*Thakur Pankaj*

# DEFORMATION IN A THIN ROTATING DISC HAVING VARIABLE THICKNESS AND EDGE LOAD WITH INCLUSION AT THE ELASTIC-PLASTIC TRANSITIONAL STRESSES

# DEFORMACIJA TANKOG ROTIRAJUĆEG DISKA PROMENLJIVE DEBLJINE I OPTEREĆENIH IVICA SA UKLJUČKOM PRI PRELAZNIM ELASTIČNO-PLASTIČNIM NAPONIMA

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**Ključne reči**
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- izotropnost
- rotirajući disk

*Abstract*

*Transition theory has been used to derive the elastic-plastic and transitional stresses. Results obtained have been discussed numerically and depicted graphically. It is observed that the rotating disc made of incompressible material with inclusion requires higher angular speed to yield at the internal surface as compared to the disc made of compressible material. It is seen that the radial and circumferential stresses are maximum at the internal surface with and without edge load (for flat disc). With the increase in thickness parameter (k = 2, 4), the circumferential stress is maximum at the external surface while the radial stress is maximum at the internal surface. From the figures drawn the disc with exponentially varying thickness (k = 2), a high angular speed is required for initial yielding at the internal surface as compared to flat disc and exponentially varying thickness for k = 4 onwards. It is concluded that the disk made of isotropic compressible material is on the safer side of the design as compared to disk of isotropic incompressible material as it requires higher increase in an angular speed to become fully plastic from its initial yielding.*

*Izvod*

*Primenjena je teorija prelaznog naponskog stanja za izračunavanje elastoplastičnih i prelaznih napona. Dobijeni rezultati su diskutovani i analizirani numerički i grafički. Primećuje se da rotirajući disk od nestišljivog materijala sa uključkom zahteva veću ugaonu brzinu za tečenje na unutrašnjoj površini, u poređenju sa diskom od stišljivog materijala. Uočeno je da su radijalni i obimni naponi maksimalni na unutrašnjoj površini, sa ili bez ivičnog opterećenja (za ravan disk). Sa porastom parametra debljine (k = 2, 4), obimni napon je najveći na spoljnoj površini, dok je radijalni napon najveći na unutrašnjoj površini. Sa prikazanih dijagrama, kod diska sa eksponencijalnom promenom debljine (k = 2) se zahteva veća ugaona brzina za početak tečenja na unutrašnjoj površini, u poređenju sa ravnim diskom i eksponencijalnom promenom debljine za k = 4 i više. Zaključuje se da je disk od izotropnog stišljivog materijala sigurniji pri projektovanju u poređenju sa diskom od izotropnog nestišljivog materijala, jer zahteva veći porast ugaone brzine da bi postigao potpunu plastičnost od početka pojave tečenja.*

## INTRODUCTION

This paper is concerned with the analysis of a rotating disk made of isotropic material with exponentially varying thickness. There are many applications of such type of rotating disks, as in turbines, rotors, flywheels and with the advent of computers, disk drives. The use of rotating disk in machinery and structural applications has generated considerable interest in many problems in the domain of solid mechanics. The analysis of stress distribution in a circular disk rotating at a high speed is important for a better understanding of the behaviour and optimum design of structures. The analysis of a thin rotating discs of isotropic material is discussed extensively by Timoshenko and Goodier, /1/. In the classical theory, solutions for such type of discs of isotropic material can be found in most of standard textbooks, /1-5/. Chakrabarty, /2/, and Heyman, /6/, solved the problem for the plastic state by utilizing the solution in the elastic range and considering the plastic state with the help of Tresca's, Von-Mises or any other classical yield condition. Han, /7/, has investigated elastic and plastic stresses for isotropic materials with a variable thickness. Eraslan, /8/, has calculated elastic and plastic stresses having variable thickness using Tresca's yield criterion, its associated flow rule and linear strain hardening. Wang, /9/, has investigated the deformation of elastic half rings.

Transition is a natural phenomenon and there is hardly any branch of science or technology in which we do not come across transition from one state to another. At transition, the fundamental structure of the medium undergoes a change. The particles constituting a medium rearrange themselves and give rise to spin, rotation, vorticity and other non-linear effects. This suggests that at transition, non-linear terms are very important and neglection of which may not represent the real physical phenomenon. Therefore transition fields are non-linear, non-conservative and irreversible in nature. Elasticity-plasticity, visco-elastic, creep, fatigue, relaxation are some examples of transition in which non-linear terms are very important. At present, such problems as elastic-plastic, creep and fatigue are treated by assuming ad-hoc, semi-empirical laws with the result that discontinuities, singular surfaces, non-differentiable regions have to be introduced over which two successive states of a medium are matched together. In a series of papers, Seth (1962-64) has given an entirely different orientation to this interesting problem of transition. He has developed a new 'transition theory' /10-12/ of elastic-plastic and creep deformation. The transition theory utilizes the concept of generalized principal strain measure and asymptotic solution at critical points or turning points of the differential system, defining the deformed field and has been successfully applied to a large number of problems, /13-19/. The generalized principal strain measure, /19/, is defined as,

$$e_{ii} = \int_0^{e_{ii}^A} \left[1-2e_{ii}^A\right]^{\frac{n}{2}-1} de_{ij}^A = \frac{1}{n}\left[1-\left(1-2e_{ii}^A\right)^{\frac{n}{2}}\right],\ (i,j=1,2,3) \qquad (1)$$

where $n$ is the measure and $e_{ij}^A$ are the principal Almansi finite strain components. For $n = -2, -1, 0, 1, 2$ it gives Cauchy, Green, Hencky, Swainger and Almansi measures respectively.

Here, an attempt is made to study the behaviour of an isotropic thin rotating disk with exponentially variable thickness and edge load using transition theory, /10/. The thickness of the disc is assumed to vary along the radius in the form

$$h = h_0 e^{-\left(\frac{r}{b}\right)^k}$$

where $h_0$ is the constant thickness at the axis, $k$ is the geometric parameter and $b$ is the radius of the disk.

*Objective of the present study*

In order to explain the elastic-plastic deformation, it is first necessary to recognise the transition state as an asymptotic one and in this work; the major aim to eliminate the need for assuming semi-empirical laws, yield condition. The constitutive equation corresponding to the transition state is also obtained.

Borah, /16/, identified the transition state in which the governing differential equation shows some criticality. The general yield condition of transition is identified from the vanishing of Jacobian of transformation, $\partial(X,Y,Z)/\partial(x,y,z) = 0$, where $(X,Y,Z)$, $(x,y,z)$ are the coordinates of a point in the undeformed and deformed state, respectively.

## GOVERNING EQUATIONS

We consider a thin disk of constant density with central bore of radius *'a'* and external radius *'b'*. The disc is rotating with angular speed *'ω'* about an axis perpendicular to its plane, passing through its centre. A case of plane stress is taken in which the axial stress $T_{zz}$ is zero. The disk is assumed to be symmetric with respect to the mid plane.

The displacement components in cylindrical polar coordinates are given by /11/.

$$u = r(1-\beta);\quad v = 0;\quad w = dz \qquad (2)$$

where $\beta$ is a function of $r=\sqrt{x^2+y^2}$ only and $d$ is a constant. The finite strain components are given as,

$$e_{rr}^A = \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-\left(\beta + r\beta'\right)^2\right]$$
$$e_{\theta\theta}^A = \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left(1-\beta^2\right) \qquad (3)$$
$$e_{zz}{}^A = \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-\left(1-d\right)^2\right]$$
$$e_{r\theta}{}^A = e_{\theta z}{}^A = e_{zr}{}^A = 0,\ \text{where}\ \beta' = \frac{d\beta}{dr}.$$

On substitution of Eq.(3) in (1), the generalised components of strain are given as

$$e_{rr} = \frac{1}{n}\left[1-\left(\beta + r\beta'\right)^n\right],\ e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right],$$
$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right],\ e_{r\theta} = e_{\theta z} = e_{zr} = 0 \qquad (4)$$

The stress-strain relations for isotropic material are given as,

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij},\ (i,j=1,2,3) \qquad (5)$$

where $T_{ij}$ and $e_{ij}$ are the stress and strain components respectively, $\lambda$ and $\mu$ are the Lame's constants, $I_k = e_{kk}$ is the first strain invariant and $\delta_{ij}$ is the Kronecker's delta.

Eq.(5) for this problem becomes

$$T_{rr} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right] + 2\mu e_{rr} \qquad (6)$$
$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right] + 2\mu e_{\theta\theta},\ T_{zz} = T_{r\theta} = T_{\theta z} = T_{zr} = 0$$

Substituting Eq.(3) in (5), the strain components in terms of stresses are obtained as

$$e_{rr} = \frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right] = \frac{1}{E}\left[T_{rr} - \left(\frac{1-C}{2-C}\right)T_{\theta\theta}\right]$$
$$e_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right] = \frac{1}{E}\left[T_{\theta\theta} - \left(\frac{1-C}{2-C}\right)T_{rr}\right] \qquad (7)$$
$$e_{zz} = \frac{1}{2}\left[1-\left(1-d\right)^2\right] = -\left(\frac{1-C}{2-C}\right)\frac{1}{E}\left(T_{rr}-T_{\theta\theta}\right)$$
$$e_{r\theta} = e_{\theta z} = e_{zr} = 0$$

where $E$ is the Young's modulus and $C$ is the compressibility factor of the material. In terms of Lame's constant they are given as $C = \frac{2\mu}{(\lambda+2\mu)}$ and $E = \frac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$

Substituting Eq.(4) in (6), we get the stresses as

$$T_{rr} = \frac{2\mu}{n}\left[3-2C-\beta^n\left\{1-C+(2-C)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right]$$
$$T_{\theta\theta} = \frac{2\mu}{n}\left[3-2C-\beta^n\left\{2-C+(1-C)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right] \qquad (8)$$
$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0$$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}\left(hrT_{rr}\right)-hT_{\theta\theta}+\rho r^2\omega^2 h=0 \tag{9}$$

where $\rho$ is density of material and $h$ is the exponentially variable thickness of the disc. Using Eq.(8) in (9), we get a non-linear differential equation in $\beta$ as

$$(2-C)nP\beta^{n+1}(P+1)^{n-1}\frac{dP}{d\beta}=\frac{n\rho\omega^2 r^2}{2\mu}+\beta^n\left\{k\left(\frac{r}{b}\right)^k-nP\right\}\times$$
$$\times\left\{1-C+(2-C)(P+1)^n\right\}+\beta^n\left\{1-(P+1)^n\right\}-k\left(3-2C\right)\left(\frac{r}{b}\right)^k \tag{10}$$

where $r\beta' = \beta P$ ($P$ is a function of $\beta$, and $\beta$ is a function of $r$). Transition or turning points of $P$ in Eq.(10) are $P \to -1$ and $P \to \pm\infty$. The boundary conditions are:

$$u=0 \text{ at } r=a, \text{ and } T_{rr}=\sigma_0 \text{ at } r=b \tag{11}$$

The edge load is attached at the boundary (i.e. at $r = b$), and because of inclusion the displacement is zero at the inner surface.

## SOLUTION THROUGH THE PRINCIPAL STRESS

It is shown /13-19/ that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point , we define the transition function $R$ as /19/

$$R=\frac{n}{2\mu}T_{\theta\theta}=\left[\left(3-2C\right)-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right] \tag{12}$$

Taking the logarithmic differentiation of Eq.(12) with respect to $r$ and using Eq.(10), we get

$$\frac{d}{dr}\left(\log R\right)=-\frac{\beta^n\left(\frac{1-C}{2-C}\right)\left[\begin{array}{l}\frac{n\rho\omega^2r^2}{2\mu\beta^n}+k\left(\frac{r}{b}\right)^k\left\{1-C+(2-C)(P+1)^n\right\}\\ -nP(1-C)+\left\{1-(P+1)^n\right\}-\frac{k(3-2C)}{\beta^n}\left(\frac{r}{b}\right)^k\end{array}\right]+nP\beta^n(2-C)}{r\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right]} \tag{13}$$

Taking the asymptotic value of Eq.(13) as $P \to \pm\infty$ and integrating, we get

$$R=A_1 r^{-\frac{1}{2-C}}e^{\left(\frac{r}{b}\right)^k} \tag{14}$$

where $A_1$ is a constant of integration, which can be determined by the boundary condition.

From Eqs.(12) and (14), we have

$$T_{\theta\theta}=\frac{2\mu}{n}A_1 r^{-\frac{1}{2-C}}e^{\left(\frac{r}{b}\right)^k} \tag{15}$$

Substituting Eq.(15) in (9) and integrating, we get

$$T_{rr}=\frac{2\mu A_1(2-C)}{n(1-C)}r^{-\frac{1}{2-C}}e^{\left(\frac{r}{b}\right)^k}-\frac{\rho\omega^2 f(r)}{r}e^{\left(\frac{r}{b}\right)^k}+\frac{B_1}{rh_0}e^{\left(\frac{r}{b}\right)^k} \tag{16}$$

where $B_1$ is constant of integration and $f(r)=\int r^2 e^{-\left(\frac{r}{b}\right)^k}dr$.

Substituting Eqs.(15) and (16) in second Eq.(7), we get

$$\beta=\sqrt{1-\frac{2}{E}\left(\frac{1-C}{2-C}\right)e^{\left(\frac{r}{b}\right)^k}\left\{\frac{\rho\omega^2}{r}f(r)-\frac{B_1}{rh_0}\right\}} \tag{17}$$

Substituting Eq.(17) in (2), we get

$$u=r-r\sqrt{1-\frac{2}{E}\left(\frac{1-C}{2-C}\right)e^{\left(\frac{r}{b}\right)^k}\left\{\frac{\rho\omega^2}{r}f(r)-\frac{B_1}{rh_0}\right\}} \tag{18}$$

where $E=\dfrac{2\mu(3-2C)}{(2-C)}$ is the Young's modulus in terms of compressibility factor. Using boundary condition (11) in Eqs.(16) and (18), we get $B_1=h_0\rho\omega^2 f(a)$, and

$$A_1=\frac{n(1-C)}{2\mu(2-C)}\left[\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]b^{\frac{1}{2-C}} \tag{19}$$

Substituting the values of constants of integration $A_1$ and $B_1$ from Eq.(19) in Eqs.(15), (16) and (18) respectively, we get the transitional stresses and displacement as

$$T_{\theta\theta}=\left[\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\frac{(1-C)}{(2-C)}\left(\frac{b}{r}\right)^{\frac{1}{2-C}}e^{\left(\frac{r}{b}\right)^k} \tag{20}$$

$$T_{rr}=\left[\left\{\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right\}\left(\frac{b}{r}\right)^{\frac{1}{2-C}}-\frac{\rho\omega^2\{f(r)-f(a)\}}{r}\right]e^{\left(\frac{r}{b}\right)^k} \tag{21}$$

$$u=r-r\sqrt{1-\frac{2}{E}\left(\frac{1-C}{2-C}\right)\frac{\rho\omega^2}{r}\{f(r)-f(a)\}e^{\left(\frac{r}{b}\right)^k}} \tag{22}$$

From Eqs.(20) and (21), we get

$$T_{rr}-T_{\theta\theta}=\left[\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\left(\frac{b}{r}\right)^{\frac{1}{2-C}}e^{\left(\frac{r}{b}\right)^k}\frac{1}{2-C}-\frac{\rho\omega^2\{f(r)-f(a)\}}{r}e^{\left(\frac{r}{b}\right)^k} \tag{23}$$

## INITIAL YIELDING

From Eq.(23), it is seen that $\left|T_{rr}-T_{\theta\theta}\right|$ is maximum at the internal surface (i.e. at $r = a$), therefore yielding will take place at the internal surface of the disc and Eq.(23) become,

$$\left|T_{rr}-T_{\theta\theta}\right|_{r=a}=$$
$$=\left|\left[\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\left(\frac{b}{a}\right)^{\frac{1}{2-C}}e^{\left(\frac{a}{b}\right)^k}\frac{1}{2-C}\right|\equiv Y(\text{say})$$

and the angular speed necessary for initial yielding is given by

$$\Omega_i^2=\frac{\rho\omega_i^2 b^2}{Y}=\left[\frac{(2-C)\left(\frac{a}{b}\right)^{\frac{1}{2-C}}}{e^{\left(\frac{a}{b}\right)^k}}-\frac{T_o}{e}\right]\frac{b^3}{\{f(b)-f(a)\}} \quad (24)$$

where $T_0=\sigma_0/Y$

FULLY PLASTIC STATE

The disc becomes fully plastic ($C\to 0$) at the external surface (i.e. at $r = b$) and Eq.(23) become

$$\left|T_{rr}-T_{\theta\theta}\right|_{r=b}=\left[\frac{\sigma_0}{2}-\left(\frac{\rho\omega^2}{2b}\right)e\{f(b)-f(a)\}\right]=Y^*$$

The angular speed required for the disc to become fully plastic is given by

$$\Omega_f^2=\frac{\rho\omega_f^2 b^2}{Y^*}=\frac{\left[-2+T_o^*\right]}{e}\frac{b^3}{\left[f(b)-f(a)\right]} \quad (25)$$

where $w_f=\frac{\Omega_f}{b}\sqrt{\frac{Y^*}{\rho}}$ and $T_0^*=\sigma_0/Y^*$.

We introduce the following non-dimensional components as

$$R=\frac{r}{b},\ R_0=\frac{a}{b},\ \sigma_r=\frac{T_{rr}}{Y},\ \sigma_\theta=\frac{T_{\theta\theta}}{Y},\ \text{and}\ \bar{u}=\frac{u}{b}$$

Transitional stresses, angular speed and displacement can be obtained from Eqs.(20)-(22) and (24) in non-dimensional form as,

$$\sigma_\theta=\left[\frac{T_o}{e}+\Omega_i^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]\frac{(1-C)}{(2-C)}R^{-\frac{1}{2-C}}e^{R^k} \quad (26)$$

$$\sigma_r=\left[\frac{T_o}{e}+\Omega_i^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2-C}}e^{R^k}-\frac{\Omega_i^2}{R}e^{R^k}\left[\int_{R_0}^{R}R^2e^{-R^k}dR\right] \quad (27)$$

$$\bar{u}=R-R\sqrt{1-\frac{2(1-C)Y}{E(2-C)}\frac{\Omega_i^2e^{R^k}}{R}\int_{R_0}^{R}R^2e^{-R^k}dR} \quad (28)$$

$$\Omega_i^2=\left[\frac{(2-C)R_0^{\frac{1}{2-C}}}{e^{R_o^k}}-\frac{T_0}{e}\right]\frac{1}{\int_{R_0}^{1}R^2e^{-R^k}dR} \quad (29)$$

Stresses, displacement and angular speed for fully-plastic state ($C\to 0$) are obtained from Eqs.(26)-(28) and (25) as

$$\sigma_\theta=\frac{1}{2}\left[\frac{T_o}{e}+\Omega_f^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2}}e^{R^k} \quad (30)$$

$$\sigma_r=\left[\frac{T_o}{e}+\Omega_f^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2}}e^{R^k}-\frac{\Omega_f^2}{R}e^{R^k}\left[\int_{R_0}^{R}R^2e^{-R^k}dR\right] \quad (31)$$

$$\bar{u}=R-R\sqrt{1-\frac{Y}{E}\frac{\Omega_f^2}{R}\int_{R_0}^{R}R^2e^{-R^k}dR\,e^{R^k}} \quad (32)$$

$$\Omega_f{}^2=\frac{\left[-2+T_o^*\right]}{e}\frac{1}{\int_{R_0}^{1}R^2e^{-R^k}dR} \quad (33)$$

NUMERICAL ILLUSTRATION AND DISCUSSION

In Fig. 1, curves are drawn between angular speed ($\Omega_i^2$) and various radii ratios $R_0 = (a/b)$ for different compressibility factors ($C = 0, 0.25, 0.5, 0.75$) and variable thickness ($k = 0, 2, 4$). It is observed that the rotating disc of incompressible material with an inclusion requires higher angular speed to yield at the internal surface as compared to the disc made of compressible material, and this behaviour remains the same with increase in edge load ($T_0 = 0.1, 0.2$). With the increase in edge load, the angular speed required for initial yielding decreases. From Table 1, it is seen that for isotropic compressible material, high percentage increase in angular speed is required to become fully plastic, as compared to rotating disk of incompressible material. In Figs. 2-5, curves are drawn between transitional stresses, displacement against the radii ratio. The parameters in Figs. 2-5 are: radii ratio ($R = r/b$), compressibility ($C = 0, 0.25, 0.5, 0.75$), variable thickness ($K = 0, 2, 4$), and edge load ($T_0 = 0, 0.1, 0.2$). The fully plastic stresses for various radii ratio ($R = r/b$) are shown in Fig. 6. From Fig. 6, it is observed that radial and circumferential stresses are maximal at the internal surface (for flat disc i.e. $k = 0$). With increase of thickness parameter ($k = 2, 4$), the circumferential stress is maximum at the external surface. With edge load, the behaviour remains the same. Similar graph is also obtained by Güven, /20/, for a rotating disc with a rigid inclusion.

CONCLUSION

It can be concluded that the disc made of isotropic compressible material is on the safer side of the design as compared to incompressible material, as it requires a higher percentage increase to become fully plastic from the initial yielding.

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Table 1. Angular speed required for initial yielding and fully plastic state with different edge loading (flat disc).
Tabela 1. Ugaona brzina za početak tečenja i stanja potpune plastičnosti sa različitim ivičnim opterećenjem (ravan disk)

| | C | $T_0 = 0$ | | Percentage increase in angular speed | $T_0 = 0.1$ | | Percentage increase in angular speed | $T_0 = 0.2$ | | Percentage increase in angular speed |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\Omega_i^2$ | $\Omega_f^2$ | | $\Omega_i^2$ | $\Omega_f^2$ | | $\Omega_i^2$ | $\Omega_f^2$ | |
| $R_0 = 0.5$ | 0 | 4.84 | 6.86 | 41.421 | 4.5059 | 6.5143 | 44.573 | 4.163 | 6.17 | 48.244 |
| | 0.25 | 4.04 | 6.86 | 69.828 | 3.6948 | 6.5143 | 76.308 | 3.352 | 6.17 | 84.112 |
| | 0.5 | 3.24 | 6.86 | 111.65 | 2.8969 | 6.5143 | 124.87 | 2.5541 | 6.17 | 141.63 |
| | 0.75 | 2.46 | 6.86 | 178.58 | 2.1186 | 6.5143 | 207.48 | 1.7758 | 6.17 | 247.53 |

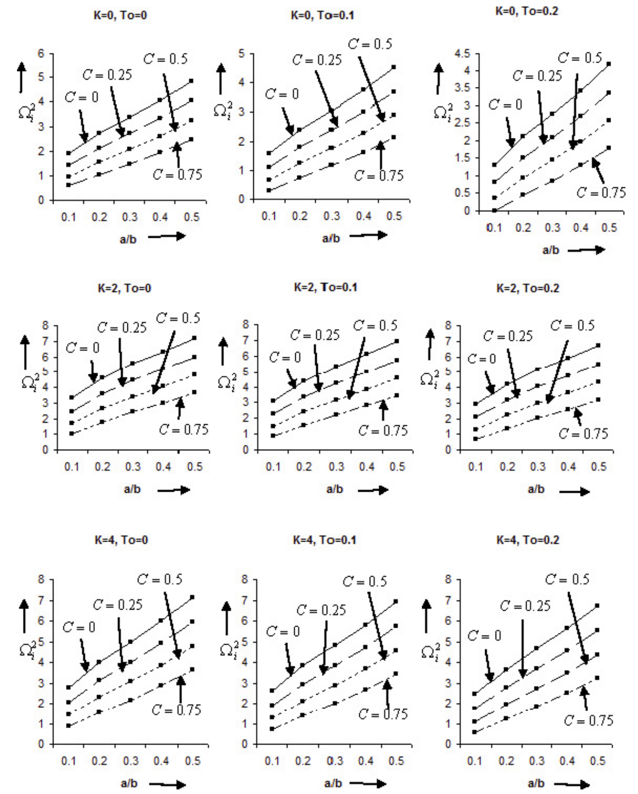


Figure 1. Angular speed for initial yielding at internal surface of rotating disc of variable thickness and edge loading.
Slika 1. Ugaona brzina za početak tečenja na unutrašnjoj površini rotirajućeg diska promenljive debljine i ivičnim opterećenjem

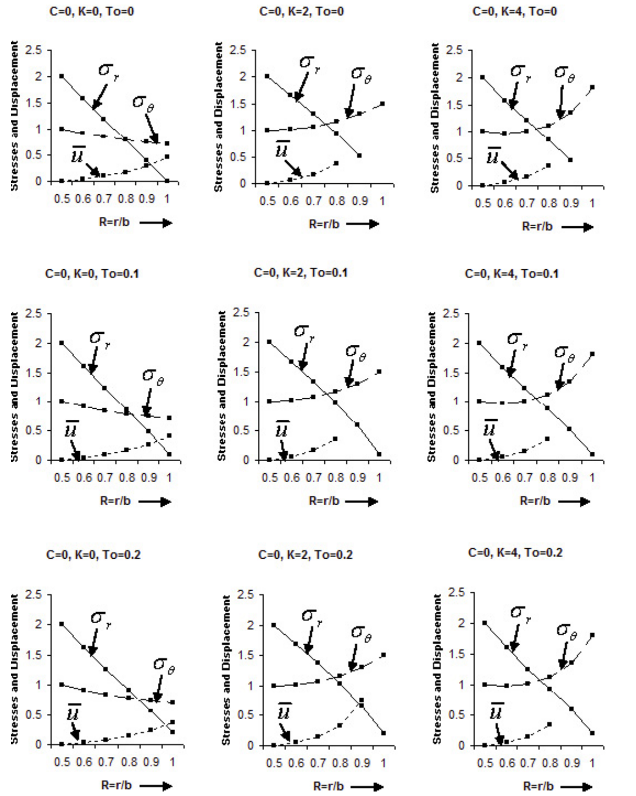


Figure 2. Transitional stresses and displacement of thin rotating disc along radii ratio and $C = 0$, for thickness and edge load.
Slika 2. Prelazni naponi i pomeranje za tanki rotirajući disk duž odnosa poluprečnika i $C = 0$, sa debljinom i ivičnim opterećenjem

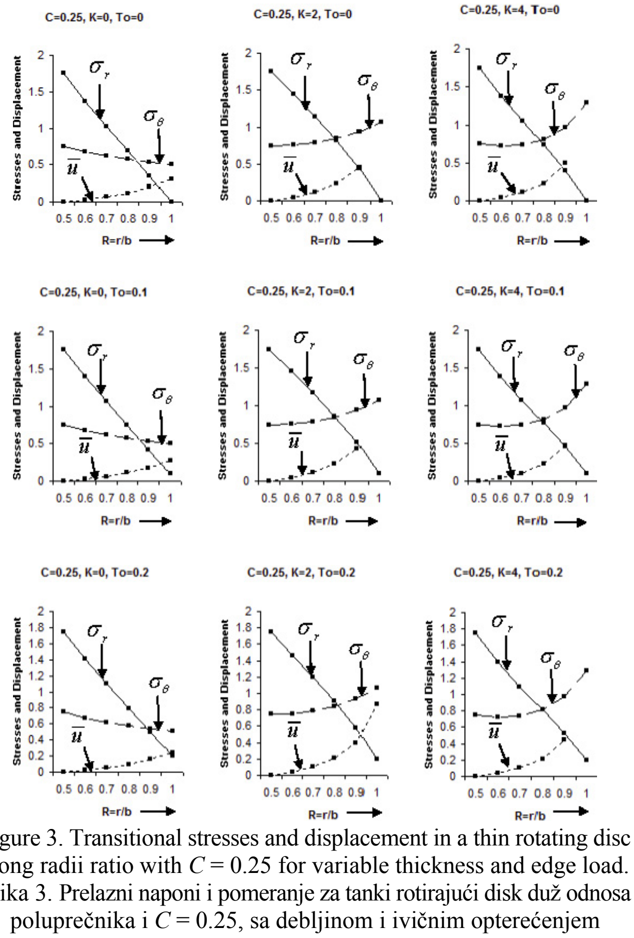


Figure 3. Transitional stresses and displacement in a thin rotating disc along radii ratio with $C = 0.25$ for variable thickness and edge load.
Slika 3. Prelazni naponi i pomeranje za tanki rotirajući disk duž odnosa poluprečnika i $C = 0.25$, sa debljinom i ivičnim opterećenjem

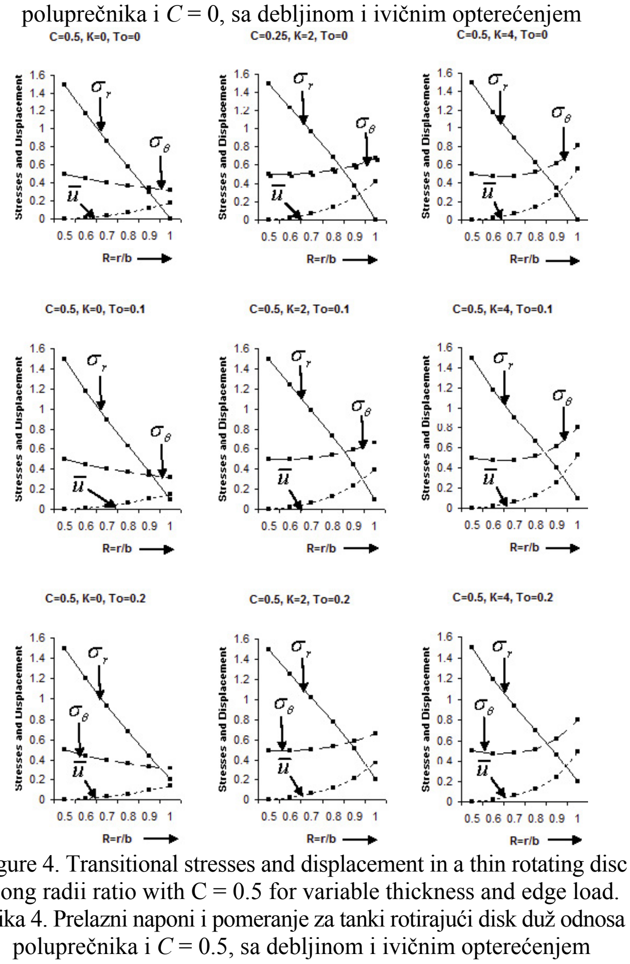


Figure 4. Transitional stresses and displacement in a thin rotating disc along radii ratio with C = 0.5 for variable thickness and edge load.
Slika 4. Prelazni naponi i pomeranje za tanki rotirajući disk duž odnosa poluprečnika i $C = 0.5$, sa debljinom i ivičnim opterećenjem

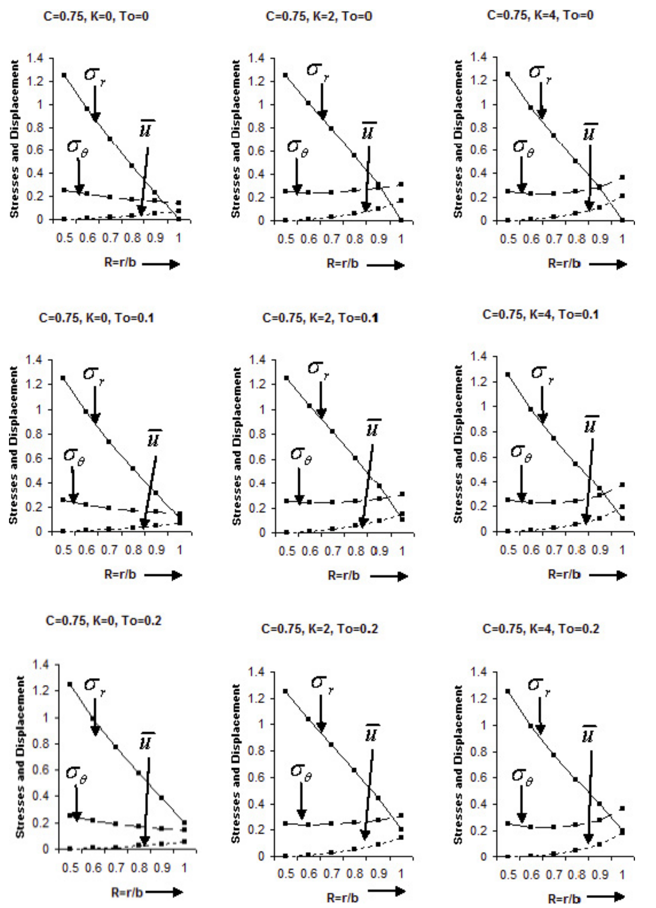


Figure 5. Transitional stresses and displacement in a thin rotating disc along radii ratio with $C = 0.75$ for variable thickness and edge load.
Slika 5. Prelazni naponi i pomeranje za tanki rotirajući disk duž odnosa poluprečnika i $C = 0.75$, sa debljinom i ivičnim opterećenjem

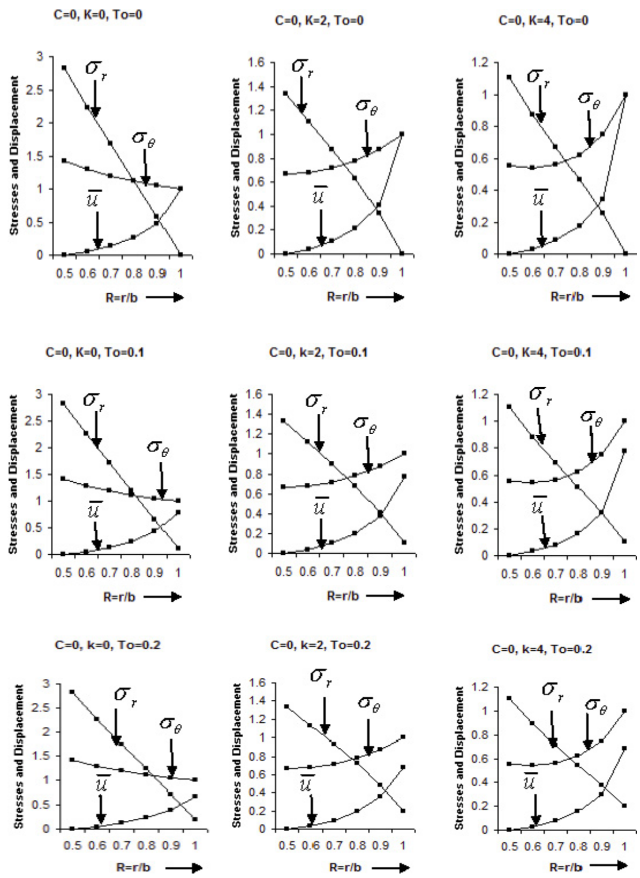


Figure 6. Transitional stresses and displacement in a thin rotating disc along radii ratio for variable thickness and edge load.
Slika 6. Prelazni naponi i pomeranje za tanki rotirajući disk duž odnosa poluprečnika sa debljinom i ivičnim opterećenjem

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