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FRACTURE CRITERIA: GLOBAL OR LOCAL?

KRITERIJUMI LOMA: GLOBALNI ILI LOKALNI?

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- constraint effects

Abstract

Three types of local fracture criterion are examined: RKR, Beremin and Volumetric method. It can be seen that these criteria are not material intrinsic but depend on constraint effects. To illustrate this problem, the following are shown: dependences of characteristic stress with stress triaxiality, Weibull stress with Q parameter, effective stress with T stress. It is concluded that the use of constraint parameters are necessary to take into account scale, geometric, loading mode and thickness effects on fracture.

Ključne reči
- lokalni kriterijum loma
- uticaji veze

Izvod

Proučavana su tri tipa lokalnih kriterijuma loma RKR, Beremin i Volumetrijska metoda. Pokazuje se da ovi kriterijumi nisu svojstveni materijalu već zavise od graničnih uticaja. Radi ilustracije problema, predstavljeno je sledeće: zavisnosti karakterističnog napona i troosnosti napona, Vejbulov napon i Q parametar, efektivni napon i T napon. Zaključuje se da je upotreba parametara veze neophodna za razmatranje kakav uticaj na lom imaju razmera, geometrija, tip opterećenja i debljina.

INTRODUCTION

Design against fracture or failure needs an accurate knowing of stress and strain distribution inside the considered structure or the component and a fracture criterion. A lot or fracture criteria are available which can be classified into several families. Some are based on stress, strain or energy. In each case, they can be divided in sub-families such as local and global criteria. Global criteria are widely used by engineers because they are connected to engineering parameters easily measurable such as gross stress or strain or work done for fracture. One can mention the critical stress intensity factor, the critical J integral, the critical gross strain or others. The most important difficulty in using such criteria is the fact that they are not intrinsic. Global fracture criteria are sensitive to geometry, scale effects, constraints, stress gradients etc. Local fracture criteria are based on an accurate description of stress-strain or strain energy density in a close volume to the defect promoting fracture. They are based on a local parameter in term of stress, strain or strain energy density. It is said that they overcome the above mentioned difficulties of the global fracture criterion but are they really independent of geometry and constraints.

Restricted to local stress fracture criteria, this paper presents the principles and discusses their intrinsic character.

CLASSIFICATION OF LOCAL STRESS FRACTURE CRITERIA

The first idea of a local stress fracture criterion has been suggested by Orowan, /1/. Local stress fracture criteria can be subdivided into three categories based on:
- an average stress over a characteristic length,
- a minimum stress over a characteristic length,
- a fictitious crack with a characteristic length.

Fracture criteria based on average stress over a characteristic length have been earlier considered by Neuber /2/, Novozhilov /3/ and other authors. It can be written in the following generalised form:

$$\frac{1}{X_c}\max_{-\pi<\theta\le\pi}\int_0^{X_c}\sigma_{\theta\theta(\rho)}d\rho=\sigma_c^* \qquad (1)$$

where X_c is characteristic length, σ_c^* characteristic stress, $\sigma_{\theta\theta}$ the circumferential stress, ρ and θ are polar coordinates.

Local fracture criterion based on a minimal stress over a characteristic length has been introduced by Whitney and Nuismer /4/:

$$\max_{-\pi<\theta\le\pi}\left[\min_{0\le\rho\le X_d}\sigma_{\theta\theta}(\rho)\right]=\sigma_c^{**} \qquad (2)$$

where X_d is another characteristic length, σ_c^{**} another characteristic stress. Criteria based on a fictitious crack were used by Waddoups et al. /5/, Cruse /6/, Caprino et al. /7/, and other authors. It is assumed that there exists a fictitious crack with a characteristic length X_e originating from the considered point y of the body. After some modifications this criterion may be represented in the following form:

$$\max_{-\pi<\theta\le\pi}\min_i K_{Ii}(y,\theta,X_e)=K_{Ic} \qquad (3)$$

where K_{Ii} is the local stress intensity factor, K_{Ic} the fracture toughness.

One notes that each of the above criteria includes two material parameters: a characteristic length and a strength parameter. These two parameters can be determined from two independent fracture tests.

RKR LOCAL FRACTURE CRITERION

Fracture tests made on notched specimens, Knott /8/, Wilshaw /9/, Tetelman et al. /10/, Griffith and Owen /11/, have shown that fracture occurs for a critical value of the stress distribution. However, the maximal stress overcomes the cleavage stress measured by extrapolation of yield stress at temperature 0 K. Rice /12/ has suggested the necessity that the stress distribution overcomes the cleavage stress over a characteristic distance X_C. This conclusion associated with the fact that the characteristic distance is connected with grain size is the basis of the Ritchie, Knott and Rice /13/ local stress fracture criterion (RKR). A picture illustrating this local fracture criterion is given in Fig. 1.

If the stress distribution at crack tip is described by the Hutchinson, Rice and Rosengreen Solution (HRR) /14/ and applying the RKR local stress fracture criterion, it has been demonstrated that the product of the fracture toughness K_{Ic} by yield stress σ_y at some power is constant. This property is valid at any temperature and particularly at 0 K. At this temperature, fracture toughness is equal to K_μ and the yield stress $\sigma_{y,0}$. One notes that at 0 K the yield stress is equal to the cleavage stress.

$$K_{Ic}\sigma_y^{(N-1/2)} = K_\mu \sigma_{y,0}^{(N-1/2)} \tag{4}$$

Validity of this relation is given in Fig. 2 where the logarithm of fracture toughness is plotted versus the logarithm of the yield stress for a nuclear pressure vessel steel 15H2NMFA. These data obtained by Krasovsky et al. /15/ lead to a minimum fracture toughness K_μ = 30.9 MPa√m and a cleavage stress σ_c^* = 1850 MPa. RKR local fracture criterion explains why high strength steels have lower fracture toughness than mild steel through relationship (4). This criterion is helpful to predict the shift of transition temperature with loading rate particularly with steel having yield stress at room temperature less than 1000 MPa.

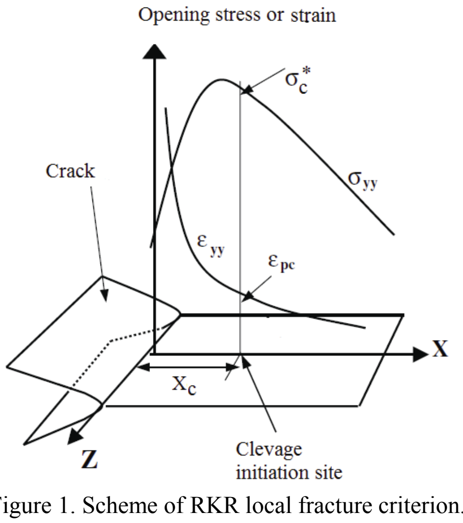

Figure 1. Scheme of RKR local fracture criterion.
Slika 1. Shema lokalnog kriterijuma loma RKR

For blunt notches, the maximum stress is often at a distance greater than the grain size. In this case, it is necessary to have a characteristic distance equal to 2 or 3 and sometimes 10 times the grain size. In this case the characteristic distance is not really a material characteristic. Cleavage stress is measured with notched specimens broken by bending or axisymmetric notched specimen loaded in tension until fracture. Cleavage stress is assumed to be equal to maximum stress. It has been seen that measured cleavage stress is sensitive to stress triaxiality.

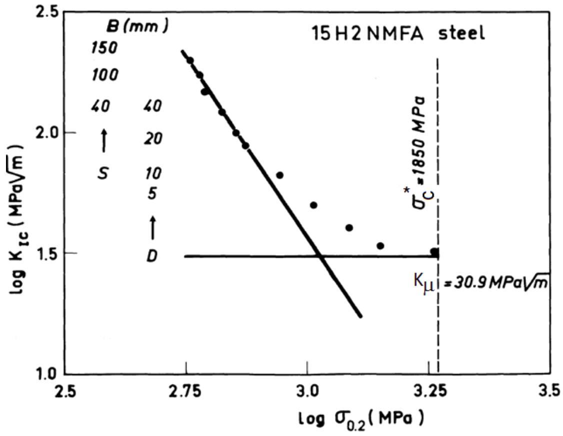

Figure 2. Evolution of fracture toughness with yield stress for a nuclear pressure vessel Soviet steel 15H2NMFA, /14/.
Slika 2. Razvoj žilavosti loma sa naponom tečenja za sovjetski čelik 15H2NMFA za nuklearne posude pod pritiskom, /14/

However the cleavage stress is not intrinsic to the material. It depends on loading mode, geometry and thickness trough the stress state. This stress state can be characterized by the stress triaxiality β which is the ratio of the hydrostatic stress σ_h and the Von Mises equivalent stress σ_{eq}.

$$\beta = \frac{\sigma_h}{\sigma_{eq}} \tag{5}$$

The effect of specimen size on the local cleavage fracture stress σ_c^* was investigated by G.Z. Wang et al. /16/ using notch flank angle Ψ = 45° single-notch and double-notch four-point pure bending specimens (4PB) with a fixed notch root radius θ of 0.25 mm and various thickness (W), width (B) and notch depth (a) (but the same $a = W$ value). Specimens were made in carbon manganese steel. Cleavage stress decreases 13% when specimen thickness increases from 12 to 16 mm (Fig. 3).

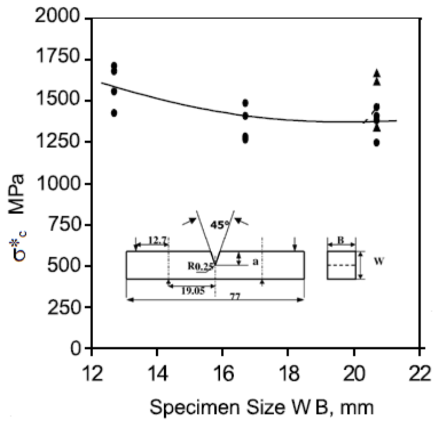

Figure 3. Influence of specimen geometry on cleavage stress for a C-Mn steel, /16/.
Slika 3. Uticaj geometrije epruvete na napon cepanja kod C-Mn čelika, /16/

BEREMIN LOCAL FRACTURE CRITERION

In this method /17/, the fracture process volume is the plastic zone volume. This plastic zone is divided into n elementary volumes V_i and one considers a small reference volume V_0, Fig. 4.

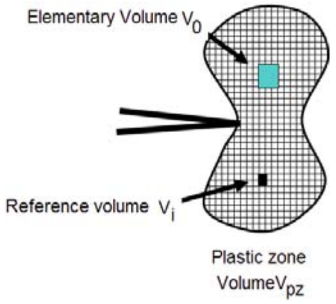

Figure 4. Elementary, reference and plastic zone volume in the Beremin method.
Slika 4. Elementarna, referentna i zapremina plastične zone kod metode Beremina

In each elementary volume, the stress state is assumed homogenous and equal to σ_i. Failure probability P_f is given by:

$$\ln(1-P_f)=-\sum_{i=1}^{n}\frac{V_i}{V_0}\left(\frac{\sigma_I^i}{\sigma_{no}}\right)^{m_w} \tag{6}$$

This relationship can be written as:

$$P_f=1-\exp\left[-\left(\frac{\sigma_w}{\sigma_{no}}\right)^{m_w}\right] \tag{7}$$

Where σ_w is the Weibull stress and m_w the Weibull modulus.

$$\sigma_w=\sqrt[m_w]{\sum_i(\sigma_I^i)^{m_w}\frac{V_i}{V_0}} \tag{8}$$

One notes that in the Beremin method the product of the Weibull stress at power m_w and the reference volume V_0 is constant

$$(\sigma_w)^{m_w}V_0=const. \tag{9}$$

The reference volume is chosen arbitrarily and this can lead to Weibull stress higher than the cleavage stress. For this reason, the Weibull stress has no physical meaning. Beremin model assumes that the cleavage fracture is controlled by the fracture process volume i.e. the plastic zone volume. However this plastic zone volume is controlled by constraint.

In Dodds-Anderson methodology /18/, crack tip stress distribution $\sigma_{yy}(x)$ is scaled to a reference solution: the small scale yielding conditions (ssy) and constraint is defined by the Q parameter:

$$Q=\frac{\sigma_{yy}-(\sigma_{yy})_{ssy}}{R_e} \tag{10}$$

Figure 5 shows the influence of the constraint parameter Q on fracture toughness J_{IC} of a low carbon-manganese cast steel through the ligament size.

According to Eq. (7), the Weibull stress depends on the Weibull modulus and decreases when m_w increases. It has been shown that the Weibull stress depends also on constraint (Fig. 6).

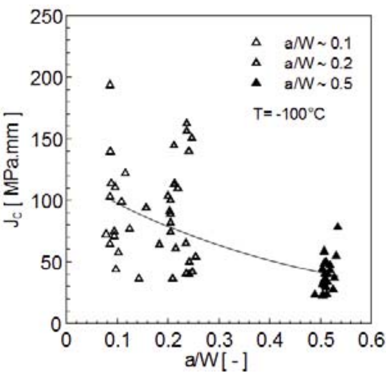

Figure 5. Influence of ligament size on fracture toughness J_{Ic} for a low carbon-manganese cast steel, /19/.
Slika 5. Uticaj veličine ligamenta na žilavost loma J_{Ic} kod niskolegiranog C-Mn čeličnog liva, /19/

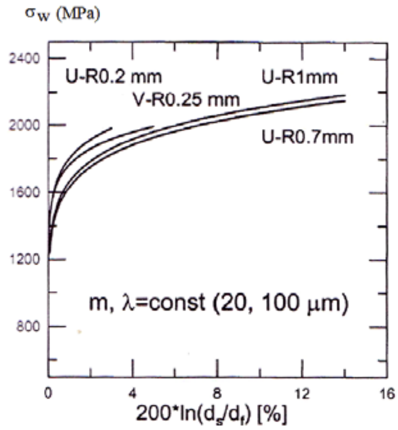

Figure 6. Influence of Weibull modulus and deformation level on Weibull stress, /20/.
Slika 6. Uticaj Vejbulovog modula i nivoa deformacije na Vejbulov napon, /20/

THE VOLUMETRIC METHOD

The volumetric method, /21/, is a local fracture criterion, that assumes that the fracture process requires a certain volume. This volume is assumed as a cylindrical volume with effective distance as its diameter. The physical meaning of this fracture process volume is "the high stressed region" where the necessary fracture energy release rate is stored. The difficulty is to find the limit of this "high stressed region". This limit is *a priori* not a material constant but depends on loading mode, structural geometry and load level. The size of the fracture process reduced to the effective distance, according to the above mentioned assumptions, is obtained by examination of the stress distribution.

The bi-logarithmic elastic-plastic stress distribution (Fig. 7) along the ligament exhibits three distinct zones that can be easily distinguished. The elasto-plastic stress primarily increases and it attains a peak value (zone I) then it gradually drops to the elastic-plastic regime (zone II). Zone III represents linear behaviour in the bi-logarithmic diagram. It has been proved by examination of fracture initiation sites that the effective distance corresponds to the beginning of zone III which is in fact an inflexion point on this bi-logarithmic stress distribution. A graphical method based on the relative stress gradient χ associated the effective distance to the minimum of χ. The relative stress gradient is given by:

$$\chi(r)=\frac{1}{\sigma_{yy}(r)}\frac{\partial\sigma_{yy}(r)}{\partial r} \qquad (11)$$

where $\chi(r)$ and $\sigma_{yy}(r)$ are the relative stress gradient and maximal principal stress or crack opening stress, in respect. The effective stress for fracture is then considered as the average volume of the stress distribution over the effective distance. However stresses are multiplied by a weight function in order to take into account the stress gradient due to geometry and loading mode.

$$\sigma_{eff}=\frac{1}{X_{eff}}\int_0^{X_{eff}}\sigma_{yy}(r)\times(1-r\times\chi(r))dr \qquad (12)$$

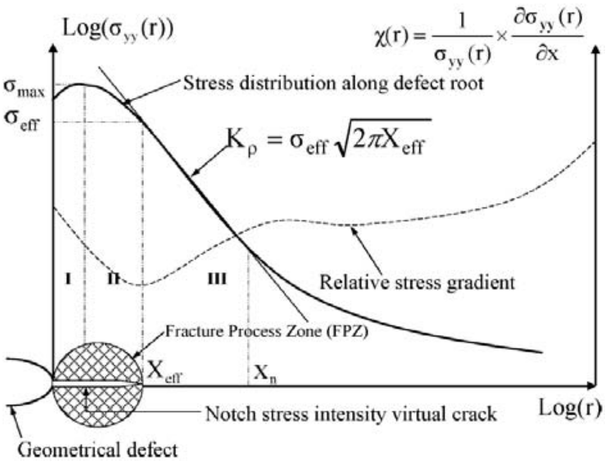

Figure 7. Principle of the Volumetric Method.
Slika 7. Princip volumetrijske metode

Similarly, the effective stress is also affected by constraint. In this case the constraint parameter is T stress defined by the second term which characterizes the stress distribution at the crack tip. The stress intensity factor is then given by the following formula:

$$K\approx\sigma_{ij}\sqrt{2\pi r}f_{ij}(\theta)-T\sqrt{2\pi r}\delta_{1i}\delta_{1j} \qquad (13)$$

where σ_{ij} are the components of the stress tensor, r and θ are polar coordinates and f_{ij} is an angular function. Numerical results obtained by Hadj Méliani et al. /22/ on a pipe submitted to internal pressure shows a linear decrease of effective stress with T stress, Fig. 8.

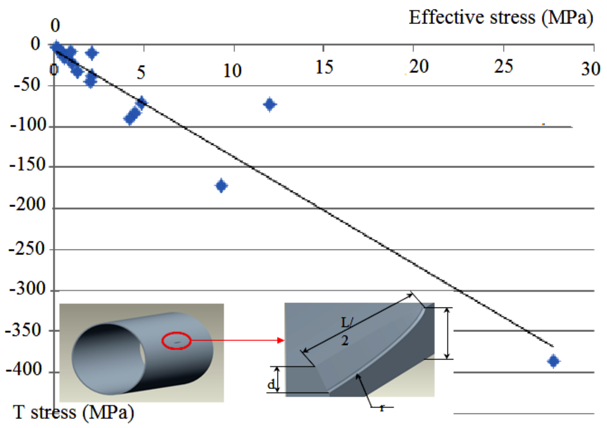

Figure 8. Effective vs. T stress for pipe with internal pressure, /22/.
Slika 8. Efektivni prema T naponu za cev pod pritiskom, /22/

CONCLUSION

The local fracture criteria as the global increase their accuracy by taking into account the constraint effect. This effect can be represented by stress triaxiality, Q parameter or T stress. Generally, local fracture criteria need two parameters: effective or characteristic stress and distance. Both parameters are not intrinsic to the material and sensitive to the constraint effect. By taking this effect into account, one can represent the scale, geometric, loading mode, and thickness effects.

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