theta} \) – circumferential stress component \( (T_{\theta\theta}/Y) \)
- \( \theta \) – temperature
- \( R = r/b, R_0 = a/b \) (radii ratio)
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