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STRUCTURAL INTEGRITY ANALYSIS OF MULTI-BOLTED CONNECTIONS USING THE INNOVATIVE BEAM MODEL

ANALIZA INTEGRITETA KONSTRUKCIJSKOG SPOJA SA VIŠE VIJAKA PRIMENOM POBOLJŠANOG MODELA GREDE

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Abstract

Some available analyses of bolted joint behaviour under axial load usually begin from a single bolt connection, loaded by a force introduced near the bolt axis. The applied axial loads act in this way rarely or not at all. The disadvantage of these simplified calculations is that they produce a linear dependence of additional stresses (caused by tensile load and bending moment) on external load, thus overestimating them at low loads, and neglecting the danger induced at higher loads by such an approach. This can be avoided using a beam model. The solution of this model is based on solid mechanics, allowing analytical calculation of the joint. Although the presented model corresponds essentially to models previously developed by the author, some improvements and changes are introduced, that also contribute to finer final results. The beam model has enabled to achieve more reasonable treatment of many important behavioural characteristics and problems in the analysis of bolted joints, such as: response of multi-bolted connections, effect of preload level on the reduction of additional loads in the connection, accurate determination of the eccentricity of the external force. It also allows necessary correction of important effects, i.e. substituting the position of the force by introducing a more relevant shearing resilience of the connected parts and the lever principle for accurate edge effects. It is expected that the uniform use of the beam model will help to optimize the structural safety, and reduce the time and cost of developing structures and mechanical systems.

Izvod

Dostupne analize ponašanja vijčanih spojeva pri aksijalnom opterećenju obično polaze od spoja sa jednim vijkom, opterećenim silom koja deluje u blizini ose vijka. Realna aksijalna opterećenja retko ili nikada ne deluju na taj način. Nedostatak tako uprošćenih proračuna se ogleda u tome što ona uvode linearnu zavisnost dodatnih napona (unetih aksijalnim opterećenjem ili momentom savijanja) od spoljnjeg opterećenja, preuveličavajući ih pri malim opterećenjima, a zanemarujući takvim pristupom unetu opasnost pri velikim opterećenjima. Ovo se može izbeći primenom modela grede. Rešenje ovog modela se zasniva na mehanici čvrstog tela, čime je omogućen analitički proračun spoja. Iako prikazani model u osnovi odgovara modelima koje je autor ranije uveo, uneta su neka poboljšanja i promene, koje takođe doprinose tačnijim konačnim rezultatima. Model grede omogućava mnogo pogodniju obradu značajnih karakteristika ponašanja i problema u analizi vijčanih spojeva, kao što su spojevi sa više vijaka, uticaj nivoa predopterećenja na smanjenje dodatnih opterećenja u spoju, precizno određivanje ekscentričnosti spoljnje sile. Model dopušta i potrebne korekcije značajnih uticaja, npr. zamenu položaja sile uvođenjem merodavnije otpornosti na smicanje spojenih delova i princip poluge kod preciznih uticaja ivice. Očekuje se da jednoznačna upotreba modela grede može pomoći u postizanju optimalne sigurnosti konstrukcije i u smanjenju vremena i troškova u procesu razvoja konstrukcija i mehaničkih sistema.

INTRODUCTION

As in the case of many structural groups composed of mechanical parts, the benefits of research results are not fully applied to justify the importance of bolt connections. The designer usually applies only approximate calculation, with high safety margins, mistakenly expecting that the size of the connection will protect it from any risk of failure. While that may work at static loads, with fatigue it may lead more to critical situations and failure.

The simple analysis of the behaviour of the bolted joint under axial loading usually begins from a connection with a single bolt, loaded by a force introduced in the bolt axis. This is a useful simplification, because it results in a linear behaviour and leads to a simple analytical calculation. Applied axial loads act rarely, or never, in this way. More commonly, the bolt connection is performed by several bolts so that the total external loads are acting outside of the bolt axis. In typical components in machines and structures the multi-bolted connection is the basic form of connection. This type of connection, through multiple nonlinear behaviour, depends on many factors, complex in nature. This often leads to the drastic increase of tension and bending loads of the bolt through acting external force. However, the behaviour of multi-bolted joints is not as well understood as the behaviour of joint with only one bolt and for that, much more efforts are needed to create an effective tool for the designer.

The existing methods are limited mostly to linear analysis. Except for bolt connections, in most design problems, linear analysis offers a reasonable approximation of real properties. Due to the complicated problem formulation and long-term solutions, designers have used the non-linear analysis in the past only unwillingly. This behaviour is changing now, as the powerful desktop computer and appropriate software are available.

In addition, the currently best-known analytical method of design of bolted joints, VDI /1/, has been developed only for connections with one bolt. In the case of highly eccentric load, the linear model of VDI 2230 is no longer applicable. It follows that the VDI model is not directly transferable to multi-bolted connections. In his thesis, Massol confirmed similar findings /2/. The disadvantage is that the VDI model gives a linear variation of additional stresses (tensile load and bending moment) as a dependence on external load and overestimates these for low loads, while at higher loading, misjudges how they are dangerous.

Because of this, the structure of a highly loaded bolted connection reaches very often the limits, especially when an "oversizing" due to economic, weight limits, and other reasons, is not a viable option.

Additionally, for a calculation of the connection, the fastening parts are not only in question, but also the connected components. The adequate function of a bolted connection is dependent on all parts in the assembly. This means that parts cannot be designed independently. The parts such as flange, cap, lid, are by the eccentric loading of the connection, exposed to high bending stresses. Then how should they be calculated within the VDI procedure?

Perhaps the most significant shortcoming of such methods is that they reduce the function of the connection to only two controlling parameters: compliance of parts in the direction of the bolt axis, and the guaranteed sufficient minimal preloading. All other influencing variables, that will be treated here, remain practically ignored.

Numerous experimental studies have clearly shown that the level of preloading and bending stiffness of parts in connection are the two most important parameters. Firstly proposed by Agatonovic /3, 4/, the beam model, based on basic formulas of solid mechanics, takes into account these two parameters. The effect of these parameters is crucial for variable loading and fatigue. Using the beam model, the bolted joint can be considered accounting for nonlinear behaviour and the conditions for adequate function in a more general way. This offers safe and conservative methods in the design of bolted connections.

However, the beam model of the bolted connection is, as models in general are, one simplification of the real connection behaviour that allows the calculation of stresses and deformations of the connection. It covers the case of the pre-stressed bolt connection exposed to eccentric external loading. The analytical solution of the model is sufficiently summarized in /3-7/ several years ago.

Based on typical structure forms of multiple bolt connections and the complexity of the problem, the approach that allows the development and implementation of each individual module for the solution is necessary. The implementation of a module based on the beam model of the bolted joint, in a multi-body system must be done in a modular and easy to implement systematic way. The method must be suitable for both static and dynamic problems. With the development of computer technology the mastery of these problems in applied economy and programming technology has become much clearer today.

Currently, the FEM (Finite Element Method) is a very extended technique that can be used to solve almost all design problems. However, for their appropriate use, it is more than ever necessary to understand how to model the problem. The required analytical mind is the prerequisite for obtaining reliable and useful results, so analytical solutions can be most easily obtained for simple geometries. Therefore, the analytical technique can be applied to problems that can be approximated by analytical body. Nevertheless, also on hand of such more or less exact solution for a less exact geometry the information can be gained that is very useful for the designer.

Consequently, the two analytical and numerical methods are not competitive to each other, but complementary. In addition, concerning the support of the modern computer technology, it is clear that both benefit enormously.

BEAM MODEL OF THE BOLTED JOINT

The basic idea of the model is shown in Fig. 1. Although this model corresponds essentially to previous models /3, 7/. Based on application experiences some improvements and changes are introduced that also play a role regarding the results. Details regarding this will be explained later.

It is assumed that each multi-bolted connection can be broken down into individual connections, that consist of plate segments or beams and a connecting element pair.

Former calculation methods for bolted joints indicated that the determination of the eccentricity is risky if the bolt connection is treated as disconnected from the rest of the structure (plate, flange). At the clamping position, the connection to the rest of the structure (Fig. 1), a bending moment must be considered in addition to the force.

$$F_B \cdot a - M_A = F_K \cdot s_K \tag{1}$$

For the analytical treatment of the model, the following assumptions are made:
- The influence of the bolt hole on the bending rigidity of the parts is neglected.
- Both bending stiffness and compressive stiffness of the assembled parts remain constant under load change.
- All loads in the joint are implied as concentrated forces.
- Shear deformations of the model beam are not considered.

Under eccentric external load the reaction force shifts with increasing load from the position of the connection axis, balancing at the same time the moment caused by external force. Determination of the clamping force eccentricity s_K is very important in the evaluation of the design. When approaching the edge of the beam, the additional forces increase excessively and this should be avoided to maintain structural integrity.

Usually, during assembly of the connection, both forces bolt preload F_V and reaction force in the separation surface F_K act along the bolt axis (Fig. 2a). The clamped parts are pressed together where the counting compliance is δ_P. Under the external load this effect separates (Fig. 2b), so that the compliance is divided into the effect based on bolt force and effect based on the force in the separation surface.

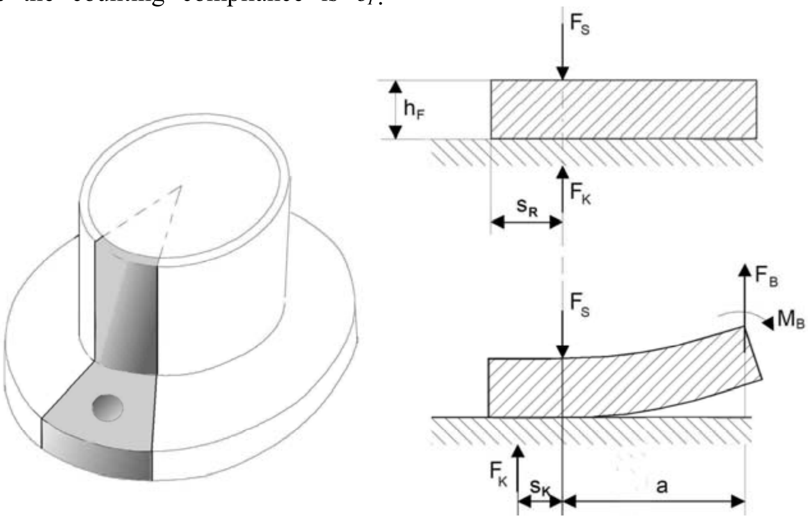


Figure 1. Beam model of the bolt connection.
Slika 1. Model grede za vijčani spoj

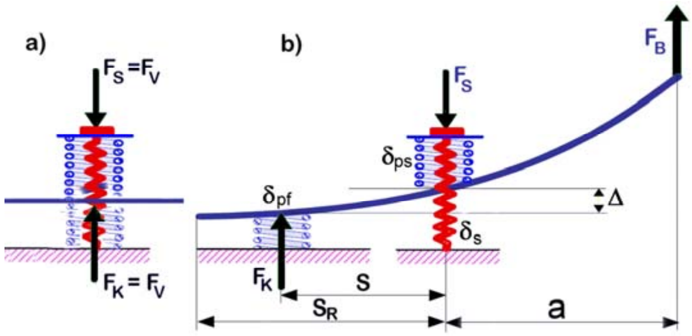


Figure 2. Compliances in connection under loading.
Slika 2. Popustljivost u spoju pod opterećenjem

$$\delta_P = \delta_{PS} + \delta_{PF} \tag{2}$$

It is very important to not confuse this effect with the position of the external force introduction along bolt axis.

By determining the division /2/, it should be taken into account that the loading surface on the bolt side is much smaller than at the interface (therefore $\delta_{PS} \geq 2\delta_{PF}$).

The part of the plate between the two positions is modelled as part of the beam that expands, however, onto the remaining structure. The differential equation of the beam has the following form:

$$EJ\frac{d^2y}{dx^2} = F_K x - F_S(x - s_K) \tag{3}$$

This equation applies to the two fields of the beam, if the expression in brackets, if not greater than zero, is not taken into consideration. The integration yields:

$$F_K\frac{x^2}{2} - F_S\frac{(x-s_K)^2}{2} + C_1 \tag{4}$$

$$EJ \cdot y = F_K\frac{x^3}{6} - F_S\frac{(x-s_K)^3}{6} + C \tag{5}$$

At the location of the clamping force, a yet unknown deflection angle α_0 of the beam develops

$$x = 0\,;\quad \frac{dy}{dx} = \alpha_0 = \frac{1}{EJ}C_1\,;\quad C_1 = \alpha_0 EJ$$

The beam can bend at this point only to the extent allowed by the elastic flexibility in the joint (Fig. 2). So,

$$x = 0\,;\quad y = -F_K\delta_{pf}\,;\quad C_2 = -EJF_K\delta_{pf}$$

Under the bolt ($x = s_K$) the deflection of the beam equals the amount between initial deformation after preloading of the connection and the bolt extension by the additional force FSA:

$$x = s_K\,;$$

$$y = (F_S - F_V)(\delta_S + \delta_{PS}) - F_V\delta_{pf} = F_S(\delta_S + \delta_{PS}) - f$$

Or, respectively, after the introduction of the equilibrium condition: $F_S = F_K + F_A$

$$F_K(\delta_S + \delta_{PS}) + F_A(\delta_S + \delta_{PS}) - F_V\delta = \frac{1}{EJ}\left[\frac{F_K s_K^3}{6} + \alpha_0 EJs_K - EJF_K\delta_{pf}\right] \tag{6}$$

Solving for F_K this yields:

$$F_K = \frac{F_V\delta - F_A(\delta_S+\delta_{PS})+\alpha_0 s_K}{\delta_S+\delta_{PS}+\delta_{PF}-\dfrac{s_K^3}{6EJ}} = \frac{\left(\dfrac{F_V}{F_A}-\dfrac{\delta_S+\delta_{PS}}{\delta}\right)F_A+\dfrac{\alpha_0 s_K}{\delta}}{1-\dfrac{s_K^3}{6EJ\delta}} \quad (7)$$

The expression in the first parentheses is based on important relationships that determine the load conditions in a connection: between preload and working load and the ratio of the stiffness in the joint. After inserting

$$\delta_S+\delta_{PS}=\delta_{SE} \quad \text{and} \quad \Phi=\frac{F_V}{F_A}-\frac{\delta_{SE}}{\delta}$$

the relationship (7) simplifies in:

$$F_K=\frac{\Phi F_A+\dfrac{\alpha_0 s_K}{\delta}}{1-\dfrac{s_K^3}{6EJ\delta}} \quad (7')$$

The conditions at the connection of the model to the rest of the structure ($x = s_K + a$) depend on the force relationships at this point and can be written in general as:

$$\alpha=\sum\beta_i L_i=\beta_F F_A+\beta_M M_A+\beta_F p\ldots \quad (8)$$

We consider the vicinity of the connection, which is in equilibrium under the influence of external forces L_i, whereas by "force" also a moment (M_A) or pressure (p) is to be understood. Assuming linear elastic behaviour, the rotation angle of the connection is determined by superposition of rotating parts originating from individual load components.

The angular position of the beam (see /2/) is for $x = s_K + a$:

$$x=0$$

$$\frac{dy}{dx}=\frac{1}{EJ}\left[F_K\frac{(s_K+a)^2}{2}-F_S\frac{a^2}{2}+\alpha_0 EJ\right]$$

and because $F_S = F_K + F_A$

$$\frac{dy}{dx}=\frac{1}{2EJ}\left[F_K(s_K^2+2s_K a)-F_A a^2\right]+\alpha_0 \quad (9)$$

The introduction of relation (8) in equation (9) results in:

$$\beta_F F_A+\beta_M M_A+\beta_p p= =\frac{1}{2EJ}\left[F_K(s_K^2+2s_K a)-F_A a^2\right]+\alpha_0$$

The solution of this equation for M_A is

$$M_A=\frac{1}{2EJ\beta_M}\left[F_K(s_K^2+2s_K a)-F_A a^2\right]+ +\frac{\alpha_0}{\beta_M}-\frac{\beta_F}{\beta_M}F_A-\frac{\beta_p}{\beta_M}p \quad (10)$$

and leads, after consideration of /1/ the simplifications in the form of influence numbers:

$$b_M=2EJ\beta_M\,;\quad b_F=2EJ\frac{\beta_F}{a}\,;\quad b_p=2EJ\frac{\beta_p}{a}$$

to the second relationship for the clamping force

$$F_K=\frac{\left(1+\dfrac{a}{b_M}+\dfrac{b_F}{b_M}+\dfrac{b_p}{b_M}\dfrac{1}{F_A}\right)aF_A-\dfrac{\alpha_0 2EJ}{b_M}}{s_K+\dfrac{s_K}{b_M}(s_K+2a)}$$

that after multiplication by

$$F_K=\frac{\left(a+b_M+b_F+b_p\dfrac{1}{F_A}\right)aF_A-2\alpha_0 EJ}{s_K(s_K+2a+b_M)}$$

and simplification

$$B=(a+b_M+b_F+b_p)$$

can be written

$$F_K=\frac{BaF_A-2\alpha_0 EJ}{s_K(b_M+2a+s_K)} \quad (10')$$

The system is uniform after equating the two expressions (7') and (10') for clamping force:

$$\frac{BaF_A-2EJ\alpha_0}{s_K(2a+s_K+b_M)}=F_K=\frac{\Phi F_A+\alpha_0\dfrac{s_K}{\delta}}{1-\dfrac{s_K^3}{6EJ\delta}}$$

so that the rotation of the beam can be determined at the point of clamping force:

$$\alpha_0=\frac{\dfrac{Ba}{s_K(2a+s_K+b_M)}-\dfrac{\Phi}{1-\dfrac{s_K^3}{6EJ\delta}}}{\dfrac{\dfrac{s_K}{\delta}}{1-\dfrac{s_K^3}{6EJ\delta}}+\dfrac{2EJ}{s_K(2a+s_K+b_M)}}F_A \quad (11)$$

or writing this in a more general way

$$\alpha_0=\xi(s_K)F_A \quad (11')$$

The simplest condition for a solution is met when the position of the clamping force is far from the edge so that no edge effects are expected. Under these conditions, the pressure distribution in the interface, based on FEM calculations (Fig. 3), is approximately symmetric so that it can be assumed that the position of the resulting force corresponds to the maximum where the angle α_0 equals zero. It follows:

$$\frac{Ba}{s_K(2a+s_K+b_M)}=\frac{\Phi}{1-\dfrac{s_K^3}{6EJ\delta}}$$

and after arranging in accords to s_K

$$\frac{s_K^3}{6EJ\delta}+\frac{\Phi}{Ba}s_K^2+\frac{\Phi}{Ba}(b_M+2a)s_K-1=0 \quad (12)$$

and introducing $x=\dfrac{s_K}{a}$ as a new unknown, the characteristic equation of the connection is obtained as

$$C_1x^3+C_2x^2+C_3x+C_4=0 \quad (13)$$

Here the constants are

$$C_1 = \frac{a^3}{6EJ\delta};\ C_2 = \frac{\Phi a}{B};\ C_3 = \frac{\Phi(b_M + 2a)}{B};\ C_4 = -1$$

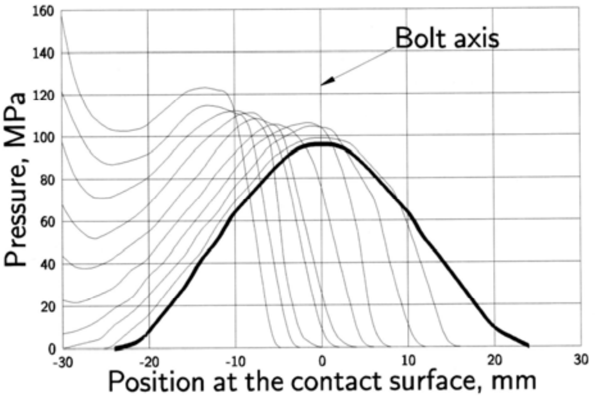


Figure 3. Pressure distribution in the separation surface (FEM analysis).
Slika 3. Raspodela pritiska na površini razdvajanja (analiza MKE)

A reasonable solution to Eq. (13) presumes that the s_K distance is not too close to the edge. Thereafter the forces in the connection can be determined starting with the clamping force (7'). If the position of the clamping force to the edge is so close that the pressure distribution in the joint is not symmetric, the beam tilts at the edge and the conditions $\alpha_0 = 0$ become increasingly inaccurate. The so-called lever principle must not be valid for this case, because the clamping is not free and when turning around the edge, it can not happen without the influence of the restraint of the remaining structure.

The lever principle is in its primitive form when applied to multi-bolted connection, just a rough simplification, that is also on the unsafe side, and therefore really should not be used. The procedures based on this assumption ("circle arc", /1/) are also vague and on the unsafe side. Moreover, the characteristic Eq. (12) clearly shows that the forces do not track changes in a circular arc, but in a parabolic curve.

Nevertheless, a reasonable solution of the system is still possible. Putting in the relationship (11) for α_0 as an effective clamping force eccentricity the value that edge approaches (for example $0.8S_R$) α_0 may be determined. Adopting this value in (10') results in an F_K evaluation that can be used in the determination of other forces in the connection.

Shifting the bolt axis position and approaching the edge has an additional effect – the reduction of the effective preloading force due to the additional embedding at the new loaded separation surfaces caused by the change in the position of the clamping force. For the present, the effective reduction of the initial preload can be approximated by the following relationship:

$$\Delta F_V = \frac{f_x}{\delta} \tanh\left(\frac{1}{\Phi}\right) \quad (14)$$

Therefore, the bolt additional force is to be calculated based on

$$F_Z = F_K + F_A - F_V + \Delta F_V \quad (15)$$

Analysis of results

An important feature of connection in function is the possible disconnection at separation surface. The shifting of the clamping force axis from the bolt position can cause the separation in the joint already at very low operating forces and then slowly rises, reaching in short the bolt holes. The gaping in the connection on this site may endanger the function (sealing, increase in load, fretting). The analytical determination of the lift-off at bolt hole can be derived from the condition of the deformation at this point:

$$y = \frac{1}{EJ}\left[\frac{F_K s_K^3}{6} - EJF_K\delta_{pf}\right] = F_K\delta_{pf}$$

$$F_K s_K^3 = 12EJF_K\delta_{pf} \quad (16)$$

$$s_K = \sqrt[3]{12EJ\delta_{pf}}$$

Compliance of the clamped parts is determined for the condition of the assembly (preload), which means that the forces work in the same axis against each other on both sides of the plate. Its dividing according to (2) requires that the sum of both components remains unchanged. In this respect, the proved data are lacking. However, the influence of the plate flexibility is noticeable only at the lowest loads and becomes negligible with the increase of the bending deformation in the connection.

One of the most important influences is likely based on the effect of shear deformations on the connection. It is well known, that for very short beams behind the bending deformation, shear deformations should be considered. The prerequisite is, however, the case of so-called parallel shear stress that arises when between two surfaces (Fig. 4a) under the action of two opposing forces, the relative movement (sliding) occurs, or may occur. In the presented model, resulting shear forces and deformations develop only due to the non-uniform distribution of pressure, which gradually changes along the beam (Fig. 4b). With this type of *pinch shear stress***,** its intensity is proportional to the gradient of the pressure distribution and not to the magnitude of the lateral force.

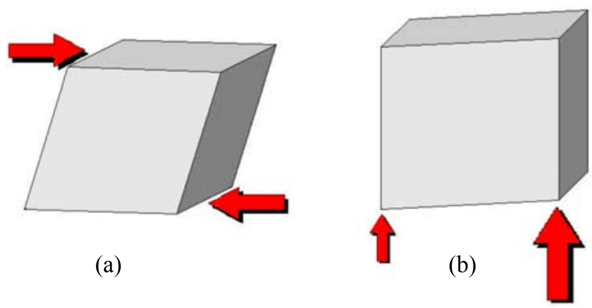


Figure 4. Shear forces effect on deformations.
Slika 4. Uticaj sile smicanja na deformaciju

Overall, shear deformations occur in the area around the bolt already during preloading (assembly) of the connection. As part of the compliance of clamped parts, they are also considered. For that purpose the shear stress distribution under the action of the pressure load (bolt head) is very informative, as presented in Fig. 5.

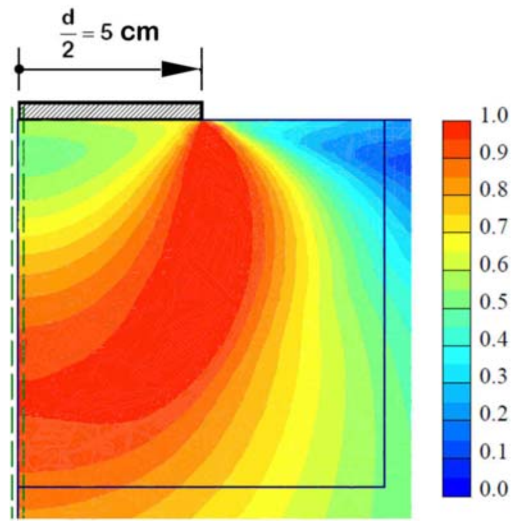


Figure 5. Shear stress distribution under compression load, /10/.
Slika 5. Raspodela napona smicanja od pritisnog opterećenja, /10/

Under the influence of the operating load, redistribution of shear deformations starts first. The force in the bolt and clamping force are distributed loads, that are overlapped. After assembling, these two forces are in balance. This means the resulting mean shear is zero. Under operating load, after shifting of the clamping force, it does not mean that in the "*s*" field the resulting shear forces will immediately achieve the amount of clamping force. The shear force in the beam grows together with *s* very slowly from zero to maximal values that will almost never reach the level of the clamping force. Consequently, additional shear deformations with the smallest *s* are also very small and cannot have a significant influence.

The level of the external moment load ($F_B a \pm M_B$) will not be affected in this way, so that its effect to the connection parts remains unchanged. Fixed clamping does not act at the position of connection. Inclusion of the bending deformation of the clamped parts, which are modelled here as a beam, is necessary to guarantee the balance with the moment loading. An assumption $\alpha = 0$ at the position of the resulting clamping force is coupled with the bending line of the beam. It corresponds to the typical pressure distribution in the joint (Fig. 3).

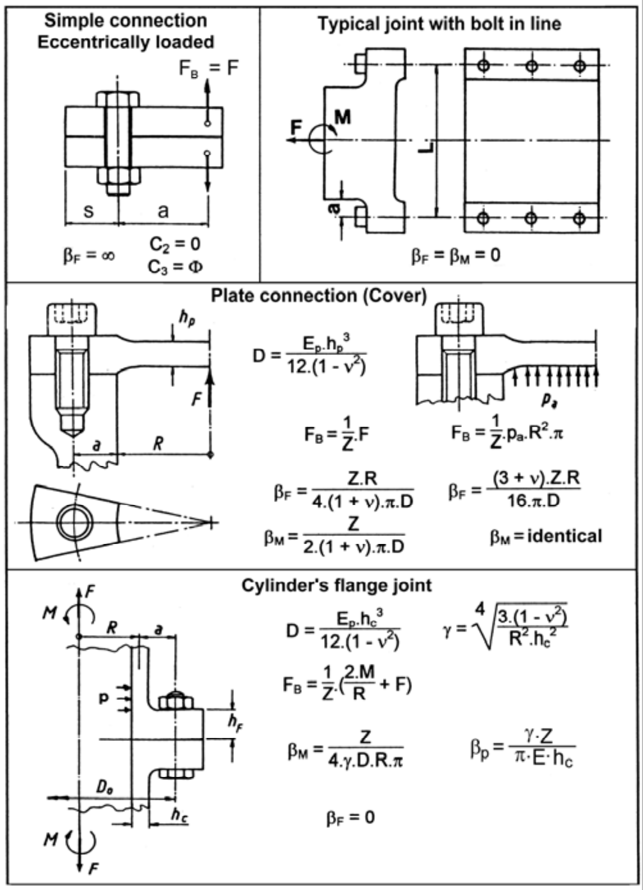
Experience has shown that the bending deformation generally corresponds with great accuracy to the real situation, while theoretical calculations for shear deformations are less accurate. For that we consider our model here as a *shear-stiff beam* (Bernoulli theory). This assumption is also on the safe side, because therewith, the overall stiffness of the connection is higher and accordingly, the loads.

APPLICATION TO MULTI-THREADED CONNECTIONS

Determination of influence coefficients

The influence numbers for a series of typical forms of connection can be summarized in tabular form (Table 1). For the connection listed in Table 1 the calculation of the connection or the estimation of values is simplified. The joining stiffness can also be found by a FEM calculation. The significance of this possibility is underestimated. Compared to usual finite element calculations with inappropriate boundary conditions (at the joining) and linear behaviour, a combined analysis delivers more exact information about the effect of preloading of the connection, eccentricity of the force introduction and the separation of interface surfaces. Failure to consider nonlinear effects, however, particular in a finite element analysis, can lead to inaccurate results.

Table 1. Influence numbers for some typical forms of connections.
Tabela 1. Uticajni koeficijenti za neke tipične oblike spojeva



The physical significance of influence coefficients b_i can be shown in the example of a bolted round plate (cover). In the relationship for bending moment

$$b_M = 2EJ\beta_M$$

in account of the number of bolts, the known conditions are introduced

$$\beta_M = \frac{z}{2(1+\nu)\pi D}$$

$$\text{with } D = \frac{EH_p^3}{12(1-\nu^2)} \text{ and } J = \frac{1}{12}\frac{\pi D_0}{z}H_F^3$$

the following relationship is obtained

$$b_M = (1-\nu)D_0\left(\frac{H_F}{H_P}\right)^3 \qquad (17)$$

showing that this figure is based on the influence of pitch diameter, corrected by the ratio of plate and flange thickness.

Similarly, the following expression is produced

$$b_F = \frac{(1-\nu)}{2}\frac{D_0 R}{a}\left(\frac{H_F}{H_P}\right)^3 \quad (18)$$

This shows that b_F includes once more the correction by *R/a* ratio.

An important task for all multi-bolted connections is to determine the load distribution onto the individual joint. In the simplest cases, such as disk or cylinder flange, for the overall structure loaded by bending moment M_B and a total axial force F_B (assuming the linear distribution of normal stress, undisturbed by the connection area) for the highest loaded bolted joint segment, the nominal operating force F_A can be determined from the following formula:

$$F_{A\max} = \frac{1}{Z}\left(\frac{2M_B}{R} + F_B\right) \quad (19)$$

For all segments, the stiffening effect due to the curvature of the flange and the weakening effect by the bolt hole are neglected. The resulting errors are usually small. The two effects cancel out each other.

Examples of calculation

The manual calculations of bolted joint based on the derived relationships can be performed in a simple way using appropriate software. Calculations here are carried out using the programme *MathCAD.*

A typical result for the simplest connection of the free beam is shown in Fig. 6. Comparison of results with FEM analysis shows that the transition to, for this case valid, the lever principle, has a parabolic character. However, such a relative abrupt transition is only true if the edge distance is too small, so that the maximal value for clamping force distance cannot be achieved ($s_R < s_{\max}$). Here arises the dilemma for the designer, whether he should avoid the load levels within the transition or to increase the edge distance.

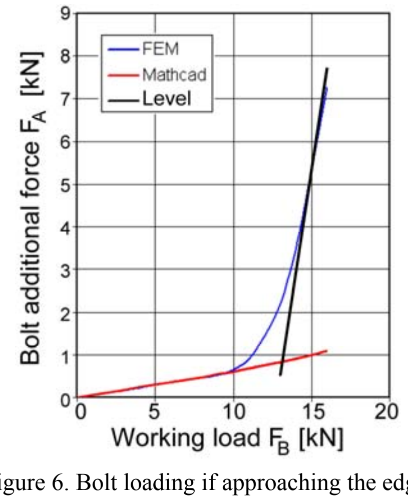


Figure 6. Bolt loading if approaching the edge.
Slika 6. Opterećenje vijka približavajući se ivici

The so-called lever principle is not applicable to a connecting segment that is attached to a structure because at this position, in addition to the operational force, a connecting bending moment acts. As already indicated, for the calculation of corresponding limit state, the change in angle α_0 at the edge (11) must firstly be calculated. The following calculation is made using relation (10'). Results of such calculations for the plate with several bolts are shown in Fig. 7. For security reasons, the further course may be scaled by some reserve factor (dashed lines). The calculated straight line has nothing to do with the line based on the level principle.

An additional effect arises when, based on design, through smaller edge distances or other measures, the areas of influence (not only reduced to the calculation influence cone) of preload forces in the clamped parts are cut (Fig. 8). In this case, in addition to the preload force, an initial moment also develops in the assembly. Depending on its direction, this moment can have a positive or negative effect to additional forces in the connection and must be considered in the calculation ($S_{eff} = S_K + S_O$).

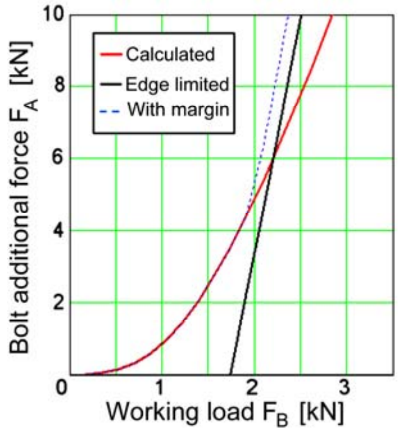


Figure 7. Influence of the edge (example of circular plate).
Slika 7. Uticaj ivice (primer za kružnu ploču)

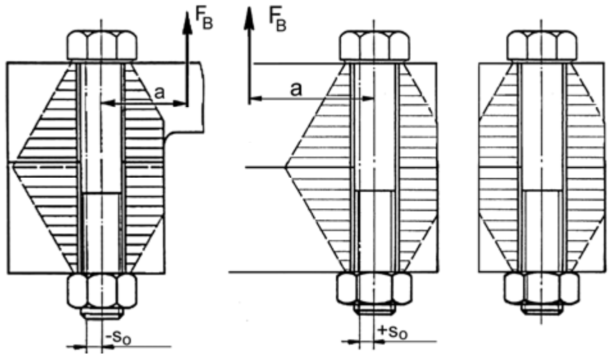


Figure 8. Limiting the external dimensions of the connection.
Slika 8. Ograničenje spoljnih dimenzija spoja

A major advantage of the relationships derived based on the beam model is the possibility of taking into account the preloading effects. Numerous experimental studies have clearly shown that the level of preloading force is a very important parameter for the integrity of the structures with bolted joints. Especially in cases of fatigue loading, by a suitable choice of the preload, it is possible to optimally adjust the connection to the requirements of the application. Thus, i.e. Fig. 9 presents results of one calculation, and confirms that depending on the connection preload the additional stresses, responsible for fatigue failure, could be significantly reduced.

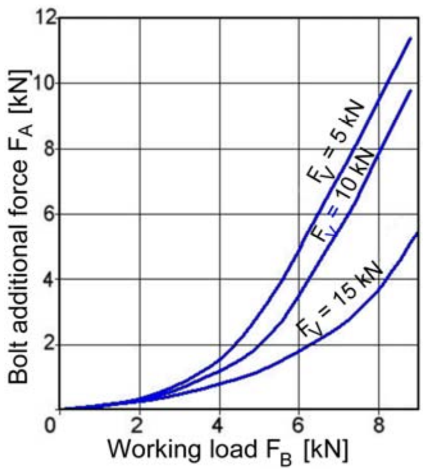


Figure 9. Calculation taking into account the effects of different preload force.
Slika 9. Proračun sa uticajem različitih sila predopterećenja

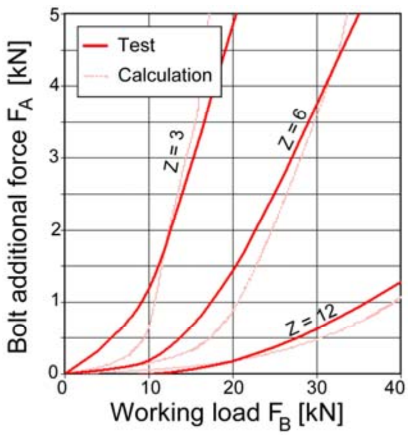


Figure 10. Effect of different number of bolts.
Slika 10. Uticaj broja vijaka

Figure 10 shows the comparison between results of experimental investigations in /3/ and calculation for the connection of two circular plates using a different number of bolts. At a minimal number of bolts, the calculation underestimates the amount of additional force at the start of the applied load. That is not very surprising since only 3 bolts are insufficient to charge the loading uniformly around the plate. Conditions for assumed axisymmetric loading of the plate are not applicable.

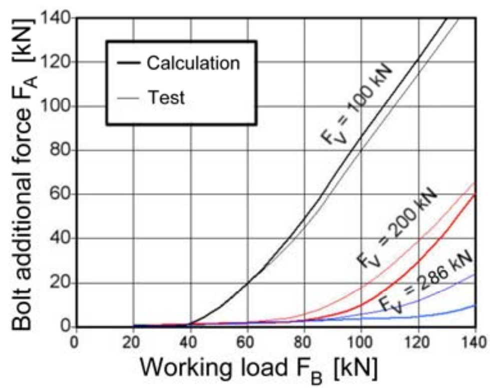


Figure 11. Comparison of calculation results from investigations /2/.
Slika 11. Poređenje rezultata proračuna sa istraživanjem iz (2)

Regardless of that, the comparison shows relatively good agreement with measurements. Similar results are shown in Fig. 11 based on experimental investigations of Massol /2/. However, also here are worth noting larger deviations for maximal preload (286 kN). If you yet consider that the maximal preload here is 90% of yield point, these differences do not appear too surprising.

Clearly, the calculations cannot cover the omissions and errors in input data and/or conditions. The designer must be committed in the unavoidable limits in this respect.

Evaluation of a_{eff}

An analysis of existing methods for calculation of bolted joints has shown that the determination of eccentric position a of the operational force is associated with a large uncertainty, since the bolt connection cannot be easily treated separately from the rest of the structure (plate, flange). Usually, at the position of a clamped connection to the rest of the structure there is, in addition to the force, a moment that must be considered, and stiffness relationships in the connection are not independent in respect to the remaining structure. By less accurate determination of the relevant parameter of the load, calculation results will be questionable. By using relationships developed in the beam model, this problem can be solved fairly.

From the general Eq. (12) and the equation for the free beam ($b_F = \infty$, $C_2 = 0$, $C_3 = \Phi$)

$$\frac{1}{6EJ\delta}s_K^3 + \frac{\Phi}{a_{eff}}s_K - 1 = 0$$

it follows $$a_{eff} = \frac{a(a+b_M+b_F)}{2a+b_M+s_K}$$

and since $$s_K \ll 2a+b_M$$

it is $$a_{eff} = \frac{a(a+b_M+b_F)}{2a+b_M} .$$

Figure 12 shows results of calculations for a pipe flange. The agreement is very good. One should, however not forget that the level principle is not applicable for the pipe flange, and therefore when approaching the edge the results split. Regardless of this, the application of a_{eff} can be very helpful for optimising the connection and verification tests performance.

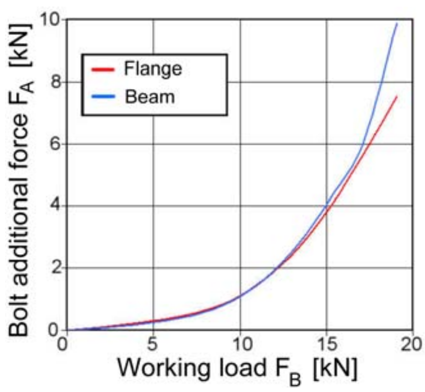


Figure 12. Proof for the applicability of a_{eff}.
Slika 12. Dokaz primenljivosti a_{eff}

DISCUSSION OF RESULTS

Problems of a bolted joint, as the opening at separation surface, that raise tensile and bending stresses, can be avoided in the design through the level of preload. A corresponding preload of the connection enables that under external load, the bolt has to carry only a minor part of the additional load. This is why the appropriate preload of the connection is so important. Effects of preload can be involved in the calculation at present only with the help of the beam model. Beyond it, it is also shown that the inclusion of initial moment could be also important. The initial moment develops through the geometry of clamped parts and follows the same pattern as the preloading force and, if properly adjusted, acts against the external moment.

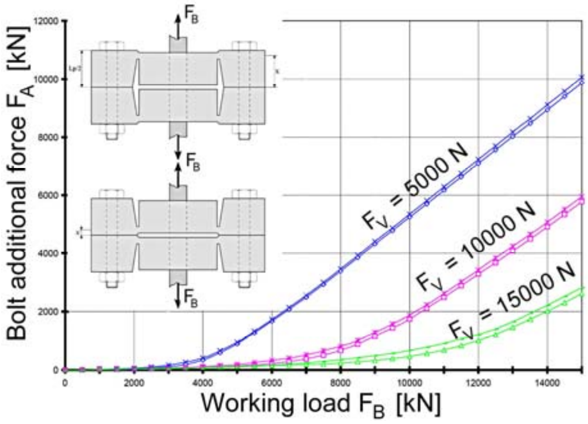


Figure 13. Results of FEM calculation for different positions of introduced force /2/.
Slika 13. Rezultati proračuna FEM za različite položaje napadne linije sile /2/

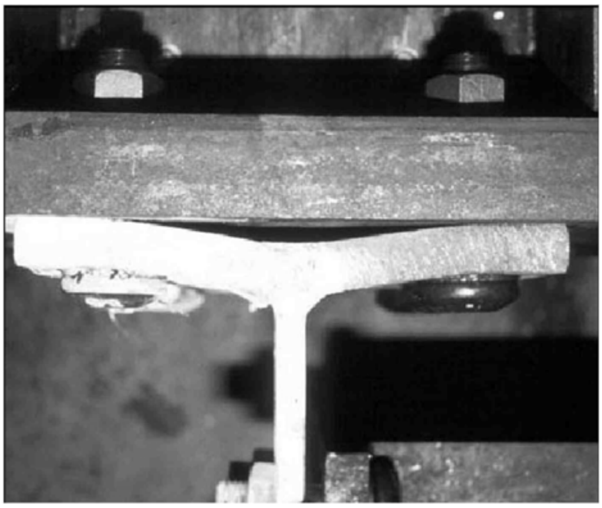


Figure 14. Phenomenon of “prying” – forces caused by deformation near bolt.
Slika 14. Fenomen “prajing” – sile od deformacije spoja blizu vijka

The consideration of the currently preferred position factor of introduced load does not make any sense when the external effective force is eccentric to the bolt axis. This is a case for a majority of bolted connections. FEM calculations given in /2/ for two extreme cases of introduced load have shown almost no differences (Fig. 13). The result was, moreover, regardless of the level of preload. The corresponding behaviour is dominated by a different mechanism, that is previously described by derivation of relationships of the beam model.

In English-based literature, the increased stressing in the connection by eccentric external load is known as so-called “prying”. By definition, prying is a phenomenon in which additional forces in the bolt are developed as a result of deformations in the connection near the bolt (Fig. 14).

CONCLUSIONS

Using the beam model has enabled to deal in the purpose of many important behavioural characteristics and problems of analysis of bolted joints such as: treatment of multi-bolted connections, importance of preload height concerning additional loads in the connection, accurate determination of the eccentricity of external force for free connection, shearing of clamped part resilience instead of the vertical position of introduced force, and the correction of the lever principle by accurate edge effects.

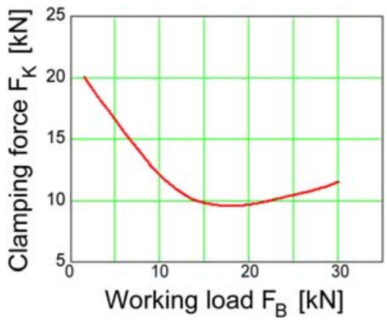


Figure 15. Typical change of force in separation surface F_K.
Slika 15. Tipične promene sile pri razdvajanju površina F_K

Application of beam models establishes as a better tool for design optimisation. Determination of clamping force eccentricity s is important for the evaluation in the design. Approaching the edge, additional forces strongly increase and in order to save the structural integrity this has to be avoided. Resulting non-linear behaviour of the connection and growing one-sided gaping of connections are not to be neglected and it is important as we search for, by material-saving, to achieve high reliabilities.

The cited lever principle is a rough simplification, which is also on the unsafe side. This principle in its primitive form is not applicable for multi-bolted joints. Procedures based on this assumption (“circle arc”) are just as vague and on the unsafe side.

The beam model can be considered as a tool for effective design of connections. Although this model is designed to support a full analysis, it provides methods for answering questions also in different phases of design development. It is expected that the use of the beam model in a uniform way will result in the optimisation of structural safety, reduction of the time and cost of the development cycle of structures and more simple mechanical systems.

However, there are obviously conditions in which this model is at its limits. Benefits are high in complementing such an analysis by experimental studies and in a way that they support each other.

Complex calculations can hardly be made without the available computational resources. Expenses in the context of an overall calculation of the bolt connection are in any way acceptable. The example of complete analysis of a typical connection can be downloaded free from the author’s homepage:
http://pagatonovic.homepage.t-online.de/index.htm

Symbols and notation not explained in the text

D_0	pitch diameter of the bolt
E	Young’s modulus
F_A	bolts additional force
F_B	working load of the connection
F_K	reaction force at separation surface
F_S	bolt total force
F_V	bolt preload (assembly) force
f_z	amount of embedding
H_i	wall thickness: $_F$ – flange; $_S$ – shell; $_P$ – plate
J	moment of inertia
M_A	external moment
p	internal pressure

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