Patrik Gubeljak [1]

EXPERIMENTAL DETERMINATION OF OSCILLATION BEHAVIOUR OF DOUBLE PENDULUM

EKSPERIMENTALNO ODREĐIVANJE OSCILATORNOG PONAŠANJA DVOSTRUKOG KLATNA

Originalni naučni rad / Original scientific paper
UDK /UDC: 531.53
Rad primljen / Paper received: 07.04.2013.

Adresa autora / Author's address:
International Baccalaureate, II. gimnazija Maribor, Slovenia pgubeljak@gmail.com

Keywords
- double pendulum
- Lissajous-like curves
- oscillation behaviour

Ključne reči
- dvostruko klatno
- Lisažuove krive
- oscilatorno ponašanje

Abstract

In this paper the experimental motion of the double pendulum is investigated. The influence of initial orientation and energy of the system on the oscillation is analysed. Angular displacement sensors which contributed to the damping of the pendulum were attached to the pivots. The crank angles of both limbs were measured as a function of time. On the basis of the analysis it is shown that the motion of the system is quite unpredictable unless initial conditions are known to a very high degree of accuracy. Even very small differences in the initial condition cause a completely different oscillation of the pendulum.

Izvod

U radu je predstavljeno eksperimentalno oscilovanje dvostrukog klatna. Analiziran je efekat početnog položaja i energije sistema na oscilovanje. Senzori za merenje ugla, koji izrazito prigušuju oscilovanje klatna su bili ugrađeni u oba zgloba klatna. Ugaona rotacija oba kraka klatna je merena u zavisnosti od vremena. Na osnovu analize može se zaključiti da je oscilovanje sistema prilično nepredvidljivo, osim ako su početni uslovi vrlo precizno definisani, odnosno, izmereni. Dakle, vrlo mala odstupanja u početnim uslovima pokazuju potpuno drugačije oscilovanje klatna.

INTRODUCTION

The determination of the magnitudes of forces or strains in the limbs of the double pendulum is necessary for ensuring the integrity of oscillating mechanisms. In the case of structural collapse where a part loses support, the rest of the structure can oscillate in modes similar to that of the double pendulum. During the collapse under the weight of structural components angular accelerations arise, and rotational forces in the system can cause strains multiple times above allowed in the structure. The aim of the work is to find laws governing the behaviour of dynamic systems, such as the double pendulum. This paper presents a two-dimensional dynamic loading on the double pendulum. This device can be analysed in two dimensions since all elements move in parallel planes. The presence of significant accelerations on the moving elements in the system requires that dynamic analysis should be performed.

DESCRIPTION OF DEMONSTRATOR MODEL

Figure 1 shows the double pendulum demonstrator model. It consists of two rotating limbs plus the ground link. The fixed and movable pivots are instrumented with inductive angle transducers to measure the crank angle around the pivots vs. time. A CCD camera is used for filming the experiments, as shown in Fig. 2.

The sensors themselves are modified. The rigid stock wires are replaced with new, more flexible ones. The wired connection from the lower sensor is set up orthogonally to the direction of rotation, so the wires would not be pulled by the pendulum or get entangled. Wires connecting the upper sensor did rotate with the pendulum, but due to their flexibility the effect on motion is characterized as negligible.

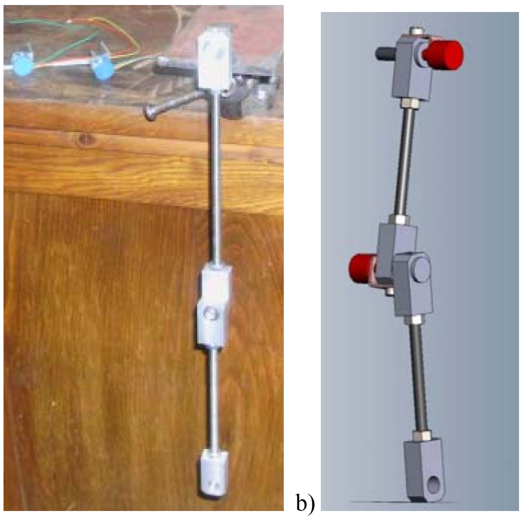
a)  b)

Figure 1. a) The pendulum model. Two small blue cylinders are inductive angle measurement sensors; b) SolidWorks model.
Slika 1. a) Model klatna. Dva mala plava cilindra su indukcioni senzori za merenje ugla; b) SolidWorks model.

Figure 2. The experiments are also filmed by CCD camera and recorded for correlation with measured data.
Slika 2. Eksperimenti su snimljeni CCD kamerom za korelacije sa izmerenim podacima

The sensors are connected to the computer interface via a commercial National Instruments signal relaying chipset (Fig. 3) and the analog/digital converter. The sensors are inductive; the programme calculated angular displacement from the voltage signal. The sensor is essentially a pointer sliding on a resistor. The chipset monitors the current and keeps it constant, so the measured voltage obeys Ohm's law. Angular displacement can then be calculated. This is performed by the programme with a measuring frequency of 200 Hz. The pendulum in motion and as assembled with sensors is shown in Figs. 4-5.

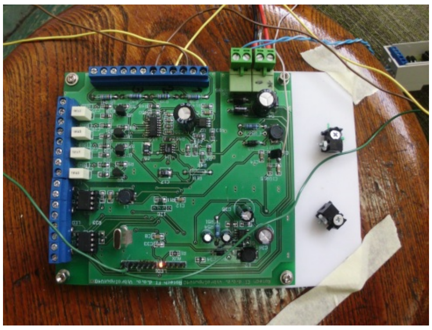


Figure 3. Signal relaying chipset.
Slika 3. Čipset za obradu signala

Fig. 4. Double pendulum.
Slika 4. Dvostruko klatno

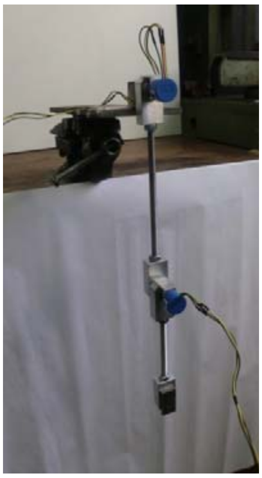


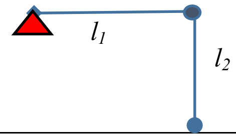
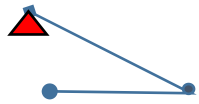
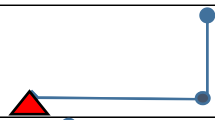
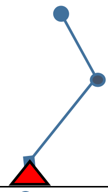
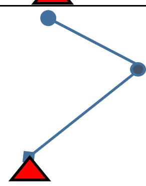
Fig. 5. Assembled pendulum.
Slika 5. Sastavljeno klatno

EXPERIMENTAL PROCEDURE

The initial angular displacement of the limbs from the vertical line is shown in Table 2. Noteworthy is the procedure used to calculate the angular- with respect to vertical displacement. The positive direction of oscillation for the measuring gear is clockwise. The positive direction is reserved using the transformation $360° - \varphi_1 = \text{ENK1}$, where φ_1 is the measured angular displacement of the upper limb and ENK1 is the angle of the first limb with respect to the vertical line. The second sensor measures the angular displacement of the lower limb relative to the first limb. Therefore, the actual angle of the second limb with respect to the vertical is the sum of the angle of the first limb with respect to the vertical and the second limb with respect to the first limb:

$$\text{ENK2} = 360° - \varphi_1 - \varphi_2 \qquad (1)$$

Table 2. Initial angular displacement of limbs from vertical line.
Tabela 2. Početno ugaono pomeranje krakova u odnosu na vertikalu

No.	ENK-1, °	ENK-2, °	Position of limbs
1	90.91	5.30	
2	91.72	89.94	
3	64.22	–91.87	
4	89.54	–179.80	
5	182.72	4.43	
6	178.6	–179.8	
7	139.61	–165.55	
8	128.40	–125.56	

MEASUREMENTS

Experimental measurements of angular displacements as a function of time are shown in Figs. 6-9, with data from two experiments on a single plot.

From Figure 6 it can be seen that in experiment 1 the lower limb makes a full revolution around the pivot, while the upper limb oscillates chaotically until about 4 seconds, when the motion of both limbs becomes periodical. In experiment 2, where both limbs are parallel, the pendulum first exhibits quasi-periodical motion which quickly becomes periodical, although the initial potential energy of the system is larger. From all of the experimental data it is seen that the damping of the system is so strong that all the energy is dissipated in 10 seconds maximum, if not sooner, due to the acceleration forces on the joints.

In Figure 7, data for experiments 3 and 4 are plotted against time. Especially in Fig. 3, differences in resonant frequencies between the limbs are apparent. It can also be seen that these differences are gradually reduced as the system dissipates energy. Further evaluation of the motion exhibited in experiment 3 is discussed below. In experiment 4 the transition from quasi-periodical to periodical motion is even more evident, as the curve becomes a smooth "sine" for low energies. The large difference between the experiments can be explained by the rapidity of motion in experiment 3, causing large dissipation forces.

In both experiments 5 and 6 the lower limb performs a full revolution around its pivot. In experiment 6 enough energy is transferred to the lower limb for it to perform three revolutions. This can be seen from the graph as the angular displacement of the lower limb goes from approximately –180° to –800° and from there it oscillates until coming to rest at 0°. The effect of large initial energies on the duration of the chaotic phase of motion can be seen as the irregularities in the curves persist for twice as long as in previous experiments (in experiments 5 and 6 the duration is approximately 6 s, and in run 4 the time is approximately 3 seconds).

Discrepancies in the motion of the system between experiments 7 and 8 serve to show the very high sensitivity to initial conditions. Differences between the initial angular displacements are only a few degrees. Very interesting is the relatively small size of irregularities in experiment 7 for such high energies of the system. This is discussed further below. On the other hand, the shape of the plots for experiment 8 exhibits no similarities to periodic motion, except right at the very end.

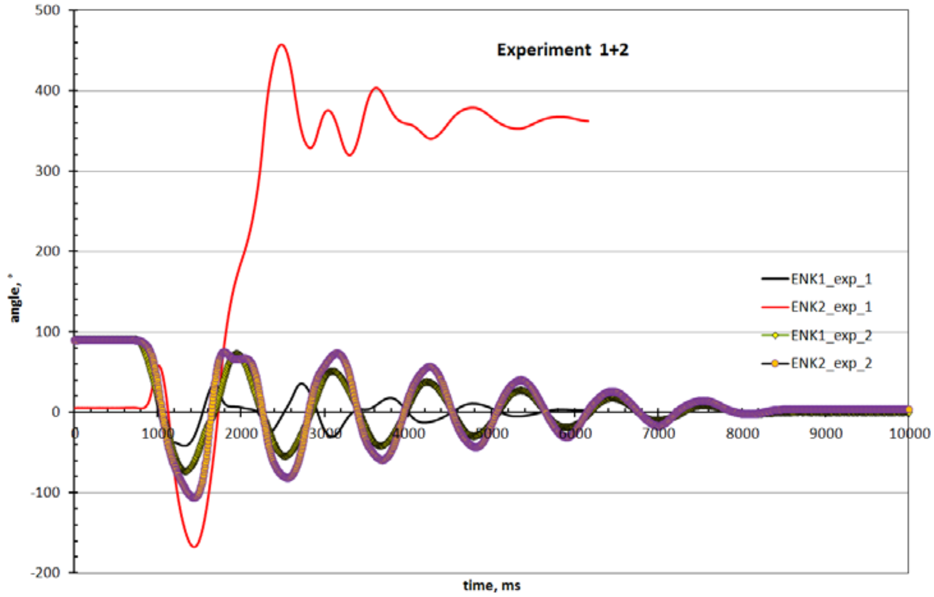


Figure 6. Plot of crank angles vs. time for tests 1 and 2.
Slika 6. Dijagram pomeranja uglova sa vremenom za ispitivanja 1 i 2

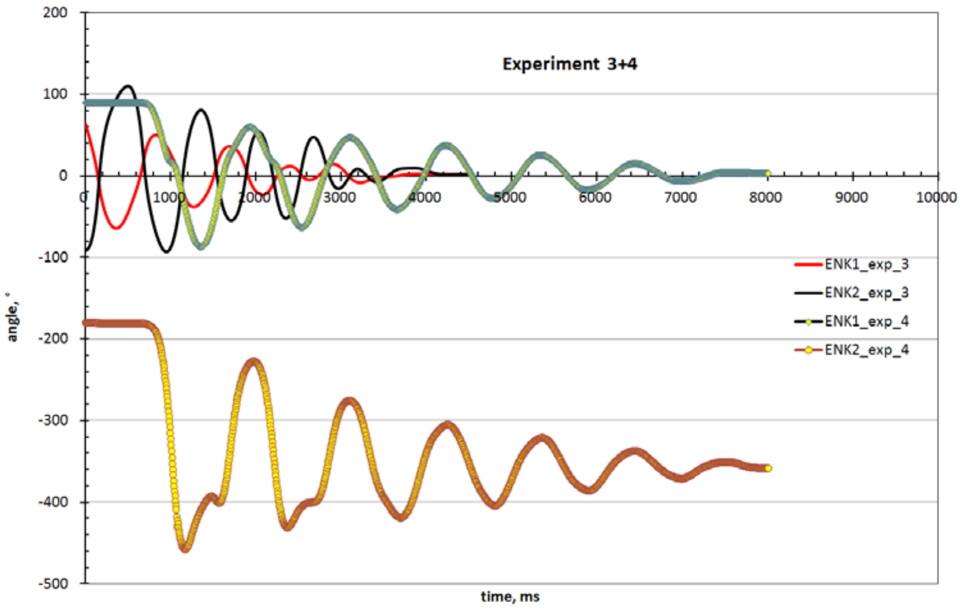


Figure 7. Plot of crank angles vs. time for tests 3 and 4.
Slika 7. Dijagram pomeranja uglova u vremenu za ispitivanja 3 i 4

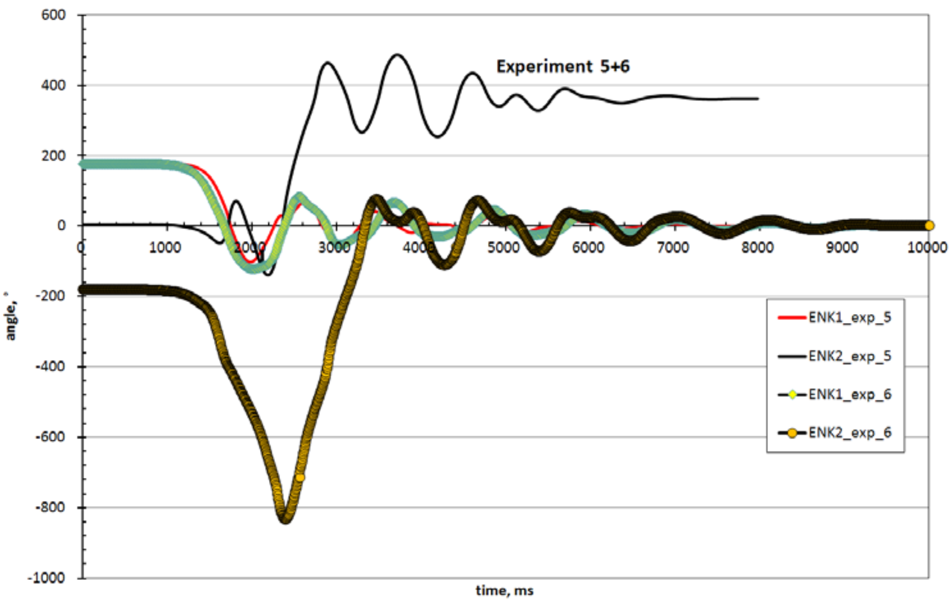


Figure 8. Plot of crank angles vs. time for tests 5 and 6.
Slika 8. Dijagram pomeranja uglova u vremenu za ispitivanja 5 i 6

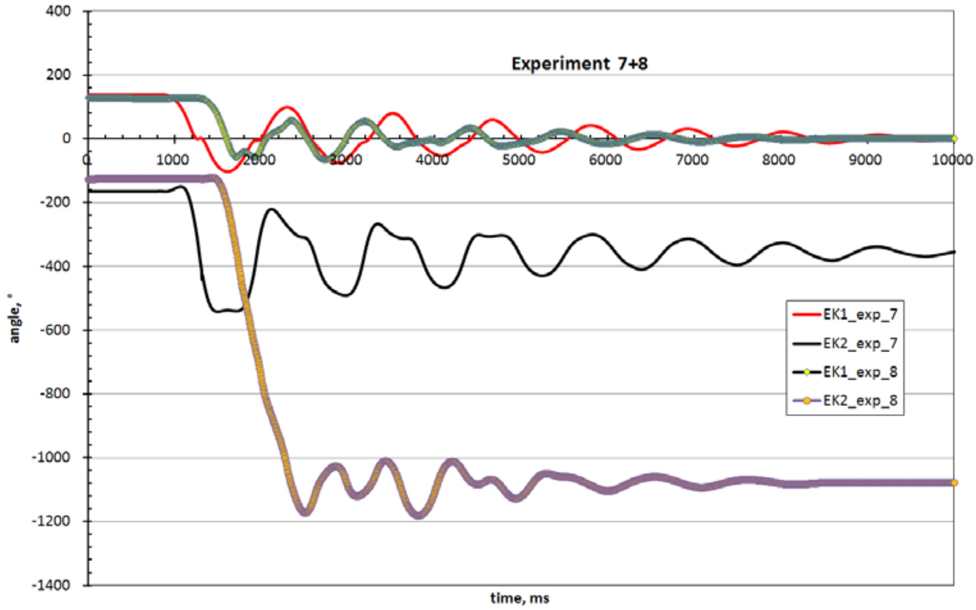


Figure 9. Plot of crank angles vs. time for tests 7 and 8.
Figure 9. Dijagram pomeranja uglova u vremenu za ispitivanja 7 i 8

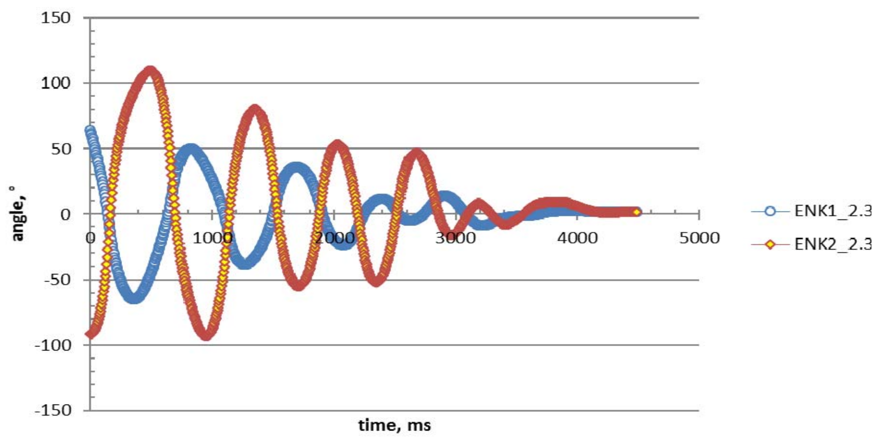


Figure 10. Graph of angles vs. time.
Slika 10. Dijagram uglovi–vreme

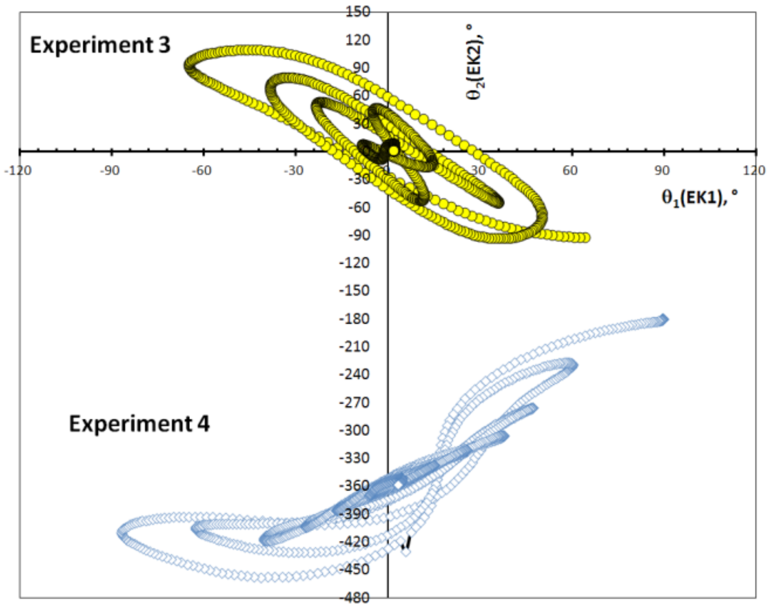


Figure 11. Parametric graph of angles for 3 and 4.
Slika 11. Parametarski dijagram za uglove za 3 i 4

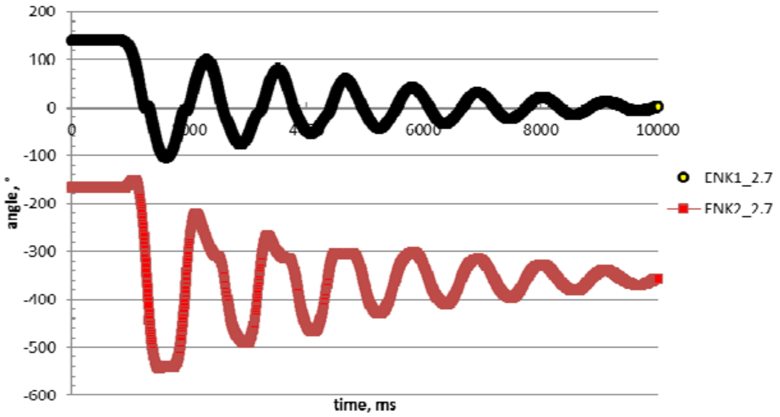


Figure 12. Graph for experiment 7.
Slika 12. Dijagram za eksperiment 7

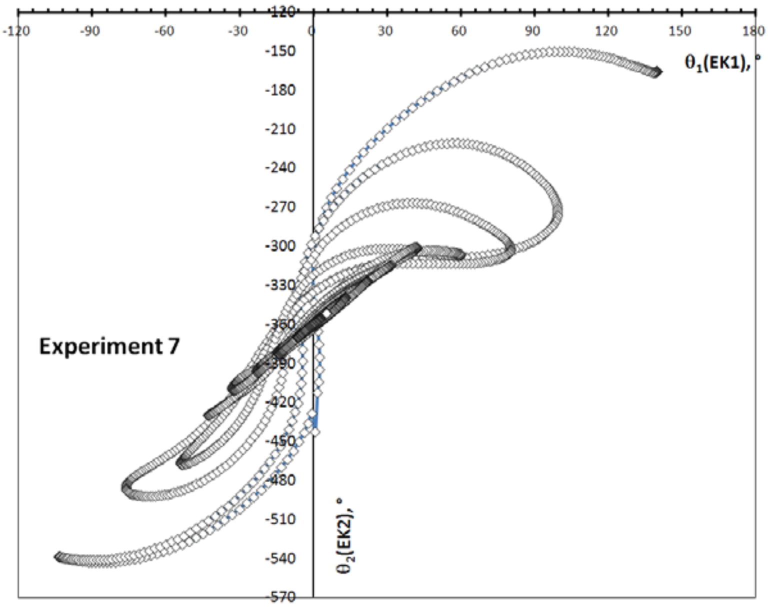


Figure 13. Parametric graph of angles.
Slika 13. Parametarski dijagram uglova

Experiments with the built model are performed at a number of initial positions. Some of them are chosen in pairs, where the setup resulted in the system having the same potential energy, but the initial angles are different, with a relative orientation of the limbs to each other. The pairs themselves had different initial energies, so that the aspect is examined as well. It is found that at low initial energies (angles) the data agrees well, even with the previous "frictionless" model. For higher initial energies, chaotic behaviour started to emerge, with the lower limb rotating full circles around the upper one, until enough energy is lost for it to transit to less eccentric motion. The behaviour of the physical system showed the inadequacies of the numerical model. The system loses energy very fast already at the very beginning of the experiment, while the weight-induced friction-only model has a much lower rate of energy loss.

Some of the experiments performed had very similar initial conditions, the angles of the limbs differed for a few degrees. Although the energy of the system was practically the same, the behaviour of the system between the experiments was radically different, especially that of the lower limb. While in experiment 6 the lower limb turned over itself and significant deformations to the "sine" curve were exhibited, in experiment 7, with very similar initial energy the motion was radically different.

The limbs in experiment 3 rotate in opposite phase, therefore a large force is exerted on both joints, greatly increasing friction and energy loss. The temporary stability of the motion is best seen in the parametric graph, where the periodic mode produces Lissajous-like curves. Additionally, experiment 3 proves wrong the assumption that friction is produced mainly by weight. This can be deduced from the time the pendulum is in motion, as this experiment has the shortest oscillation time.

Interesting, however, is that although the system is chaotic and the failure in achieving the aim (producing a viable simulation of the double pendulum), stable high energy modes of oscillation are found. These can be seen in graphs of experiments 3 and 7. Here, the two limbs rotate in phase, and centripetal forces on the second limb cancel out, resulting in less friction and, hence, less energy loss experienced by the system. Again symmetry is seen in the parametric graph of angles.

CONCLUSIONS

The motion of the double pendulum is investigated experimentally. The modes of oscillation are shown to be very dependent on both the initial orientation of the limbs and the initial energy of the system. Experiments, where initial conditions did not differ significantly, were conducted, their results showing vastly different time evolutions of the system.

A previously ignored source of energy dissipation of the system is the vibration of the pendulum. The bearings still have some spacing, allowing the limbs lower to the joints to oscillate. These vibrations went unnoticed when no additional mass is attached to the lower limb, presumably because they are small. When an additional mass of 30 g is attached to the lower limb, however, and the initial angle of the first limb is greater than approximately 90° the pendulum begins to shake rapidly and noisily. The shaking impacts the sensors, resulting in data with much signal noise, rendering the data unreliable.

Coupled temporary modes of quasi-periodic oscillation at high energies of the system are found for certain initial angles, their origin and persistence are evaluated.

In all experiments, once the system had dissipated enough energy, chaotic motion becomes the unfavoured mode of oscillation and the system quickly begins to exhibit: first quasi-periodic, and later, harmonic motion. This is deduced from the shape of the graphs, where the irregularities in the peaks disappear over time.

Future efforts are focused into producing a reliable mathematical model.

ACKNOWLEDGEMENTS

The author wishes to express gratitude to Prof. Aleksandar Veg for his support in the development of the electronics and the paper itself.

Additionally, the author would like to extend thanks to RoTech Inc. for the support in the building of the model.

REFERENCES

1. Calkin, M.G., Lagrangian and Hamiltonian Mechanics, World Scientific Publishing Co., Singapore, 1998.
2. Korsch, H.-J., Jodl, H.-J., Chaos: a program collection for the PC, Springer-Verlag, Berlin, 1994.
3. Spiegel, M.R., Schaum's outline of theory and problems of theoretical mechanics: with an introduction to Lagrange's equations and Hamiltonian theory, New York, McGraw-Hill, 1967.
4. Bazjanac, D., Tehnička mehanika III: Dinamika (in Croatian), Sveučilište u Zagrebu, Zagreb, 1980.
5. Neill, H., Quadling, D., Higher level 2, Mathematics for the IB diploma, series ed. Hugh Neill, Cambridge University Press, 2008.
6. Oertel-Jaeger, T., Topological Characterisation of Elliptic Islands in a Chaotic Sea, 2000.
7. Shinbrot, T., Grebogi, C., Wisdom, J., Yorke, J.A., *Chaos in a double pendulum*, Am. J Phys. 60, 491, 1992.
8. Buresh, T.R., Aliaga, D., Madsen, M.J., *Chaos of the double pendulum*, Wabash Journal of Physics, PHY381, 2011.
9. Brannan, J.R., Boyce, W.E., Differential equations with boundary value problems: Modern methods and applications, 2nd ed., John Wiley & Sons Inc., 2011.
10. Richter, P.H., Scholz, H.-J., Das ebene Doppelpendel, Film C1574, Publ. Wiss. Film., Sekt. Techn. Wiss./Naturw., Ser. 9, Nr.7/C1574, 1986.
11. Levien, R.B., Tan, S.M., *Double pendulum: An experiment in chaos*, Am. J Phys. 61, 1038, 1993.
12. Eckmann, J.-P., Ruele, D., *Ergodic theory of chaos and strange attractors*, Rev. Mod. Phys. 57: 617-656, 1985.
13. Gleick, J., Chaos: Making a new Science; New York: Penguin, p.368, 1988.