ASSESSMENT OF PRESSURE VESSEL LOAD CAPACITY IN THE PRESENCE OF CRACKS

INTRODUCTION

Pressure vessels are fabricated in various geometrical forms (cylindrical, cone, spherical, or combined), volumes and sizes, and are used for different purposes.

Although high safety factor values are implemented in the design of pressure vessels, the insufficient knowledge of operating conditions and how they change, and the poor quality control in fabrication, diminish the pre-designed safety factor to a large extent. The assessment of pressure vessel safety and the risk in exploitation includes the consideration of implications of leak-before-break and total collapse due to brittle fracture.

Cracks in components exposed to static and variable loads grow in a stable manner to a certain period (subcritically) and may develop as instable – critical, depending on the operating conditions. The analysis of the behaviour of pressure vessels exposed to internal pressure is aimed in assessing the structural limit load capacity when the structure contains crack-like defects. Assessment of detected or assumed defects safely, i.e. the assessment on whether defects will become critical within the observed operation period, the so-called conservative assessment, is based on linear elastic (LEFM)- or elastic-plastic (EPFM) fracture mechanics.

Pressure vessels considered are defined as stationary vessels used for liquefied gas storage under pressure, made of NIONIKRAL 70 (NN 70) produced by Ironworks Jese-nice, low-alloyed high strength steel (HSLA), specified for operating temperatures ranging from –40 °C to ambient temperature. These vessels are usually components of pressure equipment in the processing-, petro-chemical and oil industries.

SAFE OPERATION AND DESIGN PROBLEMS

The design and fabrication of pressure equipment strict conforms regulations and codes, to the aim to assure the required safety during the expected exploitation period. Technical regulations for calculating pressure vessels are aimed at defining the difference between the operating pressure and one or more characteristic pressures that may lead to fracture. Pressure vessels and pipelines are mostly designed for an operating life from 10 to 25 years, and because of the devastating effects that can occur at structural failure – catastrophic failure, the investors do not allow the operation of a component having a detected...
crack, thus emphasizing the efforts for devoting much attention to the problem of assuring the integrity of the pressure vessel.

The concept of leak-before-break (LBB) is a widely accepted method for determining the pressure components’ susceptibility to fracture, due to the stable development of cracks. Because of the fatigue loading and/or stress corrosion, the initial crack in the pressure vessel wall tends to grow through the thickness in a stable manner and ruptures the wall of the vessel and so the fluid begins to leak. The leak that precedes the global failure of the component is easily noticeable, and so the local wall through-thickness fracture is considered controlled. In the mid 20th century, Irwin had suggested adopting the leak-before-break criterion for designing pressure vessels /1-6/.

Alternatively, the surface crack can lead to catastrophic failure with an absence of leakage. The possibility for a sudden and unexpected brittle fracture is an important problem in the safety analysis and risk of failure assessment of pressure components. Brittle fracture occurs when the crack or defect experiences high stresses and low toughness of the material. In fact, the initial defect can remain undetected, and high stress values may result from a geometrical stress concentration or from residual stresses, often created in the welding process. Material toughness is a measure of the susceptibility of materials to brittle fracture and it decreases at low temperature.

Proof pressure testing is obligatory before putting pressure vessels into service. Overpressure values required for testing have been the matter of debate in the scientific and professional community, since overpressure must not be the cause of a functional damage and result in the decrease of the safety of the component. At least 200 times higher deformation energy is released in the air than in water for the same overpressure which must be accounted for when performing air pressure tests. Conditions of brittle fracture (low temperature operation, non heat-treated welded joints, incomplete inspection and low material toughness) give an explanation to why this type of fracture sometime occurs after the hydro test: – when the fluid operating temperature is much lower than the hydro test temperature; cracks created during the pressure test may develop in exploitation and if they cannot be detected, failure is most certain.

Pressure tests up to explosive fracture are applied in exceptional cases since they are costly and demanding. The aim of this test is to determine the degree of plasticity of the vessel or pipe, defined by the percentual plastic deformation at fracture.

HISTORICAL BACKGROUND

Since 1828, the French Conseil général des mines has adopted a relation between the pressure responsible for vessel fracture and the ultimate tensile strength of the material used for determining the boiler wall thickness /3/ as:

$$t = \frac{9 \rho R_i \varepsilon}{\sigma_M} + 3$$

(1)

In Eq. (1) $$t = R_0 - R_i$$ is the wall thickness, where $$R_0$$ and $$R_i$$ are the external and internal radii of the cylindrical boiler shell, $$\rho$$ is operating pressure, while 9 is the safety factor, and 3 mm is a thickness correction due to corrosion. $$\sigma_M$$ is the value of ultimate strength, with a limit up to 260 MPa. The hydro test has required a pressure 3 times higher than the operating pressure.

In 1911, while performing experiments on pressure vessels, Cook and Robertson noticed that cylindrical vessels made of soft steel deform at the diameter along with the increasing pressure, slowly at first, then rapidly, and finally they bulge before cracking. They had observed that the “highest pressure in the cylinder was larger than the pressure at the instant of fracture, owing to the fast dilatations that had preceded” /3/.

For ideal plastic materials that do not strengthen by deformation, in the region of small deformations, with sufficient accuracy, the tensile properties are known not to exhibit any changes with the increasing deformation. When plastic deformation starts, at first it is limited to the material that is in the zone of elasticity until plasticity is reached throughout the thickness, or a plastic joint develops. This type of load is the threshold, above which the structure cannot resist deformations if material behaviour remains plastic. When deformations overcome those at threshold loads, most materials tend to strengthen by deformation, which is manifested in the increase of tensile properties. The internal pressure in vessels increases the material tensile properties, and the stress increases as a result of the reduction of the net section. As a result of these two processes, a maximal resisting pressure develops in the cylinder. This phenomena is called plastic instability, and does not represent the fracture but it advances rapidly to fracture with a persisting load.

Early pressure vessel design required the determination of the safety factor with respect to fracture, due from the increased pressure for plastic instability. Later, the safety factor was associated with the onset of large deformations, or the limited pressure, that precedes the fracture. These two approaches have limited the allowable stresses to a fraction of the ultimate strength, or to the yield strength of the material. In certain countries, the safety factor with respect to the limiting pressure is considered as a sufficient criterion in the design, while elsewhere, two safety factors are applied: with respect to rupture pressure, and a sufficient deformation with values between the onset of large deformations and burst fracture.

EXPERIMENTS

A procedure is considered for the limit load assessment of the remaining ligament in a cylindrical wall of a test vessel with an initial crack as shown in Fig. 1. The crack is located on the outer vessel wall surface, of length $$2a = 108$$ mm and initial depth $$d_0 = 5$$ mm. The vessel is used for the storage of liquefied gas under pressure. The shown prototype is fabricated by welding NN 70 steel of nominal yield strength $$R_{y,2} = 780$$ MPa, ultimate strength $$R_m = 820$$ MPa (Table 1), with chemical contents given in Table 2.
The vessel is produced as a one-piece, stationary, horizontal, with torispherical heads welded to the cylindrical shell by shielded metal arc welding (SMAW) of segments 700 mm in length. The outer diameter of the vessel is \( D_s = 1200 \) mm, and wall thickness \( s = 16 \) mm. Table 3 shows the geometrical measurements of the tested vessel and the investigated surface crack. The operating temperature in the vessel is \( t = -40^\circ\text{C} \).

According to design, the allowable stress is calculated as the lesser of the two quotients:

\[
\sigma_{z允} = \min \left[ \frac{R_{p0.2}}{2}, \frac{R_m}{1.5} \right] = 342 \text{ MPa}
\]

The allowable operating pressure for the vessel shell with no defects is determined based on the boiler formula and equals \( p_{允} = 9.243 \text{ MPa} = 92.43 \text{ bar} \), with a weld factor \( \varphi = 1 \) and corrosion correction \( c = 0 \). The longitudinal stress is \( \sigma_z = 342 \text{ MPa} \) and it is twice the value of the circumferential stress \( \sigma_\theta = 170.94 \text{ MPa} \).

Thermal stresses in the pressure vessel wall /4, 7/ are calculated in the case of outer surface shell temperature \( t_1 = 40^\circ\text{C} \), inner surface temperature \( t_2 = -40^\circ\text{C} \), with the assumption of linear change in temperature through wall thickness, according to Eq.(2):

\[
\sigma_z = \sigma_\theta = \mp \frac{E\alpha (t_1 - t_2)}{2(1-\nu)}
\]

and are \( \sigma_z = \sigma_\theta = \pm 143.28 \text{ MPa} \), where the ‘+’ sign refers to the outer surface with tensile stress if \( t_1 > t_2 \).

The crack opening mode of fracture, known as mode I, is defined as the separation of fractured surfaces by tensile stress symmetrical to the initial crack plane /8/, is a characteristic of axial surface cracks in vessels or pipes exposed to internal pressure. For this case the critical value of the stress intensity factor \( K_Ic \), or plane strain fracture toughness, is calculated as:

\[
K_{Ic} = \sigma_z \sqrt{a} ,
\]

where fracture occurs at stress \( \sigma_z \) in a specimen with crack length \( a \). Apparently, the plane stress state is dominant for the investigated thin-walled pressure vessel, because of the ratio \( s/D_s = 0.0135 \) and material properties. Hence, the fracture mechanics parameter COD (crack opening displacement) is used for the vessel limit load assessment, since it includes a more pronounced plastic behaviour. The COD is directly proportional to the \( J \)-integral that may be used, as its critical value \( J_{Kc} \), for determining \( K_{Ic} \) on samples that are not required to fracture in conditions of plane strain.

**REMAINING STRENGTH ASSESSMENT BY R-CURVE**

During the 60s, Irwin, Krafft, et al., /9, 10/, have introduced the fracture criterion based on material resistance to crack growth. According to this criterion the crack grows in a stable manner as long as the increase in resistance to crack growth, \( R \), is greater than the increase of the acting stress intensity factor \( K \), or

\[
\frac{\partial R}{\partial a} > \frac{\partial K}{\partial a}
\]
Fast fracture develops when
\[ K = R \text{ and } \frac{\partial K}{\partial a} > \frac{\partial R}{\partial a} \quad (5) \]

In order to include the influence of plasticity, this concept can be extended to the use of the J-integral instead of the stress intensity factor K.

The remaining load capacity of the structure in the presence of a crack can be determined when the following is known, /11/:
- \( J_R \) material curve for the material of the structure, from curves and curve graphs, and
- driving force as a cylindrical shell, /7, 12-15/.

The CDF curves, representing the J-integral dependence on crack extension at a constant value of the load (stress), are determined by the shell parameter \( \lambda \):
\[ \lambda = \left[ 12(1-\nu^2) \right]^{1/4} \frac{a}{\sqrt{R_s}}, \quad (8) \]
where, \( a \) is the half length of the outer surface crack, Fig. 1. From Eq.(8), the proportionality is evident between the crack half length \( a \) and shell parameter \( \lambda \).

From the contributing compliance and from the slope of the unloading curve, an instantaneous crack length is determined for all calculated J-integral values, that is necessary for plotting the \( J_R \)-curve, Fig.5.

At the intersection of the \( J_R \)-curve and the crack tip blunting line given by Eq.(6):
\[ J = 2\sigma \Delta a, \quad (6) \]
the value \( J_R = 341.8 \text{ N/mm} \) is calculated, and the fictive crack propagation value from the crack blunting \( \Delta a = 0.214 \text{ mm} \). Here, \( \bar{\sigma} \) is the mean plastic strengthening stress of the specimen, calculated from Eq.(7):
\[ \bar{\sigma} = \frac{R_{p,0.2} + R_e}{2}. \quad (7) \]

Calculated values make the basic initial data on the behaviour of the tested NN 70 material in the presence of a crack. In order to apply these values to the investigated pressure vessel structure, it is necessary to plot the crack driving force (CDF) curves for the shell structure, assumed as a cylindrical shell, /7, 12-15/.

The values \( \delta_d/d_1 \) and \( \theta_2/d_1 \) depend on the shell parameter \( \lambda \) and on the fraction of crack depth \( d/s \), /12, 14/.

\[ J = 716.2 \Delta a^{0.534} \]

\[ J^* = \frac{JE}{4aR_{p,0.2}} = \frac{2\sqrt{3}}{3} \left[ \delta_d + \theta_2 \frac{0.5 - d}{s} \right] \]

For the crack shown in Fig. 1, the relation holds:

\[ J^* = \frac{JE}{4aR_{p,0.2}} = \frac{2\sqrt{3}}{3} \left[ \delta_d + \theta_2 \frac{0.5 - d}{s} \right] \]

where, \( J^* \) is the normalized value of J-integral, \( J \) is current value of J-integral, and \( d_1 \) and \( d_2 \) are determined from
\[ d_1 = \frac{4aR_{p,0.2}}{E} \quad \text{and} \quad d_2 = \frac{4aR_{p,0.2}}{sE} \]

The testing temperature was –40°C.

Three-point bend specimens, shown in Fig. 3, with introduced fatigue pre-cracks were loaded by constantly increasing bending forces and were successively unloaded as shown in the diagram load–CMOD (crack mouth opening displacement), Fig. 4.

From the contributing compliance and from the slope of the unloading curve, an instantaneous crack length is determined for all calculated J-integral values, that is necessary for plotting the \( J_R \)-curve, Fig.5.

The necessary data for limit load capacity assessment is the material resistance curve for crack growth (\( J-\Delta a \) or \( J_R \)-curve), where \( \Delta a \) is crack extension. Tests are performed at the Bay-Logi Institute for Logistics and Production Systems, at Miskolctapolca, Hungary, in the low temperature chamber, using a servo-hydraulic INSTRON 8803 machine with the compliance method that gives average values of the crack length. The testing temperature was –40°C.

The values \( \delta_d/d_1 \) and \( \theta_2/d_1 \) depend on the shell parameter \( \lambda \) and on the fraction of crack depth \( d/s \), /12, 14/.

\[ J^* = \frac{JE}{4aR_{p,0.2}} = \frac{2\sqrt{3}}{3} \left[ \delta_d + \theta_2 \frac{0.5 - d}{s} \right] \]

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\[ d_1 = \frac{4aR_{p,0.2}}{E} \quad \text{and} \quad d_2 = \frac{4aR_{p,0.2}}{sE} \]

The testing temperature was –40°C.
The plot in Fig. 6 shows the change of $\sqrt{J^*}$ for shell parameter $\lambda = 1$ with fraction of crack depth $d/s$ at constant values of $(pr/sR_{p0.2})$ and $J^*_{R0.5}$ for the resistance curve of the specimen made of parent metal and transferred from Fig. 5, with an initial crack depth of $d_0 = 5$ mm.

**DISCUSSION**

A crack length of $2a = 108$ mm is calculated from Eq.(8) and for shell parameter $\lambda = 1$. Plots on the diagram in Fig. 6 show that for the surface crack of initial depth $d_0 = 5$ mm and length $2a = 108$ mm, the common tangent on the CDF curve gives $(pr/sR_{p0.2}) = 0.691$. The remaining load carrying ligament at initial crack depth $d_0$ equals $b_0 = s - d_0 = 16 - 5 = 11$ mm.

The crack shall continue to grow in a stable manner in the plastic zone up to the point of instability A, whose coordinates are $(0.406; 0.5895)$, so that up to the depth of $d \leq 0.406 = 16 - 0.406 = 6.496$ mm there is no risk of fast fracture. The limit pressure of the vessel with a 6.4 mm crack depth is then

$$p = 0.691 \times \frac{16 \times 780}{592} = 14.567 \text{ MPa} = 145.67 \text{ bar},$$

$$p > p_{\text{allow}} = 92.43 \text{ bar},$$

$$p > p_I = 120.16 \text{ bar}.$$  

This pressure value is higher than the calculated values of the operating and testing pressures for a vessel having no defects in the shell. The remaining ligament at the length of 108 mm is then

$$b = s - d = 16 - 6.496 = 9.504 \text{ mm}.$$  

**PRESSURE VESSEL CALCULATION BY APPLYING THE LINEAR FINITE ELEMENT THEORY**

The first calculation model is formed based on 751 nodal points that define 708 plate finite elements. The model is presented in Fig. 7 with appropriate boundary conditions. The calculation is done for one quarter of the pressure-vessel, using the program package KOMIPS /16, 17/.  

Three cases of the loading are investigated: the first (I) case represents a steady internal pressure of 100 bar, the second (II) case of loading is thermal loading, and the third (III) case represents their combination. In the II case, a temperature of $-40^\circ\text{C}$ is taken at the inner surface of the vessel, and $+40^\circ\text{C}$ for the outer surface. In order to simulate this loading, the plates are assumed to be at a mean temperature of $0^\circ\text{C}$, the ambient temperature is $40^\circ\text{C}$ and the gradient of temperature change across the plate thickness is $5^\circ\text{C/mm}$. The influence of dead weight of the structure is negligible. Figure 8 shows the deformed vessel under pressure for loading cases I and II.
The maximal calculated deformation in the case of internal pressure of 100 bar is 7.05 mm, and 0.75 mm in the second case, Fig. 8. As deformation fields in these cases have opposite directions, in the third loading case, the total maximal calculated deformation is 6.61 mm.

The equivalent stress is calculated by using the Huber-Hancky-Mises hypothesis. The obtained results are presented in Fig. 9. In all loading cases the maximal stresses act in the toroidal part of the head. Values determined are: in the first loading case 913 MPa; in the second 142 MPa; and in the third case 890 MPa. The weakest part of the vessel is the toroidal section of the head.

The linear finite element method has been applied in this calculation, where simple calculations have obtained allowable limit pressure loading values that are close to those calculated with the appropriate boiler formula.

The influence of the crack located in the cylindrical shell is investigated, and thus the equivalent stresses for this model are: 324 MPa (I loading case), 137 MPa (II loading case), and 448 MPa (III loading case). A comparative view of the obtained results is given in Table 4.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Maximal deformation, mm</th>
<th>Maximal equivalent stresses, MPa</th>
<th>Equivalent stresses in the shell, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Internal pressure 10 MPa</td>
<td>7.048</td>
<td>913</td>
<td>324</td>
</tr>
<tr>
<td>II Thermal load -40 to +40°C</td>
<td>0.747</td>
<td>142</td>
<td>137</td>
</tr>
<tr>
<td>III Combined loading</td>
<td>6.609</td>
<td>890</td>
<td>448</td>
</tr>
</tbody>
</table>
Individual stresses required for a detailed calculation at a pressure load of 100 bar are the longitudinal stress of 187 MPa, and circumferential stress of 374 MPa.

CRACK CALCULATION BY APPLYING THE LINEAR FEM THEORY

A detailed calculation of the crack influence, located in the cylindrical shell, required a new FEM model of 14495 nodal points. A part of the cylinder, 708 mm in length, is simulated with 10500 volume finite elements. The precise loading data input had required a generation of an 3800 element mesh, constituted by thin plate type elements, as shown in Fig. 10. The model does not include crack growth, so all calculated values relate to the configuration with the initial fatigue crack of the size $2a$ and $d_0$.

The three previously described loading cases are also considered for this model. In order to achieve the I case, a corresponding axial pressure of 187 MPa is added to the internal radial pressure of 10 MPa. The temperature range from –40 to +40°C is defined for all nodes depending on the radius.

The deformation field is similar for all three loading cases and is shown in Fig. 10b. The displacement of the shell in the first case is 1.27 mm (outer surface), and 0.29 mm (inner surface) in the second case, and finally 1.32 mm on the pressure side in the third case.

The crack influence can be defined based on the obtained stress fields. The stress in the I loading case is approximately constant throughout the shell thickness, the axial stress is 187 MPa and the radial stress is 361 MPa. Equivalent stresses for loading cases are shown in Fig. 11, while Fig. 12 shows the distribution of equivalent stresses in the vicinity of the crack. It is evident that a crack of 108 mm in length and 5 mm deep causes only a local stress concentration and does not substantially increase the stress concentration magnitude.

Temperature distribution is such that the stress increases on the inner surface of the vessel, and decreases on the outer surface that contains the crack. The maximal stress
from thermal loading is 121 MPa. At a pressure of 10 MPa, the stress remains below 280 MPa at almost up to half of the shell thickness. In this way the previously concluded remark is confirmed: crack propagation is not likely at operating pressures and temperatures. At higher pressure values, the stress does not exceed the allowed value, and so the linear finite element theory is not capable of giving a precise calculation.

![Equivalent stress, MPa for \( p = 10 \) MPa and \( \Delta T = 80^\circ C \)](image)

**Figure 12. Equivalent stress distribution around the crack.**

**Slika 12. Raspodela ekvivalentnog napona oko prsline**

**CONCLUSION**

The investigated pressure vessel in plane stress conditions with an outer surface crack of 5 mm depth located on the shell, can continue with operation even if the pressure jumps to 120.2 bar, as is the testing pressure. The safe operation of vessels of this type may be expected also for proportionally larger defects, since there is no risk of a leak-before-break, and no risk of catastrophic failure.

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**REFERENCES**